SinhIntegral

Notations

Traditional name

Hyperbolic sine integral

Traditional notation

$\text{Shi}(z)$

Mathematica StandardForm notation

`SinhIntegral[z]`

Primary definition

06.39.02.0001.01

$$\text{Shi}(z) == \int_0^z \frac{\sinh(t)}{t}\, dt$$

Specific values

Values at fixed points

06.39.03.0001.01

$$\text{Shi}(0) == 0$$

Values at infinities

06.39.03.0002.01

$$\text{Shi}(\infty) == \infty$$

06.39.03.0003.01

$$\text{Shi}(-\infty) == -\infty$$

06.39.03.0004.01

$$\text{Shi}(i\,\infty) == \frac{i\,\pi}{2}$$

06.39.03.0005.01

$$\text{Shi}(-i\,\infty) == -\frac{i\,\pi}{2}$$

06.39.03.0006.01

$$\text{Shi}(\tilde{\infty}) == ¿$$

General characteristics

Domain and analyticity

Shi(z) is an entire function of z which is defined in the whole complex z-plane.

06.39.04.0001.01

$$z \longrightarrow \text{Shi}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

Shi(z) is an odd function.

06.39.04.0002.01

$$\text{Shi}(-z) = -\text{Shi}(z)$$

Mirror symmetry

06.39.04.0003.01

$$\text{Shi}(\bar{z}) = \overline{\text{Shi}(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function Shi(z) has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

06.39.04.0004.01

$$\mathcal{S}ing_z(\text{Shi}(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function Shi(z) does not have branch points.

06.39.04.0005.01

$$\mathcal{BP}_z(\text{Shi}(z)) = \{\}$$

Branch cuts

The function Shi(z) does not have branch cuts.

06.39.04.0006.01

$$\mathcal{BC}_z(\text{Shi}(z)) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

06.39.06.0010.01

$$\mathrm{Shi}(z) \propto \mathrm{Shi}(z_0) + \frac{\sinh(z_0)}{z_0}(z - z_0) + \frac{z_0 \cosh(z_0) - \sinh(z_0)}{2 z_0^2}(z - z_0)^2 + \ldots \;/;\; (z \to z_0)$$

06.39.06.0011.01

$$\mathrm{Shi}(z) \propto \mathrm{Shi}(z_0) + \frac{\sinh(z_0)}{z_0}(z - z_0) + \frac{z_0 \cosh(z_0) - \sinh(z_0)}{2 z_0^2}(z - z_0)^2 + O\left((z - z_0)^3\right)$$

06.39.06.0012.01

$$\mathrm{Shi}(z) = \mathrm{Shi}(z_0) - \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \frac{i^j (-1)^k z_0^{j-k}}{k\, j!} \sinh\left(\frac{i \pi j}{2} + z_0\right)(z - z_0)^k$$

06.39.06.0013.01

$$\mathrm{Shi}(z) = \pi \sum_{k=0}^{\infty} \frac{2^{k-2} z_0^{1-k}}{k!}\, {}_2\tilde{F}_3\left(\frac{1}{2}, 1; \frac{3}{2}, 1 - \frac{k}{2}, \frac{3-k}{2}; \frac{z_0^2}{4}\right)(z - z_0)^k$$

06.39.06.0014.01

$$\mathrm{Shi}(z) \propto \mathrm{Shi}(z_0)\,(1 + O(z - z_0))$$

Expansions at $z = 0$

For the function itself

06.39.06.0001.02

$$\mathrm{Shi}(z) \propto z\left(1 + \frac{z^2}{18} + \frac{z^4}{600} + \ldots\right) /;\; (z \to 0)$$

06.39.06.0015.01

$$\mathrm{Shi}(z) \propto z\left(1 + \frac{z^2}{18} + \frac{z^4}{600} + O\left(z^6\right)\right)$$

06.39.06.0002.01

$$\mathrm{Shi}(z) = z \sum_{k=0}^{\infty} \frac{z^{2k}}{(1 + 2k)^2 (2k)!}$$

06.39.06.0003.01

$$\mathrm{Shi}(z) = z\, {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{z^2}{4}\right)$$

06.39.06.0004.02

$$\mathrm{Shi}(z) \propto z\left(1 + O\left(z^2\right)\right)$$

06.39.06.0016.01

$$\mathrm{Shi}(z) = F_\infty(z) \;/;$$
$$\left(\left(F_n(z) = z \sum_{k=0}^{n} \frac{z^{2k}}{(2k+1)^2 (2k)!} = \mathrm{Shi}(z) - \frac{z^{2n+3}}{(2n+3)^2 (2n+2)!}\, {}_2F_3\left(1, n + \frac{3}{2}; n+2, n+\frac{5}{2}, n+\frac{5}{2}; \frac{z^2}{4}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

06.39.06.0005.01

$$\operatorname{Shi}(z) \propto -\frac{\pi\sqrt{-z^2}}{2\,z} + \frac{\cosh(z)}{z}\,{}_3F_0\!\left(\frac{1}{2}, 1, 1;\, ;\, \frac{4}{z^2}\right) + \frac{\sinh(z)}{z^2}\,{}_3F_0\!\left(1, 1, \frac{3}{2};\, ;\, \frac{4}{z^2}\right) /;\, (|z| \to \infty)$$

06.39.06.0006.01

$$\operatorname{Shi}(z) \propto -\frac{\pi\sqrt{-z^2}}{2\,z} + \frac{\cosh(z)}{z}\left(1 + O\!\left(\frac{1}{z^2}\right)\right) + \frac{\sinh(z)}{z^2}\left(1 + O\!\left(\frac{1}{z^2}\right)\right) /;\, (|z| \to \infty)$$

Residue representations

06.39.06.0007.01

$$\operatorname{Shi}(z) = \frac{\sqrt{\pi}}{4}\, z \sum_{j=0}^{\infty} \operatorname{res}_s\!\left(\frac{\Gamma\!\left(\frac{1}{2} - s\right)\left(-\frac{z^2}{4}\right)^{-s}}{\Gamma\!\left(\frac{3}{2} - s\right)^2}\, \Gamma(s)\right)(-j)$$

06.39.06.0008.01

$$\operatorname{Shi}(z) = -\frac{i}{2}\sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{res}_s\!\left(\frac{\Gamma(-s)\left(\frac{i\,z}{2}\right)^{-2\,s}}{\Gamma(1-s)^2}\, \Gamma\!\left(s + \frac{1}{2}\right),\, \left\{s, -\frac{1}{2} - j\right\}\right)$$

Other series representations

06.39.06.0009.01

$$\operatorname{Shi}(z) = \pi \sum_{k=0}^{\infty} (-1)^k\, I_{k+\frac{1}{2}}\!\left(\frac{z}{2}\right)^2$$

Integral representations

On the real axis

Of the direct function

06.39.07.0001.01

$$\operatorname{Shi}(z) = \int_0^z \frac{\sinh(t)}{t}\, dt$$

Contour integral representations

06.39.07.0002.01

$$\operatorname{Shi}(z) = \frac{\sqrt{\pi}\, z}{8\,\pi\, i} \int_{\mathcal{L}} \frac{\Gamma(s)\,\Gamma\!\left(\frac{1}{2} - s\right)}{\Gamma\!\left(\frac{3}{2} - s\right)^2} \left(-\frac{z^2}{4}\right)^{-s} ds$$

06.39.07.0003.01

$$\operatorname{Shi}(z) = \frac{\sqrt{\pi}}{4\,\pi} \int_{\mathcal{L}} \frac{\Gamma\!\left(s + \frac{1}{2}\right)\Gamma(-s)}{\Gamma(1-s)^2} \left(\frac{i\,z}{2}\right)^{-2\,s} ds$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

06.39.13.0001.01

$$z\,w^{(3)}(z) + 2\,w''(z) - z\,w'(z) = 0\,/;\, w(z) = c_1\,\text{Shi}(z) + c_2\,\text{Chi}(z) + c_3$$

06.39.13.0004.01

$$W_z(1, \text{Shi}(z), \text{Chi}(z)) = -\frac{1}{z^2}$$

06.39.13.0002.01

$$z\,w^{(3)}(z) + 2\,w''(z) - z\,w'(z) = 0\,/;\, w(z) = c_1\,\text{Shi}(z) + c_2\,\text{Ei}(z) + c_3$$

06.39.13.0005.01

$$W_z(1, \text{Shi}(z), \text{Ei}(z)) = -\frac{1}{z^2}$$

06.39.13.0003.01

$$z\,w^{(3)}(z) + 2\,w''(z) - z\,w'(z) = 0\,/;\, w(z) = c_1\,\text{Shi}(z) + c_2\,\text{Ei}(-z) + c_3$$

06.39.13.0006.01

$$W_z(1, \text{Shi}(z), \text{Ei}(-z)) = -\frac{1}{z^2}$$

06.39.13.0007.01

$$w^{(3)}(z) + \left(\frac{2\,g'(z)}{g(z)} - \frac{3\,g''(z)}{g'(z)}\right) w''(z) + \left(-g'(z)^2 + \frac{3\,g''(z)^2}{g'(z)^2} - \frac{2\,g''(z)}{g(z)} - \frac{g^{(3)}(z)}{g'(z)}\right) w'(z) = 0\,/;\, w(z) = c_1\,\text{Shi}(g(z)) + c_2\,\text{Chi}(g(z)) + c_3$$

06.39.13.0008.01

$$W_z(\text{Shi}(g(z)), \text{Chi}(g(z)), 1) = -\frac{g'(z)^3}{g(z)^2}$$

06.39.13.0009.01

$$w^{(3)}(z) + \left(\frac{2\,g'(z)}{g(z)} - \frac{3\,h'(z)}{h(z)} - \frac{3\,g''(z)}{g'(z)}\right) w''(z) +$$
$$\left(-g'(z)^2 - \frac{4\,h'(z)\,g'(z)}{g(z)\,h(z)} + \frac{6\,h'(z)^2}{h(z)^2} + \frac{3\,g''(z)^2}{g'(z)^2} + \frac{6\,h'(z)\,g''(z)}{h(z)\,g'(z)} - \frac{2\,g''(z)}{g(z)} - \frac{3\,h''(z)}{h(z)} - \frac{g^{(3)}(z)}{g'(z)}\right) w'(z) +$$
$$\left(-\frac{6\,h'(z)^3}{h(z)^3} + \frac{4\,g'(z)\,h'(z)^2}{g(z)\,h(z)^2} - \frac{6\,g''(z)\,h'(z)^2}{h(z)^2\,g'(z)} + \frac{6\,h''(z)\,h'(z)}{h(z)^2} - \frac{3\,g''(z)^2\,h'(z)}{h(z)\,g'(z)^2} + \frac{2\,h'(z)\,g''(z) - 2\,g'(z)\,h''(z)}{g(z)\,h(z)} + \right.$$
$$\left.\frac{3\,g''(z)\,h''(z) + h'(z)\,g^{(3)}(z)}{h(z)\,g'(z)} + \frac{g'(z)^2\,h'(z) - h^{(3)}(z)}{h(z)}\right) w(z) = 0\,/;\, w(z) = c_1\,h(z)\,\text{Shi}(g(z)) + c_2\,\text{Chi}(g(z))\,h(z) + c_3\,h(z)$$

06.39.13.0010.01

$$W_z(h(z)\,\text{Shi}(g(z)), h(z)\,\text{Chi}(g(z)), h(z)) = -\frac{h(z)^3\,g'(z)^3}{g(z)^2}$$

06.39.13.0011.01

$$z^3\, w^{(3)}(z) - (r + 3\, s - 3)\, z^2\, w''(z) - \left(a^2\, r^2\, z^{2\, r} - 3\, s^2 + r - 2\, r\, s + 3\, s - 1\right) z\, w'(z) - s \left(-a^2\, r^2\, z^{2\, r} + s^2 + r\, s\right) w(z) = 0\; /;\; w(z) = c_1\, z^s\, \mathrm{Shi}(a\, z^r) + c_2\, z^s\, \mathrm{Chi}(a\, z^r) + c_3\, z^s$$

06.39.13.0012.01

$$W_z(z^s\, \mathrm{Shi}(a\, z^r),\, z^s\, \mathrm{Chi}(a\, z^r),\, z^s) = -a\, r^3\, z^{r+3\, s-3}$$

06.39.13.0013.01

$$w^{(3)}(z) - (\log(r) + 3\log(s))\, w''(z) + \left(-a^2 \log^2(r)\, r^{2\, z} + 3 \log^2(s) + 2 \log(r) \log(s)\right) w'(z) - \log(s) \left(-a^2 \log^2(r)\, r^{2\, z} + \log^2(s) + \log(r) \log(s)\right) w(z) = 0\; /;\; w(z) = c_1\, s^z\, \mathrm{Shi}(a\, r^z) + c_2\, s^z\, \mathrm{Chi}(a\, r^z) + c_3\, s^z$$

06.39.13.0014.01

$$W_z(s^z\, \mathrm{Shi}(a\, r^z),\, s^z\, \mathrm{Chi}(a\, r^z),\, s^z) = -a\, r^z\, s^{3\, z} \log^3(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.39.16.0001.01

$$\mathrm{Shi}(-z) = -\mathrm{Shi}(z)$$

06.39.16.0002.01

$$\mathrm{Shi}(i\, z) = i\, \mathrm{Si}(z)$$

06.39.16.0003.01

$$\mathrm{Shi}(-i\, z) = -i\, \mathrm{Si}(z)$$

06.39.16.0004.01

$$\mathrm{Shi}(a\, (b\, z^c)^m) = \frac{(b\, z^c)^m}{b^m\, z^{m\, c}}\, \mathrm{Shi}(a\, b^m\, z^{m\, c})\; /;\; 2\, m \in \mathbb{Z}$$

06.39.16.0005.01

$$\mathrm{Shi}\left(\sqrt{z^2}\right) = \frac{\sqrt{z^2}}{z}\, \mathrm{Shi}(z)$$

Complex characteristics

Real part

06.39.19.0001.01

$$\mathrm{Re}(\mathrm{Shi}(x + i\, y)) = \sum_{k=0}^{\infty} \frac{x^{2\, k+1}}{(2\, k + 1)!\, (2\, k + 1)}\, {}_1F_2\!\left(k + \frac{1}{2};\, \frac{1}{2},\, k + \frac{3}{2};\, -\frac{y^2}{4}\right)$$

06.39.19.0002.01

$$\mathrm{Re}(\mathrm{Shi}(x + i\, y)) = \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{(-1)^j\, x^{2\, k-2\, j+1}\, y^{2\, j}}{(2\, k + 1)\, (2\, j)!\, (2\, k - 2\, j + 1)!}$$

06.39.19.0003.01

$$\operatorname{Re}(\operatorname{Shi}(x+i\,y)) = \frac{1}{2}\left(\operatorname{Shi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

06.39.19.0004.01

$$\operatorname{Im}(\operatorname{Shi}(x+i\,y)) = y\sum_{k=0}^{\infty}\frac{x^{2k}}{(2k+1)!}\,{}_1F_2\left(k+\frac{1}{2};\frac{3}{2},k+\frac{3}{2};-\frac{y^2}{4}\right)$$

06.39.19.0005.01

$$\operatorname{Im}(\operatorname{Shi}(x+i\,y)) = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(-1)^j\,x^{2k-2j}\,y^{2j+1}}{(2k+1)\,(2k-2j)!\,(2j+1)!}$$

06.39.19.0006.01

$$\operatorname{Im}(\operatorname{Shi}(x+i\,y)) = \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Shi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Absolute value

06.39.19.0007.01

$$|\operatorname{Shi}(x+i\,y)| = \sqrt{\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Shi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)}$$

Argument

06.39.19.0008.01

$$\arg(\operatorname{Shi}(x+i\,y)) = \tan^{-1}\left(\frac{1}{2}\left(\operatorname{Shi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right), \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Shi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

Conjugate value

06.39.19.0009.01

$$\overline{\operatorname{Shi}(x+i\,y)} = \frac{1}{2}\left(\operatorname{Shi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)-\frac{i\,x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Shi}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Signum value

06.39.19.0010.01

$$\operatorname{sgn}(\operatorname{Shi}(x+i\,y)) = \left(\frac{1}{y}\left(i\sqrt{-\frac{y^2}{x^2}}\;x\left(\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Shi}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)\right)+\operatorname{Shi}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right) \Big/ \left(2\sqrt{\operatorname{Shi}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Shi}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)}\right)$$

Differentiation

Low-order differentiation

06.39.20.0001.01

$$\frac{\partial \operatorname{Shi}(z)}{\partial z} = \frac{\sinh(z)}{z}$$

06.39.20.0002.01

$$\frac{\partial^2 \operatorname{Shi}(z)}{\partial z^2} = \frac{\cosh(z)}{z} - \frac{\sinh(z)}{z^2}$$

Symbolic differentiation

06.39.20.0006.01

$$\frac{\partial^n \operatorname{Shi}(z)}{\partial z^n} = \delta_n \operatorname{Shi}(z) - \sum_{k=0}^{n-1} \frac{i^k (-1)^n (n-1)!\, z^{k-n}}{k!} \sinh\left(\frac{i\,\pi\,k}{2} + z\right) /;\, n \in \mathbb{N}$$

06.39.20.0003.01

$$\frac{\partial^n \operatorname{Shi}(z)}{\partial z^n} = \delta_n \operatorname{Shi}(z) - \operatorname{Boole}\left(n \neq 0,\, (n-1)! \sum_{k=0}^{n-1} \frac{(-1)^n (-i)^k z^{k-n}}{k!} \sinh\left(z - \frac{i\,k\,\pi}{2}\right)\right) /;\, n \in \mathbb{N}$$

06.39.20.0004.02

$$\frac{\partial^n \operatorname{Shi}(z)}{\partial z^n} = 2^{n-2}\,\pi\,z^{1-n}\,{}_2\tilde{F}_3\left(\frac{1}{2}, 1; \frac{3}{2}, 1-\frac{n}{2}, \frac{3-n}{2}; \frac{z^2}{4}\right) /;\, n \in \mathbb{N}$$

Fractional integro-differentiation

06.39.20.0005.01

$$\frac{\partial^\alpha \operatorname{Shi}(z)}{\partial z^\alpha} = 2^{\alpha-2}\,\pi\,z^{1-\alpha}\,{}_2\tilde{F}_3\left(\frac{1}{2}, 1; \frac{3}{2}, 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; \frac{z^2}{4}\right)$$

Integration

Indefinite integration

Involving only one direct function

06.39.21.0001.01

$$\int \mathrm{Shi}(b + a z)\, dz = \frac{(b + a z)\, \mathrm{Shi}(b + a z) - \cosh(b + a z)}{a}$$

06.39.21.0002.01

$$\int \mathrm{Shi}(a z)\, dz = z\, \mathrm{Shi}(a z) - \frac{\cosh(a z)}{a}$$

06.39.21.0003.01

$$\int \mathrm{Shi}(z)\, dz = z\, \mathrm{Shi}(z) - \cosh(z)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear argument

06.39.21.0004.01

$$\int z^{\alpha-1}\, \mathrm{Shi}(a z)\, dz = \frac{z^{\alpha}}{2\, \alpha} \left(\Gamma(\alpha, -a z)\, (-a z)^{-\alpha} - (a z)^{-\alpha}\, \Gamma(\alpha, a z) + 2\, \mathrm{Shi}(a z)\right)$$

06.39.21.0005.01

$$\int z^{\alpha-1}\, \mathrm{Shi}(z)\, dz = \frac{z^{\alpha}}{\alpha}\, \mathrm{Shi}(z) + \frac{1}{2\, \alpha} \left((-z)^{-\alpha}\, z^{\alpha}\, \Gamma(\alpha, -z) - \Gamma(\alpha, z)\right)$$

06.39.21.0006.01

$$\int z\, \mathrm{Shi}(a z)\, dz = \frac{a^2\, \mathrm{Shi}(a z)\, z^2 - a \cosh(a z)\, z + \sinh(a z)}{2\, a^2}$$

06.39.21.0007.01

$$\int \frac{\mathrm{Shi}(a z)}{z}\, dz = \frac{1}{2} \left(a z\, {}_3F_3(1, 1, 1; 2, 2, 2; -a z) + a z\, {}_3F_3(1, 1, 1; 2, 2, 2; a z) + \log(z) \left(\Gamma(0, -a z) - \Gamma(0, a z) + \log(-a z) - \log(a z) + 2\, \mathrm{Shi}(a z)\right)\right)$$

06.39.21.0008.01

$$\int \frac{\mathrm{Shi}(a z)}{z^2}\, dz = -\frac{-a z\, \mathrm{Chi}(a z) + \sinh(a z) + \mathrm{Shi}(a z)}{z}$$

06.39.21.0009.01

$$\int \frac{\mathrm{Shi}(b + a z)}{z^2}\, dz = \frac{1}{b z} \left(a z\, \mathrm{Chi}(a z) \sinh(b) + a z \cosh(b)\, \mathrm{Shi}(a z) - (b + a z)\, \mathrm{Shi}(b + a z)\right)$$

Power arguments

06.39.21.0010.01

$$\int z^{\alpha-1}\, \mathrm{Shi}(a z^r)\, dz = \frac{1}{2\, \alpha} \left(z^{\alpha} \left(\Gamma\!\left(\frac{\alpha}{r}, -a z^r\right) (-a z^r)^{-\frac{\alpha}{r}} - (a z^r)^{-\frac{\alpha}{r}}\, \Gamma\!\left(\frac{\alpha}{r}, a z^r\right) + 2\, \mathrm{Shi}(a z^r)\right)\right)$$

Involving exponential function

Involving exp

06.39.21.0011.01

$$\int e^{b z}\, \text{Shi}(a z)\, dz = \frac{\text{Ei}((b-a) z) - \text{Ei}((a+b) z) + 2 e^{b z}\, \text{Shi}(a z)}{2 b}$$

06.39.21.0012.01

$$\int e^{a z}\, \text{Shi}(a z)\, dz = \frac{-\text{Ei}(2 a z) + \log(a z) + 2 e^{a z}\, \text{Shi}(a z)}{2 a}$$

06.39.21.0013.01

$$\int e^{-a z}\, \text{Shi}(a z)\, dz = -\frac{\text{Ei}(-2 a z) - \log(a z) + 2 e^{-a z}\, \text{Shi}(a z)}{2 a}$$

Involving exponential function and a power function

Involving exp and power

06.39.21.0014.01

$$\int z^n e^{b z}\, \text{Shi}(a z)\, dz = \frac{1}{2} n!\, (-b)^{-n-1} \left(-\text{Ei}((b-a) z) + \text{Ei}((a+b) z) - 2 e^{b z}\, \text{Shi}(a z) \sum_{k=0}^{n} \frac{(-b z)^k}{k!} - e^{(a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(-a-b)^k z^k}{k!} + e^{(b-a) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(a-b)^k z^k}{k!}\right) /;\, n \in \mathbb{N}$$

06.39.21.0015.01

$$\int z^n e^{a z}\, \text{Shi}(a z)\, dz = \frac{(-a)^{-n}}{2 a} \left(2 \Gamma(n+1, -a z)\, \text{Shi}(a z) + n! \left(-\text{Ei}(2 a z) + \log(z) + 2 \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(-a z)^k}{2 k} + 2^{-k-1} \Gamma(k, -2 a z)\right)\right)\right) /;\, n \in \mathbb{N}$$

06.39.21.0016.01

$$\int z^n e^{-a z}\, \text{Shi}(a z)\, dz = \frac{1}{2} a^{-n-1} \left(-2 \Gamma(n+1, a z)\, \text{Shi}(a z) - n! \left(\text{Ei}(-2 a z) - \log(z) - 2 \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(a z)^k}{2 k} + 2^{-k-1} \Gamma(k, 2 a z)\right)\right)\right) /;\, n \in \mathbb{N}$$

06.39.21.0017.01

$$\int z\, e^{b z}\, \text{Shi}(a z)\, dz = \frac{1}{2 b^2} \left(-\text{Ei}((b-a) z) + \text{Ei}((a+b) z) + 2 e^{b z} (b z - 1)\, \text{Shi}(a z) - \frac{2 b\, e^{b z} (b \sinh(a z) - a \cosh(a z))}{(b-a)(a+b)}\right)$$

06.39.21.0018.01

$$\int z^2 e^{b z}\, \text{Shi}(a z)\, dz = \frac{1}{b^3} \left(\text{Ei}((b-a) z) - \text{Ei}((a+b) z) - \frac{1}{(a-b)^2 (a+b)^2} \left(b\, e^{b z} \left(a \left((b z - 2) a^2 + b^2 (4 - b z)\right) \cosh(a z) + b \left((1 - b z) a^2 + b^2 (b z - 3)\right) \sinh(a z)\right)\right) + e^{b z} \left(b^2 z^2 - 2 b z + 2\right) \text{Shi}(a z)\right)$$

06.39.21.0019.01

$$\int z^3 e^{bz}\,\mathrm{Shi}(az)\,dz = \frac{1}{b^4}\left(3\,(\mathrm{Ei}((a+b)z)-\mathrm{Ei}((b-a)z)) - \frac{1}{(b-a)^3\,(a+b)^3}\left(b\,e^{bz}\left(b\left(\left(b^2z^2-bz+3\right)a^4-2b^2\left(b^2z^2-3bz+3\right)a^2+b^4\left(b^2z^2-5bz+11\right)\right)\sinh(az)-a\left(\left(b^2z^2-3bz+6\right)a^4-2b^2\left(b^2z^2-5bz+8\right)a^2+b^4\left(b^2z^2-7bz+18\right)\right)\cosh(az)\right)\right)+e^{bz}\left(b^3z^3-3b^2z^2+6bz-6\right)\mathrm{Shi}(az)\right)$$

Involving trigonometric functions

Involving sin

06.39.21.0020.01

$$\int \sin(bz)\,\mathrm{Shi}(az)\,dz = \frac{-2\cos(bz)\,\mathrm{Shi}(az)+\mathrm{Shi}((a+b\,i)z)+\mathrm{Shi}((a-i\,b)z)}{2b}$$

Involving cos

06.39.21.0021.01

$$\int \cos(bz)\,\mathrm{Shi}(az)\,dz = -\frac{i\,(-\mathrm{Chi}((a+b\,i)z)+\mathrm{Chi}((a-i\,b)z)+2\,i\sin(bz)\,\mathrm{Shi}(az))}{2b}$$

Involving trigonometric functions and a power function

Involving sin and power

06.39.21.0022.01

$$\int z^n\sin(bz)\,\mathrm{Shi}(az)\,dz = -\frac{i}{4}(i\,b)^{-n-1}\,n!\left(\mathrm{Ei}(-(a+b\,i)z)-(-1)^{-n}\,\mathrm{Ei}((a+b\,i)z)-\mathrm{Ei}((a-i\,b)z)+(-1)^n\,\mathrm{Ei}(i\,bz-az)+\frac{1}{\Gamma(n+2)}(2\,(n+1)\,((-1)^n\,\Gamma(n+1,-i\,bz)+\Gamma(n+1,i\,bz))\,\mathrm{Shi}(az))+e^{(a-i\,b)z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{i\,b}{i\,b-a}\right)^m\sum_{k=0}^{m-1}\frac{(i\,b-a)^k z^k}{k!}-e^{-(a+b\,i)z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{i\,b}{a+b\,i}\right)^m\sum_{k=0}^{m-1}\frac{(a+b\,i)^k z^k}{k!}+(-1)^n\,e^{(a+b\,i)z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{i\,b}{a+b\,i}\right)^m\sum_{k=0}^{m-1}\frac{(-a-i\,b)^k z^k}{k!}-(-1)^n\,e^{i\,bz-az}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{i\,b}{i\,b-a}\right)^m\sum_{k=0}^{m-1}\frac{(a-i\,b)^k z^k}{k!}\right)\,/;\,n\in\mathbb{N}$$

06.39.21.0023.01

$$\int z\sin(bz)\,\mathrm{Shi}(az)\,dz = \frac{1}{4b^2\left(a^2+b^2\right)}\left(4\sin(bz)\sinh(az)\,b^2+4a\cos(bz)\cosh(az)\,b-i\left(a^2+b^2\right)(-\mathrm{Ei}(-(a+b\,i)z)-\mathrm{Ei}((a+b\,i)z)+\mathrm{Ei}((a-i\,b)z)+\mathrm{Ei}(i\,bz-az)+2\,(\Gamma(2,-i\,bz)-\Gamma(2,i\,bz))\,\mathrm{Shi}(az))\right)$$

06.39.21.0024.01

$$\int z^2 \sin(b z) \operatorname{Shi}(a z) \, dz = \frac{1}{4 b^3} \left(\frac{1}{(a^2+b^2)^2} \left(4 \cos(b z) \left(a (a^2+b^2) z \cosh(a z) + (a^2+3 b^2) \sinh(a z)\right) b^2 - 2 (a^2+b^2)^2 (-\operatorname{Ei}(-(a+b i) z) + \operatorname{Ei}((a+b i) z) + \operatorname{Ei}((a-i b) z) - \operatorname{Ei}(i b z - a z)) + 4 \sin(b z) \left(b^3 (a^2+b^2) z \sinh(a z) - 2 a b (a^2+2 b^2) \cosh(a z)\right)\right) + 2 (\Gamma(3, -i b z) + \Gamma(3, i b z)) \operatorname{Shi}(a z) \right)$$

06.39.21.0025.01

$$\int z^3 \sin(b z) \operatorname{Shi}(a z) \, dz = -\frac{1}{4 b^4} \left(i \left(\frac{1}{(a^2+b^2)^3} \left(-6 (-\operatorname{Ei}(-(a+b i) z) - \operatorname{Ei}((a+b i) z) + \operatorname{Ei}((a-i b) z) + \operatorname{Ei}(i b z - a z)) (a^2+b^2)^3 + 4 b i \cos(b z) \left((a^2+b^2)(a^2+5 b^2) z \sinh(a z) b^2 + a \left(b^2 (a^2+b^2)^2 z^2 - 2 (3 a^4 + 8 b^2 a^2 + 9 b^4)\right) \cosh(a z)\right) - 4 i b^2 \sin(b z) \left(a (a^2+b^2)(3 a^2+7 b^2) z \cosh(a z) - \left(-3 a^4 - 6 b^2 a^2 - 11 b^4 + b^2 (a^2+b^2)^2 z^2\right) \sinh(a z)\right)\right) + 2 (\Gamma(4, i b z) - \Gamma(4, -i b z)) \operatorname{Shi}(a z) \right) \right)$$

Involving cos and power

06.39.21.0026.01

$$\int z^n \cos(b z) \operatorname{Shi}(a z) \, dz = \frac{1}{4} (i b)^{-n-1} n! \left(-\operatorname{Ei}(-(a+b i) z) - (-1)^{-n} \operatorname{Ei}((a+b i) z) + \operatorname{Ei}((a-i b) z) + (-1)^n \operatorname{Ei}(i b z - a z) + \frac{1}{\Gamma(n+2)} (2 (n+1) ((-1)^n \Gamma(n+1, -i b z) - \Gamma(n+1, i b z)) \operatorname{Shi}(a z)) - e^{(a-i b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{i b}{i b - a}\right)^m \sum_{k=0}^{m-1} \frac{(i b - a)^k z^k}{k!} + e^{-(a+b i) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{i b}{a + b i}\right)^m \sum_{k=0}^{m-1} \frac{(a + b i)^k z^k}{k!} + (-1)^n e^{(a+b i) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{i b}{a + b i}\right)^m \sum_{k=0}^{m-1} \frac{(-a - i b)^k z^k}{k!} - (-1)^n e^{i b z - a z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{i b}{i b - a}\right)^m \sum_{k=0}^{m-1} \frac{(a - i b)^k z^k}{k!} \right) /; n \in \mathbb{N}$$

06.39.21.0027.01

$$\int z \cos(b z) \operatorname{Shi}(a z) \, dz = -\frac{1}{4 b^2} \left(\operatorname{Ei}((a - i b) z) + \frac{1}{a^2+b^2} \left(-4 \cos(b z) \sinh(a z) b^2 + 4 a \cosh(a z) \sin(b z) b - (a^2+b^2) (\operatorname{Ei}(-(a+b i) z) - \operatorname{Ei}((a+b i) z) + \operatorname{Ei}(i b z - a z))\right) - 2 (\Gamma(2, -i b z) + \Gamma(2, i b z)) \operatorname{Shi}(a z) \right)$$

06.39.21.0028.01

$$\int z^2 \cos(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{1}{4\,b^3}\left(i\left(\frac{1}{\left(a^2+b^2\right)^2}\left(4\,i\sin(b\,z)\left(a\left(a^2+b^2\right)z\cosh(a\,z)+\left(a^2+3\,b^2\right)\sinh(a\,z)\right)b^2+ 2\left(a^2+b^2\right)^2\left(-\mathrm{Ei}(-(a+b\,i)\,z)-\mathrm{Ei}((a+b\,i)\,z)+\mathrm{Ei}((a-i\,b)\,z)+\mathrm{Ei}(i\,b\,z-a\,z)\right)+ 4\,i\cos(b\,z)\left(2\,a\,b\left(a^2+2\,b^2\right)\cosh(a\,z)-b^3\left(a^2+b^2\right)z\sinh(a\,z)\right)\right)+2\left(\Gamma(3,-i\,b\,z)-\Gamma(3,i\,b\,z)\right)\mathrm{Shi}(a\,z)\right)\right)$$

06.39.21.0029.01

$$\int z^3 \cos(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{1}{4\,b^4}\left(\frac{1}{\left(a^2+b^2\right)^3}\left(6\left(-\mathrm{Ei}(-(a+b\,i)\,z)+\mathrm{Ei}((a+b\,i)\,z)+\mathrm{Ei}((a-i\,b)\,z)-\mathrm{Ei}(i\,b\,z-a\,z)\right)\left(a^2+b^2\right)^3+4\,b^2\cos(b\,z) \left(\left(-3\,a^4-6\,b^2\,a^2-11\,b^4+b^2\left(a^2+b^2\right)^2z^2\right)\sinh(a\,z)-a\left(a^2+b^2\right)\left(3\,a^2+7\,b^2\right)z\cosh(a\,z)\right)+ 4\,b\sin(b\,z)\left(-\left(a^2+b^2\right)\left(a^2+5\,b^2\right)z\sinh(a\,z)\,b^2-a\left(b^2\left(a^2+b^2\right)^2z^2-2\left(3\,a^4+8\,b^2\,a^2+9\,b^4\right)\right)\cosh(a\,z)\right)\right)-2 \left(\Gamma(4,-i\,b\,z)+\Gamma(4,i\,b\,z)\right)\mathrm{Shi}(a\,z)\right)$$

Involving hyperbolic functions

Involving sinh

06.39.21.0030.01

$$\int \sinh(b\,z)\,\mathrm{Shi}(a\,z)\,dz = -\frac{-2\cosh(b\,z)\,\mathrm{Shi}(a\,z)+\mathrm{Shi}((a-b)\,z)+\mathrm{Shi}((a+b)\,z)}{2\,b}$$

Involving cosh

06.39.21.0031.01

$$\int \cosh(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{\mathrm{Chi}((a-b)\,z)-\mathrm{Chi}((a+b)\,z)+2\sinh(b\,z)\,\mathrm{Shi}(a\,z)}{2\,b}$$

06.39.21.0032.01

$$\int \cosh(a\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{-\mathrm{Chi}(2\,a\,z)+\log(a\,z)+2\sinh(a\,z)\,\mathrm{Shi}(a\,z)}{2\,a}$$

Involving hyperbolic functions and a power function

Involving sinh and power

06.39.21.0033.01

$$\int z^n \sinh(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{1}{4}\left(n!\,b^{-n-1}\right) \left(\mathrm{Ei}((-a-b)\,z) - \mathrm{Ei}((a-b)\,z) + (-1)^n\,\mathrm{Ei}((b-a)\,z) + (-1)^{-n-1}\,\mathrm{Ei}((a+b)\,z) + 2\,(-1)^n\,e^{b\,z}\,\mathrm{Shi}(a\,z)\sum_{k=0}^{n}\frac{(-b\,z)^k}{k!} + 2\,e^{-b\,z}\,\mathrm{Shi}(a\,z)\sum_{k=0}^{n}\frac{(b\,z)^k}{k!} + (-1)^n\,e^{(a+b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{a+b}\right)^m\sum_{k=0}^{m-1}\frac{(-a-b)^k\,z^k}{k!} - (-1)^n\,e^{(b-a)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b-a}\right)^m\sum_{k=0}^{m-1}\frac{(a-b)^k\,z^k}{k!} + e^{(a-b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{b-a}\right)^m\sum_{k=0}^{m-1}\frac{(b-a)^k\,z^k}{k!} - e^{(-a-b)\,z}\sum_{m=1}^{n}\frac{1}{m}\left(\frac{b}{a+b}\right)^m\sum_{k=0}^{m-1}\frac{(a+b)^k\,z^k}{k!}\right) /;\ n\in\mathbb{N}$$

06.39.21.0034.01

$$\int z^n \sinh(a\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{(-a)^{-n}}{4\,a}\left(2\,(\Gamma(n+1,\,-a\,z) + (-1)^n\,\Gamma(n+1,\,a\,z))\,\mathrm{Shi}(a\,z) + n!\left((-1)^n\,\mathrm{Ei}(-2\,a\,z) - \mathrm{Ei}(2\,a\,z) + (1-(-1)^n)\log(z) + 2\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(-a\,z)^k}{2\,k} + 2^{-k-1}\,\Gamma(k,\,-2\,a\,z)\right) - 2\,(-1)^n\sum_{k=1}^{n}\frac{1}{k!}\left(\frac{(a\,z)^k}{2\,k} + 2^{-k-1}\,\Gamma(k,\,2\,a\,z)\right)\right)\right) /;\ n\in\mathbb{N}$$

06.39.21.0035.01

$$\int z\sinh(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{a\cosh(a\,z)\cosh(b\,z)}{b^3 - a^2\,b} + \frac{\sinh(a\,z)\sinh(b\,z)}{a^2-b^2} + \frac{1}{4\,b^2}\left(-\mathrm{Ei}((a-b)\,z) - \mathrm{Ei}((b-a)\,z) + \mathrm{Ei}(-(a+b)\,z) + \mathrm{Ei}((a+b)\,z) + 4\,(b\,z\cosh(b\,z) - \sinh(b\,z))\,\mathrm{Shi}(a\,z)\right)$$

06.39.21.0036.01

$$\int z^2 \sinh(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{1}{2\,b^3}\left(-\mathrm{Ei}((a-b)\,z) + \frac{1}{(a-b)^2\,(a+b)^2}\left(2\cosh(b\,z)\left(a\left(b^2-a^2\right)z\cosh(a\,z) - \left(a^2-3\,b^2\right)\sinh(a\,z)\right)b^2 + 2\left((a-b)\,(a+b)\,z\sinh(a\,z)\,b^2 + 2\,a\left(a^2-2\,b^2\right)\cosh(a\,z)\right)\sinh(b\,z)\,b + \left(a^2-b^2\right)^2\left(\mathrm{Ei}((b-a)\,z) + \mathrm{Ei}(-(a+b)\,z) - \mathrm{Ei}((a+b)\,z)\right)\right) + 2\left(\left(b^2\,z^2+2\right)\cosh(b\,z) - 2\,b\,z\sinh(b\,z)\right)\mathrm{Shi}(a\,z)\right)$$

06.39.21.0037.01

$$\int z^3 \sinh(b\,z)\,\mathrm{Shi}(a\,z)\,dz = -\frac{1}{4\,b^4}\left(\frac{1}{(a-b)^3\,(a+b)^3}\left(6\left(\mathrm{Ei}((a-b)\,z) + \mathrm{Ei}((b-a)\,z) - \mathrm{Ei}(-(a+b)\,z) - \mathrm{Ei}((a+b)\,z)\right)\left(a^2-b^2\right)^3 + 4\,b\cosh(b\,z)\left(\left(a^4-6\,b^2\,a^2+5\,b^4\right)z\sinh(a\,z)\,b^2 + a\left(b^2\left(a^2-b^2\right)^2 z^2 + 2\left(3\,a^4-8\,b^2\,a^2+9\,b^4\right)\right)\cosh(a\,z)\right) + 4\,b^2\left(a\left(-3\,a^4+10\,b^2\,a^2-7\,b^4\right)z\cosh(a\,z) - \left(3\,a^4-6\,b^2\,a^2+11\,b^4+b^2\left(a^2-b^2\right)^2 z^2\right)\sinh(a\,z)\right)\sinh(b\,z)\right) + 4\left(3\left(b^2\,z^2+2\right)\sinh(b\,z) - b\,z\left(b^2\,z^2+6\right)\cosh(b\,z)\right)\mathrm{Shi}(a\,z)\right)$$

Involving cosh and power

06.39.21.0038.01

$$\int z^n \cosh(b z)\,\text{Shi}(a z)\,dz = \frac{1}{4}\left(n!\, b^{-n-1}\right)\left(-\text{Ei}((-a-b) z) + \text{Ei}((a-b) z) + (-1)^n\,\text{Ei}((b-a) z) + (-1)^{-n-1}\,\text{Ei}((a+b) z) + 2 (-1)^n e^{b z}\,\text{Shi}(a z) \sum_{k=0}^{n} \frac{(-b z)^k}{k!} - 2\,\text{Shi}(a z)\, e^{-b z} \sum_{k=0}^{n} \frac{(b z)^k}{k!} + (-1)^n e^{(a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(-a-b)^k z^k}{k!} - (-1)^n e^{(b-a) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(a-b)^k z^k}{k!} - e^{(a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a}\right)^m \sum_{k=0}^{m-1} \frac{(b-a)^k z^k}{k!} + e^{(-a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a+b}\right)^m \sum_{k=0}^{m-1} \frac{(a+b)^k z^k}{k!}\right) /;\ n \in \mathbb{N}$$

06.39.21.0039.01

$$\int z^n \cosh(a z)\,\text{Shi}(a z)\,dz = \frac{(-a)^{-n}}{4 a}\left(2\left(\Gamma(n+1, -a z) - (-1)^n\,\Gamma(n+1, a z)\right)\text{Shi}(a z) - n!\left((-1)^n\,\text{Ei}(-2 a z) + \text{Ei}(2 a z) - \left(1 + (-1)^n\right)\log(z) - 2 \sum_{k=1}^{n} \frac{1}{k!}\left(\frac{(-a z)^k}{2 k} + 2^{-k-1}\,\Gamma(k, -2 a z)\right) - 2 (-1)^n \sum_{k=1}^{n} \frac{1}{k!}\left(\frac{(a z)^k}{2 k} + 2^{-k-1}\,\Gamma(k, 2 a z)\right)\right)\right) /;\ n \in \mathbb{N}$$

06.39.21.0040.01

$$\int z \cosh(b z)\,\text{Shi}(a z)\,dz = \frac{1}{4 b^2}\left(\text{Ei}((a-b) z) + \frac{1}{b^2 - a^2}\left(-4 \cosh(b z) \sinh(a z)\, b^2 + 4 a \cosh(a z) \sinh(b z)\, b + (a-b)(a+b)\left(\text{Ei}((b-a) z) + \text{Ei}(-(a+b) z) - \text{Ei}((a+b) z)\right)\right) + 4 \left(b z \sinh(b z) - \cosh(b z)\right)\text{Shi}(a z)\right)$$

06.39.21.0041.01

$$\int z^2 \cosh(b z)\,\text{Shi}(a z)\,dz = \frac{1}{2 b^3}\left(\text{Ei}((a-b) z) + \text{Ei}((b-a) z) + \frac{1}{(a-b)^2 (a+b)^2}\left(2\left(a\left(b^2 - a^2\right) z \cosh(a z) - \left(a^2 - 3 b^2\right)\sinh(a z)\right)\sinh(b z)\, b^2 + 2 \cosh(b z)\left((a-b)(a+b) z \sinh(a z)\, b^2 + 2 a \left(a^2 - 2 b^2\right)\cosh(a z)\right) b - \left(a^2 - b^2\right)^2 \left(\text{Ei}(-(a+b) z) + \text{Ei}((a+b) z)\right)\right) + 2\left(\left(b^2 z^2 + 2\right)\sinh(b z) - 2 b z \cosh(b z)\right)\text{Shi}(a z)\right)$$

06.39.21.0042.01

$$\int z^3 \cosh(b z)\, \text{Shi}(a z)\, dz = \frac{1}{4 b^4}\left(\frac{1}{(a-b)^3 (a+b)^3}\left(6 (\text{Ei}((a-b) z) - \text{Ei}((b-a) z) - \text{Ei}(-(a+b) z) + \text{Ei}((a+b) z)) \left(a^2-b^2\right)^3 + 4 b^2 \cosh(b z) \left(a \left(3 a^4 - 10 b^2 a^2 + 7 b^4\right) z \cosh(a z) + \left(3 a^4 - 6 b^2 a^2 + 11 b^4 + b^2 \left(a^2-b^2\right)^2 z^2\right) \sinh(a z)\right) + 4 b \left(-\left(a^4 - 6 b^2 a^2 + 5 b^4\right) z \sinh(a z) b^2 - a \left(b^2 \left(a^2-b^2\right)^2 z^2 + 2 \left(3 a^4 - 8 b^2 a^2 + 9 b^4\right)\right) \cosh(a z)\right) \sinh(b z)\right) + 4 \left(b z \left(b^2 z^2 + 6\right) \sinh(b z) - 3 \left(b^2 z^2 + 2\right) \cosh(b z)\right) \text{Shi}(a z)\right)$$

Involving logarithm

Involving log

06.39.21.0043.01

$$\int \log(b z)\, \text{Shi}(a z)\, dz = \frac{\text{Chi}(a z) - (\log(b z) - 1) (\cosh(a z) - a z\, \text{Shi}(a z))}{a}$$

Involving logarithm and a power function

Involving log and power

06.39.21.0044.01

$$\int z^{\alpha-1} \log(b z)\, \text{Shi}(a z)\, dz = \frac{1}{2 \alpha^3}\left(z^\alpha \left(-a^2 z^2\right)^{-\alpha} \left({}_2F_2(\alpha, \alpha; \alpha+1, \alpha+1; a z) \left(-a^2 z^2\right)^\alpha - \left(-a^2 z^2\right)^\alpha {}_2F_2(\alpha, \alpha; \alpha+1, \alpha+1; -a z) + \alpha \left(\Gamma(\alpha, a z) (-a z)^\alpha + \Gamma(\alpha+1) \log(z) (-a z)^\alpha - \alpha\, \Gamma(\alpha, a z) \log(b z) (-a z)^\alpha - (a z)^\alpha\, \Gamma(\alpha+1) \log(z) + (a z)^\alpha\, \Gamma(\alpha, -a z) (\alpha \log(b z) - 1) + 2 \left(-a^2 z^2\right)^\alpha (\alpha \log(b z) - 1)\, \text{Shi}(a z)\right)\right)\right)$$

06.39.21.0045.01

$$\int z \log(b z)\, \text{Shi}(a z)\, dz = \frac{1}{4 a^2}\left(-a z \cosh(a z) (2 \log(b z) - 1) + (2 \log(b z) + 1) \sinh(a z) + \left(-a^2 z^2 + 2 a^2 \log(b z) z^2 - 2\right) \text{Shi}(a z)\right)$$

06.39.21.0046.01

$$\int z^2 \log(b z)\, \text{Shi}(a z)\, dz = \frac{1}{9 a^3}\left(a^3 (3 \log(b z) - 1)\, \text{Shi}(a z) z^3 + a^2 \cosh(a z) z^2 - 3 a^2 \cosh(a z) \log(b z) z^2 + a \sinh(a z) z + 6 a \log(b z) \sinh(a z) z - 7 \cosh(a z) + 6\, \text{Chi}(a z) - 6 \cosh(a z) \log(b z)\right)$$

06.39.21.0047.01

$$\int z^3 \log(b z)\, \text{Shi}(a z)\, dz = \frac{1}{16 a^4}\left(-a z \cosh(a z) \left(-a^2 z^2 + 4 \left(a^2 z^2 + 6\right) \log(b z) + 14\right) + \left(a^2 z^2 + 12 \left(a^2 z^2 + 2\right) \log(b z) + 38\right) \sinh(a z) + \left(-a^4 z^4 + 4 a^4 \log(b z) z^4 - 24\right) \text{Shi}(a z)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

06.39.21.0048.01

$$\int \operatorname{Shi}(a z)^2 \, dz = \frac{a z \operatorname{Shi}(a z)^2 - 2 \cosh(a z) \operatorname{Shi}(a z) + \operatorname{Shi}(2 a z)}{a}$$

Involving products of the direct function

06.39.21.0049.01

$$\int \operatorname{Shi}(a z) \operatorname{Shi}(b z) \, dz = \frac{1}{2 a b} ((a - b) \operatorname{Shi}((a - b) z) - 2 b \cosh(a z) \operatorname{Shi}(b z) - 2 a \operatorname{Shi}(a z) (\cosh(b z) - b z \operatorname{Shi}(b z)) + a \operatorname{Shi}((a + b) z) + b \operatorname{Shi}((a + b) z))$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

06.39.21.0050.01

$$\int z^n \operatorname{Shi}(a z)^2 \, dz = \frac{z^{n+1} \operatorname{Shi}(a z)^2}{n + 1} - \frac{1}{2 a (n + 1)} \Bigg((-a)^{-n} \Bigg(2 (\Gamma(n + 1, -a z) + (-1)^n \Gamma(n + 1, a z)) \operatorname{Shi}(a z) + n! \Bigg((-1)^n \operatorname{Ei}(-2 a z) - \operatorname{Ei}(2 a z) + (1 - (-1)^n) \log(z) + 2 \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(-a z)^k}{2 k} + 2^{-k-1} \Gamma(k, -2 a z)\right) - 2 (-1)^n \sum_{k=1}^{n} \frac{1}{k!} \left(\frac{(a z)^k}{2 k} + 2^{-k-1} \Gamma(k, 2 a z)\right)\Bigg)\Bigg)\Bigg) /; n \in \mathbb{N}$$

06.39.21.0051.01

$$\int z \operatorname{Shi}(a z)^2 \, dz = \frac{1}{4 a^2} \left(2 a^2 z^2 \operatorname{Shi}(a z)^2 + (4 \sinh(a z) - 4 a z \cosh(a z)) \operatorname{Shi}(a z) + \cosh(2 a z) - 2 \operatorname{Chi}(2 a z) + 2 \log(z)\right)$$

06.39.21.0052.01

$$\int z^2 \operatorname{Shi}(a z)^2 \, dz = \frac{1}{12 a^3} \left(4 a^3 \operatorname{Shi}(a z)^2 z^3 + 8 a z + 2 a \cosh(2 a z) z - 5 \sinh(2 a z) - 8 \left(\left(a^2 z^2 + 2\right) \cosh(a z) - 2 a z \sinh(a z)\right) \operatorname{Shi}(a z) + 8 \operatorname{Shi}(2 a z)\right)$$

06.39.21.0053.01

$$\int z^3 \operatorname{Shi}(a z)^2 \, dz = \frac{1}{8 a^4} \left(2 a^4 \operatorname{Shi}(a z)^2 z^4 + 3 a^2 z^2 + a^2 \cosh(2 a z) z^2 - 4 a \sinh(2 a z) z + 8 \cosh(2 a z) - 12 \operatorname{Chi}(2 a z) + 12 \log(z) - 4 \left(a z \left(a^2 z^2 + 6\right) \cosh(a z) - 3 \left(a^2 z^2 + 2\right) \sinh(a z)\right) \operatorname{Shi}(a z)\right)$$

Involving products of the direct function and a power function

06.39.21.0054.01

$$\int z^n \operatorname{Shi}(a z) \operatorname{Shi}(b z) \, dz = \frac{(-1)^n a^{-n-1}}{4(n+1)} \left(n! \left(-\operatorname{Ei}((a-b) z) + \operatorname{Ei}((a+b) z) + \sum_{k=1}^{n} \frac{1}{k!} \left((-a)^k \left((b-a)^{-k} \Gamma(k, (b-a) z) - (-a-b)^{-k} \Gamma(k, -(a+b) z) \right) \right) + (-1)^n \left(\operatorname{Ei}((b-a) z) - \operatorname{Ei}(-(a+b) z) + \sum_{k=1}^{n} \frac{1}{k!} \left(a^k \left((a+b)^{-k} \Gamma(k, (a+b) z) - (a-b)^{-k} \Gamma(k, (a-b) z) \right) \right) \right) \right) - 2 \left(2 \operatorname{Shi}(a z) (-a z)^{n+1} + \Gamma(n+1, -a z) + (-1)^n \Gamma(n+1, a z) \right) \operatorname{Shi}(b z) \right) - \frac{1}{(n+1) 4} \left(\left(n! \, b^{-n-1} \right) \left(\operatorname{Ei}((-a-b) z) - \operatorname{Ei}((a-b) z) + (-1)^n \operatorname{Ei}((b-a) z) + (-1)^{-n-1} \operatorname{Ei}((a+b) z) + 2 (-1)^n e^{b z} \operatorname{Shi}(a z) \sum_{k=0}^{n} \frac{(-b z)^k}{k!} + 2 e^{-b z} \operatorname{Shi}(a z) \sum_{k=0}^{n} \frac{(b z)^k}{k!} + (-1)^n e^{(a+b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a+b} \right)^m \sum_{k=0}^{m-1} \frac{(-a-b)^k z^k}{k!} - (-1)^n e^{(b-a) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a} \right)^m \sum_{k=0}^{m-1} \frac{(a-b)^k z^k}{k!} + e^{(a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{b-a} \right)^m \sum_{k=0}^{m-1} \frac{(b-a)^k z^k}{k!} - e^{(-a-b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{b}{a+b} \right)^m \sum_{k=0}^{m-1} \frac{(a+b)^k z^k}{k!} \right) \right) /; n \in \mathbb{N}$$

06.39.21.0055.01

$$\int z \operatorname{Shi}(a z) \operatorname{Shi}(b z) \, dz = \frac{1}{8 a^2 b^2} \left(4 \operatorname{Shi}(a z) \left(b^2 \operatorname{Shi}(b z) z^2 - b \cosh(b z) z + \sinh(b z) \right) a^2 + 4 b \cosh(a z) \cosh(b z) a + \left(a^2 + b^2 \right) (\operatorname{Ei}((a-b) z) + \operatorname{Ei}((b-a) z) - \operatorname{Ei}(-(a+b) z) - \operatorname{Ei}((a+b) z)) + 4 b^2 (\sinh(a z) - a z \cosh(a z)) \operatorname{Shi}(b z) \right)$$

06.39.21.0056.01

$$\int z^2 \operatorname{Shi}(a z) \operatorname{Shi}(b z) \, dz = \frac{1}{6} \left(-\frac{2 \left(\left(b^2 z^2 + 2 \right) \cosh(b z) - 2 b z \sinh(b z) \right) \operatorname{Shi}(a z)}{b^3} + \frac{1}{a^3} \left(2 \left(a^3 \operatorname{Shi}(a z) z^3 + 2 a \sinh(a z) z - \left(a^2 z^2 + 2 \right) \cosh(a z) \right) \operatorname{Shi}(b z) \right) + \frac{1}{a^3 (a-b) b^3 (a+b)} \left(2 \left(a b \left(b \left(a^2 + 2 b^2 \right) \cosh(b z) \sinh(a z) + a \cosh(a z) \left(b \left(a^2 - b^2 \right) z \cosh(b z) - \left(2 a^2 + b^2 \right) \sinh(b z) \right) \right) + \left(a^5 - b^2 a^3 - b^3 a^2 + b^5 \right) \operatorname{Shi}((a-b) z) + \left(a^5 - b^2 a^3 + b^3 a^2 - b^5 \right) \operatorname{Shi}((a+b) z) \right) \right) \right)$$

06.39.21.0057.01

$$\int z^3\,\mathrm{Shi}(a\,z)\,\mathrm{Shi}(b\,z)\,dz = \frac{1}{8}\Big(\Big(2\left(3\left(a^4+b^4\right)\mathrm{Chi}((a-b)\,z)\left(a^2-b^2\right)^2-3\left(a^4+b^4\right)\mathrm{Chi}((a+b)\,z)\left(a^2-b^2\right)^2+a\,b\left(a\,b\,\sinh(a\,z)\left(b\left(a^4+2\,b^2\,a^2-3\,b^4\right)z\cosh(b\,z)+\left(-3\,a^4+14\,b^2\,a^2-3\,b^4\right)\sinh(b\,z)\right)+\cosh(a\,z)\left(b\left(-3\,a^4+2\,b^2\,a^2+b^4\right)z\sinh(b\,z)\,a^2+\left(\left(b^2\,z^2+6\right)a^6-2\,b^2\left(b^2\,z^2+5\right)a^4+b^4\left(b^2\,z^2-10\right)a^2+6\,b^6\right)\cosh(b\,z)\right)\right)\right)\Big)\Big/\left(a^4\,(a-b)^2\,b^4\,(a+b)^2\right)-\frac{1}{b^4}\left(2\left(b\,z\left(b^2\,z^2+6\right)\cosh(b\,z)-3\left(b^2\,z^2+2\right)\sinh(b\,z)\right)\mathrm{Shi}(a\,z)\right)+\frac{1}{a^4}\left(2\left(a^4\,\mathrm{Shi}(a\,z)\,z^4-a\left(a^2\,z^2+6\right)\cosh(a\,z)\,z+3\left(a^2\,z^2+2\right)\sinh(a\,z)\right)\mathrm{Shi}(b\,z)\right)\Big)$$

Involving direct function and Gamma-, Beta-, Erf-type functions

Involving exponential integral-type functions

Involving Ei

06.39.21.0058.01

$$\int \mathrm{Ei}(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{1}{2\,a\,b}\left(-2\,b\cosh(a\,z)\,\mathrm{Ei}(b\,z)+2\,a\,b\,z\,\mathrm{Shi}(a\,z)\,\mathrm{Ei}(b\,z)-a\,\mathrm{Ei}((b-a)\,z)+b\,\mathrm{Ei}((b-a)\,z)+a\,\mathrm{Ei}((a+b)\,z)+b\,\mathrm{Ei}((a+b)\,z)-2\,a\,e^{b\,z}\,\mathrm{Shi}(a\,z)\right)$$

06.39.21.0059.01

$$\int \mathrm{Ei}(a\,z)\,\mathrm{Shi}(a\,z)\,dz = -\frac{1}{2\,a}\left(\left(e^{-a\,z}+e^{a\,z}\right)\mathrm{Ei}(a\,z)-2\,\mathrm{Ei}(2\,a\,z)+2\left(e^{a\,z}-a\,z\,\mathrm{Ei}(a\,z)\right)\mathrm{Shi}(a\,z)\right)$$

06.39.21.0060.01

$$\int \mathrm{Ei}(-a\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{1}{2\,a}\left(-e^{a\,z}\left(1+e^{-2\,a\,z}\right)\mathrm{Ei}(-a\,z)+2\,\mathrm{Ei}(-2\,a\,z)+2\left(a\,z\,\mathrm{Ei}(-a\,z)+e^{-a\,z}\right)\mathrm{Shi}(a\,z)\right)$$

Involving Si

06.39.21.0061.01

$$\int \mathrm{Si}(b\,z)\,\mathrm{Shi}(a\,z)\,dz = -\frac{1}{2\,a\,b}\left(a\,\mathrm{Shi}((a+b\,i)\,z)+b\,i\,\mathrm{Shi}((a+b\,i)\,z)+(a-i\,b)\,\mathrm{Shi}((a-i\,b)\,z)+2\,b\cosh(a\,z)\,\mathrm{Si}(b\,z)-2\,a\,\mathrm{Shi}(a\,z)\left(\cos(b\,z)+b\,z\,\mathrm{Si}(b\,z)\right)\right)$$

Involving Ci

06.39.21.0062.01

$$\int \mathrm{Ci}(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{1}{2\,a\,b}\left((b-i\,a)\,\mathrm{Chi}((a+b\,i)\,z)+(b+a\,i)\,\mathrm{Chi}((a-i\,b)\,z)-2\left(a\sin(b\,z)\,\mathrm{Shi}(a\,z)+b\,\mathrm{Ci}(b\,z)\left(\cosh(a\,z)-a\,z\,\mathrm{Shi}(a\,z)\right)\right)\right)$$

Involving exponential integral-type functions and a power function

Involving Ei and power

06.39.21.0063.01

$$\int z^n \operatorname{Ei}(b z) \operatorname{Shi}(a z) \, dz = \frac{\left(\Gamma(n+1, -b z) (-b)^{-n-1} + z^{n+1} \operatorname{Ei}(b z)\right) \operatorname{Shi}(a z)}{n+1} - \frac{a^{-n-1}}{2 (n+1)} \left(-\operatorname{Ei}((b-a) z) \, n! + n! \sum_{k=1}^n \frac{a^k (a-b)^{-k} \Gamma(k, (a-b) z)}{k!} + \operatorname{Ei}(b z) \Gamma(n+1, a z) + (-1)^n \left(-\operatorname{Ei}((a+b) z) \, n! + n! \sum_{k=1}^n \frac{a^k (a+b)^{-k} \Gamma(k, -(a+b) z)}{k!} + \operatorname{Ei}(b z) \Gamma(n+1, -a z)\right) + \frac{(-b)^{-n-1} n!}{2} \left(-\operatorname{Ei}(b z - a z) + \operatorname{Ei}(a z + b z) + \sum_{k=1}^n \frac{1}{k!} \left(b^k \left((b-a)^{-k} \Gamma(k, (a-b) z) - (a+b)^{-k} \Gamma(k, -(a+b) z)\right)\right)\right)\right) /; n \in \mathbb{N}$$

06.39.21.0064.01

$$\int z^n \operatorname{Ei}(a z) \operatorname{Shi}(a z) \, dz = \frac{\left(\Gamma(n+1, -a z) (-a)^{-n-1} + z^{n+1} \operatorname{Ei}(a z)\right) \operatorname{Shi}(a z)}{n+1} - \frac{(-a)^{-n-1} n!}{2 (n+1)} \left(\operatorname{Ei}(2 a z) - \log(z) - \sum_{k=1}^n \frac{1}{k!} \left(\frac{(-a z)^k}{k} + 2^{-k} \Gamma(k, -2 a z)\right)\right) - \frac{1}{2 (n+1)} \left(a^{-n-1} \left(-(-1)^n \operatorname{Ei}(2 a z) \, n! - \log(z) \, n! + (-1)^n n! \sum_{k=1}^n \frac{2^{-k} \Gamma(k, -2 a z)}{k!} - n! \sum_{k=1}^n \frac{(a z)^k}{k \, k!} + \operatorname{Ei}(a z) \left((-1)^n \Gamma(n+1, -a z) + \Gamma(n+1, a z)\right)\right)\right) /; n \in \mathbb{N}$$

06.39.21.0065.01

$$\int z \operatorname{Ei}(b z) \operatorname{Shi}(a z) \, dz = \frac{1}{4 a^2 b^2} \left(-\operatorname{Ei}((a+b) z) a^2 + 2 \left(b^2 \operatorname{Ei}(b z) z^2 + \Gamma(2, -b z)\right) \operatorname{Shi}(a z) a^2 + b e^{(b-a) z} a + b e^{(a+b) z} a + \left(a^2 + b^2\right) \operatorname{Ei}((b-a) z) - b^2 \operatorname{Ei}((a+b) z) + b^2 \operatorname{Ei}(b z) \Gamma(2, -a z) - b^2 \operatorname{Ei}(b z) \Gamma(2, a z)\right)$$

06.39.21.0066.01

$$\int z^2 \operatorname{Ei}(b z) \operatorname{Shi}(a z) \, dz = \frac{1}{3} \left(\frac{1}{2 b^2} \left(\frac{b \Gamma(2, (a-b) z)}{(a-b)^2} + \frac{2 e^{(b-a) z}}{b-a} - \frac{2 e^{(a+b) z}}{a+b} - \frac{b \Gamma(2, -(a+b) z)}{(a+b)^2}\right) + \frac{\operatorname{Ei}((a+b) z) - \operatorname{Ei}((b-a) z)}{b^3} - \frac{1}{2 a^3} \left(\frac{\Gamma(2, (a-b) z) a^2}{(a-b)^2} + \frac{\Gamma(2, -(a+b) z) a^2}{(a+b)^2} + \frac{2 e^{(b-a) z} a}{a-b} + \frac{2 e^{(a+b) z} a}{a+b} - 2 \operatorname{Ei}((b-a) z) - 2 \operatorname{Ei}((a+b) z) + \operatorname{Ei}(b z) \Gamma(3, -a z) + \operatorname{Ei}(b z) \Gamma(3, a z)\right) + \left(z^3 \operatorname{Ei}(b z) - \frac{\Gamma(3, -b z)}{b^3}\right) \operatorname{Shi}(a z)\right)$$

06.39.21.0067.01

$$\int z^3 \operatorname{Ei}(b z) \operatorname{Shi}(a z) \, dz = \frac{1}{4}\left(-\frac{1}{2 b^3}\left(\frac{\Gamma(3, (a-b) z) b^2}{(b-a)^3} - \frac{\Gamma(3, -(a+b) z) b^2}{(a+b)^3} + \frac{3 \Gamma(2, (a-b) z) b}{(a-b)^2} - \frac{3 \Gamma(2, -(a+b) z) b}{(a+b)^2} + \frac{6 e^{(b-a) z}}{b-a} - \frac{6 e^{(a+b) z}}{a+b}\right) + \frac{3 (\operatorname{Ei}((b-a) z) - \operatorname{Ei}((a+b) z))}{b^4} - \frac{1}{2 a^4}\left(\frac{\Gamma(3, (a-b) z) a^3}{(a-b)^3} - \frac{\Gamma(3, -(a+b) z) a^3}{(a+b)^3} + \frac{3 \Gamma(2, (a-b) z) a^2}{(a-b)^2} - \frac{3 \Gamma(2, -(a+b) z) a^2}{(a+b)^2} + \frac{6 e^{(b-a) z} a}{a-b} - \frac{6 e^{(a+b) z} a}{a+b} - 6 \operatorname{Ei}((b-a) z) + 6 \operatorname{Ei}((a+b) z) - \operatorname{Ei}(b z) \Gamma(4, -a z) + \operatorname{Ei}(b z) \Gamma(4, a z)\right) + \left(\operatorname{Ei}(b z) z^4 + \frac{\Gamma(4, -b z)}{b^4}\right) \operatorname{Shi}(a z)\right)$$

Involving Si and power

06.39.21.0068.01

$$\int z^n \operatorname{Si}(b z) \operatorname{Shi}(a z) \, dz = \frac{n! (-i b)^{-n}}{4 b (n+1)} (\operatorname{Ei}((i b - a) z) + (-1)^n \operatorname{Ei}(-(a + b i) z) - \operatorname{Ei}((a + b i) z) - (-1)^n \operatorname{Ei}((a - i b) z)) + \frac{1}{2 (n+1)} \left(i (i b)^{-n-1} (\Gamma(n+1, i b z) + (-1)^n \Gamma(n+1, -i b z)) \operatorname{Shi}(a z)\right) + \frac{z^{n+1} \operatorname{Shi}(a z) \operatorname{Si}(b z)}{n+1} + \frac{i a^{-n-1} n!}{4 (n+1)} \left(-\operatorname{Ei}((i b - a) z) - (-1)^{-n} \operatorname{Ei}((a + b i) z) + \operatorname{Ei}((-a - i b) z) + (-1)^n \operatorname{Ei}((a - i b) z) + \frac{(2 i) ((-1)^n \Gamma(n+1, -a z) + \Gamma(n+1, a z)) \operatorname{Si}(b z)}{n!} - (-1)^n e^{(a - i b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{a}{a - i b}\right)^m \sum_{k=0}^{m-1} \frac{(i b - a)^k z^k}{k!} + (-1)^n e^{(a + b i) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{a}{a + b i}\right)^m \sum_{k=0}^{m-1} \frac{(-a - i b)^k z^k}{k!} - e^{(-a - i b) z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{a}{a + b i}\right)^m \sum_{k=0}^{m-1} \frac{(a + b i)^k z^k}{k!} + e^{i b z - a z} \sum_{m=1}^{n} \frac{1}{m} \left(\frac{a}{a - i b}\right)^m \sum_{k=0}^{m-1} \frac{(a - i b)^k z^k}{k!}\right) - \frac{i (i b)^{-n-1} n!}{4 (n+1)} \left((-1)^n \sum_{k=1}^{n} \frac{1}{k!} \left(b^k \left((b + a i)^{-k} \Gamma(k, (a - i b) z) - (b - i a)^{-k} \Gamma(k, -(a + b i) z)\right)\right) + \sum_{k=1}^{n} \frac{1}{k!} \left(b^k \left((b - i a)^{-k} \Gamma(k, (a + b i) z) - (b + a i)^{-k} \Gamma(k, -(a - i b) z)\right)\right)\right) /; n \in \mathbb{N}$$

06.39.21.0069.01

$$\int z \operatorname{Si}(b z) \operatorname{Shi}(a z) \, dz = -\frac{1}{8 a^2 b^2} \left(i \operatorname{Ei}(-(a + b i) z) a^2 + i \operatorname{Ei}((a + b i) z) a^2 - i \operatorname{Ei}((a - i b) z) a^2 - i \operatorname{Ei}(i b z - a z) a^2 - 2 i \operatorname{Shi}(a z) \left(-2 i b^2 \operatorname{Si}(b z) z^2 + \Gamma(2, -i b z) - \Gamma(2, i b z)\right) a^2 + b e^{-(a + b i) z} a + b e^{(a + b i) z} a + b e^{(a - i b) z} a + b e^{i b z - a z} a - i b^2 \operatorname{Ei}(-(a + b i) z) - i b^2 \operatorname{Ei}((a + b i) z) + b^2 i \operatorname{Ei}((a - i b) z) + b^2 i \operatorname{Ei}(i b z - a z) + 2 b^2 (\Gamma(2, a z) - \Gamma(2, -a z)) \operatorname{Si}(b z)\right)$$

06.39.21.0070.01

$$\int z^2\,\mathrm{Si}(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{1}{6}\Bigg(2\,\mathrm{Shi}(a\,z)\,\mathrm{Si}(b\,z)\,z^3 + \frac{1}{2\,b^2}\Bigg(-\frac{2\,e^{(a-i\,b)\,z}}{b+a\,i} + \frac{2\,i\,e^{-(a+b\,i)\,z}}{a+b\,i} + \frac{2\,e^{(a+b\,i)\,z}}{i\,a-b} + \frac{2\,e^{i\,b\,z-a\,z}}{b+a\,i} + \frac{b\,\Gamma(2,\,(a-i\,b)\,z)}{(b+a\,i)^2} - \frac{b\,\Gamma(2,\,-(a+b\,i)\,z)}{(b-i\,a)^2} + \frac{b\,\Gamma(2,\,(a+b\,i)\,z)}{(b-i\,a)^2} - \frac{b\,\Gamma(2,\,i\,b\,z-a\,z)}{(b+a\,i)^2}\Bigg) + \frac{1}{b^3}\,(-\mathrm{Ei}(-(a+b\,i)\,z) + \mathrm{Ei}((a+b\,i)\,z) + \mathrm{Ei}((a-i\,b)\,z) - \mathrm{Ei}(i\,b\,z-a\,z)) + \frac{i}{2\,a^3}\Bigg(-\frac{a\,e^{(a+b\,i)\,z}\left(z\,a^2+(i\,b\,z-3)\,a-2\,b\,i\right)}{(a+b\,i)^2} + \frac{a\,e^{(a-i\,b)\,z}\left(z\,a^2+(-i\,b\,z-3)\,a+2\,b\,i\right)}{(a-i\,b)^2} + \frac{a\,e^{i\,b\,z-a\,z}\left(z\,a^2+(3-i\,b\,z)\,a-2\,b\,i\right)}{(a-i\,b)^2} + 2\,\mathrm{Ei}(-(a+b\,i)\,z) - 2\,\mathrm{Ei}((a+b\,i)\,z) + 2\,\mathrm{Ei}((a-i\,b)\,z) - 2\,\mathrm{Ei}(i\,b\,z-a\,z) + 2\,i\,(\Gamma(3,\,-a\,z)+\Gamma(3,\,a\,z))\,\mathrm{Si}(b\,z) - \frac{a\,e^{-(a+b\,i)\,z}\left(z\,a^2+(b\,i\,z+3)\,a+2\,b\,i\right)}{(a+b\,i)^2}\Bigg) - \frac{(\Gamma(3,\,-i\,b\,z)+\Gamma(3,\,i\,b\,z))\,\mathrm{Shi}(a\,z)}{b^3}\Bigg)$$

06.39.21.0071.01

$$\int z^3\,\mathrm{Si}(b\,z)\,\mathrm{Shi}(a\,z)\,dz = \frac{1}{8}\Bigg(2\,\mathrm{Shi}(a\,z)\,\mathrm{Si}(b\,z)\,z^4 - \frac{i}{2\,b^3}\Bigg(\frac{\Gamma(3,\,-(a+b\,i)\,z)\,b^2}{(b-i\,a)^3} + \frac{\Gamma(3,\,(a+b\,i)\,z)\,b^2}{(b-i\,a)^3} - \frac{\Gamma(3,\,(a-i\,b)\,z)\,b^2}{(b+a\,i)^3} - \frac{\Gamma(3,\,i\,b\,z-a\,z)\,b^2}{(b+a\,i)^3} + \frac{3\,\Gamma(2,\,-(a+b\,i)\,z)\,b}{(b-i\,a)^2} + \frac{3\,\Gamma(2,\,(a+b\,i)\,z)\,b}{(b-i\,a)^2} - \frac{3\,\Gamma(2,\,(a-i\,b)\,z)\,b}{(b+a\,i)^2} - \frac{3\,\Gamma(2,\,i\,b\,z-a\,z)\,b}{(b+a\,i)^2} + \frac{6\,e^{-(a+b\,i)\,z}\,i}{a+b\,i} + \frac{6\,e^{(a+b\,i)\,z}\,i}{a+b\,i} - \frac{6\,e^{(a-i\,b)\,z}}{b+a\,i} - \frac{6\,e^{i\,b\,z-a\,z}}{b+a\,i}\Bigg) + \frac{1}{b^4}\,(3\,i\,(\mathrm{Ei}(-(a+b\,i)\,z) + \mathrm{Ei}((a+b\,i)\,z) - \mathrm{Ei}((a-i\,b)\,z) - \mathrm{Ei}(i\,b\,z-a\,z))) + \frac{i\,(\Gamma(4,\,i\,b\,z)-\Gamma(4,\,-i\,b\,z))\,\mathrm{Shi}(a\,z)}{b^4} + \frac{1}{2\,a^4}\Bigg(i\Bigg(-\frac{1}{(a+b\,i)^3}\left(a\,e^{(a+b\,i)\,z}\left(z^2\,a^4+z\,(2\,i\,b\,z-5)\,a^3-\left(b^2\,z^2+8\,b\,i\,z-11\right)a^2+3\,b\,(5\,i+b\,z)\,a-6\,b^2\right)\right) + \frac{1}{(a-i\,b)^3}\left(a\,e^{i\,b\,z-a\,z}\left(z^2\,a^4+z\,(5-2\,i\,b\,z)\,a^3-\left(b^2\,z^2+8\,b\,i\,z-11\right)a^2-3\,b\,(5\,i+b\,z)\,a-6\,b^2\right)\right) + \frac{1}{(a-i\,b)^3}\left(a\,e^{(a-i\,b)\,z}\left(z^2\,a^4+z\,(-2\,i\,b\,z-5)\,a^3+\left(-b^2\,z^2+8\,b\,i\,z+11\right)a^2+3\,b\,(-5\,i+b\,z)\,a-6\,b^2\right)\right) - \frac{1}{(a+b\,i)^3}\left(a\,e^{-(a+b\,i)\,z}\left(z^2\,a^4+z\,(2\,b\,i\,z+5)\,a^3+\left(-b^2\,z^2+8\,b\,i\,z+11\right)a^2-3\,b\,(-5\,i+b\,z)\,a-6\,b^2\right)\right) + 6\,\mathrm{Ei}(-(a+b\,i)\,z) + 6\,\mathrm{Ei}((a+b\,i)\,z) - 6\,\mathrm{Ei}((a-i\,b)\,z) - 6\,\mathrm{Ei}(i\,b\,z-a\,z) + 2\,i\,(\Gamma(4,\,a\,z)-\Gamma(4,\,-a\,z))\,\mathrm{Si}(b\,z)\Bigg)\Bigg)\Bigg)$$

Involving Ci and power

06.39.21.0072.01

$$\int z^n \operatorname{Ci}(b z) \operatorname{Shi}(a z) \, dz = \frac{(i b)^{-n-1} n!}{4 (n+1)} ((-1)^n (\operatorname{Ei}(a z + b i z) - \operatorname{Ei}(i b z - a z)) + \operatorname{Ei}(-a z - i b z) - \operatorname{Ei}(a z - i b z)) + \frac{z^{n+1} \operatorname{Ci}(b z) \operatorname{Shi}(a z)}{n+1} - \frac{1}{2 (n+1)} \left((i b)^{-n-1} ((-1)^n \Gamma(n+1, -i b z) - \Gamma(n+1, i b z)) \operatorname{Shi}(a z)\right) - \frac{a^{-n-1} n!}{4 (n+1)} \left(-\operatorname{Ei}((i b - a) z) - (-1)^n \operatorname{Ei}((a + b i) z) - \operatorname{Ei}((-a - i b) z) - (-1)^n \operatorname{Ei}((a - i b) z) + \frac{\operatorname{Ci}(b z) 2 ((-1)^n \Gamma(n+1, -a z) + \Gamma(n+1, a z))}{n!} + (-1)^n e^{(a+b i) z} \sum_{m=1}^n \frac{1}{m} \left(\frac{a}{a + b i}\right)^m \sum_{k=0}^{m-1} \frac{(-a + b(-i))^k z^k}{k!} + (-1)^n e^{(a - i b) z} \sum_{m=1}^n \frac{1}{m} \left(\frac{a}{a - i b}\right)^m \sum_{k=0}^{m-1} \frac{(i b - a)^k z^k}{k!} + e^{(i b - a) z} \sum_{m=1}^n \frac{1}{m} \left(\frac{a}{a - i b}\right)^m \sum_{k=0}^{m-1} \frac{(a - i b)^k z^k}{k!} + e^{(-a - i b) z} \sum_{m=1}^n \frac{1}{m} \left(\frac{a}{a + b i}\right)^m \sum_{k=0}^{m-1} \frac{(a + b i)^k z^k}{k!}\right) + \frac{(i b)^{-n-1} n!}{4 (n+1)} \left((-1)^n \sum_{k=1}^n \frac{1}{k!} \left(b^k \left((b + a i)^{-k} \Gamma(k, (a - i b) z) - (b - i a)^{-k} \Gamma(k, -(a + b i) z)\right)\right) - \sum_{k=1}^n \frac{1}{k!} \left(b^k \left((b - i a)^{-k} \Gamma(k, (a + b i) z) - (b + a i)^{-k} \Gamma(k, (i b - a) z)\right)\right)\right) /; n \in \mathbb{N}$$

06.39.21.0073.01

$$\int z \operatorname{Ci}(b z) \operatorname{Shi}(a z) \, dz = \frac{1}{8 a^2 b^2} \left(-\operatorname{Ei}(-(a + b i) z) a^2 + \operatorname{Ei}((a + b i) z) a^2 + \operatorname{Ei}((a - i b) z) a^2 - \operatorname{Ei}(i b z - a z) a^2 - 2 (\Gamma(2, -i b z) + \Gamma(2, i b z)) \operatorname{Shi}(a z) a^2 - i b e^{(a+b i) z} a - i b e^{i b z - a z} a + b e^{-(a+b i) z} i a + b e^{(a - i b) z} i a + b^2 \operatorname{Ei}(-(a + b i) z) - b^2 \operatorname{Ei}((a + b i) z) - b^2 \operatorname{Ei}((a - i b) z) + b^2 \operatorname{Ei}(i b z - a z) + 2 b^2 \operatorname{Ci}(b z) \left(2 a^2 \operatorname{Shi}(a z) z^2 + \Gamma(2, -a z) - \Gamma(2, a z)\right)\right)$$

06.39.21.0074.01

$$\int z^2 \operatorname{Ci}(b z) \operatorname{Shi}(a z) \, dz = \frac{1}{6} \left(2 \operatorname{Ci}(b z) \operatorname{Shi}(a z) z^3 + \frac{i}{2 b^2} \left(-\frac{b \Gamma(2, -(a + b i) z)}{(b - i a)^2} + \frac{2 e^{-(a+b i) z}}{i a - b} + \frac{2 e^{(a+b i) z}}{i a - b} + \frac{2 e^{(a - i b) z}}{b + a i} + \frac{2 e^{i b z - a z}}{b + a i} + \frac{b \Gamma(2, (a - i b) z)}{(b + a i)^2} + \frac{b \Gamma(2, i b z - a z)}{(b + a i)^2} - \frac{b \Gamma(2, (a + b i) z)}{(b - i a)^2}\right) + \frac{1}{b^3} (i (\operatorname{Ei}(-(a + b i) z) + \operatorname{Ei}((a + b i) z) - \operatorname{Ei}((a - i b) z) - \operatorname{Ei}(i b z - a z))) - \frac{1}{2 a^3} \left(-\frac{a e^{(a+b i) z} \left(z a^2 + (i b z - 3) a - 2 b i\right)}{(a + b i)^2} + \frac{a e^{-(a+b i) z} \left(z a^2 + (b i z + 3) a + 2 b i\right)}{(a + b i)^2} + \frac{a e^{i b z - a z} \left(z a^2 + (3 - i b z) a - 2 b i\right)}{(a - i b)^2} - 2 \operatorname{Ei}(-(a + b i) z) - 2 \operatorname{Ei}((a + b i) z) - 2 \operatorname{Ei}((a - i b) z) - 2 \operatorname{Ei}(i b z - a z) + 2 \operatorname{Ci}(b z) (\Gamma(3, -a z) + \Gamma(3, a z)) - \frac{a e^{(a - i b) z} \left(z a^2 + (-i b z - 3) a + 2 b i\right)}{(a - i b)^2}\right) - \frac{i (\Gamma(3, -i b z) - \Gamma(3, i b z)) \operatorname{Shi}(a z)}{b^3}\right)$$

06.39.21.0075.01

$$\int z^3 \operatorname{Ci}(b z) \operatorname{Shi}(a z) \, dz = \frac{1}{8}\left(2 \operatorname{Ci}(b z) \operatorname{Shi}(a z) z^4 + \frac{1}{2 b^3}\left(\frac{\Gamma(3, -(a+b i) z) b^2}{(b-i a)^3} + \frac{\Gamma(3, i b z - a z) b^2}{(b+a i)^3} - \frac{\Gamma(3, (a+b i) z) b^2}{(b-i a)^3} - \frac{\Gamma(3, (a-i b) z) b^2}{(b+a i)^3} + \frac{3 \Gamma(2, -(a+b i) z) b}{(b-i a)^2} + \frac{3 \Gamma(2, i b z - a z) b}{(b+a i)^2} - \frac{3 \Gamma(2, (a+b i) z) b}{(b-i a)^2} - \frac{3 \Gamma(2, (a-i b) z) b}{(b+a i)^2} + \frac{6 e^{-(a+b i) z}}{i a - b} + \frac{6 e^{(a-i b) z}}{b+a i} + \frac{6 e^{(a+b i) z} i}{a+b i} - \frac{6 e^{i b z - a z}}{b+a i}\right) + \frac{1}{b^4}(3(\operatorname{Ei}(-(a+b i) z) - \operatorname{Ei}((a+b i) z) - \operatorname{Ei}((a-i b) z) + \operatorname{Ei}(i b z - a z))) - \frac{1}{2 a^4}\left(-\frac{1}{(a+b i)^3}\left(a e^{(a+b i) z}\left(z^2 a^4 + z(2 i b z - 5) a^3 - \left(b^2 z^2 + 8 b i z - 11\right) a^2 + 3 b(5 i + b z) a - 6 b^2\right)\right) + \frac{1}{(a-i b)^3}\left(a e^{i b z - a z}\left(z^2 a^4 + z(5 - 2 i b z) a^3 - \left(b^2 z^2 + 8 b i z - 11\right) a^2 - 3 b(5 i + b z) a - 6 b^2\right)\right) - \frac{1}{(a-i b)^3}\left(a e^{(a-i b) z}\left(z^2 a^4 + z(-2 i b z - 5) a^3 + \left(-b^2 z^2 + 8 b i z + 11\right) a^2 + 3 b(-5 i + b z) a - 6 b^2\right)\right) + \frac{1}{(a+b i)^3}\left(a e^{-(a+b i) z}\left(z^2 a^4 + z(2 b i z + 5) a^3 + \left(-b^2 z^2 + 8 b i z + 11\right) a^2 - 3 b(-5 i + b z) a - 6 b^2\right)\right) - 6 \operatorname{Ei}(-(a+b i) z) + 6 \operatorname{Ei}((a+b i) z) + 6 \operatorname{Ei}((a-i b) z) - 6 \operatorname{Ei}(i b z - a z) + 2 \operatorname{Ci}(b z)(\Gamma(4, a z) - \Gamma(4, -a z))\right) + \frac{(\Gamma(4, -i b z) + \Gamma(4, i b z)) \operatorname{Shi}(a z)}{b^4}\right)$$

Definite integration

Involving the direct function

06.39.21.0076.01

$$\int_0^\infty t^{\alpha-1} e^{-z t} \operatorname{Shi}(t) \, dt = z^{-\alpha-1} \Gamma(\alpha+1) \, {}_3F_2\left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{3}{2}; \frac{1}{z^2}\right) /; \operatorname{Re}(z) > 1 \land \operatorname{Re}(\alpha) > -1$$

Integral transforms

Laplace transforms

06.39.22.0001.01

$$\mathcal{L}_t[\operatorname{Shi}(t)](z) = \frac{1}{z} \tanh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) > 1$$

Operations

Limit operation

06.39.25.0001.01

$$\lim_{x\to\infty} \text{Shi}(a+b\,x) = \begin{cases} \frac{\pi i}{2} & \arg(b) = \frac{\pi}{2} \\ -\frac{1}{2}(\pi i) & \arg(b) = -\frac{\pi}{2} \\ \infty & \text{Im}(a) = 0 \bigwedge \arg(b) = 0 \\ -\infty & \text{Im}(a) = 0 \bigwedge \arg(b) = \pi \\ \tilde{\infty} & \text{True} \end{cases}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_pF_q$

06.39.26.0001.01

$$\text{Shi}(z) = z\,{}_1F_2\left(\frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{z^2}{4}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

06.39.26.0002.01

$$\text{Shi}(z) = \frac{\sqrt{\pi}}{4}\, z\, G_{1,3}^{1,1}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, -\frac{1}{2}, -\frac{1}{2} \end{matrix}\right)$$

06.39.26.0003.01

$$\text{Shi}(z) = -\frac{\sqrt{-\pi z^2}}{2z}\, G_{1,3}^{1,1}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} 1 \\ \frac{1}{2}, 0, 0 \end{matrix}\right)$$

06.39.26.0004.01

$$\text{Shi}(z) = \frac{i\sqrt{\pi}}{2}\, G_{1,3}^{1,1}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} 1 \\ \frac{1}{2}, 0, 0 \end{matrix}\right) /;\ \text{Im}(z) > 0$$

06.39.26.0005.01

$$\text{Shi}\left(\sqrt{z}\right) = \frac{1}{4}\sqrt{z\,\pi}\; G_{1,3}^{1,1}\left(-\frac{z}{4} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, -\frac{1}{2}, -\frac{1}{2} \end{matrix}\right)$$

06.39.26.0010.01

$$\text{Shi}\left(\sqrt{z}\right) = \frac{1}{2}\pi^{3/2}\, G_{2,4}^{2,0}\left(\frac{z}{4} \,\middle|\, \begin{matrix} 1, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0, 0 \end{matrix}\right)$$

Generalized cases for the direct function itself

06.39.26.0006.01

$$\text{Shi}(z) = -\frac{i\sqrt{\pi}}{2}\, G_{1,3}^{1,1}\left(\frac{i\,z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} 1 \\ \frac{1}{2}, 0, 0 \end{matrix}\right)$$

06.39.26.0007.01

$$\text{Shi}(z) = \frac{1}{2}\pi^{3/2} G_{2,4}^{2,0}\left(\frac{z}{2}, \frac{1}{2} \middle| \begin{array}{c} 1, \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0, 0 \end{array}\right)$$

Through other functions

06.39.26.0008.01

$$\text{Shi}(z) = \frac{1}{2}(\Gamma(0, z) - \Gamma(0, -z) - \log(-z) + \log(z))$$

06.39.26.0009.01

$$\text{Shi}(z) = \frac{1}{2}(E_1(z) - E_1(-z) - \log(-z) + \log(z))$$

Representations through equivalent functions

With related functions

06.39.27.0001.01

$$\text{Shi}(z) = -i\,\text{Si}(i\,z)$$

06.39.27.0002.01

$$\text{Shi}(z) = \frac{1}{4}\left(2(\text{Ei}(z) - \text{Ei}(-z)) + \log\left(\frac{1}{z}\right) - \log\left(-\frac{1}{z}\right) + \log(-z) - \log(z)\right)$$

06.39.27.0003.01

$$\text{Shi}(z) = \frac{1}{2}(\text{li}(e^z) - \text{li}(e^{-z})) - \frac{i\,\pi}{2}\,\text{sgn}(\text{Im}(z)) \;/;\; |\text{Im}(z)| < \pi$$

Zeros

06.39.30.0001.01

$$\text{Shi}(z) = 0 \;/;\; z = 0$$

History

– C. A. Bretschneider (1843)

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