

# PartitionsP

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## Notations

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### Traditional name

Number of unrestricted partitions of an integer

### Traditional notation

$p(n)$

### Mathematica StandardForm notation

PartitionsP[ $n$ ]

## Primary definition

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$$p(n) = \left( [t^n] \prod_{k=1}^{\infty} \frac{1}{1-t^k} \right); n \in \mathbb{N}$$

$p(n)$  is the number of unrestricted partitions of the positive integer  $n$  into a sum of strictly positive numbers which add up to  $n$  independent of order, when repetitions are allowed.

For example,  $p(5) = 7$ . There are 7 possibilities to express 5 as a sum of positive integers:

$$5 = 1 + 4 = 2 + 3 = 1 + 1 + 3 = 1 + 2 + 2 = 1 + 1 + 1 + 2 = 1 + 1 + 1 + 1 + 1.$$

$$p(n) = 0; n \in \mathbb{Z} \wedge n < 0$$

## Specific values

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### Values at fixed points

$$p(0) = 1$$

$$p(1) = 1$$

$$p(2) = 2$$

04.16.03.0004.01  
 $p(3) = 3$

04.16.03.0005.01  
 $p(4) = 5$

04.16.03.0006.01  
 $p(5) = 7$

04.16.03.0007.01  
 $p(6) = 11$

04.16.03.0008.01  
 $p(7) = 15$

04.16.03.0009.01  
 $p(8) = 22$

04.16.03.0010.01  
 $p(9) = 30$

04.16.03.0011.01  
 $p(10) = 42$

04.16.03.0013.01  
 $p(10) = 42$

04.16.03.0014.01  
 $p(11) = 56$

04.16.03.0015.01  
 $p(12) = 77$

04.16.03.0016.01  
 $p(13) = 101$

04.16.03.0017.01  
 $p(14) = 135$

04.16.03.0018.01  
 $p(15) = 176$

04.16.03.0019.01  
 $p(16) = 231$

04.16.03.0020.01  
 $p(17) = 297$

04.16.03.0021.01  
 $p(18) = 385$

04.16.03.0022.01  
 $p(19) = 490$

04.16.03.0023.01  
 $p(20) = 627$

04.16.03.0024.01  
 $p(21) = 792$

04.16.03.0025.01  
 $p(22) = 1002$

04.16.03.0026.01  
 $p(23) = 1255$

04.16.03.0027.01  
 $p(24) = 1575$

04.16.03.0028.01  
 $p(25) = 1958$

04.16.03.0029.01  
 $p(26) = 2436$

04.16.03.0030.01  
 $p(27) = 3010$

04.16.03.0031.01  
 $p(28) = 3718$

04.16.03.0032.01  
 $p(29) = 4565$

04.16.03.0033.01  
 $p(30) = 5604$

04.16.03.0034.01  
 $p(31) = 6842$

04.16.03.0035.01  
 $p(32) = 8349$

04.16.03.0036.01  
 $p(33) = 10143$

04.16.03.0037.01  
 $p(34) = 12310$

04.16.03.0038.01  
 $p(35) = 14883$

04.16.03.0039.01  
 $p(36) = 17977$

04.16.03.0040.01  
 $p(37) = 21637$

04.16.03.0041.01  
 $p(38) = 26015$

04.16.03.0042.01  
 $p(39) = 31185$

04.16.03.0043.01  
 $p(40) = 37338$

04.16.03.0044.01  
 $p(41) = 44583$

04.16.03.0045.01

$$p(42) = 53\,174$$

04.16.03.0046.01

$$p(43) = 63\,261$$

04.16.03.0047.01

$$p(44) = 75\,175$$

04.16.03.0048.01

$$p(45) = 89\,134$$

04.16.03.0049.01

$$p(46) = 105\,558$$

04.16.03.0050.01

$$p(47) = 124\,754$$

04.16.03.0051.01

$$p(48) = 147\,273$$

04.16.03.0052.01

$$p(49) = 173\,525$$

04.16.03.0053.01

$$p(50) = 204\,226$$

## Values at infinities

04.16.03.0012.01

$$p(\infty) = \infty$$

## General characteristics

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### Domain and analyticity

The partitions  $p(n)$  is a nonanalytical function which is defined only for integers.

04.16.04.0001.01

$$n \rightarrow p(n) :: \mathbb{N} \rightarrow \mathbb{N}^+$$

### Symmetries and periodicities

#### Symmetry

No symmetry

#### Periodicity

No periodicity

## Series representations

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### Generalized power series

04.16.06.0001.01

$$p(n) = \frac{1}{\pi \sqrt{2}} \sum_{k=1}^{\infty} A(k, n) \sqrt{k} \frac{\partial \left( \sinh \left( \frac{1}{k} \pi \sqrt{\frac{2}{3}} \sqrt{n - \frac{1}{24}} \right) \left( n - \frac{1}{24} \right)^{-\frac{1}{2}} \right)}{\partial n} /;$$

$$A(k, n) = \sum_{h=1}^k \delta_{\gcd(h,k),1} \exp \left( \pi i \sum_{j=1}^{k-1} \frac{1}{k} j \left( \frac{h j}{k} - \left\lfloor \frac{h j}{k} \right\rfloor - \frac{1}{2} \right) - \frac{2 \pi i h n}{k} \right)$$

04.16.06.0003.01

$$p(n) = \frac{\pi^2}{9 \sqrt{3}} \sum_{k=1}^{\infty} \frac{A(k, n)}{k^{5/2}} {}_0F_1 \left( \frac{5}{2}; \frac{\left( n - \frac{1}{24} \right) \pi^2}{6 k^2} \right) /; A(k, n) = \sum_{h=1}^k \delta_{\gcd(h,k),1} \exp \left( \pi i \sum_{j=1}^{k-1} \frac{1}{k} j \left( \frac{h j}{k} - \left\lfloor \frac{h j}{k} \right\rfloor - \frac{1}{2} \right) - \frac{2 \pi i h n}{k} \right)$$

### Asymptotic series expansions

04.16.06.0002.01

$$p(n) \propto \frac{1}{4 n \sqrt{3}} \exp \left( \sqrt{\frac{2}{3}} \sqrt{n} \pi \right) \left( 1 + O \left( \frac{1}{n} \right) \right) /; (n \rightarrow \infty)$$

### Generating functions

04.16.11.0001.01

$$p(n) = \left[ t^n \right] \prod_{k=1}^{\infty} \frac{1}{1 - t^k} /; n \in \mathbb{N}$$

04.16.11.0002.01

$$p(n) = \left[ t^n \right] 1 / \left( \sum_{k=-\infty}^{\infty} (-1)^k t^{\frac{1}{2} (3k^2 + k)} \right) /; n \in \mathbb{N}$$

04.16.11.0003.01

$$p(n) = \left[ t^n \right] \sqrt[3]{\frac{2 \sqrt[8]{t}}{\vartheta_1'(0, \sqrt{t})}} /; n \in \mathbb{N}$$

### Identities

#### Functional identities

04.16.17.0001.01

$$p(n) = \frac{1}{n} \sum_{k=1}^n \sigma_1(k) p(n - k)$$

04.16.17.0002.01

$$p(n) = \sum_{k=1}^n (-1)^{k-1} \left( p \left( n - \frac{1}{2} (3k^2 - k) \right) + p \left( n - \frac{1}{2} (3k^2 + k) \right) \right)$$

04.16.17.0003.01

$$p(2n+1) = p(n) - \sum_{k=1}^{\infty} (-1)^k (p(-3k^2 - k + 2n + 1) + p(-3k^2 + k + 2n + 1)) + \sum_{k=1}^{\infty} (p(-4k^2 + 3k + n) + p(-4k^2 - 3k + n))$$

## Complex characteristics

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### Real part

04.16.19.0001.01

$$\operatorname{Re}(p(n)) = p(n)$$

### Imaginary part

04.16.19.0002.01

$$\operatorname{Im}(p(n)) = 0$$

### Absolute value

04.16.19.0003.01

$$|p(n)| = p(n)$$

### Argument

04.16.19.0004.01

$$\arg(p(n)) = 0$$

### Conjugate value

04.16.19.0005.01

$$\overline{p(n)} = p(n)$$

## Summation

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### Finite summation

04.16.23.0001.01

$$\sum_{k=\left\lfloor -\frac{1}{6}(\sqrt{24n+1}+1) \right\rfloor}^{\left\lfloor \frac{1}{6}(\sqrt{24n+1}-1) \right\rfloor} (-1)^k p\left(n - \frac{1}{2}k(3k+1)\right) = 0$$

### Infinite summation

04.16.23.0002.01

$$\sum_{k=0}^{\infty} p(k) t^k = \prod_{k=1}^{\infty} \frac{1}{1-t^k}$$

## Representations through equivalent functions

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### With related functions

04.16.27.0001.01

$$p(n) = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} q(n-2k) p(k)$$

## Inequalities

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04.16.29.0001.01

$$p(n) \leq \frac{1}{2} (p(n-1) + p(n+1)) \text{ ; } n \in \mathbb{N}^+$$

## Other identities

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### Congruence properties

04.16.32.0001.01

$$p(5n+4) \bmod 5 = 0$$

04.16.32.0002.01

$$p(7n+5) \bmod 7 = 0$$

04.16.32.0003.01

$$p(11n+6) \bmod 11 = 0$$

04.16.32.0004.01

$$p(n) \bmod (5^{k_1} 7^{k_2+1} 11^{k_3}) = 0 \text{ ; } (24n) \bmod (5^{k_1} 7^{k_2} 11^{k_3}) = 1 \bigwedge k_1 \in \mathbb{N} \bigwedge k_2 \in \mathbb{N} \bigwedge k_3 \in \mathbb{N}$$

## History

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- G. W. Leibniz (1669) investigated the number of ways a given positive integer can be decomposed into smaller ones
- L. Euler (1740)
- S. Ramanujan (1917)
- G.H. Hardy (1920) introduced the notation  $p(n)$

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