SpheroidalS2Prime

Notations

Traditional name

Derivative of the radial spheroidal function of the second kind

Traditional notation

$S_{\nu,\mu}^{(2)\prime}(\gamma, z)$

Mathematica StandardForm notation

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SpheroidalS2Prime[ν, μ, γ, z]
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Primary definition

11.15.02.0001.01

$$S_{\nu,\mu}^{(2)\prime}(\gamma, z) = \frac{\partial S_{\nu,\mu}^{(2)}(\gamma, z)}{\partial z}$$

Specific values

General characteristics

Domain and analyticity

$S_{\nu,\mu}^{(2)\prime}(\gamma, z)$ is an analytical function of ν, μ, γ, z which is defined in $\mathbb{C}^4$.

11.15.04.0001.01

$$(\nu * \mu * \gamma * z) \longrightarrow S_{\nu,\mu}^{(2)\prime}(\gamma, z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$S_{\nu,\mu}^{(2)\prime}(\gamma, z)$ is an even function with respect to μ.

11.15.04.0002.01

$$S_{\nu,-\mu}^{(2)\prime}(\gamma, z) = S_{\nu,\mu}^{(2)\prime}(\gamma, z)$$

Mirror symmetry

11.15.04.0003.01

$$S_{\bar{\nu},\bar{\mu}}^{(2)}{}'(\bar{\gamma}, \bar{z}) = \overline{S_{\nu,\mu}^{(2)}{}'(\gamma, z)}$$

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at generic point $z = z_0$

11.15.06.0001.01

$$S_{\nu,\mu}^{(2)}{}'(\gamma, z) \propto S_{\nu,\mu}^{(2)}{}'(\gamma, z_0) + \frac{1}{1-z_0^2}\left(2 S_{\nu,\mu}^{(2)}{}'(\gamma, z_0)\, z_0 + S_{\nu,\mu}^{(2)}(\gamma, z_0)\left(\left(z_0^2-1\right)\gamma^2 - \lambda_{\nu,\mu}(\gamma) + \frac{\mu^2}{1-z_0^2}\right)\right)(z-z_0) - \frac{1}{2\left(z_0^2-1\right)^3}\left(S_{\nu,\mu}^{(2)}{}'(\gamma, z_0)\left(z_0^2-1\right)\left(\gamma^2 z_0^4 - 2\left(\gamma^2+3\right)z_0^2 + \gamma^2 - \mu^2 - \lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right) - 2\right) - 2 S_{\nu,\mu}^{(2)}(\gamma, z_0)\, z_0\left(-3\mu^2 + \gamma^2\left(z_0^2-1\right)^2 - 2\lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right)\right)\right)(z-z_0)^2 + \frac{1}{6\left(z_0^2-1\right)^4}\left(4 S_{\nu,\mu}^{(2)}{}'(\gamma, z_0)\, z_0\left(z_0^2-1\right)\left(\gamma^2 z_0^4 - 2\left(\gamma^2+3\right)z_0^2 + \gamma^2 - 3\mu^2 - 2\lambda_{\nu,\mu}(\gamma)\left(z_0^2-1\right) - 6\right) + S_{\nu,\mu}^{(2)}(\gamma, z_0)\left(\gamma^4\left(z_0^2-1\right)^4 + \lambda_{\nu,\mu}(\gamma)^2\left(z_0^2-1\right)^2 - 2\gamma^2\left(\mu^2+4z_0^2+2\right)\left(z_0^2-1\right)^2 - 2\lambda_{\nu,\mu}(\gamma)\left(\gamma^2 z_0^4 - \left(2\gamma^2+9\right)z_0^2 + \gamma^2 - \mu^2 - 3\right)\left(z_0^2-1\right) + \mu^2\left(\mu^2+36z_0^2+8\right)\right)\right)(z-z_0)^3 + \ldots /; (z \to z_0)$$

11.15.06.0002.01

$$S_{\nu,\mu}^{(2)}{}'(\gamma, z) \propto S_{\nu,\mu}^{(2)}{}'(\gamma, z_0)\,(1 + O(z-z_0))$$

Expansions at $z = 0$

11.15.06.0003.01

$$S_{\nu,\mu}^{(2)}{}'(\gamma, z) \propto S_{\nu,\mu}^{(2)}{}'(\gamma, 0) - \left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma)\right) S_{\nu,\mu}^{(2)}(\gamma, 0)\, z - \frac{1}{2}\left(\gamma^2 - \mu^2 + \lambda_{\nu,\mu}(\gamma) - 2\right) S_{\nu,\mu}^{(2)}{}'(\gamma, 0)\, z^2 + \frac{1}{6}\left(\gamma^4 - 2\left(\mu^2+2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma)^2 + \mu^2\left(\mu^2+8\right) + 2\left(\gamma^2-\mu^2-3\right)\lambda_{\nu,\mu}(\gamma)\right) S_{\nu,\mu}^{(2)}(\gamma, 0)\, z^3 + \ldots /; (z \to 0)$$

11.15.06.0004.01

$$S_{\nu,\mu}^{(2)}{}'(\gamma, z) \propto S_{\nu,\mu}^{(2)}{}'(\gamma, 0)\,(1 + O(z))$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

11.15.13.0001.01

$$\left(1-z^2\right) w''(z)-2 z\left(\frac{\left(1-z^2\right)^2 \gamma^2+\mu^2}{\mu^2-\left(1-z^2\right)\left(\left(1-z^2\right)\gamma^2+\lambda_{\nu,\mu}(\gamma)\right)}+2\right) w'(z)+\left(\frac{4\left(\left(1-z^2\right)^2\gamma^2+\mu^2\right) z^2}{\left(1-z^2\right)\left(-\left(1-z^2\right)^2\gamma^2+\mu^2-\left(1-z^2\right)\lambda_{\nu,\mu}(\gamma)\right)}+\left(1-z^2\right)\gamma^2+\lambda_{\nu,\mu}(\gamma)-\frac{\mu^2}{1-z^2}-2\right) w(z)=0\;/;\; w(z)=c_1\, {S_{\nu,\mu}^{(2)}}'(\gamma, z)+c_2\, {S_{\nu,\mu}^{(1)}}'(\gamma, z)$$

11.15.13.0002.01

$$W_z\left({S_{\nu,\mu}^{(2)}}'(\gamma, z), {S_{\nu,\mu}^{(1)}}'(\gamma, z)\right)=\left(\frac{\gamma^2}{1-z^2}-\frac{\mu^2}{\left(1-z^2\right)^3}+\frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-z^2\right)^2}\right)\left({S_{\nu,\mu}^{(1)}}'(\gamma, 0)\, S_{\nu,\mu}^{(2)}(\gamma, 0)-S_{\nu,\mu}^{(1)}(\gamma, 0)\, {S_{\nu,\mu}^{(2)}}'(\gamma, 0)\right)$$

11.15.13.0003.01

$$\left(1-g(z)^2\right) w''(z)+\left(\left(g(z)^2-1\right)\frac{g''(z)}{g'(z)}-2 g(z) g'(z)\right) w'(z)+\left(\frac{\left(\gamma g(z)^2+\mu-\gamma\right)\left(-\gamma g(z)^2+\mu+\gamma\right) g'(z)^2}{g(z)^2-1}+\lambda_{\nu,\mu}(\gamma) g'(z)^2\right) w(z)=0\;/;\; w(z)=c_1\, {S_{\nu,\mu}^{(2)}}'(\gamma, g(z))+c_2\, {S_{\nu,\mu}^{(1)}}'(\gamma, g(z))$$

11.15.13.0004.01

$$W_z\left({S_{\nu,\mu}^{(2)}}'(\gamma, g(z)), {S_{\nu,\mu}^{(1)}}'(\gamma, g(z))\right)= g'(z)\left(\frac{\gamma^2}{1-g(z)^2}+\frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-g(z)^2\right)^2}-\frac{\mu^2}{\left(1-g(z)^2\right)^3}\right)\left({S_{\nu,\mu}^{(1)}}'(\gamma, 0)\, S_{\nu,\mu}^{(2)}(\gamma, 0)-S_{\nu,\mu}^{(1)}(\gamma, 0)\, {S_{\nu,\mu}^{(2)}}'(\gamma, 0)\right)$$

11.15.13.0005.01

$$\left(1-g(z)^2\right) w''(z)+\left(\left(g(z)^2-1\right)\left(\frac{2 h'(z)}{h(z)}+\frac{g''(z)}{g'(z)}\right)-2 g(z) g'(z)\right) w'(z)+\left(\frac{\left(\gamma g(z)^2+\mu-\gamma\right)\left(-\gamma g(z)^2+\mu+\gamma\right) g'(z)^2}{g(z)^2-1}+\lambda_{\nu,\mu}(\gamma) g'(z)^2+\frac{2 g(z) h'(z) g'(z)}{h(z)}+\frac{\left(g(z)^2-1\right)\left(h(z) h''(z)-2 h'(z)^2\right)}{h(z)^2}-\frac{\left(g(z)^2-1\right) h'(z) g''(z)}{h(z) g'(z)}\right) w(z)=0\;/;\; w(z)=c_1\, h(z)\, {S_{\nu,\mu}^{(2)}}'(\gamma, g(z))+c_2\, h(z)\, {S_{\nu,\mu}^{(1)}}'(\gamma, g(z))$$

11.15.13.0006.01

$$W_z\left(h(z)\, {S_{\nu,\mu}^{(2)}}'(\gamma, g(z)), h(z)\, {S_{\nu,\mu}^{(1)}}'(\gamma, g(z))\right)= h(z)^2 g'(z)\left(\frac{\gamma^2}{1-g(z)^2}+\frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-g(z)^2\right)^2}-\frac{\mu^2}{\left(1-g(z)^2\right)^3}\right)\left({S_{\nu,\mu}^{(1)}}'(\gamma, 0)\, S_{\nu,\mu}^{(2)}(\gamma, 0)-S_{\nu,\mu}^{(1)}(\gamma, 0)\, {S_{\nu,\mu}^{(2)}}'(\gamma, 0)\right)$$

11.15.13.0007.01

$$\left(1-a^2 z^{2r}\right) w''(z)-\frac{a^2 (r-2s+1) z^{2r}+r+2s-1}{z} w'(z)+\left(a^2 r^2 \lambda_{\nu,\mu}(\gamma) z^{2r-2}+\frac{-a^2 r^2\left(\left(a^2 z^{2r}-1\right)^2\gamma^2-\mu^2\right) z^{2r}-s^2\left(a^2 z^{2r}-1\right)^2+r s\left(a^4 z^{4r}-1\right)}{z^2\left(a^2 z^{2r}-1\right)}\right) w(z)=0\;/;\; w(z)=c_1 z^s\, {S_{\nu,\mu}^{(2)}}'(\gamma, a z^r)+c_2 z^s\, {S_{\nu,\mu}^{(1)}}'(\gamma, a z^r)$$

11.15.13.0008.01

$$W_z\left(z^s S_{\nu,\mu}^{(2)\prime}(\gamma, a z^r), z^s S_{\nu,\mu}^{(1)\prime}(\gamma, a z^r)\right) = a r z^{r+2s-1}\left(\frac{\gamma^2}{1-a^2 z^{2r}} - \frac{\mu^2}{\left(1-a^2 z^{2r}\right)^3} + \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-a^2 z^{2r}\right)^2}\right)\left(S_{\nu,\mu}^{(1)\prime}(\gamma, 0) S_{\nu,\mu}^{(2)}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0) S_{\nu,\mu}^{(2)\prime}(\gamma, 0)\right)$$

11.15.13.0009.01

$$\left(1-a^2 r^{2z}\right) w''(z) + \left(-a^2 \log(r) r^{2z} + 2 a^2 \log(s) r^{2z} - \log(r) - 2\log(s)\right) w'(z) + \left(a^2 \log^2(r) \lambda_{\nu,\mu}(\gamma) r^{2z} + \frac{1}{a^2 r^{2z}-1}\left(-a^2\left(\left(a^2 r^{2z}-1\right)^2 \gamma^2 - \mu^2\right)\log^2(r) r^{2z} - \left(a^2 r^{2z}-1\right)^2 \log^2(s) + \left(a^4 r^{4z}-1\right)\log(r)\log(s)\right)\right) w(z) = 0 \;/;\; w(z) = c_1 s^z S_{\nu,\mu}^{(2)\prime}(\gamma, a r^z) + c_2 s^z S_{\nu,\mu}^{(1)\prime}(\gamma, a r^z)$$

11.15.13.0010.01

$$W_z\left(s^z S_{\nu,\mu}^{(2)\prime}(\gamma, a r^z), s^z S_{\nu,\mu}^{(1)\prime}(\gamma, a r^z)\right) = a s^{2z} r^z \log(r)\left(\frac{\gamma^2}{1-a^2 r^{2z}} - \frac{\mu^2}{\left(1-a^2 r^{2z}\right)^3} + \frac{\lambda_{\nu,\mu}(\gamma)}{\left(1-a^2 r^{2z}\right)^2}\right)\left(S_{\nu,\mu}^{(1)\prime}(\gamma, 0) S_{\nu,\mu}^{(2)}(\gamma, 0) - S_{\nu,\mu}^{(1)}(\gamma, 0) S_{\nu,\mu}^{(2)\prime}(\gamma, 0)\right)$$

Differentiation

Low-order differentiation

With respect to z

11.15.20.0001.01

$$\frac{\partial S_{\nu,\mu}^{(2)\prime}(\gamma, z)}{\partial z} = \frac{1}{1-z^2}\left(2 z S_{\nu,\mu}^{(2)\prime}(\gamma, z) - \left(\left(1-z^2\right)\gamma^2 + \lambda_{\nu,\mu}(\gamma) - \frac{\mu^2}{1-z^2}\right) S_{\nu,\mu}^{(2)}(\gamma, z)\right)$$

11.15.20.0002.01

$$\frac{\partial^2 S_{\nu,\mu}^{(2)\prime}(\gamma, z)}{\partial z^2} = \frac{1}{\left(z^2-1\right)^3}\left(2 z\left(\left(z^2-1\right)^2 \gamma^2 - 3\mu^2 - 2\left(z^2-1\right)\lambda_{\nu,\mu}(\gamma)\right) S_{\nu,\mu}^{(2)}(\gamma, z) - \left(z^2-1\right)\left(\gamma^2 z^4 - 2\left(\gamma^2+3\right) z^2 + \gamma^2 - \mu^2 - \left(z^2-1\right)\lambda_{\nu,\mu}(\gamma) - 2\right) S_{\nu,\mu}^{(2)\prime}(\gamma, z)\right)$$

Integration

Indefinite integration

Involving only one direct function

11.15.21.0001.01

$$\int S_{\nu,\mu}^{(2)\prime}(\gamma, z)\, dz = S_{\nu,\mu}^{(2)}(\gamma, z)$$

Operations

Representations through equivalent functions

Theorems

History

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