Round

Notations

Traditional name

Nearest integer function

Traditional notation

$\lfloor z \rceil$

Mathematica StandardForm notation

Round[z]

Primary definition

04.03.02.0001.01

$$\lfloor x \rceil = n /; x \in \mathbb{R} \bigwedge n \in \mathbb{Z} \bigwedge |x - n| < \frac{1}{2}$$

04.03.02.0002.01

$$\lfloor z \rceil = \lfloor \mathrm{Re}(z) \rceil + i \lfloor \mathrm{Im}(z) \rceil$$

04.03.02.0003.01

$$\left\lfloor n + \frac{1}{2} \right\rceil = n /; \frac{n}{2} \in \mathbb{Z}$$

04.03.02.0004.01

$$\left\lfloor n + \frac{1}{2} \right\rceil = n + 1 /; \frac{n+1}{2} \in \mathbb{Z}$$

For real z , the function $\lfloor x \rceil$ is the integer closest to z (if $z \neq \pm \frac{1}{2}, \pm \frac{3}{2}, \ldots$).

Examples: $\lfloor 3.2 \rceil = 3$, $\lfloor 3 \rceil = 3$, $\lfloor -0.2 \rceil = 0$, $\lfloor -2.3 \rceil = -2$, $\left\lfloor \frac{2}{3} \right\rceil = 1$, $\lfloor -\pi \rceil = -3$, $\left\lfloor -4 - \frac{5}{3} i \right\rceil = -4 - 2 i$, $\left\lfloor \frac{5}{2} \right\rceil = 2$, $\left\lfloor \frac{7}{2} \right\rceil = 4$.

Specific values

Specialized values

04.03.03.0001.01

$$\lfloor x \rceil = x /; x \in \mathbb{Z}$$

04.03.03.0002.01

$\lfloor i\, x \rceil = i\, x \,/;\, x \in \mathbb{Z}$

04.03.03.0003.01

$\lfloor x + i\, y \rceil = \lfloor x \rceil + i\, \lfloor y \rceil \,/;\, x \in \mathbb{R} \bigwedge y \in \mathbb{R}$

Values at fixed points

04.03.03.0004.01

$\lfloor 0 \rceil = 0$

04.03.03.0005.01

$\lfloor 1 \rceil = 1$

04.03.03.0006.01

$\lfloor -1 \rceil = -1$

04.03.03.0007.01

$\lfloor i \rceil = i$

04.03.03.0008.01

$\lfloor -i \rceil = -i$

04.03.03.0009.01

$$\left\lfloor \frac{23}{10} \right\rceil = 2$$

04.03.03.0010.01

$\lfloor -3 \rceil = -3$

04.03.03.0011.01

$\lfloor -\pi \rceil = -3$

04.03.03.0012.01

$$\left\lfloor -\frac{27}{10} \right\rceil = -3$$

04.03.03.0013.01

$\lfloor -3.4 \rceil = -3$

04.03.03.0014.01

$$\left\lfloor \frac{23}{10} - i\, e \right\rceil = 2 - 3\, i$$

04.03.03.0015.01

$$\left\lfloor \frac{5}{2} \right\rceil = 2$$

04.03.03.0016.01

$$\left\lfloor \frac{7}{2} \right\rceil = 4$$

Values at infinities

04.03.03.0017.01

$\lfloor \infty \rceil = \infty$

04.03.03.0018.01

$\lfloor -\infty \rceil = -\infty$

04.03.03.0019.01

$\lfloor i\,\infty \rceil = i\,\infty$

04.03.03.0020.01

$\lfloor -i\,\infty \rceil = -i\,\infty$

04.03.03.0021.01

$\lfloor \tilde{\infty} \rceil = \tilde{\infty}$

General characteristics

Domain and analyticity

$\lfloor z \rceil$ is a nonanalytical function; it is a piecewise constant function which is defined over the whole complex z-plane.

04.03.04.0001.01

$z \longrightarrow \lfloor z \rceil :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

$\lfloor z \rceil$ is an odd function.

04.03.04.0002.01

$\lfloor -z \rceil = -\lfloor z \rceil$

Mirror symmetry

04.03.04.0003.01

$\lfloor \bar{z} \rceil = \overline{\lfloor z \rceil}$

Periodicity

No periodicity

Sets of discontinuity

The function $\lfloor z \rceil$ is a piecewise constant function with unit jumps on the lines $\mathrm{Re}(z) + \frac{1}{2} = k \bigvee \mathrm{Im}(z) + \frac{1}{2} = l \,/;\, k, l \in \mathbb{Z}$.
The function $\lfloor z \rceil$ is continuous from the right on the intervals $\left(2k - \frac{1}{2} - i\,\infty,\, 2k - \frac{1}{2} + i\,\infty\right)$, $k \in \mathbb{Z}$, and from the left on the intervals $\left(2k + \frac{1}{2} - i\,\infty,\, 2k + \frac{1}{2} + i\,\infty\right)$, $k \in \mathbb{Z}$.
The function $\lfloor z \rceil$ is continuous from above on the intervals $\left(-\infty + 2\,i\,k - \frac{i}{2},\, \infty + 2\,i\,k - \frac{i}{2}\right)$, $k \in \mathbb{Z}$, and from below on the intervals $\left(-\infty + 2\,i\,k + \frac{i}{2},\, \infty + 2\,i\,k + \frac{i}{2}\right)$, $k \in \mathbb{Z}$.

04.03.04.0004.01

$$\mathcal{DS}_z(\lfloor z \rceil) = \left\{\left\{\left\{\left(2k - \frac{1}{2} - i\,\infty,\, 2k - \frac{1}{2} + i\,\infty\right), -1\right\} /;\, k \in \mathbb{Z}\right\}, \left\{\left\{\left(2k + \frac{1}{2} - i\,\infty,\, 2k + \frac{1}{2} + i\,\infty\right), 1\right\} /;\, k \in \mathbb{Z}\right\}, \left\{\left\{\left(2\,i\,k - \frac{i}{2} - \infty,\, 2\,i\,k - \frac{i}{2} + \infty\right), -i\right\} /;\, k \in \mathbb{Z}\right\}, \left\{\left\{\left(2\,i\,k + \frac{i}{2} - \infty,\, 2\,i\,k + \frac{i}{2} + \infty\right), i\right\} /;\, k \in \mathbb{Z}\right\}\right\}$$

04.03.04.0005.01

$$\lim_{\epsilon\to+0} \lfloor z+\epsilon \rceil = \lfloor z \rceil \;/;\; \frac{1}{4}(2\operatorname{Re}(z)+1) \in \mathbb{Z}$$

04.03.04.0006.01

$$\lim_{\epsilon\to+0} \lfloor z-\epsilon \rceil = \lfloor z \rceil \;/;\; \frac{1}{4}(2\operatorname{Re}(z)-1) \in \mathbb{Z}$$

04.03.04.0007.01

$$\lim_{\epsilon\to+0} \lfloor z+\epsilon \rceil = \lfloor z \rceil + 1 \;/;\; \frac{1}{4}\operatorname{Re}(2z-1) \in \mathbb{Z}$$

04.03.04.0008.01

$$\lim_{\epsilon\to+0} \lfloor z-\epsilon \rceil = \lfloor z \rceil - 1 \;/;\; \frac{1}{4}(2\operatorname{Re}(z)+1) \in \mathbb{Z}$$

04.03.04.0009.01

$$\lim_{\epsilon\to+0} \lfloor z+i\epsilon \rceil = \lfloor z \rceil \;/;\; \frac{1}{4}(2\operatorname{Im}(z)+1) \in \mathbb{Z}$$

04.03.04.0010.01

$$\lim_{\epsilon\to+0} \lfloor z-i\epsilon \rceil = \lfloor z \rceil \;/;\; \frac{1}{4}(2\operatorname{Im}(z)-1) \in \mathbb{Z}$$

04.03.04.0011.01

$$\lim_{\epsilon\to+0} \lfloor z+i\epsilon \rceil = \lfloor z \rceil + i \;/;\; \frac{1}{4}(2\operatorname{Im}(z)-1) \in \mathbb{Z}$$

04.03.04.0012.01

$$\lim_{\epsilon\to+0} \lfloor z-i\epsilon \rceil = \lfloor z \rceil - i \;/;\; \frac{1}{4}(2\operatorname{Im}(z)+1) \in \mathbb{Z}$$

Series representations

Exponential Fourier series

04.03.06.0001.01

$$\lfloor x \rceil = x + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^k \sin(2\pi k x)}{k} \;/;\; x \in \mathbb{R} \bigwedge x + \frac{1}{2} \notin \mathbb{Z}$$

Other series representations

04.03.06.0002.01

$$\left\lfloor \frac{n+2m}{2n} \right\rceil = \frac{m}{n} + \frac{1}{2n}\sum_{k=1}^{n-1}\sin\left(\frac{2\pi k m}{n}\right)\cot\left(\frac{\pi k}{n}\right) + \frac{1}{2} \;/;\; m \in \mathbb{Z} \bigwedge n \in \mathbb{Z} \bigwedge \frac{m}{n} \notin \mathbb{Z} \bigwedge n > 1$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

04.03.16.0001.01

$$\lfloor -z \rceil = -\lfloor z \rceil$$

04.03.16.0002.01

$$\lfloor i\, z \rceil = i\, \lfloor z \rceil$$

04.03.16.0003.01

$$\lfloor -i\, z \rceil = -i\, \lfloor z \rceil$$

04.03.16.0004.01

$$\lfloor z + n \rceil = \lfloor z \rceil + n \,/;\, \frac{n}{2} \in \mathbb{Z} \bigvee \frac{n-1}{2} \in \mathbb{Z} \bigwedge z - \frac{1+i}{2} \notin \mathbb{Z} \bigwedge z - \frac{1}{2} \notin \mathbb{Z}$$

Argument involving related functions

04.03.16.0005.01

$$\lfloor \lfloor z \rceil \rceil = \lfloor z \rceil$$

04.03.16.0006.01

$$\lfloor z - \lfloor z \rceil \rceil = 0$$

04.03.16.0007.01

$$\lfloor \lfloor z \rfloor \rceil = \lfloor z \rfloor$$

04.03.16.0008.01

$$\lfloor \lceil z \rceil \rceil = \lceil z \rceil$$

04.03.16.0009.01

$$\lfloor \mathrm{int}(z) \rceil = \mathrm{int}(z)$$

04.03.16.0011.01

$$\lfloor \mathrm{quotient}(m,\, n) \rceil = \left\lfloor \frac{m}{n} \right\rfloor$$

Addition formulas

04.03.16.0010.01

$$\lfloor z + n \rceil = \lfloor z \rceil + n \,/;\, n \in \mathbb{Z} \bigwedge \mathrm{Re}(z) + \frac{1}{2} \notin \mathbb{Z} \bigwedge \mathrm{Im}(z) + \frac{1}{2} \notin \mathbb{Z}$$

Complex characteristics

Real part

04.03.19.0001.01

$$\mathrm{Re}(\lfloor x + i\, y \rceil) = \lfloor x \rceil$$

04.03.19.0006.01

$$\mathrm{Re}(\lfloor z \rceil) = \lfloor \mathrm{Re}(z) \rceil$$

Imaginary part

04.03.19.0002.01

$$\mathrm{Im}(\lfloor x + i\, y \rceil) = \lfloor y \rceil$$

04.03.19.0007.01

$$\mathrm{Im}(\lfloor z \rceil) = \lfloor \mathrm{Im}(z) \rceil$$

Absolute value

04.03.19.0003.01

$$\left|\lfloor x + i\, y \rceil\right| = \sqrt{\lfloor x \rceil^2 + \lfloor y \rceil^2}$$

04.03.19.0008.01

$$\left|\lfloor z \rceil\right| = \sqrt{\lfloor \mathrm{Re}(z) \rceil^2 + \lfloor \mathrm{Im}(z) \rceil^2}$$

Argument

04.03.19.0004.01

$$\arg(\lfloor x + i\, y \rceil) = \tan^{-1}(\lfloor x \rceil, \lfloor y \rceil)$$

04.03.19.0009.01

$$\arg(\lfloor z \rceil) = \tan^{-1}(\lfloor \mathrm{Re}(z) \rceil, \lfloor \mathrm{Im}(z) \rceil)$$

Conjugate value

04.03.19.0005.01

$$\overline{\lfloor x + i\, y \rceil} = \lfloor x \rceil - i\, \lfloor y \rceil$$

04.03.19.0010.01

$$\overline{\lfloor z \rceil} = \lfloor \mathrm{Re}(z) \rceil - i\, \lfloor \mathrm{Im}(z) \rceil$$

Signum value

04.03.19.0011.01

$$\mathrm{sgn}(\lceil x + i\, y \rceil) = \frac{\lfloor x \rceil + i\, \lfloor y \rceil}{\sqrt{\lfloor x \rceil^2 + \lfloor y \rceil^2}}$$

04.03.19.0012.01

$$\mathrm{sgn}(\lfloor z \rceil) = \frac{\lfloor z \rceil}{\left|\lfloor z \rceil\right|}$$

Differentiation

Low-order differentiation

04.03.20.0001.01

$$\frac{\partial \lfloor z \rceil}{\partial z} = 0$$

In a distributional sense for $x \in \mathbb{R}$.

04.03.20.0002.01

$$\frac{\partial \lfloor x \rceil}{\partial x} = \sum_{k=-\infty}^{\infty} \delta\left(-k + x + \frac{1}{2}\right)$$

Fractional integro-differentiation

04.03.20.0003.01

$$\frac{\partial^{\alpha} \lceil z \rceil}{\partial z^{\alpha}} = \frac{\lceil z \rceil z^{-\alpha}}{\Gamma(1-\alpha)}$$

Integration

Indefinite integration

Involving only one direct function

04.03.21.0001.01

$$\int \lceil z \rceil \, dz = z \lceil z \rceil$$

Involving one direct function and elementary functions

Involving power function

04.03.21.0002.01

$$\int z^{\alpha-1} \lceil z \rceil \, dz = \frac{z^{\alpha} \lceil z \rceil}{\alpha}$$

04.03.21.0003.01

$$\int \frac{\lceil z \rceil}{z} \, dz = \log(z) \lceil z \rceil$$

Definite integration

For the direct function itself

In the following formulas $a \in \mathbb{R}$.

04.03.21.0004.01

$$\int_0^n \lceil t \rceil \, dt = \frac{n^2}{2} \;/;\; n \in \mathbb{N}$$

04.03.21.0005.01

$$\int_0^a \lceil t \rceil \, dt = \frac{1}{2} (2a - \lceil a \rceil) \lceil a \rceil$$

04.03.21.0006.01

$$\int_0^a t^{\alpha-1} \lceil t \rceil \, dt = \frac{1}{\alpha} \left(\left\lfloor a + \frac{1}{2} \right\rfloor a^{\alpha} - \left(\left\lfloor a - \frac{1}{2} \right\rfloor + \frac{1}{2} \right)^{\alpha} + (1 - 2^{-\alpha}) \zeta(-\alpha) + \zeta\left(-\alpha, \left\lfloor a - \frac{1}{2} \right\rfloor + \frac{1}{2}\right) \right)$$

04.03.21.0007.01

$$\int_a^\infty t^{\alpha-1} \lceil t \rceil \, dt = -\frac{1}{\alpha} \left(\left\lfloor a + \frac{1}{2} \right\rfloor a^{\alpha} + \zeta\left(-\alpha, \left\lfloor a + \frac{1}{2} \right\rfloor + \frac{1}{2}\right) \right) \;/;\; \operatorname{Re}(\alpha) < -1$$

04.03.21.0008.01

$$\int_1^\infty t^{\alpha-1} \lceil t \rceil \, dt = -\frac{1}{\alpha} \left(\zeta\left(-\alpha, \frac{3}{2}\right) + 1 \right) \;/;\; \operatorname{Re}(\alpha) < -1$$

04.03.21.0009.01

$$\int_0^\infty t^{\alpha-1} \lfloor t \rceil \, dt = -\frac{1}{\alpha}\left(\zeta\left(-\alpha, \frac{3}{2}\right) + 2^{-\alpha}\right) /; \operatorname{Re}(\alpha) < -1$$

04.03.21.0010.01

$$\int_{-a}^{a} \lfloor t \rceil \, dt = 0$$

Integral transforms

Fourier exp transforms

04.03.22.0001.01

$$\mathcal{F}_t[\lfloor t \rceil](z) = \frac{i}{\sqrt{2\pi}} \sum_{k=1}^{\infty} \frac{(-1)^k (\delta(2k\pi - z) - \delta(2\pi k + z))}{k} - i\sqrt{2\pi}\, \delta'(z)$$

Fourier cos transforms

04.03.22.0002.01

$$\mathcal{F}c_t[\lfloor t \rceil](z) = -\frac{1}{\sqrt{2\pi}\, z} \csc\left(\frac{z}{2}\right)$$

Fourier sin transforms

04.03.22.0003.01

$$\mathcal{F}s_t[\lfloor t \rceil](z) = \frac{1}{\sqrt{2\pi}} \sum_{k=1}^{\infty} \frac{(-1)^k}{k} (\delta(2k\pi - z) - \delta(2\pi k + z)) - \sqrt{2\pi}\, \delta'(z)$$

Laplace transforms

04.03.22.0004.01

$$\mathcal{L}_t[\lfloor t \rceil](z) = \frac{e^{z/2}}{(e^z - 1)\, z} /; \operatorname{Re}(z) > 0$$

Mellin transforms

04.03.22.0005.01

$$\mathcal{M}_t[\lfloor t \rceil](z) = -\frac{1}{z}\left(\zeta\left(-z, \frac{3}{2}\right) + 2^{-z}\right) /; \operatorname{Re}(z) < -1$$

Representations through equivalent functions

With related functions

With Floor

For real arguments

04.03.27.0008.01

$$\lfloor x \rceil = \left\lfloor x + \frac{1}{2} \right\rfloor /; x \in \mathbb{R} \bigwedge \frac{2x-1}{4} \notin \mathbb{Z}$$

04.03.27.0009.01

$$\lfloor x \rceil = \left\lfloor x - \frac{1}{2} \right\rfloor /; \frac{2x-1}{4} \in \mathbb{Z}$$

04.03.27.0010.01

$$\lfloor x \rceil = \left\lfloor x + \frac{1}{2} \right\rfloor - \chi_{\mathbb{Z}}\left(\frac{2x-1}{4}\right) /; x \in \mathbb{R}$$

For complex arguments

04.03.27.0001.01

$$\lfloor z \rceil = \left\lfloor \frac{1+i}{2} + z \right\rfloor - i\,\chi_{\mathbb{Z}}\left(\frac{2\,\mathrm{Im}(z)-1}{4}\right) - \chi_{\mathbb{Z}}\left(\frac{2\,\mathrm{Re}(z)-1}{4}\right)$$

With Ceiling

For real arguments

04.03.27.0011.01

$$\lfloor x \rceil = \left\lceil x - \frac{1}{2} \right\rceil /; x \in \mathbb{R} \bigwedge \frac{2x+1}{4} \notin \mathbb{Z}$$

04.03.27.0012.01

$$\lfloor x \rceil = \left\lceil x + \frac{1}{2} \right\rceil /; \frac{2x+1}{4} \in \mathbb{Z}$$

04.03.27.0013.01

$$\lfloor x \rceil = \left\lceil x - \frac{1}{2} \right\rceil + \chi_{\mathbb{Z}}\left(\frac{2x+1}{4}\right) /; x \in \mathbb{R}$$

For complex arguments

04.03.27.0002.01

$$\lfloor z \rceil = \left\lceil z - \frac{1+i}{2} \right\rceil + \chi_{\mathbb{Z}}\left(\frac{2\,\mathrm{Re}(z)+1}{4}\right) + i\,\chi_{\mathbb{Z}}\left(\frac{2\,\mathrm{Im}(z)+1}{4}\right)$$

With IntegerPart

For real arguments

04.03.27.0014.01

$$\lfloor x \rceil = \mathrm{int}\left(x + \frac{1}{2}\right) /; x \in \mathbb{R} \bigwedge x \geq -\frac{1}{2} \bigvee \frac{2x+1}{4} \in \mathbb{Z}$$

04.03.27.0015.01

$$\lfloor x \rceil = \mathrm{int}\left(x - \frac{1}{2}\right) /; x \in \mathbb{R} \bigwedge x < \frac{1}{2} \bigwedge \frac{2x+1}{4} \notin \mathbb{Z}$$

04.03.27.0016.01

$$\lfloor x \rceil = \operatorname{int}\left(x+\frac{1}{2}\right)-1-\chi_{\mathbb{Z}}\left(\frac{2x-1}{4}\right)+\operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(x+\frac{1}{2}\right)+\theta\left(x+\frac{1}{2}\right)\right)\; /;\; x\in\mathbb{R}$$

For complex arguments

04.03.27.0003.01

$$\lfloor z \rceil = \operatorname{int}\left(z+\frac{1+i}{2}\right)-1-i-\chi_{\mathbb{Z}}\left(\frac{2\operatorname{Re}(z)-1}{4}\right)-i\,\chi_{\mathbb{Z}}\left(\frac{2\operatorname{Im}(z)-1}{4}\right)+ \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Re}(z)+\frac{1}{2}\right)+\theta\left(\operatorname{Re}(z)+\frac{1}{2}\right)\right)+i\operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Im}(z)+\frac{1}{2}\right)+\theta\left(\operatorname{Im}(z)+\frac{1}{2}\right)\right)$$

With FractionalPart

For real arguments

04.03.27.0017.01

$$\lfloor x \rceil = x-\operatorname{frac}\left(x+\frac{1}{2}\right)+\frac{1}{2}\; /;\; x\in\mathbb{R}\bigwedge x\geq -\frac{1}{2}\bigvee \frac{2x+1}{4}\in\mathbb{Z}$$

04.03.27.0018.01

$$\lfloor x \rceil = x-\operatorname{frac}\left(x-\frac{1}{2}\right)-\frac{1}{2}\; /;\; x\in\mathbb{R}\bigwedge x<\frac{1}{2}\bigwedge \frac{2x+1}{4}\notin\mathbb{Z}$$

04.03.27.0019.01

$$\lfloor x \rceil = x-\operatorname{frac}\left(x+\frac{1}{2}\right)-\frac{1}{2}-\chi_{\mathbb{Z}}\left(\frac{2x-1}{4}\right)+\operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(x+\frac{1}{2}\right)+\theta\left(x+\frac{1}{2}\right)\right)\; /;\; x\in\mathbb{R}$$

For complex arguments

04.03.27.0004.01

$$\lfloor z \rceil = z-\frac{1+i}{2}-\operatorname{frac}\left(z+\frac{1+i}{2}\right)-\chi_{\mathbb{Z}}\left(\frac{2\operatorname{Re}(z)-1}{4}\right)-i\,\chi_{\mathbb{Z}}\left(\frac{2\operatorname{Im}(z)-1}{4}\right)+ \operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Re}(z)+\frac{1}{2}\right)+\theta\left(\operatorname{Re}(z)+\frac{1}{2}\right)\right)+i\operatorname{sgn}\left(\chi_{\mathbb{Z}}\left(\operatorname{Im}(z)+\frac{1}{2}\right)+\theta\left(\operatorname{Im}(z)+\frac{1}{2}\right)\right)$$

With Mod

For real arguments

04.03.27.0020.01

$$\lfloor x \rceil = x-\operatorname{frac}\left(x+\frac{1}{2}\right)+\frac{1}{2}\; /;\; x\in\mathbb{R}\bigwedge \frac{2x-1}{4}\notin\mathbb{Z}$$

04.03.27.0021.01

$$\lfloor x \rceil = x-\operatorname{frac}\left(x+\frac{1}{2}\right)-\frac{1}{2}\; /;\; \frac{2x-1}{4}\in\mathbb{Z}$$

04.03.27.0022.01

$$\lfloor x \rceil = x-\left(x+\frac{1}{2}\right)\bmod 1+\frac{1}{2}-\chi_{\mathbb{Z}}\left(\frac{2x-1}{4}\right)\; /;\; x\in\mathbb{R}$$

For complex arguments

04.03.27.0005.01

$$\lfloor z \rceil = \frac{1+i}{2} + z - i\,\chi_{\mathbb{Z}}\left(\frac{2\operatorname{Im}(z)-1}{4}\right) - \chi_{\mathbb{Z}}\left(\frac{2\operatorname{Re}(z)-1}{4}\right) - \left(\frac{1+i}{2}+z\right) \bmod 1$$

With Quotient

For real arguments

04.03.27.0023.01

$$\lfloor x \rceil = \operatorname{quotient}\left(x+\frac{1}{2}, 1\right) /; x \in \mathbb{R} \bigwedge \frac{2x-1}{4} \notin \mathbb{Z}$$

04.03.27.0024.01

$$\lfloor x \rceil = \operatorname{quotient}\left(x+\frac{1}{2}, 1\right) - 1 /; \frac{2x-1}{4} \in \mathbb{Z}$$

04.03.27.0025.01

$$\lfloor x \rceil = \operatorname{quotient}\left(x+\frac{1}{2}, 1\right) - \chi_{\mathbb{Z}}\left(\frac{2x-1}{4}\right) /; x \in \mathbb{R}$$

For complex arguments

04.03.27.0006.01

$$\lfloor z \rceil = \operatorname{quotient}\left(z+\frac{1+i}{2}, 1\right) - \chi_{\mathbb{Z}}\left(\frac{2\operatorname{Re}(z)-1}{4}\right) - i\,\chi_{\mathbb{Z}}\left(\frac{2\operatorname{Im}(z)-1}{4}\right)$$

With elementary functions

04.03.27.0007.01

$$\lfloor z \rceil = z - \frac{\tan^{-1}(\tan(\pi z))}{\pi} /; z \in \mathbb{R} \bigwedge z+\frac{1}{2} \notin \mathbb{Z}$$

Zeros

04.03.30.0001.01

$$\lfloor z \rceil = 0 /; -\frac{1}{2} \leq \operatorname{Re}(z) \leq \frac{1}{2} \bigwedge -\frac{1}{2} \leq \operatorname{Im}(z) \leq \frac{1}{2}$$

History

– C. F. Gauss (1808)

– J. Liouville (1838)

– J. Hastad (1988) suggested the notation $\lfloor z \rceil$

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