

MathieuCharacteristicA

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Notations

Traditional name

Characteristic value of an even Mathieu function

Traditional notation

$$a_r(q)$$

Mathematica StandardForm notation

MathieuCharacteristicA[r , q]

Primary definition

11.05.02.0001.01

$$a_r(q)$$

$a_r(q)$ is the characteristic value a for even Mathieu functions $w(z) = \text{Ce}(a, q, z)$ with characteristic exponent r and parameter q , such that there exists a solution of the corresponding Mathieu differential equation $w''(z) + (a - 2q \cos(2z))w(z) = 0$ that is of the form $w(z) = e^{izr} f(z)$, where $f(z)$ is an even function of z with period 2π .

Specific values

Specialized values

11.05.03.0001.01

$$a_r(0) = r^2$$

Values at fixed points

11.05.03.0002.01

$$a_0(0) = 0$$

General characteristics

Domain and analyticity

$a_r(q)$ is an analytical function of r, q which is defined in \mathbb{C}^2 .

11.05.04.0001.01

$$(r * q) \rightarrow a_r(q) :: (\mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$a_r(q)$ is an even function.

11.05.04.0002.01

$$a_{-r}(q) = a_r(q)$$

11.05.04.0003.01

$$a_r(-q) = a_r(q)$$

11.05.04.0004.01

$$a_{-r}(-q) = a_r(q)$$

Mirror symmetry

11.05.04.0005.01

$$a_r(\bar{q}) = \overline{a_r(q)}$$

Periodicity

No periodicity

Branch points

Branch points locations: complicated

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

Expansions at $q = 0$

11.05.06.0001.01

$$a_r(q) \propto r^2 + \frac{q^2}{2(r-1)(r+1)} + \frac{(5r^2+7)q^4}{32(r-2)(r-1)^3(r+1)^3(r+2)} + \frac{(9r^4+58r^2+29)q^6}{64(r-3)(r-2)(r-1)^5(r+1)^5(r+2)(r+3)} + \frac{(1469r^{10}+9144r^8-140354r^6+64228r^4+827565r^2+274748)q^8}{8192(r-4)(r-3)(r-2)^3(r-1)^7(r+1)^7(r+2)^3(r+3)(r+4)} + \frac{((4471r^{12}+69361r^{10}-1039598r^8-2844430r^6+13541915r^4+20651309r^2+4453452)q^{10})}{(16384(r-5)(r-4)(r-3)(r-2)^3(r-1)^9(r+1)^9(r+2)^3(r+3)(r+4)(r+5))} + O(q^{11}); - (r \in \mathbb{Z} \wedge -10 \leq r \leq 10)$$

11.05.06.0002.01

$$a_0(q) \propto -\frac{q^2}{2} + \frac{7q^4}{128} - \frac{29q^6}{2304} + \frac{68687q^8}{18874368} - \frac{123707q^{10}}{104857600} + \frac{8022167579q^{12}}{19568944742400} - \frac{286241141477q^{14}}{1917756584755200} + \frac{7534554811777337q^{16}}{134060901907751239680} - \frac{63642189915976296887q^{18}}{2931911924722519611801600} + \frac{4011632808829219892175301q^{20}}{469105907955603137888256000000} + O(q^{21})$$

11.05.06.0003.01

$$a_1(q) \propto 1 + q - \frac{q^2}{8} - \frac{q^3}{64} - \frac{q^4}{1536} + \frac{11q^5}{36864} + \frac{49q^6}{589824} + \frac{55q^7}{9437184} - \frac{83q^8}{35389440} - \frac{12121q^9}{15099494400} - \frac{114299q^{10}}{1630745395200} + \frac{192151q^{11}}{7827577896960} + \frac{83513957q^{12}}{8766887244595200} + \frac{944750239q^{13}}{981891371394662400} - \frac{27587714461q^{14}}{94261571653887590400} - \frac{45487147753q^{15}}{361964435150928347136} - \frac{11583279236477q^{16}}{814419979089588781056000} + \frac{4401918060178787q^{17}}{1172764769889007844720640000} + \frac{20737942737397933q^{18}}{1172764769889007844720640000} + \frac{206649295526133419q^{19}}{938211815911206275776512000000} - \frac{230932630430735533q^{20}}{4586813322232564014907392000000} + O(q^{21})$$

11.05.06.0004.01

$$a_2(q) \propto 4 + \frac{5q^2}{12} - \frac{763q^4}{13824} + \frac{1002401q^6}{79626240} - \frac{1669068401q^8}{458647142400} + \frac{4363384401463q^{10}}{3698530556313600} - \frac{40755179450909507q^{12}}{99416501353709568000} + \frac{170942293775248009327q^{14}}{1145278095594734223360000} - \frac{11586143933886768007817q^{16}}{206150057207052160204800000} + \frac{27218491783251329740936233551q^{18}}{125392091965327187575308288000000} - \frac{475590687175353210308450391084589q^{20}}{55613863918846191423340073189376000000} + O(q^{21})$$

11.05.06.0005.01

$$a_3(q) \propto 9 + \frac{q^2}{16} + \frac{q^3}{64} + \frac{13q^4}{20480} - \frac{5q^5}{16384} - \frac{1961q^6}{23592960} - \frac{609q^7}{104857600} + \frac{4957199q^8}{2113929216000} + \frac{872713q^9}{1087163596800} + \frac{421511q^{10}}{6012954214400} - \frac{16738435813q^{11}}{681869007912960000} - \frac{572669780189q^{12}}{60115798248652800000} - \frac{27992567161q^{13}}{29093077670952960000} + \frac{110350873865291q^{14}}{377046286615550361600000} + \frac{10234605999596669q^{15}}{81441997908958878105600000} + \frac{704720978382089561q^{16}}{49548909345104858185728000000} - \frac{195640795481512957q^{17}}{52122878661733681987584000000} - \frac{40878632822977874980039q^{18}}{2311753914405212263513325568000000} - \frac{38723118606468500148773q^{19}}{175807458181927253620272660480000000} + \frac{96836518225571706158506019q^{20}}{192337925678513660324308687257600000000} + O(q^{21})$$

11.05.06.0006.01

$$a_4(q) \propto 16 + \frac{q^2}{30} + \frac{433 q^4}{864000} - \frac{5701 q^6}{2721600000} - \frac{112236997 q^8}{2006581248000000} + \frac{8417126443 q^{10}}{3160365465600000000} + \frac{2887659548698709 q^{12}}{27184199588904960000000000} - \frac{1362879008360033 q^{14}}{27461589380628480000000000} - \frac{85043641535997859212637 q^{16}}{347234002608128482349600000000000000} + \frac{100385830833154261150792187 q^{18}}{93846933884898884923372339200000000000000} + \frac{108310756056687781830924018371231 q^{20}}{172978668536645624690759895613440000000000000000} + O(q^{21})$$

11.05.06.0007.01

$$a_5(q) \propto 25 + \frac{q^2}{48} + \frac{11 q^4}{774144} + \frac{q^5}{147456} + \frac{37 q^6}{891813888} - \frac{7 q^7}{339738624} + \frac{63439 q^8}{201364441399296} - \frac{q^9}{2130840649728} - \frac{60609509 q^{10}}{5799295912299724800} - \frac{6655 q^{11}}{88370223425519616} + \frac{105674803279 q^{12}}{2400630141488295680409600} - \frac{5574517 q^{13}}{4988322371923731283968} - \frac{16772318839 q^{14}}{395075131856359517690265600} + \frac{65998078949 q^{15}}{2394394738523391016304640000} - \frac{670043657712623 q^{16}}{3247054871864845067146864392929280} - \frac{6770772278721223 q^{17}}{49062642294661120520076247695360000} + \frac{240333014990212253 q^{18}}{56109108185824522760297816709817958400} + \frac{120574357589460233 q^{19}}{452161311387596886713022698760437760000} - \frac{50669327402196350835443 q^{20}}{565579810513111189423801992434965020672000000} + O(q^{21})$$

11.05.06.0008.01

$$a_6(q) \propto 36 + \frac{q^2}{70} + \frac{187 q^4}{43904000} + \frac{6743617 q^6}{92935987200000} - \frac{2337184771 q^8}{23315780468736000000} + \frac{107856094183 q^{10}}{1595835640971264000000000} - \frac{6219042272496549577 q^{12}}{6566958136066968588288000000000000} + \frac{176848789897619589410449 q^{14}}{9370261225190848413925638144000000000000} - \frac{1312104713817372060914843821 q^{16}}{43098204163224467584903841788723200000000000000} + \frac{159171985780417056808424886521009 q^{18}}{7389652564389797916816806361679108505600000000000000} - \frac{(352531861885330332479936898959174099749 q^{20})}{66277498263909520801681326358564545702238617600000000000000000} + O(q^{21})$$

11.05.06.0009.01

$$a_7(q) \propto 49 + \frac{q^2}{96} + \frac{7q^4}{4423680} + \frac{17q^6}{20384317440} + \frac{q^7}{2123366400} + \frac{80617q^8}{103324028239872000} - \frac{q^9}{2174327193600} +$$

$$\frac{22381q^{10}}{19044684885173207040} + \frac{121q^{11}}{1502894956216320000} + \frac{1585697167q^{12}}{475355334733923247718400000} -$$

$$\frac{169q^{13}}{4155203974946881536000} - \frac{4087866435403q^{14}}{107331431740241997948832972800000} - \frac{619q^{15}}{1177060798609528345600000} +$$

$$\frac{127886416305104603q^{16}}{2393782869301730012493396119126016000000} - \frac{710653159q^{17}}{5337927405856933273185288192000000} -$$

$$\frac{4615810827596713259q^{18}}{165458271926135578463543539753990225920000000} - \frac{8903677513487q^{19}}{13856405477219661667865298500321280000000} +$$

$$\frac{6648543660666518664511q^{20}}{64806695948028783372600733650842891688345600000000} + O(q^{21})$$

11.05.06.0010.01

$$a_8(q) \propto 64 + \frac{q^2}{126} + \frac{109q^4}{160030080} + \frac{2707q^6}{13973506525440} +$$

$$\frac{56675690063q^8}{22716763094823469056000} - \frac{8826844303q^{10}}{6088918573525228742246400} +$$

$$\frac{96800089911697819q^{12}}{343105787905984995846250718822400000} + \frac{1515253244196940621q^{14}}{19065016210783962279192767442085478400000} -$$

$$\frac{718193843284873708135309579q^{16}}{793447490813706069985104693586949714944720896000000} +$$

$$(2003770466274941097606322564005809q^{18}) /$$

$$252637717342469782006035348665661066051599057081597952000000 -$$

$$(279965926267161319306810784467204616783q^{20}) /$$

$$83426229131004245389858597664653529760411881908730940987801600000000 + O(q^{21})$$

11.05.06.0011.01

$$a_9(q) \propto 81 + \frac{q^2}{160} + \frac{103 q^4}{315 392 000} + \frac{1993 q^6}{36 333 158 400 000} + \frac{425 125 339 q^8}{28 676 616 703 967 232 000 000} + \frac{q^9}{106 542 032 486 400} +$$

$$\frac{1 130 345 443 q^{10}}{203 922 607 672 655 872 000 000 000} - \frac{11 q^{11}}{2 727 476 031 651 840 000} + \frac{62 629 265 104 555 151 q^{12}}{22 563 907 986 167 495 394 538 291 200 000 000 000} +$$

$$\frac{31 447 q^{13}}{68 997 143 004 432 039 936 000 000} + \frac{80 555 146 157 400 451 q^{14}}{41 259 717 460 420 563 007 155 732 480 000 000 000 000} -$$

$$\frac{7 339 639 q^{15}}{171 686 970 880 788 333 613 547 520 000 000} + \frac{405 059 575 281 583 710 161 972 141 q^{16}}{167 926 821 562 095 099 968 656 489 164 199 428 096 000 000 000 000 000} -$$

$$\frac{15 120 710 271 341 q^{17}}{2 201 990 611 321 866 125 684 585 325 920 256 000 000 000} - (11 523 405 880 011 634 059 270 211 650 029 q^{18}) /$$

$$\frac{940 175 254 416 133 078 096 516 459 013 386 622 069 637 120 000 000 000 000 000 -}{145 650 578 734 543 q^{19}}$$

$$\frac{31 317 199 805 466 540 454 180 769 079 754 752 000 000 000 000}{(15 089 819 507 835 055 103 567 092 482 603 681 329 q^{20}) /}$$

$$1 936 670 767 272 810 192 103 326 639 987 511 156 347 733 782 036 480 000 000 000 000 000 + O(q^{21})$$

11.05.06.0012.01

$$a_{10}(q) \propto 100 + \frac{q^2}{198} + \frac{169 q^4}{993 586 176} + \frac{31 943 q^6}{1 772 341 136 197 632} + \frac{1 704 670 559 q^8}{569 203 604 713 406 176 690 176} +$$

$$\frac{370 335 410 859 899 q^{10}}{12 496 432 546 743 250 420 538 529 546 240} - \frac{214 594 017 197 038 697 689 q^{12}}{24 728 051 091 987 796 768 549 356 462 392 664 391 680} +$$

$$\frac{5 861 959 225 549 071 902 627 q^{14}}{6 849 020 157 994 773 168 515 969 871 284 328 000 745 635 840} -$$

$$\frac{8 114 398 180 685 822 279 346 163 q^{16}}{673 635 344 764 279 156 614 477 007 813 514 698 348 562 609 464 147 968} +$$

$$(1 938 126 664 701 022 749 795 786 101 196 163 q^{18}) /$$

$$\frac{71 231 950 759 419 817 062 813 360 338 411 255 588 508 033 607 893 280 150 026 977 280 -}{(241 356 573 945 261 403 675 626 142 142 516 673 875 393 q^{20}) /}$$

$$2 234 061 918 057 835 606 504 427 982 965 659 891 273 495 159 651 078 524 001 326 093 828 096 000 000 + O(q^{21})$$

Representations through more general functions

Through other functions

11.05.26.0001.01

$$a_r(q) = 2q + \lambda_{r-\frac{1}{2}, \frac{1}{2}} \left(2\sqrt{q} \right) + \frac{1}{4}$$

Representations through equivalent functions

With related functions

11.05.27.0001.01

$$a_r(q) = b_r(q) /; r \notin \mathbb{Z} \wedge q \in \mathbb{R}$$

History

- E. L. Mathieu (1868, 1873)
- H. Weber (1869)
- G. W. Hill (1877)
- E. Heine (1878)
- G. Floquet (1883)
- R. C. Maclaurin (1898)
- J. Dougall (1916, 1926)

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