LegendreQ3General

Notations

Traditional name

Assosiated Legendre function of the second kind of type 3

Traditional notation

$\mathfrak{Q}_{\nu}^{\mu}(z)$

Mathematica StandardForm notation

```
LegendreQ[ν, μ, 3, z]
```

Primary definition

07.12.02.0001.01

$$\mathfrak{Q}_{\nu}^{\mu}(z) = \frac{\pi \csc(\mu \pi)}{2} e^{\pi i \mu} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right) - \frac{\Gamma(\mu+\nu+1)}{\Gamma(-\mu+\nu+1)} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; \mu+1; \frac{1-z}{2}\right)\right) /; \mu \notin \mathbb{Z}$$

For $\mu \in \mathbb{Z}$ the above definition becomes indeterminate, and the function LegendreQ[ν, μ, 3, z] is defined by taking a limit. Series expansions for this case can be found in the *Series representations* section.

07.12.02.0002.01

$$\mathfrak{Q}_{\nu}^{\mu}(z) = \lim_{\tilde{\mu} \to \mu} \mathfrak{Q}_{\nu}^{\tilde{\mu}}(z) /; \mu \in \mathbb{Z}$$

Specific values

Specialized values

For fixed ν, μ

07.12.03.0001.01

$$\mathfrak{Q}_{\nu}^{\mu}(0) = \frac{1}{2} e^{i \pi \mu} \pi \csc(\pi \mu) \left(\frac{\left(\frac{i}{2}\right)^{-\mu} \sqrt{\pi} \cos(\pi \mu)}{\Gamma\left(-\frac{\mu}{2} - \frac{\nu}{2} + \frac{1}{2}\right) \Gamma\left(-\frac{\mu}{2} + \frac{\nu}{2} + 1\right)} - \frac{\left(\frac{i}{2}\right)^{\mu} \sqrt{\pi} \, (-\mu+\nu+1)_{2\mu}}{\Gamma\left(\frac{\mu}{2} - \frac{\nu}{2} + \frac{1}{2}\right) \Gamma\left(\frac{\mu}{2} + \frac{\nu}{2} + 1\right)} \right)$$

07.12.03.0002.01

$$\mathfrak{Q}_{\nu}^{\mu}(1) = ¿ /; \mu \notin \mathbb{Z}$$

07.12.03.0003.01

$$\mathfrak{Q}_{\nu}^{\mu}(1) = \tilde{\infty} /; \mu \in \mathbb{Z}$$

07.12.03.0004.01

$$\mathfrak{Q}_\nu^\mu(-1) = ¿ /; \mu \notin \mathbb{Z}$$

07.12.03.0005.01

$$\mathfrak{Q}_\nu^\mu(-1) = \tilde{\infty} /; \mu \in \mathbb{Z}$$

For fixed ν, z

07.12.03.0006.01

$$\mathfrak{Q}_\nu^m(z) = (z-1)^{m/2}(z+1)^{m/2}\left(\frac{\partial^m Q_\nu(z)}{\partial z^m} - \frac{\pi}{2}\frac{\sqrt{z-1}}{\sqrt{1-z}}\frac{\partial^m P_\nu(z)}{\partial z^m}\right) /; m \in \mathbb{N}$$

07.12.03.0007.01

$$Q_\nu^0(z) = Q_\nu(z) - \frac{\pi\sqrt{z-1}\,P_\nu(z)}{2\sqrt{1-z}}$$

07.12.03.0008.01

$$\mathfrak{Q}_n^0(z) = \left(\frac{\log(z+1)-\log(z-1)}{2} - \psi(n+1)\right)P_n(z) + \sum_{k=0}^{n}\frac{(n+k)!\,\psi(k+1)}{k!^2\,(n-k)!}\left(\frac{z-1}{2}\right)^k /; n \in \mathbb{N}$$

07.12.03.0009.01

$$\mathfrak{Q}_\nu^{\frac{1}{2}}(z) = i\sqrt{\frac{\pi}{2}}\,(z-1)^{-1/4}(z+1)^{-1/4}\left(-\frac{\sqrt{z-1}}{\sqrt{1-z}}\sin\left((\nu+1/2)\cos^{-1}(z)\right) + \cos\left((\nu+1/2)\cos^{-1}(z)\right)\right)$$

07.12.03.0010.01

$$\mathfrak{Q}_\nu^{-\nu-n}(z) = \tilde{\infty} /; n \in \mathbb{N}^+$$

07.12.03.0011.01

$$\mathfrak{Q}_\nu^{-\nu}(z) = \frac{e^{-i\pi\nu}}{2^{\nu+1}}(z+1)^{\nu/2}(z-1)^{\nu/2}\left(\Gamma(-\nu) - \frac{\Gamma(\nu+1)}{\Gamma(2\nu+1)}\frac{(1-z)^\nu}{(z-1)^\nu}\mathrm{B}_{\frac{1-z}{2}}(-\nu,-\nu)\right) /; \nu \notin \mathbb{Z}$$

07.12.03.0012.01

$$\mathfrak{Q}_\nu^{\nu}(z) = -\frac{e^{i\pi\nu}}{2\sqrt{\pi}}\Gamma(\nu+1)(z+1)^{\nu/2}(z-1)^{\nu/2}\left(2^{-\nu}\sqrt{\pi}\,\frac{(1-z)^\nu}{(z-1)^\nu}\mathrm{B}_{\frac{1-z}{2}}(-\nu,-\nu) - 2^\nu\,\Gamma(-\nu)\,\Gamma\left(\nu+\frac{1}{2}\right)\right) /; \nu \notin \mathbb{Z}$$

07.12.03.0013.01

$$\mathfrak{Q}_\nu^{\nu+1}(z) = \frac{2^\nu\,\pi\,(i+\cot(\pi\nu))}{\Gamma(-\nu)}(z-1)^{-\frac{\nu+1}{2}}(z+1)^{-\frac{\nu+1}{2}}$$

For fixed μ, z

07.12.03.0014.01

$$\mathfrak{Q}_0^\mu(z) = \frac{1}{2}e^{i\pi\mu}\,\Gamma(\mu)\,\frac{(1+z)^\mu-(z-1)^\mu}{(z-1)^{\mu/2}(z+1)^{\mu/2}} /; \mu \notin \mathbb{Z}$$

07.12.03.0015.01

$$\mathfrak{Q}_0^m(z) = \frac{(-1)^m(m-1)!}{2}\,\frac{(1+z)^m-(z-1)^m}{(z-1)^{m/2}(z+1)^{m/2}} /; m \in \mathbb{N}^+$$

07.12.03.0016.01

$$\mathfrak{Q}_0^{-m}(z) = \tilde{\infty} /; m \in \mathbb{N}^+$$

07.12.03.0018.01

$$\mathfrak{Q}_1^{\mu}(z) = -e^{i\pi\mu} \frac{\pi \csc(\pi\mu)}{2\,\Gamma(2-\mu)} (z-1)^{-\mu/2} (z+1)^{-\mu/2} \left((z+1)^{\mu} (-z+\mu) + (z-1)^{\mu} (z+\mu)\right)$$

07.12.03.0019.01

$$\mathfrak{Q}_2^{\mu}(z) = \frac{\pi \csc(\pi\mu)\, e^{i\pi\mu}}{2\,\Gamma(3-\mu)} (z-1)^{-\mu/2} (z+1)^{-\mu/2} \left((z+1)^{\mu} \left(3 z^2 - 3\mu z + \mu^2 - 1\right) - (z-1)^{\mu} \left(3 z^2 + 3\mu z + \mu^2 - 1\right)\right)$$

07.12.03.0020.01

$$\mathfrak{Q}_3^{\mu}(z) = \frac{\pi \csc(\pi\mu)}{2\,\Gamma(4-\mu)} e^{i\pi\mu} (z-1)^{-\mu/2} (z+1)^{-\mu/2} \left((z+1)^{\mu} \left(15 z^3 - 15\mu z^2 + \left(6\mu^2 - 9\right) z - \mu^3 + 4\mu\right) - (z-1)^{\mu} \left(15 z^3 + 15\mu z^2 + \left(6\mu^2 - 9\right) z + \mu^3 - 4\mu\right)\right)$$

07.12.03.0021.01

$$\mathfrak{Q}_4^{\mu}(z) = \frac{\pi \csc(\pi\mu)}{2\,\Gamma(5-\mu)} e^{i\pi\mu} (z-1)^{-\mu/2} (z+1)^{-\mu/2} \left((z+1)^{\mu} \left(105 z^4 - 105\mu z^3 + 45\left(\mu^2 - 2\right) z^2 + \left(55\mu - 10\mu^3\right) z + \mu^4 - 10\mu^2 + 9\right) - (z-1)^{\mu} \left(105 z^4 + 105\mu z^3 + 45\left(\mu^2 - 2\right) z^2 + 5\mu\left(2\mu^2 - 11\right) z + \mu^4 - 10\mu^2 + 9\right)\right)$$

07.12.03.0022.01

$$\mathfrak{Q}_5^{\mu}(z) = \frac{\pi \csc(\pi\mu)}{2\,\Gamma(6-\mu)} e^{i\pi\mu} (z-1)^{-\mu/2} (z+1)^{-\mu/2} \left((z+1)^{\mu} \left(945 z^5 - 945\mu z^4 + 210\left(2\mu^2 - 5\right) z^3 - 105\mu\left(\mu^2 - 7\right) z^2 + 15\left(\mu^4 - 13\mu^2 + 15\right) z - (\mu-4)(\mu-2)\mu(\mu+2)(\mu+4)\right) - (z-1)^{\mu} \left(945 z^5 + 945\mu z^4 + 210\left(2\mu^2 - 5\right) z^3 + 105\mu\left(\mu^2 - 7\right) z^2 + 15\left(\mu^4 - 13\mu^2 + 15\right) z + \mu^5 - 20\mu^3 + 64\mu\right)\right)$$

07.12.03.0023.01

$$\mathfrak{Q}_6^{\mu}(z) = \frac{\pi \csc(\pi\mu)}{2\,\Gamma(7-\mu)} e^{i\pi\mu} (z-1)^{-\mu/2} (z+1)^{-\mu/2} \left((z+1)^{\mu} \left(10\,395 z^6 - 10\,395\mu z^5 + 4725\left(\mu^2 - 3\right) z^4 - 630\mu\left(2\mu^2 - 17\right) z^3 + 105\left(2\mu^4 - 32\mu^2 + 45\right) z^2 - 21\mu\left(\mu^4 - 25\mu^2 + 99\right) z + \mu^6 - 35\mu^4 + 259\mu^2 - 225\right) + (z-1)^{\mu} \left(-10\,395 z^6 - 10\,395\mu z^5 - 4725\left(\mu^2 - 3\right) z^4 - 630\mu\left(2\mu^2 - 17\right) z^3 - 105\left(2\mu^4 - 32\mu^2 + 45\right) z^2 - 21\mu\left(\mu^4 - 25\mu^2 + 99\right) z - \mu^6 + 35\mu^4 - 259\mu^2 + 225\right)\right)$$

07.12.03.0024.01

$$\mathfrak{Q}_7^{\mu}(z) = \frac{\pi \csc(\pi\mu)}{2\,\Gamma(8-\mu)} e^{i\pi\mu} (z-1)^{-\mu/2} (z+1)^{-\mu/2} \left((z+1)^{\mu} \left(135\,135 z^7 - 135\,135\mu z^6 + 31\,185\left(2\mu^2 - 7\right) z^5 - 17\,325\mu\left(\mu^2 - 10\right) z^4 + 1575\left(2\mu^4 - 38\mu^2 + 63\right) z^3 - 189\mu\left(2\mu^4 - 60\mu^2 + 283\right) z^2 + 7\left(4\mu^6 - 170\mu^4 + 1516\mu^2 - 1575\right) z - \mu\left(\mu^6 - 56\mu^4 + 784\mu^2 - 2304\right)\right) + (z-1)^{\mu} \left(-135\,135 z^7 - 135\,135\mu z^6 - 31\,185\left(2\mu^2 - 7\right) z^5 - 17\,325\mu\left(\mu^2 - 10\right) z^4 - 1575\left(2\mu^4 - 38\mu^2 + 63\right) z^3 - 189\mu\left(2\mu^4 - 60\mu^2 + 283\right) z^2 - 7\left(4\mu^6 - 170\mu^4 + 1516\mu^2 - 1575\right) z - \mu\left(\mu^6 - 56\mu^4 + 784\mu^2 - 2304\right)\right)\right)$$

07.12.03.0025.01

$$\mathfrak{Q}_8^{\mu}(z) = \frac{\pi \csc(\pi\mu)}{2\,\Gamma(9-\mu)} e^{i\pi\mu} (z-1)^{-\mu/2} (z+1)^{-\mu/2} \left((z+1)^{\mu} \left(2\,027\,025 z^8 - 2\,027\,025\mu z^7 + 945\,945\left(\mu^2 - 4\right) z^6 - 135\,135\mu\left(2\mu^2 - 23\right) z^5 + 51\,975\left(\mu^4 - 22\mu^2 + 42\right) z^4 - 3465\mu\left(2\mu^4 - 70\mu^2 + 383\right) z^3 + 315\left(2\mu^6 - 100\mu^4 + 1043\mu^2 - 1260\right) z^2 - 9\mu\left(4\mu^6 - 266\mu^4 + 4396\mu^2 - 15\,159\right) z + \mu^8 - 84\mu^6 + 1974\mu^4 - 12\,916\mu^2 + 11\,025\right) + (z-1)^{\mu} \left(-2\,027\,025 z^8 - 2\,027\,025\mu z^7 - 945\,945\left(\mu^2 - 4\right) z^6 - 135\,135\mu\left(2\mu^2 - 23\right) z^5 - 51\,975\left(\mu^4 - 22\mu^2 + 42\right) z^4 - 3465\mu\left(2\mu^4 - 70\mu^2 + 383\right) z^3 - 315\left(2\mu^6 - 100\mu^4 + 1043\mu^2 - 1260\right) z^2 - 9\mu\left(4\mu^6 - 266\mu^4 + 4396\mu^2 - 15\,159\right) z - \mu^8 + 84\mu^6 - 1974\mu^4 + 12\,916\mu^2 - 11\,025\right)\right)$$

07.12.03.0026.01

$$\mathfrak{Q}_9^\mu(z) = \frac{\pi\csc(\pi\mu)}{2\,\Gamma(10-\mu)}\,e^{i\pi\mu}\,(z-1)^{-\mu/2}\,(z+1)^{-\mu/2}$$
$$\Big((z+1)^\mu\big(34\,459\,425\,z^9 - 34\,459\,425\,\mu\,z^8 + 8\,108\,100\,(2\,\mu^2-9)\,z^7 - 4\,729\,725\,\mu\,(\mu^2-13)\,z^6 +$$
$$945\,945\,(\mu^4-25\,\mu^2+54)\,z^5 - 135\,135\,\mu\,(\mu^4-40\,\mu^2+249)\,z^4 +$$
$$6930\,(2\,\mu^6-115\,\mu^4+1373\,\mu^2-1890)\,z^3 - 495\,\mu\,(2\,\mu^6-154\,\mu^4+2933\,\mu^2-11\,601)\,z^2 +$$
$$45\,(\mu^8-98\,\mu^6+2674\,\mu^4-20\,217\,\mu^2+19\,845)\,z - \mu\,(\mu^8-120\,\mu^6+4368\,\mu^4-52\,480\,\mu^2+147\,456)\big) +$$
$$(z-1)^\mu\big(-34\,459\,425\,z^9 - 34\,459\,425\,\mu\,z^8 - 8\,108\,100\,(2\,\mu^2-9)\,z^7 - 4\,729\,725\,\mu\,(\mu^2-13)\,z^6 -$$
$$945\,945\,(\mu^4-25\,\mu^2+54)\,z^5 - 135\,135\,\mu\,(\mu^4-40\,\mu^2+249)\,z^4 -$$
$$6930\,(2\,\mu^6-115\,\mu^4+1373\,\mu^2-1890)\,z^3 - 495\,\mu\,(2\,\mu^6-154\,\mu^4+2933\,\mu^2-11\,601)\,z^2 -$$
$$45\,(\mu^8-98\,\mu^6+2674\,\mu^4-20\,217\,\mu^2+19\,845)\,z - \mu\,(\mu^8-120\,\mu^6+4368\,\mu^4-52\,480\,\mu^2+147\,456)\big)\Big)$$

07.12.03.0027.01

$$\mathfrak{Q}_{10}^\mu(z) = \frac{\pi\csc(\pi\mu)}{2\,\Gamma(11-\mu)}\,e^{i\pi\mu}\,(z-1)^{-\mu/2}\,(z+1)^{-\mu/2}$$
$$\Big((z+1)^\mu\big(654\,729\,075\,z^{10} - 654\,729\,075\,\mu\,z^9 + 310\,134\,825\,(\mu^2-5)\,z^8 - 45\,945\,900\,\mu\,(2\,\mu^2-29)\,z^7 +$$
$$9\,459\,450\,(2\,\mu^4-56\,\mu^2+135)\,z^6 - 2\,837\,835\,\mu\,(\mu^4-45\,\mu^2+314)\,z^5 +$$
$$315\,315\,(\mu^6-65\,\mu^4+874\,\mu^2-1350)\,z^4 - 12\,870\,\mu\,(2\,\mu^6-175\,\mu^4+3773\,\mu^2-16\,830)\,z^3 +$$
$$1485\,(\mu^8-112\,\mu^6+3479\,\mu^4-29\,828\,\mu^2+33\,075)\,z^2 - 55\,\mu\,(\mu^8-138\,\mu^6+5754\,\mu^4-78\,877\,\mu^2+251\,865)\,z +$$
$$\mu^{10}-165\,\mu^8+8778\,\mu^6-172\,810\,\mu^4+1\,057\,221\,\mu^2-893\,025\big) +$$
$$(z-1)^\mu\big(-654\,729\,075\,z^{10} - 654\,729\,075\,\mu\,z^9 - 310\,134\,825\,(\mu^2-5)\,z^8 - 45\,945\,900\,\mu\,(2\,\mu^2-29)\,z^7 -$$
$$9\,459\,450\,(2\,\mu^4-56\,\mu^2+135)\,z^6 - 2\,837\,835\,\mu\,(\mu^4-45\,\mu^2+314)\,z^5 -$$
$$315\,315\,(\mu^6-65\,\mu^4+874\,\mu^2-1350)\,z^4 - 12\,870\,\mu\,(2\,\mu^6-175\,\mu^4+3773\,\mu^2-16\,830)\,z^3 -$$
$$1485\,(\mu^8-112\,\mu^6+3479\,\mu^4-29\,828\,\mu^2+33\,075)\,z^2 - 55\,\mu\,(\mu^8-138\,\mu^6+5754\,\mu^4-78\,877\,\mu^2+251\,865)\,z -$$
$$\mu^{10}+165\,\mu^8-8778\,\mu^6+172\,810\,\mu^4-1\,057\,221\,\mu^2+893\,025\big)\Big)$$

07.12.03.0028.01

$$\mathfrak{Q}_n^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\,e^{\pi i\mu}\left(\frac{1}{\Gamma(1-\mu)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{n}\frac{(-n)_k\,(n+1)_k}{(1-\mu)_k\,k!}\left(\frac{1-z}{2}\right)^k - \frac{(n-\mu+1)_{2\mu}}{\Gamma(\mu+1)}\,\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\sum_{k=0}^{n}\frac{(-n)_k\,(n+1)_k}{(\mu+1)_k\,k!}\left(\frac{1-z}{2}\right)^k\right)\,/;\ n\in\mathbb{N}\bigwedge\mu\notin\mathbb{N}$$

07.12.03.0029.01

$$\mathfrak{Q}_{-n}^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\,e^{\pi i\mu}\left(\frac{1}{\Gamma(1-\mu)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{n-1}\frac{(n)_k\,(1-n)_k}{(1-\mu)_k\,k!}\left(\frac{1-z}{2}\right)^k - \frac{(1-n-\mu)_{2\mu}}{\Gamma(\mu+1)}\,\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\sum_{k=0}^{n-1}\frac{(n)_k\,(1-n)_k}{(\mu+1)_k\,k!}\left(\frac{1-z}{2}\right)^k\right)\,/;$$
$$n\in\mathbb{N}^+\bigwedge\mu\notin\mathbb{N}$$

07.12.03.0030.01

$$\mathfrak{Q}_n^m(z) = 2^{-n-1}\,(m+n)!\,n!\,(z-1)^{|m|/2}\,(z+1)^{n-|m|/2}\left(\sum_{k=0}^{\text{Abs}[m]-1}\frac{(-1)^{|m|-k}\,(|m|-k-1)!}{k!\,(n-k)!\,(|m|+n-k)!}\left(\frac{z-1}{z+1}\right)^{k-|m|}+\sum_{k=0}^{n-|m|}\frac{1}{k!\,(k+|m|)!\,(n-k)!\,(n-|m|-k)!}\right.$$
$$(\psi(k+1)+\psi(k+|m|+1)-\psi(n-k+1)-\psi(n-|m|-k+1)+\log(z+1)-\log(z-1))\left(\frac{z-1}{z+1}\right)^k+$$
$$\left.\sum_{k=n-|m|+1}^{n}\frac{(-1)^{n-k+|m|-1}\,(k+|m|-n-1)!}{k!\,(k+|m|)!\,(n-k)!}\left(\frac{z-1}{z+1}\right)^k\right)/;\, n\in\mathbb{N}\land m\in\mathbb{Z}\land|m|\leq n$$

07.12.03.0155.01

$$\mathfrak{Q}_n^m(z) = \frac{(m+n)!}{2}\,(1-z)^{m+n}\,(z-1)^{-\frac{m}{2}-n}\,(z+1)^{-\frac{m}{2}}$$
$$\sum_{k=0}^{n}\frac{(-n)_k\,(n+1)_k}{k!}\left(\frac{(-1)^n\,(m-n-1)!}{(k+m)!}\left(\frac{z+1}{z-1}\right)^m+\frac{(-1)^{k-1}\,(m-k-1)!}{(m+n)!}\right)\left(\frac{z+1}{2}\right)^k /;\, n\in\mathbb{N}\bigwedge m\in\mathbb{N}^+\bigwedge m>n$$

07.12.03.0032.01

$$\mathfrak{Q}_{-n}^m(z) = (m-n)!\left((-1)^{m-n-1}\,(m+n-1)!\;\mathfrak{P}_{-n}^{-m}(z)+\frac{2^{n-1}\,(-1)^m}{(n-1)!}\,\frac{(z-1)^{m/2}}{(z+1)^{m/2+n}}\sum_{k=0}^{m-n}\frac{(k+n-1)!\,(m-k-1)!}{k!\,(m-n-k)!}\left(\frac{z-1}{z+1}\right)^{k-m}\right)/;$$
$$n\in\mathbb{N}^+\bigwedge m\in\mathbb{N}^+\bigwedge m\geq n$$

07.12.03.0033.01

$$\mathfrak{Q}_{-n-\mu}^{\mu}(z) = \tilde{\infty}\,/;\, n\in\mathbb{N}^+$$

For fixed *z*

07.12.03.0034.01

$$\mathfrak{Q}_0^0(z) = \frac{\log(z+1)-\log(z-1)}{2}$$

07.12.03.0035.01

$$\mathfrak{Q}_0^1(z) = -\frac{1}{\sqrt{z-1}\,\sqrt{z+1}}$$

07.12.03.0036.01

$$\mathfrak{Q}_0^2(z) = \frac{2\,z}{z^2-1}$$

07.12.03.0037.01

$$\mathfrak{Q}_0^3(z) = \frac{2+6\,z^2}{(z+1)^{3/2}\,(z-1)^{3/2}}$$

07.12.03.0038.01

$$\mathfrak{Q}_0^4(z) = \frac{24\left(z+z^3\right)}{\left(z^2-1\right)^2}$$

07.12.03.0039.01

$$\mathfrak{Q}_0^5(z) = \frac{24\left(1+5\,z^2\left(2+z^2\right)\right)}{(z-1)^{5/2}\,(z+1)^{5/2}}$$

07.12.03.0040.01

$$\mathfrak{Q}_0^6(z) = \frac{240\, z\,(3 + z^2)\,(1 + 3\, z^2)}{(z^2 - 1)^3}$$

07.12.03.0041.01

$$\mathfrak{Q}_0^7(z) = -\frac{720\,(1 + 21\, z^2 + 35\, z^4 + 7\, z^6)}{(z - 1)^{7/2}\,(z + 1)^{7/2}}$$

07.12.03.0042.01

$$\mathfrak{Q}_0^8(z) = \frac{40\,320\,(z + 7\, z^3 + 7\, z^5 + z^7)}{(z^2 - 1)^4}$$

07.12.03.0043.01

$$\mathfrak{Q}_0^9(z) = -\frac{40\,320\,(1 + 36\, z^2 + 126\, z^4 + 84\, z^6 + 9\, z^8)}{(z - 1)^{9/2}\,(z + 1)^{9/2}}$$

07.12.03.0044.01

$$\mathfrak{Q}_0^{10}(z) = \frac{725\,760\, z\,(5 + 10\, z^2 + z^4)\,(1 + 5\, z^2\,(2 + z^2))}{(z^2 - 1)^5}$$

07.12.03.0045.01

$$\mathfrak{Q}_1^0(z) = \frac{1}{2}\, z\,(\log(z + 1) - \log(z - 1)) - 1$$

07.12.03.0046.01

$$\mathfrak{Q}_1^1(z) = \frac{(z^2 - 1)\,(\log(z + 1) - \log(z - 1)) - 2\, z}{2\,\sqrt{z - 1}\,\sqrt{z + 1}}$$

07.12.03.0047.01

$$\mathfrak{Q}_1^2(z) = \frac{2}{z^2 - 1}$$

07.12.03.0048.01

$$\mathfrak{Q}_1^3(z) = -\frac{8\, z}{(z - 1)^{3/2}\,(z + 1)^{3/2}}$$

07.12.03.0049.01

$$\mathfrak{Q}_1^4(z) = \frac{8\,(1 + 5\, z^2)}{(z^2 - 1)^2}$$

07.12.03.0050.01

$$\mathfrak{Q}_1^5(z) = -\frac{48\, z\,(3 + 5\, z^2)}{(z - 1)^{5/2}\,(z + 1)^{5/2}}$$

07.12.03.0051.01

$$\mathfrak{Q}_1^6(z) = \frac{48\,(3 + 42\, z^2 + 35\, z^4)}{(z^2 - 1)^3}$$

07.12.03.0052.01

$$\mathfrak{Q}_1^7(z) = \frac{1920\,z\,(3+7\,z^2\,(2+z^2))}{(-1-z)^{7/2}\,(1-z)^{7/2}}$$

07.12.03.0053.01

$$\mathfrak{Q}_1^8(z) = \frac{5760\,(1+3\,z^2\,(9+7\,z^2\,(3+z^2)))}{(z^2-1)^4}$$

07.12.03.0054.01

$$\mathfrak{Q}_1^9(z) = -\frac{80\,640\,z\,(5+45\,z^2+63\,z^4+15\,z^6)}{(z-1)^{9/2}\,(z+1)^{9/2}}$$

07.12.03.0055.01

$$\mathfrak{Q}_1^{10}(z) = \frac{80\,640\,(5+11\,z^2\,(20+90\,z^2+84\,z^4+15\,z^6))}{(z^2-1)^5}$$

07.12.03.0056.01

$$\mathfrak{Q}_2^0(z) = \frac{1}{4}\left((3\,z^2-1)\,(\log(z+1)-\log(z-1))-6\,z\right)$$

07.12.03.0057.01

$$\mathfrak{Q}_2^1(z) = \frac{4-6\,z^2+3\,z\,(z^2-1)\,(\log(z+1)-\log(z-1))}{2\,\sqrt{z-1}\,\sqrt{z+1}}$$

07.12.03.0058.01

$$\mathfrak{Q}_2^2(z) = \frac{1}{2\,(z^2-1)}\left(3\,(z^2-1)^2\,(\log(z+1)-\log(z-1))+2\,z(5-3\,z^2)\right)$$

07.12.03.0059.01

$$\mathfrak{Q}_2^3(z) = -\frac{8}{(z-1)^{3/2}\,(z+1)^{3/2}}$$

07.12.03.0060.01

$$\mathfrak{Q}_2^4(z) = \frac{48\,z}{(z^2-1)^2}$$

07.12.03.0061.01

$$\mathfrak{Q}_2^5(z) = -\frac{48\,(1+7\,z^2)}{(z-1)^{5/2}\,(z+1)^{5/2}}$$

07.12.03.0062.01

$$\mathfrak{Q}_2^6(z) = \frac{384\,z\,(3+7\,z^2)}{(z^2-1)^3}$$

07.12.03.0063.01

$$\mathfrak{Q}_2^7(z) = -\frac{1152\,(1+18\,z^2+21\,z^4)}{(z-1)^{7/2}\,(z+1)^{7/2}}$$

07.12.03.0064.01

$$\mathfrak{Q}_2^8(z) = \frac{11520\, z\left(5+30\, z^2+21\, z^4\right)}{\left(z^2-1\right)^4}$$

07.12.03.0065.01

$$\mathfrak{Q}_2^9(z) = -\frac{11520\left(5+33\, z^2\left(5+15\, z^2+7\, z^4\right)\right)}{(z-1)^{9/2}\,(z+1)^{9/2}}$$

07.12.03.0066.01

$$\mathfrak{Q}_2^{10}(z) = \frac{967680\, z\left(5+55\, z^2+99\, z^4+33\, z^6\right)}{\left(z^2-1\right)^5}$$

07.12.03.0067.01

$$\mathfrak{Q}_3^0(z) = \frac{1}{12}\left(8-30\, z^2+3\, z\left(-3+5\, z^2\right)(\log(z+1)-\log(z-1))\right)$$

07.12.03.0068.01

$$\mathfrak{Q}_3^1(z) = \frac{\left(26\, z-30\, z^3+3\left(1-6\, z^2+5\, z^4\right)(\log(z+1)-\log(z-1))\right)}{4\sqrt{z-1}\,\sqrt{z+1}}$$

07.12.03.0069.01

$$\mathfrak{Q}_3^2(z) = \frac{-8+25\, z^2-15\, z^4}{z^2-1}+\frac{15}{2}\, z\left(z^2-1\right)(\log(z+1)-\log(z-1))$$

07.12.03.0070.01

$$\mathfrak{Q}_3^3(z) = \frac{\left(-66\, z+80\, z^3-30\, z^5+15\left(-1+z^2\right)^3(\log(z+1)-\log(z-1))\right)}{2\,(z-1)^{3/2}\,(z+1)^{3/2}}$$

07.12.03.0071.01

$$\mathfrak{Q}_3^4(z) = \frac{48}{\left(z^2-1\right)^2}$$

07.12.03.0072.01

$$\mathfrak{Q}_3^5(z) = -\frac{384\, z}{(z-1)^{5/2}\,(z+1)^{5/2}}$$

07.12.03.0073.01

$$\mathfrak{Q}_3^6(z) = \frac{384\left(1+9\, z^2\right)}{\left(z^2-1\right)^3}$$

07.12.03.0074.01

$$\mathfrak{Q}_3^7(z) = -\frac{11520\left(z+3\, z^3\right)}{(z-1)^{7/2}\,(z+1)^{7/2}}$$

07.12.03.0075.01

$$\mathfrak{Q}_3^8(z) = \frac{11520\left(1+22\, z^2+33\, z^4\right)}{\left(z^2-1\right)^4}$$

07.12.03.0076.01

$$\mathfrak{Q}_3^9(z) = -\frac{46\,080\,z\left(15 + 110\,z^2 + 99\,z^4\right)}{(z-1)^{9/2}\,(z+1)^{9/2}}$$

07.12.03.0077.01

$$\mathfrak{Q}_3^{10}(z) = \frac{138\,240\left(5 + 195\,z^2 + 715\,z^4 + 429\,z^6\right)}{\left(z^2-1\right)^5}$$

07.12.03.0078.01

$$\mathfrak{Q}_4^0(z) = \frac{1}{48}\left(110\,z - 210\,z^3 + 3\left(3 - 30\,z^2 + 35\,z^4\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0079.01

$$\mathfrak{Q}_4^1(z) = -\frac{32 - 230\,z^2 + 210\,z^4 - 15\,z\left(3 - 10\,z^2 + 7\,z^4\right)(\log(z+1) - \log(z-1))}{12\sqrt{z-1}\,\sqrt{z+1}}$$

07.12.03.0080.01

$$\mathfrak{Q}_4^2(z) = \frac{1}{2}\,z\left(-105\,z^2 + \frac{4}{z^2-1} + 85\right) + \frac{15}{4}\left(7\,z^4 - 8\,z^2 + 1\right)(\log(z+1) - \log(z-1))$$

07.12.03.0081.01

$$\mathfrak{Q}_4^3(z) = \frac{96 - 462\,z^2 + 560\,z^4 - 210\,z^6 + 105\,z\left(-1 + z^2\right)^3(\log(z+1) - \log(z-1))}{2\,(z-1)^{3/2}\,(z+1)^{3/2}}$$

07.12.03.0082.01

$$\mathfrak{Q}_4^4(z) = \frac{558\,z - 1022\,z^3 + 770\,z^5 - 210\,z^7 + 105\left(-1 + z^2\right)^4(\log(z+1) - \log(z-1))}{2\left(z^2-1\right)^2}$$

07.12.03.0083.01

$$\mathfrak{Q}_4^5(z) = -\frac{384}{(z-1)^{5/2}\,(z+1)^{5/2}}$$

07.12.03.0084.01

$$\mathfrak{Q}_4^6(z) = \frac{3840\,z}{\left(z^2-1\right)^3}$$

07.12.03.0085.01

$$\mathfrak{Q}_4^7(z) = -\frac{3840\left(1 + 11\,z^2\right)}{(z-1)^{7/2}\,(z+1)^{7/2}}$$

07.12.03.0086.01

$$\mathfrak{Q}_4^8(z) = \frac{46\,080\,z\left(3 + 11\,z^2\right)}{\left(z^2-1\right)^4}$$

07.12.03.0087.01

$$\mathfrak{Q}_4^9(z) = -\frac{46\,080\left(3 + 78\,z^2 + 143\,z^4\right)}{(z-1)^{9/2}\,(z+1)^{9/2}}$$

07.12.03.0088.01

$$\mathfrak{Q}_4^{10}(z) = \frac{645120\, z\left(15 + 130 z^2 + 143 z^4\right)}{\left(z^2 - 1\right)^5}$$

07.12.03.0089.01

$$\mathfrak{Q}_5^{0}(z) = -\frac{8}{15} - \frac{7}{8} z^2 \left(-7 + 9 z^2\right) + \frac{1}{16} z \left(15 - 70 z^2 + 63 z^4\right) (\log(z+1) - \log(z-1))$$

07.12.03.0090.01

$$\mathfrak{Q}_5^{1}(z) = -\frac{226 z - 840 z^3 + 630 z^5 - 15 \left(-1 + 15 z^2 - 35 z^4 + 21 z^6\right) (\log(z+1) - \log(z-1))}{16 \sqrt{z-1} \sqrt{z+1}}$$

07.12.03.0091.01

$$\mathfrak{Q}_5^{2}(z) = \frac{1}{4\left(z^2 - 1\right)} \left(64 - 14 z^2 \left(49 + 45 z^2 \left(-2 + z^2\right)\right) + 105 z \left(-1 + z^2\right)^2 \left(-1 + 3 z^2\right) (\log(z+1) - \log(z-1))\right)$$

07.12.03.0092.01

$$\mathfrak{Q}_5^{3}(z) = \frac{105 \left(9 z^2 - 1\right) (\log(z+1) - \log(z-1)) \left(z^2 - 1\right)^3 + 2 z \left(-945 z^6 + 2625 z^4 - 2359 z^2 + 663\right)}{4 (z-1)^{3/2} (z+1)^{3/2}}$$

07.12.03.0093.01

$$\mathfrak{Q}_5^{4}(z) = \frac{945}{2} z (\log(z+1) - \log(z-1)) \left(z^2 - 1\right)^2 + \frac{9 z^2 \left(279 - 7 z^2 \left(15 z^4 - 55 z^2 + 73\right)\right) - 384}{\left(z^2 - 1\right)^2}$$

07.12.03.0094.01

$$\mathfrak{Q}_5^{5}(z) = \frac{3 \left(315 \left(z^2 - 1\right)^5 (\log(z+1) - \log(z-1)) - 2 z \left(3 \left(105 z^6 - 490 z^4 + 896 z^2 - 790\right) z^2 + 965\right)\right)}{2 (z-1)^{5/2} (z+1)^{5/2}}$$

07.12.03.0095.01

$$\mathfrak{Q}_5^{6}(z) = \frac{3840}{\left(z^2 - 1\right)^3}$$

07.12.03.0096.01

$$\mathfrak{Q}_5^{7}(z) = -\frac{46080\, z}{(z-1)^{7/2} (z+1)^{7/2}}$$

07.12.03.0097.01

$$\mathfrak{Q}_5^{8}(z) = \frac{46080 \left(1 + 13 z^2\right)}{\left(z^2 - 1\right)^4}$$

07.12.03.0098.01

$$\mathfrak{Q}_5^{9}(z) = -\frac{645120\, z \left(3 + 13 z^2\right)}{(z-1)^{9/2} (z+1)^{9/2}}$$

07.12.03.0099.01

$$\mathfrak{Q}_5^{10}(z) = \frac{1935360 \left(1 + 30 z^2 + 65 z^4\right)}{\left(z^2 - 1\right)^5}$$

07.12.03.0100.01

$$\mathfrak{Q}_6^0(z) = \frac{1}{160}\left(5\left(21 z^2 \left(11 z^4 - 15 z^2 + 5\right) - 5\right)(\log(z+1) - \log(z-1)) - 14 z \left(165 z^4 - 170 z^2 + 33\right)\right)$$

07.12.03.0101.01

$$\mathfrak{Q}_6^1(z) = \frac{1}{80\sqrt{z-1}\,\sqrt{z+1}}\left(256 - 42 z^2 \left(103 - 260 z^2 + 165 z^4\right) + 105 z \left(-5 + 35 z^2 - 63 z^4 + 33 z^6\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0102.01

$$\mathfrak{Q}_6^2(z) = 525 z^3 - \frac{3465 z^5}{8} + z\left(-\frac{903}{8} + \frac{2}{-1+z^2}\right) + \frac{105}{16}\left(-1+z^2\right)\left(1 - 18 z^2 + 33 z^4\right)(\log(z+1) - \log(z-1))$$

07.12.03.0103.01

$$\mathfrak{Q}_6^3(z) = \frac{1}{4 (z-1)^{3/2} (z+1)^{3/2}}\left(315 z \left(11 z^2 - 3\right)(\log(z+1) - \log(z-1))\left(z^2-1\right)^3 + 6 z^2 \left(1221 - 7 z^2 \left(165 z^4 - 485 z^2 + 483\right)\right) - 512\right)$$

07.12.03.0104.01

$$\mathfrak{Q}_6^4(z) = \frac{1}{4\left(z^2-1\right)^2}\left(3\left(-6930 z^9 + 26\,040 z^7 - 36\,036 z^5 + 21\,480 z^3 - 4490 z + 315\left(z^2-1\right)^4 \left(11 z^2 - 1\right)(\log(z+1) - \log(z-1))\right)\right)$$

07.12.03.0105.01

$$\mathfrak{Q}_6^5(z) = \frac{1}{2 (z-1)^{5/2} (z+1)^{5/2}}\left(3\left(3465 z (\log(z+1) - \log(z-1))\left(z^2-1\right)^5 - 22 z^2 \left(3\left(105 z^6 - 490 z^4 + 896 z^2 - 790\right) z^2 + 965\right) + 2560\right)\right)$$

07.12.03.0106.01

$$\mathfrak{Q}_6^6(z) = \frac{1}{2\left(z^2-1\right)^3}\left(3\left(-6930 z^{11} + 39\,270 z^9 - 91\,476 z^7 + 111\,276 z^5 - 73\,370 z^3 + 23\,790 z + 3465\left(z^2-1\right)^6 (\log(z+1) - \log(z-1))\right)\right)$$

07.12.03.0107.01

$$\mathfrak{Q}_6^7(z) = -\frac{46\,080}{(z-1)^{7/2} (z+1)^{7/2}}$$

07.12.03.0108.01

$$\mathfrak{Q}_6^8(z) = \frac{645\,120 z}{\left(z^2-1\right)^4}$$

07.12.03.0109.01

$$\mathfrak{Q}_6^9(z) = -\frac{645\,120\left(1 + 15 z^2\right)}{(z-1)^{9/2} (z+1)^{9/2}}$$

07.12.03.0110.01

$$\mathfrak{Q}_6^{10}(z) = \frac{30\,965\,760\left(z + 5 z^3\right)}{\left(z^2-1\right)^5}$$

07.12.03.0111.01

$$\mathfrak{Q}_7^0(z) = \frac{1}{1120}\left(512 - 14 z^2 \left(849 + 55 z^2 \left(-50 + 39 z^2\right)\right) + 35 z \left(-35 + 315 z^2 - 693 z^4 + 429 z^6\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0112.01

$$\mathfrak{Q}_7^1(z) = \frac{1}{160\sqrt{z-1}\,\sqrt{z+1}} \left(3746\,z - 28\,546\,z^3 + 54\,670\,z^5 - 30\,030\,z^7 + 35\left(5 - 140\,z^2 + 630\,z^4 - 924\,z^6 + 429\,z^8\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0113.01

$$\mathfrak{Q}_7^2(z) = \frac{1}{80\left(z^2-1\right)} \left(6\left(7641 - 7\,z^2\left(55\left(39\,z^2 - 95\right)z^2 + 4119\right)\right)z^2 + 315\left(z^2-1\right)^2\left(143\,z^4 - 110\,z^2 + 15\right)(\log(z+1) - \log(z-1))\,z - 2048\right)$$

07.12.03.0114.01

$$\mathfrak{Q}_7^3(z) = \frac{1}{16\,(z-1)^{3/2}\,(z+1)^{3/2}} \left(\left(-2\,z\left(9295 + 3\,z^2\left(-22\,950 + 7\,z^2\left(7404 - 6710\,z^2 + 2145\,z^4\right)\right)\right)\right) + 315\left(-1+z^2\right)^3\left(3 - 66\,z^2 + 143\,z^4\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0115.01

$$\mathfrak{Q}_7^4(z) = \frac{1}{4\left(z^2-1\right)^2} \left(5120 - 22\,z^2\left(4175 + 3\,z^2\left(-5160 + 7798\,z^2 - 5320\,z^4 + 1365\,z^6\right)\right) + 3465\,z\left(-1+z^2\right)^4\left(-3 + 13\,z^2\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0116.01

$$\mathfrak{Q}_7^5(z) = \frac{3}{4\,(z-1)^{5/2}\,(z+1)^{5/2}} \left(54\,510\,z - 328\,130\,z^3 + 736\,956\,z^5 - 801\,108\,z^7 + 427\,350\,z^9 - 90\,090\,z^{11} + 3465\left(-1+z^2\right)^5\left(-1 + 13\,z^2\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0117.01

$$\mathfrak{Q}_7^6(z) = \frac{135\,135}{2}\,z\left(z^2-1\right)^3(\log(z+1) - \log(z-1)) - \frac{3\left(13\left(11\,z^2\left(315\,z^8 - 1785\,z^6 + 4158\,z^4 - 5058\,z^2 + 3335\right) - 11\,895\right)z^2 + 15\,360\right)}{\left(z^2-1\right)^3}$$

07.12.03.0118.01

$$\mathfrak{Q}_7^7(z) = \frac{1}{2\,(z-1)^{7/2}\,(z+1)^{7/2}} \left(3\left(45\,045\left(z^2-1\right)^7(\log(z+1) - \log(z-1)) - 2\,z\left(13\left(11\,z^2\left(315\,z^8 - 2100\,z^6 + 5943\,z^4 - 9216\,z^2 + 8393\right) - 48\,580\right)z^2 + 169\,995\right)\right)\right)$$

07.12.03.0119.01

$$\mathfrak{Q}_7^8(z) = \frac{645\,120}{\left(z^2-1\right)^4}$$

07.12.03.0120.01

$$\mathfrak{Q}_7^9(z) = -\frac{10\,321\,920\,z}{(z-1)^{9/2}\,(z+1)^{9/2}}$$

07.12.03.0121.01

$$\mathfrak{Q}_7^{10}(z) = \frac{10\,321\,920\left(1 + 17\,z^2\right)}{\left(z^2-1\right)^5}$$

07.12.03.0122.01

$$\mathfrak{Q}_8^0(z) = \frac{1}{8960}\left(-450\,450\,z^7 + 690\,690\,z^5 - 294\,910\,z^3 + 30\,318\,z + 35\left(6435\,z^8 - 12\,012\,z^6 + 6930\,z^4 - 1260\,z^2 + 35\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0123.01

$$\mathfrak{Q}_8^1(z) = \frac{1}{1120\,\sqrt{z-1}\,\sqrt{z+1}}\left(6\left(20\,901 - 385\,z^2\left(195\,z^4 - 403\,z^2 + 261\right)\right)z^2 + 315\,(z-1)\,(z+1)\left(11\,z^2\left(65\,z^4 - 91\,z^2 + 35\right) - 35\right)(\log(z+1) - \log(z-1))\,z - 4096\right)$$

07.12.03.0124.01

$$\mathfrak{Q}_8^2(z) = \frac{1}{32\left(z^2-1\right)}\left(-90\,090\,z^9 + 240\,240\,z^7 - 218\,988\,z^5 + 76\,464\,z^3 - 7562\,z + 315\left(z^2-1\right)^2\left(143\,z^6 - 143\,z^4 + 33\,z^2 - 1\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0125.01

$$\mathfrak{Q}_8^3(z) = \frac{1}{16\,(z-1)^{3/2}\,(z+1)^{3/2}}\left(3465\,z\left(39\,z^4 - 26\,z^2 + 3\right)(\log(z+1) - \log(z-1))\left(z^2-1\right)^3 - 66\,z^2\left(4095\,z^8 - 13\,650\,z^6 + 16\,604\,z^4 - 8718\,z^2 + 1733\right) + 4096\right)$$

07.12.03.0126.01

$$\mathfrak{Q}_8^4(z) = \frac{1}{16\left(z^2-1\right)^2}\left(10\,395\left(z^2-1\right)^4\left(65\,z^4 - 26\,z^2 + 1\right)(\log(z+1) - \log(z-1)) - 6\,z\left(11\,z^2\left(3\left(35\,z^2\left(195\,z^4 - 793\,z^2 + 1238\right) - 31\,806\right)z^2 + 31\,615\right) - 37\,495\right)\right)$$

07.12.03.0127.01

$$\mathfrak{Q}_8^5(z) = -\frac{1}{4\,(z-1)^{5/2}\,(z+1)^{5/2}}\left(6\left(13\left(11\,z^2\left(3\left(35\,z^2\left(15\,z^4 - 73\,z^2 + 142\right) - 4846\right)z^2 + 7195\right) - 17\,015\right)z^2 + 10\,240\right) - 135\,135\,z\left(z^2-1\right)^5\left(5\,z^2 - 1\right)(\log(z+1) - \log(z-1))\right)$$

07.12.03.0128.01

$$\mathfrak{Q}_8^6(z) = \frac{3}{4\left(z^2-1\right)^3}\left(45\,045\left(z^2-1\right)^6\left(15\,z^2 - 1\right)(\log(z+1) - \log(z-1)) - 2\,z\left(13\left(33\,z^2\left(\left(35\,z^2\left(45\,z^4 - 258\,z^2 + 611\right) - 26\,676\right)z^2 + 18\,361\right) - 215\,110\right)z^2 + 385\,035\right)\right)$$

07.12.03.0129.01

$$\mathfrak{Q}_8^7(z) = \frac{45}{2\,(z-1)^{7/2}\,(z+1)^{7/2}}\left(45\,045\,z\,(\log(z+1) - \log(z-1))\left(z^2-1\right)^7 - 2\,z^2\left(13\left(11\,z^2\left(315\,z^8 - 2100\,z^6 + 5943\,z^4 - 9216\,z^2 + 8393\right) - 48\,580\right)z^2 + 169\,995\right) + 28\,672\right)$$

07.12.03.0130.01

$$\mathfrak{Q}_8^8(z) = \frac{45}{2\left(z^2-1\right)^4}\left(-90\,090\,z^{15} + 690\,690\,z^{13} - 2\,300\,298\,z^{11} + 4\,335\,474\,z^9 - 5\,036\,174\,z^7 + 3\,663\,478\,z^5 - 1\,603\,070\,z^3 + 368\,662\,z + 45\,045\left(z^2-1\right)^8(\log(z+1) - \log(z-1))\right)$$

07.12.03.0131.01

$$\mathfrak{Q}_8^9(z) = -\frac{10\,321\,920}{(z-1)^{9/2}\,(z+1)^{9/2}}$$

07.12.03.0132.01

$$\mathfrak{Q}_8^{10}(z) = \frac{185\,794\,560\,z}{\left(z^2-1\right)^5}$$

07.12.03.0133.01

$$\mathfrak{Q}_9^0(z) = \frac{1}{80\,640}\left(315\,z\left(11\left(13\,z^2\left(85\,z^4-180\,z^2+126\right)-420\right)z^2+315\right)(\log(z+1)-\log(z-1)) - 2\left(165\left(91\,z^2\left(255\,z^4-455\,z^2+249\right)-3867\right)z^2+16\,384\right)\right)$$

07.12.03.0134.01

$$\mathfrak{Q}_9^1(z) = \frac{1}{1792\sqrt{z-1}\,\sqrt{z+1}}\left(315\,(z-1)\,(z+1)\left(11\left(221\,z^6-364\,z^4+182\,z^2-28\right)z^2+7\right)(\log(z+1)-\log(z-1)) - 2\,z\left(33\left(91\,z^2\left(255\,z^4-590\,z^2+456\right)-11\,910\right)z^2+30\,563\right)\right)$$

07.12.03.0135.01

$$\mathfrak{Q}_9^2(z) = \frac{1}{224\left(z^2-1\right)}\left(8192-22\left(3\left(91\,z^2\left(255\,z^4-740\,z^2+766\right)-30\,180\right)z^2+14\,179\right)z^2 + 3465\left(z^2-1\right)^2\left(221\,z^6-273\,z^4+91\,z^2-7\right)(\log(z+1)-\log(z-1))\,z\right)$$

07.12.03.0136.01

$$\mathfrak{Q}_9^3(z) = \left(2\,z\left(45\,687-11\,z^2\left(52\,411+3\,z^2\left(-66\,678+91\,z^2\left(1206-905\,z^2+255\,z^4\right)\right)\right)\right) + 3465\left(-1+z^2\right)^3\left(-1+39\,z^2-195\,z^4+221\,z^6\right)(\log(z+1)-\log(z-1))\right)\Big/\left(32\,(z-1)^{3/2}\,(z+1)^{3/2}\right)$$

07.12.03.0137.01

$$\mathfrak{Q}_9^4(z) = \frac{1}{16\left(z^2-1\right)^2}\left(135\,135\,z\left(z^2-1\right)^4\left(17\,z^4-10\,z^2+1\right)(\log(z+1)-\log(z-1)) - 6\left(13\left(11\,z^2\left(5355\,z^8-22\,785\,z^6+37\,926\,z^4-30\,714\,z^2+12\,079\right)-21\,111\right)z^2+8192\right)\right)$$

07.12.03.0138.01

$$\mathfrak{Q}_9^5(z) = -\left(6\,z\left(528\,395+13\,z^2\left(-453\,320+11\,z^2\left(155\,813+15\,z^2\left(-18\,904+18\,193\,z^2-8960\,z^4+1785\,z^6\right)\right)\right)\right) - 135\,135\left(-1+z^2\right)^5\left(1-30\,z^2+85\,z^4\right)(\log(z+1)-\log(z-1))\right)\Big/\left(16\,(z-1)^{5/2}\,(z+1)^{5/2}\right)$$

07.12.03.0139.01

$$\mathfrak{Q}_9^6(z) = \frac{15}{4\left(z^2-1\right)^3}\left(45\,045\,z\left(17\,z^2-3\right)(\log(z+1)-\log(z-1))\left(z^2-1\right)^6 - 2\,z^2\left(13\left(11\,z^2\left(5355\,z^8-31\,290\,z^6+76\,041\,z^4-98\,460\,z^2+71\,869\right)-312\,270\right)z^2+725\,025\right)+57\,344\right)$$

07.12.03.0140.01

$$\mathfrak{Q}_9^7(z) = \left(45\left(2\,z\left(413\,707+z^2\left(-3\,521\,455-13\,z^2\left(-918\,183+11\,z^2\left(151\,897+3\,z^2\left(-54\,205+7\,z^2\left(4911-1715\,z^2+255\,z^4\right)\right)\right)\right)\right)\right) + 45\,045\left(-1+z^2\right)^7\left(-1+17\,z^2\right)(\log(z+1)-\log(z-1))\right)\right)\Big/\left(4\,(z-1)^{7/2}\,(z+1)^{7/2}\right)$$

07.12.03.0141.01

$$\mathfrak{Q}_9^8(z) == \frac{45}{2\left(z^2-1\right)^4}\left(765\,765\, z\left(z^2-1\right)^8 (\log(z+1)-\log(z-1)) - 2\left(17\left(z^2\left(13\left(11\, z^2\left(315\, z^8-2415\, z^6+8043\, z^4-15\,159\, z^2+17\,609\right)-140\,903\right) z^2+801\,535\right)-184\,331\right) z^2+229\,376\right)\right)$$

07.12.03.0142.01

$$\mathfrak{Q}_9^9(z) == \left(45\left(-2\, z\left(3\,363\,003+17\, z^2\left(-985\,866+z^2\left(2\,633\,274+13\, z^2\left(-334\,602+11\, z^2\left(32\,768+3\, z^2\left(-7734+3486\, z^2-910\, z^4+105\, z^6\right)\right)\right)\right)\right)\right)+765\,765\left(-1+z^2\right)^9 (\log(z+1)-\log(z-1))\right)\right) \Big/ \left(2\,(z-1)^{9/2}\,(z+1)^{9/2}\right)$$

07.12.03.0143.01

$$\mathfrak{Q}_9^{10}(z) == \frac{185\,794\,560}{\left(z^2-1\right)^5}$$

07.12.03.0144.01

$$\mathfrak{Q}_{10}^0(z) == \frac{1}{161\,280}\left(315\left(11\, z^2\left(13\left(323\, z^6-765\, z^4+630\, z^2-210\right) z^2+315\right)-63\right)(\log(z+1)-\log(z-1)) - 22\, z\left(39\left(7\, z^2\left(4845\, z^4-9860\, z^2+6594\right)-11\,220\right) z^2+27\,985\right)\right)$$

07.12.03.0145.01

$$\mathfrak{Q}_{10}^1(z) == \frac{1}{16\,128\sqrt{z-1}\,\sqrt{z+1}}\left(65\,536-22\, z^2\left(143\,995+39\, z^2\left(-28\,650+78\,008\, z^2-86\,870\, z^4+33\,915\, z^6\right)\right)+3465\,(-1+z)\, z\,(1+z)\left(63+13\, z^2\left(-84+378\, z^2-612\, z^4+323\, z^6\right)\right)(\log(z+1)-\log(z-1))\right)$$

07.12.03.0146.01

$$\mathfrak{Q}_{10}^2(z) == \frac{1}{1792\left(z^2-1\right)}\left(2\, z\left(368\,961-11\, z^2\left(537\,025+39\, z^2\left(-63\,762+7\, z^2\left(17\,634+85\, z^2\left(-179+57\, z^2\right)\right)\right)\right)\right)+3465\left(z^2-1\right)^2\left(7+13\, z^2\left(-28+210\, z^2-476\, z^4+323\, z^6\right)\right)(\log(z+1)-\log(z-1))\right)$$

07.12.03.0147.01

$$\mathfrak{Q}_{10}^3(z) == -\frac{1}{224\,(z-1)^{3/2}\,(z+1)^{3/2}}\left(2\left(13\left(11\, z^2\left(3\left(7\, z^2\left(85\left(57\, z^2-215\right) z^2+26\,514\right)-128\,106\right) z^2+124\,885\right)-172\,353\right) z^2+49\,152\right)-45\,045\, z\left(z^2-1\right)^3\left(323\, z^6-357\, z^4+105\, z^2-7\right)(\log(z+1)-\log(z-1))\right)$$

07.12.03.0148.01

$$\mathfrak{Q}_{10}^4(z) == \frac{1}{32\left(z^2-1\right)^2}\left(45\,045\left(z^2-1\right)^4\left(323\, z^6-255\, z^4+45\, z^2-1\right)(\log(z+1)-\log(z-1)) - 2\, z\left(13\left(11\, z^2\left(3\left(7\, z^2\left(85\left(57\, z^2-254\right) z^2+38\,279\right)-237\,852\right) z^2+324\,919\right)-748\,874\right) z^2+643\,083\right)\right)$$

07.12.03.0149.01

$$\mathfrak{Q}_{10}^5(z) == -\frac{1}{16\,(z-1)^{5/2}\,(z+1)^{5/2}}\left(6\left(z^2\left(13\left(11\, z^2\left(3\left(7\, z^2\left(85\left(57\, z^2-296\right) z^2+53\,469\right)-414\,840\right) z^2+754\,915\right)-2\,609\,040\right) z^2+4\,485\,285\right)-114\,688\right)-135\,135\, z\left(z^2-1\right)^5\left(323\, z^4-170\, z^2+15\right)(\log(z+1)-\log(z-1))\right)$$

07.12.03.0150.01

$$\mathfrak{Q}_{10}^{6}(z) = \frac{1}{16\left(z^2-1\right)^3}\left(15\left(-29\,099\,070\,z^{15}+174\,083\,910\,z^{13}-436\,450\,014\,z^{11}+590\,076\,630\,z^9-459\,200\,170\,z^7+201\,522\,594\,z^5-44\,329\,530\,z^3+3\,399\,746\,z+45\,045\left(z^2-1\right)^6\left(323\,z^4-102\,z^2+3\right)(\log(z+1)-\log(z-1))\right)\right)$$

07.12.03.0151.01

$$\mathfrak{Q}_{10}^{7}(z) = -\frac{1}{4\,(z-1)^{7/2}\,(z+1)^{7/2}}\left(15\left(765\,765\,z\left(z^2-1\right)^7\left(19\,z^2-3\right)(\log(z+1)-\log(z-1))-2\left(17\left(z^2\left(13\left(11\,z^2\left(3\left(7\,z^2\left(285\,z^4-1945\,z^2+5677\right)-64\,311\right)z^2+187\,115\right)-1\,199\,989\right)z^2+5\,124\,525\right)-782\,369\right)z^2+458\,752\right)\right)\right)$$

07.12.03.0152.01

$$\mathfrak{Q}_{10}^{8}(z) = \frac{1}{4\left(z^2-1\right)^4}\left(45\left(765\,765\left(z^2-1\right)^8\left(19\,z^2-1\right)(\log(z+1)-\log(z-1))-2\,z\left(17\left(z^2\left(13\left(11\,z^2\left(3\left(7\,z^2\left(285\,z^4-2200\,z^2+7392\right)-98\,688\right)z^2+349\,730\right)-2\,870\,856\right)z^2+17\,060\,904\right)-4\,303\,824\right)z^2+7\,491\,771\right)\right)\right)$$

07.12.03.0153.01

$$\mathfrak{Q}_{10}^{9}(z) = \frac{1}{2\,(z-1)^{9/2}\,(z+1)^{9/2}}\left(45\left(14\,549\,535\,z\,(\log(z+1)-\log(z-1))\left(z^2-1\right)^9-38\,z^2\left(17\left(z^2\left(13\left(11\,z^2\left(3\left(105\,z^6-910\,z^4+3486\,z^2-7734\right)z^2+32\,768\right)-334\,602\right)z^2+2\,633\,274\right)-985\,866\right)z^2+3\,363\,003\right)+8\,257\,536\right)\right)$$

07.12.03.0154.01

$$\mathfrak{Q}_{10}^{10}(z) = \frac{654\,729\,075}{2}\left(z^2-1\right)^5(\log(z+1)-\log(z-1))-\frac{1}{\left(z^2-1\right)^5}\left(45\,z\left(19\,z^2\left(17\left(z^2\left(13\left(11\,z^2\left(315\,z^8-3045\,z^6+13\,188\,z^4-33\,660\,z^2+55\,970\right)-695\,050\right)z^2+6\,983\,100\right)-3\,619\,140\right)z^2+20\,122\,725\right)-68\,025\,825\right)\right)$$

General characteristics

Domain and analyticity

$\mathfrak{Q}_\nu^\mu(z)$ is an analytical function of ν, μ and z which is defined over $\mathbb{C}^3$. For integer ν and noninteger μ, $\mathfrak{Q}_\nu^\mu(z)$ degenerates to a sum of two polynomial in z multiplied on functions $\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$and $\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}$correspondently. For integers ν, $\frac{\mu}{2}$ /; $\mu > -\nu$, $\mathfrak{Q}_\nu^\mu(z)$ becomes to meromorthic function.

07.12.04.0001.01

$$(\nu * \mu * 3 * z) \longrightarrow \mathfrak{Q}_\nu^\mu(z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \{3\} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.12.04.0002.01

$$\mathfrak{Q}_{\bar{\nu}}^{\bar{\mu}}(\bar{z}) = \overline{\mathfrak{Q}_{\nu}^{\mu}(z)} \;/;\; z \notin (-\infty, 1)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν, μ (except $\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu$), the function $\mathfrak{Q}_{\nu}^{\mu}(z)$ does not have poles and essential singularities.

07.12.04.0003.01

$$\mathcal{S}ing_z\left(\mathfrak{Q}_{\nu}^{\mu}(z)\right) = \{\} \;/;\; \neg\left(\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu\right)$$

For integer ν and integer $\frac{\mu}{2}$ /; $\mu > 1 \wedge \mu > -\nu$, the function $Q_{\nu}^{\mu}(z)$ is a meromorthic function with poles at points $z = \pm 1$ of orders $\frac{\mu}{2}$and at (maybe) $z = \tilde{\infty}$ (for $\nu < -1$of order $-\nu - 1$).

07.12.04.0004.01

$$\mathcal{S}ing_z\left(\mathfrak{Q}_{\nu}^{\mu}(z)\right) = \left\{\left\{1, \frac{\mu}{2}\right\}, \left\{-1, \frac{\mu}{2}\right\}\right\} \;/;\; \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu \bigwedge \nu \geq 0$$

07.12.04.0005.01

$$\mathcal{S}ing_z\left(\mathfrak{Q}_{\nu}^{\mu}(z)\right) = \left\{\left\{1, \frac{\mu}{2}\right\}, \left\{-1, \frac{\mu}{2}\right\}, \{\tilde{\infty}, -\nu - 1\}\right\} \;/;\; \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu \bigwedge \nu < -1$$

With respect to μ

For fixed ν, z, the function $\mathfrak{Q}_{\nu}^{\mu}(z)$ has an infinite set of singular points:

a) $\mu = -\nu - k \;/;\; k \in \mathbb{N}^+$, are the simple poles with residues $\frac{i\,(-1)^{k-1}\,\pi}{\left(e^{2 i \pi \nu}-1\right)(k-1)!\,\Gamma(k+2\nu+1)}\,\mathfrak{P}_{\nu}^{k+\nu}(z)$;

b) $\mu = \tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

07.12.04.0006.01

$$\mathcal{S}ing_{\mu}\left(\mathfrak{Q}_{\nu}^{\mu}(z)\right) = \left\{\left\{\{-\nu - k, 1\} \;/;\; k \in \mathbb{N}^+\right\}, \{\tilde{\infty}, \infty\}\right\}$$

07.12.04.0007.01

$$\operatorname{res}_{\mu}\left(\mathfrak{Q}_{\nu}^{\mu}(z)\right)(-\nu - k) = \frac{i\,(-1)^{k-1}\,\pi}{\left(e^{2 i \pi \nu} - 1\right)(k-1)!\,\Gamma(k + 2\nu + 1)}\,\mathfrak{P}_{\nu}^{k+\nu}(z) \;/;\; k \in \mathbb{N}^+$$

With respect to ν

For fixed μ, z, the function $\mathfrak{Q}_{\nu}^{\mu}(z)$ has an infinite set of singular points:

a) $\nu = -\mu - k \;/;\; k \in \mathbb{N}^+$ are the simple poles with residues $\frac{(-1)^k\,e^{i \pi \mu}\,\pi \csc(\pi \mu)}{2\,(k-1)!\,\Gamma(1-k-2\mu)}\,\mathfrak{P}_{-k-\mu}^{-\mu}(z)$;

b) $\nu = \tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

07.12.04.0008.01

$$\mathcal{S}ing_{\nu}\left(\mathfrak{Q}_{\nu}^{\mu}(z)\right) = \left\{\left\{\{-\mu - k, 1\} \;/;\; k \in \mathbb{N}^+\right\}, \{\tilde{\infty}, \infty\}\right\}$$

07.12.04.0009.01

$$\text{res}_\nu\left(\mathfrak{Q}_\nu^\mu(z)\right)(-\mu-k) = \frac{(-1)^k e^{i\pi\mu}\,\pi\csc(\pi\mu)}{2\,(k-1)!\,\Gamma(1-k-2\mu)}\,\mathfrak{P}_{-k-\mu}^{-\mu}(z)\;/;\;k\in\mathbb{N}^+$$

Branch points

With respect to z

For fixed ν, μ (except $\nu\in\mathbb{Z}\bigwedge\frac{\mu}{2}\in\mathbb{N}^+\bigwedge\mu>-\nu$), the function $\mathfrak{Q}_\nu^\mu(z)$ has three singular branch points: $z=-1$, $z=1$and $z=\tilde{\infty}$.

For integer ν and integer $\frac{\mu}{2}$ /; $\mu>1\land\mu>-\nu$, the function $\mathfrak{Q}_\nu^\mu(z)$ does not have branch points.

07.12.04.0010.01

$$\mathcal{BP}_z\left(\mathfrak{Q}_\nu^\mu(z)\right) = \{-1, 1, \tilde{\infty}\}\;/;\;\neg\left(\nu\in\mathbb{Z}\bigwedge\frac{\mu}{2}\in\mathbb{N}^+\bigwedge\mu>-\nu\right)$$

07.12.04.0011.01

$$\mathcal{BP}_z\left(\mathfrak{Q}_\nu^\mu(z)\right) = \{\}\;/;\;\nu\in\mathbb{Z}\bigwedge\frac{\mu}{2}\in\mathbb{N}^+\bigwedge\mu>-\nu$$

07.12.04.0012.01

$$\mathcal{R}_z\left(\mathfrak{Q}_\nu^\mu(z), -1\right) = \log\;/;\;\nu\in\mathbb{Z}\bigwedge\neg\left(\nu\in\mathbb{Z}\bigwedge\frac{\mu}{2}\in\mathbb{Z}\bigwedge\frac{\mu}{2}>0\bigwedge\mu>-\nu\right)$$

07.12.04.0013.01

$$\mathcal{R}_z\left(\mathfrak{Q}_\nu^\mu(z), -1\right) = s\;/;\;\mu=\frac{2r}{s}\bigwedge\{r, s\}\in\mathbb{Z}\bigwedge s>1\bigwedge\gcd(r, s)=1$$

07.12.04.0014.01

$$\mathcal{R}_z\left(\mathfrak{Q}_\nu^\mu(z), 1\right) = \log\;/;\;\nu\in\mathbb{Z}\bigwedge\neg\left(\nu\in\mathbb{Z}\bigwedge\frac{\mu}{2}\in\mathbb{Z}\bigwedge\frac{\mu}{2}>0\bigwedge\mu>-\nu\right)$$

07.12.04.0015.01

$$\mathcal{R}_z\left(\mathfrak{Q}_\nu^\mu(z), 1\right) = s\;/;\;\mu=\frac{2r}{s}\bigwedge\{r, s\}\in\mathbb{Z}\bigwedge s>1\bigwedge\gcd(r, s)=1$$

07.12.04.0016.01

$$\mathcal{R}_z\left(\mathfrak{Q}_\nu^\mu(z), \tilde{\infty}\right) = \log\;/;\;2\nu\in\mathbb{Z}\bigvee a\notin\mathbb{Q}$$

07.12.04.0017.01

$$\mathcal{R}_z\left(Q_\nu^\mu(z), \tilde{\infty}\right) = s\;/;\;\nu=\frac{r}{s}\bigwedge\{r, s\}\in\mathbb{Z}\bigwedge s>1\bigwedge\gcd(r, s)=1$$

With respect to μ

For fixed ν, z, the function $\mathfrak{Q}_\nu^\mu(z)$ does not have branch points.

07.12.04.0018.01

$$\mathcal{BP}_\mu\left(\mathfrak{Q}_\nu^\mu(z)\right) = \{\}$$

With respect to ν

For fixed μ, z, the function $\mathfrak{Q}_\nu^\mu(z)$ does not have branch points.

07.12.04.0019.01

$$\mathcal{BP}_\nu\left(\mathfrak{Q}_\nu^\mu(z)\right) = \{\}$$

Branch cuts

With respect to *z*

For fixed ν, μ (except $\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu$), the function $\mathfrak{Q}_\nu^\mu(z)$ is a single-valued function on the z-plane cut along the intervals $(-\infty, -1)$ and $(-1, 1)$.

The function $\mathfrak{Q}_\nu^\mu(z)$ is continuous from above on the interval $(-\infty, -1)$ and on the interval $(-1, 1)$.

For integer ν and integer $\frac{\mu}{2}$ /; $\mu > 1 \bigwedge \mu > -\nu$, the function $\mathfrak{Q}_\nu^\mu(z)$ is a meromorthic function and does not have branch cuts.

07.12.04.0020.01

$$\mathcal{BC}_z\left(\mathfrak{Q}_\nu^\mu(z)\right) = \{\{(-\infty, -1), -i\}, \{(-1, 1), -i\}\} /; \neg\left(\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu\right)$$

07.12.04.0021.01

$$\mathcal{BC}_z\left(\mathfrak{Q}_\nu^\mu(z)\right) = \{\} /; \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu$$

07.12.04.0022.01

$$\lim_{\epsilon \to +0} \mathfrak{Q}_\nu^\mu(x + i\,\epsilon) = \mathfrak{Q}_\nu^\mu(x) /; x < -1$$

07.12.04.0023.01

$$\lim_{\epsilon \to +0} \mathfrak{Q}_\nu^\mu(x - i\,\epsilon) = -i\,e^{i\pi\nu}\left(1 + e^{2i\pi\mu}\right)\pi\,\mathfrak{P}_\nu^\mu(-x) + \left(1 + e^{2i\pi\mu}\right)i\,\pi\,\mathfrak{P}_\nu^\mu(x) + e^{-2i\pi\mu}\,\mathfrak{Q}_\nu^\mu(x) + 2\,e^{i\pi\mu}\,i\,\mathfrak{Q}_\nu^\mu(-x)\sin(\pi(\mu+\nu)) /; x < -1$$

07.12.04.0024.01

$$\lim_{\epsilon \to +0} \mathfrak{Q}_\nu^\mu(x + i\,\epsilon) = \mathfrak{Q}_\nu^\mu(x) /; -1 < x < 1$$

07.12.04.0025.01

$$\lim_{\epsilon \to +0} \mathfrak{Q}_\nu^\mu(x - i\,\epsilon) = e^{\pi i\mu}\,i\,\pi\,\mathfrak{P}_\nu^\mu(x) + e^{-\pi i\mu}\,\mathfrak{Q}_\nu^\mu(x) /; -1 < x < 1$$

With respect to μ

For fixed ν, z, the function $\mathfrak{Q}_\nu^\mu(z)$ does not have branch cuts.

07.12.04.0026.01

$$\mathcal{BC}_\mu\left(\mathfrak{Q}_\nu^\mu(z)\right) = \{\}$$

With respect to ν

For fixed μ, z, the function $\mathfrak{Q}_\nu^\mu(z)$ does not have branch cuts.

07.12.04.0027.01

$$\mathcal{BC}_\nu\left(\mathfrak{Q}_\nu^\mu(z)\right) = \{\}$$

Series representations

Generalized power series

Expansions at $z == 0$

07.12.06.0001.01

$$\mathfrak{Q}_\nu^\mu(z) == \frac{\pi \csc(\pi \mu)}{2} e^{\pi i \mu} \left(\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k \left(\frac{\mu}{2}-k\right)_m \left(-\frac{\mu}{2}\right)_j (-1)^j z^{j+m}}{\Gamma(1-\mu+k)\, k!\, 2^k\, m!\, j!} - (\nu-\mu+1)_{2\mu} \frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}} \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k \left(-k-\frac{\mu}{2}\right)_m \left(\frac{\mu}{2}\right)_j (-1)^j z^{j+m}}{\Gamma(1+\mu+k)\, k!\, 2^k\, m!\, j!} \right) /; |z| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.12.06.0002.01

$$\mathfrak{Q}_\nu^\mu(z) == \frac{\pi \csc(\mu \pi)}{2} e^{\pi i \mu} \left(\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} \left(\frac{\mu z}{2} \sum_{k=0}^{\infty} (k+1)! \left(\frac{\mu}{2}+1\right)_k z^k \, {}_1F_0\left(-\frac{\mu}{2};;-z\right) {}_3\tilde{F}_2\left(-\nu, \nu+1, 1-\frac{\mu}{2}; k+2, 1-\mu; -\frac{z}{2}\right) + \Gamma\left(1-\frac{\mu}{2}\right) \sum_{k=0}^{\infty} \left(-\frac{\mu}{2}\right)_k (-z)^k \tilde{F}_{2\times1\times0}^{3\times1\times0}\left(\begin{matrix} -\nu, \nu+1, 1-\frac{\mu}{2}; 1;; \\ 1, 1-\mu; 1-\frac{\mu}{2};; \end{matrix} \frac{1}{2}, -\frac{z}{2} \right) \right) - (\nu-\mu+1)_{2\mu} \frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}} \left(-\frac{\mu z}{2} \sum_{k=0}^{\infty} (k+1)! \left(1-\frac{\mu}{2}\right)_k z^k \, {}_1F_0\left(\frac{\mu}{2};;-z\right) {}_3\tilde{F}_2\left(-\nu, \nu+1, 1+\frac{\mu}{2}; k+2, 1+\mu; -\frac{z}{2}\right) + \Gamma\left(1+\frac{\mu}{2}\right) \sum_{k=0}^{\infty} \left(\frac{\mu}{2}\right)_k (-z)^k \tilde{F}_{2\times1\times0}^{3\times1\times0}\left(\begin{matrix} -\nu, \nu+1, 1+\frac{\mu}{2}; 1;; \\ 1, 1+\mu; 1+\frac{\mu}{2};; \end{matrix} \frac{1}{2}, -\frac{z}{2} \right) \right) \right) /; \mu \notin \mathbb{Z}$$

07.12.06.0003.01

$$\mathfrak{Q}_\nu^\mu(z) \propto \frac{\pi^{3/2} \csc(\pi \mu)}{2} e^{\pi i \mu} \left(\frac{2^\mu}{\Gamma\left(\frac{1-\mu-\nu}{2}\right) \Gamma\left(1-\frac{\mu-\nu}{2}\right)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} (1+O(z)) - \frac{2^{-\mu} (\nu-\mu+1)_{2\mu}}{\Gamma\left(\frac{\mu-\nu+1}{2}\right) \Gamma\left(1+\frac{\mu+\nu}{2}\right)} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} (1+O(z)) \right) /; (z \to 0)$$

Expansions at $z == 1$

07.12.06.0004.01

$$\mathfrak{Q}_\nu^\mu(z) == \frac{\pi \csc(\mu \pi)}{2} e^{\pi i \mu} \left(\frac{1}{\Gamma(1-\mu)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \left(1 + \frac{\nu(1+\nu)}{2(1-\mu)} (z-1) - \frac{(1-\nu)\nu(1+\nu)(2+\nu)}{8(1-\mu)(2-\mu)} (z-1)^2 + \ldots \right) - \frac{(\nu-\mu+1)_{2\mu}}{\Gamma(\mu+1)} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \left(1 + \frac{\nu(1+\nu)}{2(1+\mu)} (z-1) - \frac{(1-\nu)\nu(1+\nu)(2+\nu)}{8(1+\mu)(2+\mu)} (z-1)^2 + \ldots \right) \right) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.12.06.0005.01

$$\mathfrak{Q}_\nu^\mu(z) == \frac{\pi \csc(\mu \pi)}{2} e^{\pi i \mu} \left(\frac{1}{\Gamma(1-\mu)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{(1-\mu)_k\, k!} \left(\frac{1-z}{2}\right)^k - \frac{(\nu-\mu+1)_{2\mu}}{\Gamma(\mu+1)} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{(\mu+1)_k\, k!} \left(\frac{1-z}{2}\right)^k \right) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.12.06.0006.01

$$\mathfrak{Q}_\nu^\mu(z) == \frac{\pi \csc(\mu \pi)}{2} e^{\pi i \mu} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right) - (\nu-\mu+1)_{2\mu} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; \mu+1; \frac{1-z}{2}\right) \right) /; \mu \notin \mathbb{Z}$$

07.12.06.0007.01

$$\mathfrak{Q}_{\nu}^{\mu}(z) = \frac{\pi \csc(\pi \mu)}{2} e^{\pi i \mu} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right) - (\nu-\mu+1)_{2\mu} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; \mu+1; \frac{1-z}{2}\right)\right) /; \mu \notin \mathbb{Z}$$

07.12.06.0008.01

$$\mathfrak{Q}_{\nu}^{\mu}(z) \propto \frac{2^{-\frac{\mu}{2}-1} e^{\pi i \mu} \Gamma(\mu+\nu+1)\Gamma(-\mu)}{\Gamma(\nu-\mu+1)} (z-1)^{\mu/2} (1+O(z-1)) + 2^{\frac{\mu}{2}-1} e^{\pi i \mu} \Gamma(\mu) (z-1)^{-\mu/2} (1+O(z-1)) /; (z \to 1) \bigwedge \mu \notin \mathbb{Z}$$

07.12.06.0009.01

$$\mathfrak{Q}_{\nu}^{m}(z) = \frac{(-1)^m \Gamma(m+\nu+1)}{2\, m!\, \Gamma(\nu-m+1)} (z-1)^{-m/2} (z+1)^{-m/2} \left(\left(\frac{1}{2}(\log(z+1)-\log(z-1)) - \psi(m) - \psi(\nu-m+1) - \psi(m+\nu+1)\right)(1-z)^m \, {}_2F_1\left(-\nu, \nu+1; m+1; \frac{1-z}{2}\right) + (1-z)^m \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k \psi(k+m+1)}{k!\,(m+1)_k} \left(\frac{1-z}{2}\right)^k + \frac{(z+1)^m m!\, \Gamma(\nu-m+1)}{\Gamma(m+\nu+1)} \sum_{k=0}^{m-1} \frac{(m-k-1)!\,(-\nu)_k (\nu+1)_k}{k!} \left(\frac{z-1}{2}\right)^k + 2^{-m} \left(1-z^2\right)^m \sum_{k=0}^{\infty} \frac{(m-\nu)_k (m+\nu+1)_k \psi(k+1)}{k!\,(m+1)_k} \left(\frac{1-z}{2}\right)^k\right) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge m \in \mathbb{N}^+ \bigwedge (\nu \notin \mathbb{Z} \bigvee \nu \in \mathbb{Z} \land \nu \geq m)$$

07.12.06.0010.01

$$\frac{(-1)^{n-m-1} (m+n)!\,(m-n-1)!}{2\, m!} \frac{(z-1)^{m/2}}{(z+1)^{m/2}} \sum_{k=0}^{n} \frac{(-n)_k (n+1)_k}{(m+1)_k\, k!} \left(\frac{1-z}{2}\right)^k + \frac{(-1)^m}{2} \frac{(z+1)^{m/2}}{(z-1)^{m/2}} \sum_{k=0}^{m-1} \frac{(m-k-1)!\,(-n)_k (n+1)_k}{k!} \left(\frac{z-1}{2}\right)^k /; \left|\frac{1-z}{2}\right| < 1 \bigwedge n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+ \bigwedge m > n$$

07.12.06.0011.01

$$\mathfrak{Q}_{\nu}^{m}(z) \propto (-1)^m 2^{m/2-1} (m-1)! \left(\frac{1}{z-1}\right)^{m/2} (1+O(z-1)) + \frac{i^{-m} 2^{-\frac{m}{2}-2} \Gamma(m+\nu+1)}{m\, m!\, \Gamma(\nu-m+1)} (1-z)^{m/2} (m(\log(2)-\log(z-1)-2\psi(\nu-m+1)-2\psi(m+\nu+1)+2\gamma)+2)(1+O(z-1)) /; (z \to 1) \bigwedge m \in \mathbb{N}^+$$

07.12.06.0012.01

$$\mathfrak{Q}_{\nu}^{0}(z) = \left(\frac{1}{2}(\log(z+1)-\log(z-1)) - \psi(\nu+1)\right) {}_2F_1\left(-\nu, \nu+1; 1; \frac{1-z}{2}\right) + \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k \psi(k+1)}{k!^2} \left(\frac{1-z}{2}\right)^k /; \left|\frac{1-z}{2}\right| < 1$$

07.12.06.0013.01

$$\mathfrak{Q}_{\nu}^{0}(z) = \left(\frac{1}{2}(\log(2)-\log(z-1)) - \psi(\nu+1) - \gamma\right)(1+O(z-1)) /; (z \to 1) \bigwedge \nu \notin \mathbb{Z}$$

07.12.06.0014.01

$$\mathfrak{Q}_{n}^{\mu}(z) = \frac{\pi \csc(\mu \pi)}{2} e^{\pi i \mu} \left(\frac{1}{\Gamma(1-\mu)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{n} \frac{(-n)_k (n+1)_k}{(1-\mu)_k\, k!} \left(\frac{1-z}{2}\right)^k - \frac{(n-\mu+1)_{2\mu}}{\Gamma(\mu+1)} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \sum_{k=0}^{n} \frac{(-n)_k (n+1)_k}{(\mu+1)_k\, k!} \left(\frac{1-z}{2}\right)^k\right) /; n \in \mathbb{N} \land \mu \notin \mathbb{Z}$$

07.12.06.0015.01

$$\mathfrak{Q}_{-n}^{\mu}(z) = \frac{\pi\csc(\mu\pi)}{2}\, e^{\pi i\mu}\left(\frac{1}{\Gamma(1-\mu)}\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{n-1}\frac{(n)_k\,(1-n)_k}{(1-\mu)_k\,k!}\left(\frac{1-z}{2}\right)^k - \frac{(1-n-\mu)_{2\mu}}{\Gamma(\mu+1)}\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\sum_{k=0}^{n-1}\frac{(n)_k\,(1-n)_k}{(\mu+1)_k\,k!}\left(\frac{1-z}{2}\right)^k\right)/;\ n\in\mathbb{N}^+\bigwedge\mu\notin\mathbb{Z}$$

Expansions at $z = -1$

07.12.06.0033.01

$$\mathfrak{Q}_{\nu}^{\mu}(z) = -\frac{\pi}{2}\csc^2(\mu\pi)\, e^{\pi i\mu}\left(\left(\sin(\nu\pi)\frac{(1-z)^{\mu}}{(z-1)^{\mu}}+\sin((\mu-\nu)\pi)\right)\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\frac{1}{\Gamma(1-\mu)}\left(1-\frac{\nu(1+\nu)}{2(1-\mu)}(z+1)-\frac{(1-\nu)\nu(1+\nu)(2+\nu)}{8(1-\mu)(2-\mu)}(z+1)^2+\ldots\right)+\frac{\pi}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\left(1-\csc((\mu+\nu)\pi)\sin(\nu\pi)\frac{(z-1)^{\mu}}{(1-z)^{\mu}}\right)\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\frac{1}{\Gamma(1+\mu)}\left(1-\frac{\nu(1+\nu)}{2(1+\mu)}(z+1)-\frac{(1-\nu)\nu(1+\nu)(2+\nu)}{8(1+\mu)(2+\mu)}(z+1)^2+\ldots\right)\right)/;\ \left|\frac{z+1}{2}\right|<1\bigwedge\mu\notin\mathbb{Z}$$

07.12.06.0017.01

$$\mathfrak{Q}_{\nu}^{\mu}(z) = -\frac{\pi}{2}\csc^2(\mu\pi)\, e^{\pi i\mu}\left(\left(\sin(\nu\pi)\frac{(1-z)^{\mu}}{(z-1)^{\mu}}+\sin((\mu-\nu)\pi)\right)\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\frac{1}{\Gamma(1-\mu)}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k}{(1-\mu)_k\,k!}\left(\frac{z+1}{2}\right)^k+\frac{\pi}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\left(1-\csc((\mu+\nu)\pi)\sin(\nu\pi)\frac{(z-1)^{\mu}}{(1-z)^{\mu}}\right)\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\frac{1}{\Gamma(1+\mu)}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k}{(\mu+1)_k\,k!}\left(\frac{z+1}{2}\right)^k\right)/;\ \left|\frac{z+1}{2}\right|<1\bigwedge\mu\notin\mathbb{Z}$$

07.12.06.0018.01

$$\mathfrak{Q}_{\nu}^{\mu}(z) = -\frac{\pi}{2}\csc^2(\mu\pi)\, e^{\pi i\mu}\left(\left(\sin(\nu\pi)\frac{(1-z)^{\mu}}{(z-1)^{\mu}}+\sin((\mu-\nu)\pi)\right)\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\frac{1}{\Gamma(1-\mu)}\,{}_2F_1\!\left(-\nu,\nu+1;1-\mu;\frac{z+1}{2}\right)+\frac{\pi}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\left(1-\csc((\mu+\nu)\pi)\sin(\nu\pi)\frac{(z-1)^{\mu}}{(1-z)^{\mu}}\right)\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\frac{1}{\Gamma(1+\mu)}\,{}_2F_1\!\left(-\nu,\nu+1;\mu+1;\frac{z+1}{2}\right)\right)/;\ \mu\notin\mathbb{Z}$$

07.12.06.0034.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{\pi}{2}\csc(\mu\pi)\, e^{\pi i \mu}\Bigg(\frac{2^{-\frac{\mu}{2}}\,\Gamma(-\mu)}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\,\frac{(1-z)^{\mu/2}\,(1+z)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{\left(\frac{\mu}{2}\right)_{k-j}(-\nu)_j\,(\nu+1)_j\,2^{-k}}{(k-j)!\,j!\,(\mu+1)_j}(z+1)^k - \frac{2^{\frac{\mu}{2}}\sin(\nu\pi)\,\Gamma(\mu)}{\pi}\,\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}\,(1+z)^{\mu/2}}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{\left(\frac{\mu}{2}\right)_{k-j}(-\mu-\nu)_j\,(-\mu+\nu+1)_j\,2^{-k}}{(k-j)!\,j!\,(1-\mu)_j}(z+1)^k - (-\mu+\nu+1)_{2\mu}\Bigg(\frac{2^{\mu/2}\,\Gamma(\mu)}{\Gamma(\mu-\nu)\,\Gamma(\mu+\nu+1)}\,\frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}\,(1+z)^{\mu/2}}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{\left(-\frac{\mu}{2}\right)_{k-j}(-\nu)_j\,(\nu+1)_j\,2^{-k}}{(k-j)!\,j!\,(1-\mu)_j}(z+1)^k - \frac{2^{-\frac{\mu}{2}}\sin(\nu\pi)\,\Gamma(-\mu)}{\pi}\,\frac{(z-1)^{\mu/2}\,(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{\left(-\frac{\mu}{2}\right)_{k-j}(\mu-\nu)_j\,(\mu+\nu+1)_j\,2^{-k}}{(k-j)!\,j!\,(\mu+1)_j}(z+1)^k\Bigg)\Bigg)\;/;\;\mu\notin\mathbb{Z}$$

07.12.06.0020.01

$$\mathfrak{Q}_\nu^\mu(z) \propto \frac{1}{2\,\Gamma(-\mu-\nu)\,\Gamma(-\mu+\nu+1)}\Big(\left(\csc(\pi\mu)\, e^{\pi i\mu}\right)\Big(-2^{\mu/2}\sin(\pi\nu)\,\Gamma(\mu)\,\Gamma(-\mu-\nu)\,\Gamma(-\mu+\nu+1)\,(1-z)^{\mu/2}\,(z+1)^{-\frac{\mu}{2}}\,(O(z+1)+1)\,(z-1)^{-\frac{\mu}{2}} + 2^{-\frac{\mu}{2}}\pi\,(1-z)^{\mu/2}\,(z+1)^{\mu/2}\,\Gamma(-\mu)\,(O(z+1)+1)\,(z-1)^{-\frac{\mu}{2}} - (-\mu+\nu+1)_{2\mu}\Big(2^{\mu/2}\,\Gamma(\mu)\,\pi\,(z-1)^{\mu/2}\,(1-z)^{-\frac{\mu}{2}}\,(z+1)^{-\frac{\mu}{2}}\,(O(z+1)+1) - 2^{-\frac{\mu}{2}}\sin(\pi\nu)\,\Gamma(-\mu)\,\Gamma(-\mu-\nu)\,\Gamma(-\mu+\nu+1)\,(z-1)^{\mu/2}\,(z+1)^{\mu/2}\,(1-z)^{-\frac{\mu}{2}}\,(O(z+1)+1)\Big)\Big)\Big)\;/;\;(z\to -1)\bigwedge \mu\notin\mathbb{Z}$$

07.12.06.0021.01

$$\mathfrak{Q}_\nu^m(z) = \frac{1}{4\,m!\,\Gamma(-m-\nu)\,\Gamma(\nu-m+1)}\Bigg(\frac{2\,m!\,\Gamma(-m-\nu)}{\Gamma(m-\nu)}(\pi\cot(\pi\nu)+\log(-z-1)-\log(z+1))\,\frac{(z-1)^{m/2}}{(z+1)^{m/2}}\sum_{k=0}^{m-1}\frac{(m-k-1)!\,(-\nu)_k\,(\nu+1)_k}{k!}\left(-\frac{z+1}{2}\right)^k + (-1)^m\Big(2\pi^2\cot^2(\pi\nu)+\pi^2-\log(-z-1)-\log(z+1)\,(4\pi\cot(\pi\nu)-4\log(z+1)+1) - 2\,(\pi\cot(\pi\nu)+\log(-z-1)-\log(z+1))\,(\psi(m)+\psi(m-\nu))\Big)\,\frac{(z+1)^{m/2}}{(z-1)^{m/2}}\,{}_2F_1\!\left(-\nu,\,\nu+1;\,m+1;\,\frac{z+1}{2}\right) + 2\,(-1)^m\,(\pi\cos(\pi\nu)+\log(-z-1)-\log(z+1))\,\frac{(z+1)^{m/2}}{(z-1)^{m/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k\,\psi(k+m+1)}{k!\,(m+1)_k}\left(\frac{z+1}{2}\right)^k + (-1)^m\,2^{1-m}\,(\pi\cos(\pi\nu)+\log(-z-1)-\log(z+1))\,(z-1)^{m/2}\,(z+1)^{m/2}\sum_{k=0}^{\infty}\frac{(m+1)_k\,(m-\nu)_k\,\psi(k+1)}{k!\,(m+\nu+1)_k}\left(\frac{z+1}{2}\right)^k\Bigg)\;/;\;\left|\frac{z+1}{2}\right|<1\bigwedge m\in\mathbb{N}^+$$

07.12.06.0022.01

$$\mathfrak{Q}_\nu^m(z) \propto \frac{(-1)^m 2^{-\frac{m}{2}}}{4\, m!\, \Gamma(-m-\nu)\, \Gamma(\nu-m+1)} \left(2\pi^2 \cot^2(\pi\nu) + \pi^2 - \log(-z-1) - (4\pi\cot(\pi\nu) - 4\log(z+1) + 1)\log(z+1) - 2(\pi\cos(\pi\nu) + \log(-z-1) - \log(z+1)) (\gamma - \psi(m+1)) - 2(\pi\cot(\pi\nu) + \log(-z-1) - \log(z+1))(\psi(m) + \psi(m-\nu))\right)(z+1)^{m/2}(1+O(z+1)) - \frac{(-1)^m 2^{\frac{m}{2}-1}(m-1)!\sin(\pi\nu)}{\pi}(\pi\cot(\pi\nu) + \log(-z-1) - \log(z+1))(z+1)^{-\frac{m}{2}}(1+O(z+1)) \;/;\; (z\to -1) \bigwedge m\in\mathbb{N}^+$$

07.12.06.0023.01

$$\mathfrak{Q}_\nu^0(z) = -\frac{\sin(\pi\nu)}{2\pi}\Bigg(\Big(2\pi^2\cot^2(\pi\nu) + \pi(\log(-z-1) + \log(1-z) - 2\log(z+1))\cot(\pi\nu) + \pi^2 + (\log(-z-1) - \log(z+1))(\log(1-z) - \log(z+1)) - 2(\pi\cot(\pi\nu) + \log(-z-1) - \log(z+1))\psi(-\nu)\Big)\, {}_2F_1\left(-\nu, \nu+1; 1; \frac{z+1}{2}\right) + 2(\pi\cot(\pi\nu) + \log(-z-1) - \log(z+1))\sum_{k=0}^{\infty}\frac{(-\nu)_k(\nu+1)_k\,\psi(k+1)}{k!^2}\left(\frac{z+1}{2}\right)^k\Bigg) \;/;\; \left|\frac{z+1}{2}\right| < 1 \bigwedge \nu\notin\mathbb{Z}$$

07.12.06.0024.01

$$\mathfrak{Q}_\nu^0(z) \propto \frac{1}{2\pi}\Big(2(\pi\cos(\pi\nu) + (\log(-z-1) - \log(z+1))\sin(\pi\nu))\psi(-\nu) + \big((2(\log(z+1)+\gamma) - \log(-2(z+1)))\pi\cos(\pi\nu) - 2\pi^2\csc(\pi\nu) + \left(-\log^2(z+1) + (\log(-2(z+1)) - 2\gamma)\log(z+1) + \pi^2 + (2\gamma - \log(2))\log(-z-1)\right)\sin(\pi\nu)\big)\Big)(1+O(z+1)) \;/;\; (z\to -1) \bigwedge \nu\notin\mathbb{Z}$$

07.12.06.0025.01

$$\mathfrak{Q}_n^0(z) = (-1)^n\left(\frac{1}{2}\log\left(\frac{z+1}{z-1}\right) + \psi(n+1)\right){}_2F_1\left(-n, n+1; 1; \frac{z+1}{2}\right) - (-1)^n\sum_{k=0}^{n}\frac{(k+n)!\,\psi(k+1)}{k!^2\,(n-k)!}\left(-\frac{z+1}{2}\right)^k \;/;\; n\in\mathbb{N}$$

07.12.06.0026.01

$$\mathfrak{Q}_n^0(z) \propto (-1)^n\left(\frac{1}{2}\log\left(-\frac{z+1}{2}\right) + \psi(n+1) + \gamma\right)(1+O(z+1)) \;/;\; (z\to -1)\land n\in\mathbb{N}$$

Expansions at $z = \infty$

07.12.06.0027.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{2^{-\nu-1}e^{i\pi\mu}\sqrt{\pi}\,\Gamma(\mu+\nu+1)}{\Gamma\left(\nu+\frac{3}{2}\right)} z^{-\mu-\nu-1}(z-1)^{\mu/2}(z+1)^{\mu/2}\left(1 + \frac{(1+\mu+\nu)(2+\mu+\nu)}{2(3+2\nu)z^2} + \frac{(1+\mu+\nu)(2+\mu+\nu)(3+\mu+\nu)(4+\mu+\nu)}{8(3+2\nu)(5+2\nu)z^4} + \ldots\right) \;/;\; |z| > 1$$

07.12.06.0028.01

$$\mathfrak{Q}_\nu^\mu(z) = 2^{-\nu-1}e^{i\pi\mu}\sqrt{\pi}\, z^{-\mu-\nu-1}(z-1)^{\mu/2}(z+1)^{\mu/2}\,\Gamma(\mu+\nu+1)\sum_{k=0}^{\infty}\frac{\left(\frac{\mu+\nu+1}{2}\right)_k\left(\frac{\mu+\nu}{2}+1\right)_k z^{-2k}}{\Gamma\left(k+\nu+\frac{3}{2}\right)k!} \;/;\; |z| > 1$$

07.12.06.0029.01

$$\mathfrak{Q}_\nu^\mu(z) = 2^{-\nu-1}\sqrt{\pi}\, e^{i\pi\mu}\,\Gamma(\mu+\nu+1)\, z^{-\mu-\nu-1}(z-1)^{\mu/2}(z+1)^{\mu/2}\, {}_2\tilde{F}_1\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1; \nu+\frac{3}{2}; \frac{1}{z^2}\right) /;\; z\notin(-1, 0)$$

07.12.06.0030.01

$$\mathfrak{Q}_\nu^\mu(z) = -\frac{2^{-\nu-1} e^{i\pi\mu}\cos(\pi\nu)}{\sqrt{\pi}}\Gamma\left(-\nu-\frac{1}{2}\right)\Gamma(\mu+\nu+1)(z-1)^{-\mu/2-\nu-1}(z+1)^{\mu/2}\sum_{k=0}^{\infty}\frac{(\nu+1)_k(\mu+\nu+1)_k}{(2(\nu+1))_k\,k!}\left(\frac{2}{1-z}\right)^k /; \left|\frac{1-z}{2}\right|>1$$

07.12.06.0031.01

$$\mathfrak{Q}_\nu^\mu(z) = -\frac{2^{-\nu-1} e^{i\pi\mu}\cos(\pi\nu)}{\sqrt{\pi}}\Gamma\left(-\nu-\frac{1}{2}\right)\Gamma(\mu+\nu+1)(z-1)^{-\mu/2-\nu-1}(z+1)^{\mu/2}\,{}_2F_1\left(\nu+1,\,\mu+\nu+1;\,2(\nu+1);\,\frac{2}{1-z}\right)/; z\notin(-1,1)$$

07.12.06.0032.01

$$\mathfrak{Q}_\nu^\mu(z) \propto \frac{2^{-\nu-1} e^{i\pi\mu}\sqrt{\pi}\,\Gamma(\mu+\nu+1)}{\Gamma\left(\nu+\frac{3}{2}\right)} z^{-1-\nu}\left(1+O\left(\frac{1}{z}\right)\right)/; (|z|\to\infty)$$

Integral representations

On the real axis

Of the direct function

07.12.07.0001.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{\Gamma(\mu+\nu+1)\,e^{\pi i\mu}\,2^{-\nu-1}}{\Gamma(\nu+1)}\left(z^2-1\right)^{\mu/2}\int_{-1}^{1}\left(1-t^2\right)^{\nu}(z-t)^{-\mu-\nu-1}\,dt /; \operatorname{Re}(\nu)>-1\land z\notin(-\infty,1)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

07.12.13.0001.01

$$\left(1-z^2\right)w''(z)-2zw'(z)+\left(\nu(\nu+1)-\frac{\mu^2}{1-z^2}\right)w(z)=0 /; w(z)= c_1\,\mathfrak{P}_\nu^\mu(z)+c_2\,\mathfrak{Q}_\nu^\mu(z)$$

07.12.13.0002.02

$$W_z\left(\mathfrak{P}_\nu^\mu(z),\,\mathfrak{Q}_\nu^\mu(z)\right) = \frac{e^{i\pi\mu}\,\Gamma(\mu+\nu+1)}{\left(1-z^2\right)\Gamma(-\mu+\nu+1)}$$

07.12.13.0003.01

$$g'(z)\,w''(z)-\left(\frac{2g(z)\,g'(z)^2}{1-g(z)^2}+g''(z)\right)w'(z)-\frac{\left(\mu^2-\nu(\nu+1)\left(1-g(z)^2\right)\right)g'(z)^3}{\left(1-g(z)^2\right)^2}w(z)=0 /; w(z)=c_1\,P_\nu^\mu(g(z))+c_2\,Q_\nu^\mu(g(z))$$

07.12.13.0004.01

$$W_z\left(P_\nu^\mu(g(z)),\,Q_\nu^\mu(g(z))\right) = \frac{e^{i\pi\mu}\,\Gamma(\mu+\nu+1)}{\Gamma(1-\mu+\nu)}g'(z)\,(-g(z)-1)^{-\frac{\mu}{2}}\,(g(z)-1)^{-\frac{\mu}{2}}\left(1-g(z)^2\right)^{\frac{\mu}{2}-1}$$

07.12.13.0005.01

$$g'(z)\,h(z)^2\,w''(z) - \left(\left(\frac{2\,g(z)\,g'(z)^2}{1-g(z)^2} + g''(z)\right)h(z)^2 + 2\,g'(z)\,h'(z)\,h(z)\right)w'(z) + \left(-\frac{\mu^2 - \nu\,(\nu+1)\left(1-g(z)^2\right)}{\left(1-g(z)^2\right)^2}\,h(z)^2\,g'(z)^3 + 2\,h'(z)^2\,g'(z) + h(z)\left(h'(z)\left(\frac{2\,g(z)\,g'(z)^2}{1-g(z)^2} + g''(z)\right) - g'(z)\,h''(z)\right)\right)w(z) = 0 \;/;\; w(z) = c_1\,h(z)\,P_\nu^\mu(g(z)) + c_2\,h(z)\,Q_\nu^\mu(g(z))$$

07.12.13.0006.01

$$W_z\left(h(z)\,P_\nu^\mu(g(z)),\,h(z)\,Q_\nu^\mu(g(z))\right) = \frac{e^{i\pi\mu}\,\Gamma(\mu+\nu+1)}{\Gamma(1-\mu+\nu)}\,h(z)^2\,g'(z)\,(-g(z)-1)^{-\frac{\mu}{2}}\,(g(z)-1)^{-\frac{\mu}{2}}\left(1-g(z)^2\right)^{\frac{\mu}{2}-1}$$

07.12.13.0007.01

$$z^2\,w''(z) - z\left(2\,s + \frac{r\left(a^2\,z^{2r}+1\right)}{1-a^2\,z^{2r}} - 1\right)w'(z) + \left(-\frac{a^2\,r^2\left(\mu^2 + \left(a^2\,z^{2r}-1\right)\nu\,(\nu+1)\right)z^{2r}}{\left(1-a^2\,z^{2r}\right)^2} + s^2 + \frac{r\,s\left(a^2\,z^{2r}+1\right)}{1-a^2\,z^{2r}}\right)w(z) = 0 \;/;\; w(z) = c_1\,z^s\,P_\nu^\mu(a\,z^r) + c_2\,z^s\,Q_\nu^\mu(a\,z^r)$$

07.12.13.0008.01

$$W_z\left(z^s\,P_\nu^\mu(a\,z^r),\,z^s\,Q_\nu^\mu(a\,z^r)\right) = \frac{a\,e^{i\pi\mu}\,r\,z^{r+2s-1}\,\Gamma(\mu+\nu+1)}{\Gamma(-\mu+\nu+1)}\,(-a\,z^r-1)^{-\frac{\mu}{2}}\,(a\,z^r-1)^{-\frac{\mu}{2}}\left(1-a^2\,z^{2r}\right)^{\frac{\mu}{2}-1}$$

07.12.13.0009.01

$$w''(z) - \frac{a^2\,(\log(r) - 2\log(s))\,r^{2z} + \log(r) + 2\log(s)}{1-a^2\,r^{2z}}\,w'(z) + \left(-\frac{a^2\left(\mu^2 - \left(1-a^2\,r^{2z}\right)\nu\,(\nu+1)\right)\log^2(r)\,r^{2z}}{\left(1-a^2\,r^{2z}\right)^2} + \log^2(s) + \frac{\left(a^2\,r^{2z}+1\right)\log(r)\log(s)}{1-a^2\,r^{2z}}\right)w(z) = 0 \;/;\; w(z) = c_1\,s^z\,P_\nu^\mu(a\,r^z) + c_2\,s^z\,Q_\nu^\mu(a\,r^z)$$

07.12.13.0010.01

$$W_z\left(s^z\,P_\nu^\mu(a\,r^z),\,s^z\,Q_\nu^\mu(a\,r^z)\right) = \frac{a\,e^{i\pi\mu}\,r^z\,(-a\,r^z-1)^{-\frac{\mu}{2}}\,(a\,r^z-1)^{-\frac{\mu}{2}}\left(1-a^2\,r^{2z}\right)^{\frac{\mu}{2}-1}\,s^{2z}\,\Gamma(\mu+\nu+1)\log(r)}{\Gamma(-\mu+\nu+1)}$$

Identities

Recurrence identities

Consecutive neighbors

07.12.17.0001.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{(2\,\nu+3)\,z}{\mu+\nu+1}\,\mathfrak{Q}_{\nu+1}^\mu(z) + \frac{\mu-\nu-2}{\mu+\nu+1}\,\mathfrak{Q}_{\nu+2}^\mu(z)$$

07.12.17.0002.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{(2\,\nu-1)\,z}{\nu-\mu}\,\mathfrak{Q}_{\nu-1}^\mu(z) - \frac{\mu+\nu-1}{\nu-\mu}\,\mathfrak{Q}_{\nu-2}^\mu(z)$$

07.12.17.0003.02

$$\mathfrak{Q}_\nu^\mu(z) = \frac{2(\mu+1)z}{(\mu(\mu+1)-\nu(\nu+1))\sqrt{-1-z}\,\sqrt{1-z}}\,\mathfrak{Q}_\nu^{\mu+1}(z) - \frac{1}{\mu(\mu+1)-\nu(\nu+1)}\,\mathfrak{Q}_\nu^{\mu+2}(z)$$

07.12.17.0004.02

$$\mathfrak{Q}_\nu^\mu(z) = \frac{2(\mu-1)z}{\sqrt{-1-z}\,\sqrt{1-z}}\,\mathfrak{Q}_\nu^{\mu-1}(z) - ((\mu-2)(\mu-1)-\nu(\nu+1))\,\mathfrak{Q}_\nu^{\mu-2}(z)$$

Distant neighbors

07.12.17.0013.01

$$\mathfrak{Q}_\nu^\mu(z) = C_n(\nu,\mu,z)\,\mathfrak{Q}_{\nu+n}^\mu(z) + \frac{\mu-\nu-n-1}{n+\mu+\nu}\,C_{n-1}(\nu,\mu,z)\,\mathfrak{Q}_{\nu+n+1}^\mu(z) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{(2\nu+3)z}{\mu+\nu+1} \bigwedge C_n(\nu,\mu,z) = \frac{z(2n+2\nu+1)}{n+\mu+\nu}\,C_{n-1}(\nu,\mu,z) + \frac{\mu-\nu-n}{n+\mu+\nu-1}\,C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.12.17.0014.01

$$\mathfrak{Q}_\nu^\mu(z) = C_n(\nu,\mu,z)\,\mathfrak{Q}_{\nu-n}^\mu(z) - \frac{\mu+\nu-n}{\nu-\mu-n+1}\,C_{n-1}(\nu,\mu,z)\,\mathfrak{Q}_{\nu-n-1}^\mu(z) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{(2\nu-1)z}{\nu-\mu} \bigwedge C_n(\nu,\mu,z) = \frac{z(2n-2\nu-1)}{n+\mu-\nu-1}\,C_{n-1}(\nu,\mu,z) - \frac{\mu+\nu-n+1}{\nu-\mu-n+2}\,C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.12.17.0015.01

$$\mathfrak{Q}_\nu^\mu(z) = C_n(\nu,\mu,z)\,\mathfrak{Q}_\nu^{\mu+n}(z) - \frac{1}{(n+\mu-1)(n+\mu)-\nu(\nu+1)}\,C_{n-1}(\nu,\mu,z)\,\mathfrak{Q}_\nu^{\mu+n+1}(z) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{2(\mu+1)z}{(\mu(\mu+1)-\nu(\nu+1))\sqrt{-z-1}\,\sqrt{1-z}} \bigwedge C_n(\nu,\mu,z) = \frac{2z(n+\mu)}{\sqrt{-z-1}\,\sqrt{1-z}\,((n+\mu-1)(n+\mu)-\nu(\nu+1))}\,C_{n-1}(\nu,\mu,z) - \frac{1}{(n+\mu-2)(n+\mu-1)-\nu(\nu+1)}\,C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.12.17.0016.01

$$\mathfrak{Q}_\nu^\mu(z) = C_n(\nu,\mu,z)\,\mathfrak{Q}_\nu^{\mu-n}(z) - ((\mu-n-1)(\mu-n)-\nu(\nu+1))\,C_{n-1}(\nu,\mu,z)\,\mathfrak{Q}_\nu^{\mu-n-1}(z) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{2(\mu-1)z}{\sqrt{-z-1}\,\sqrt{1-z}} \bigwedge C_n(\nu,\mu,z) = \frac{2z(\mu-n)}{\sqrt{-z-1}\,\sqrt{1-z}}\,C_{n-1}(\nu,\mu,z) - ((\mu-n)(\mu-n+1)-\nu(\nu+1))\,C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

07.12.17.0005.01

$$(\mu+\nu)\,\mathfrak{Q}_{\nu-1}^\mu(z) + (\nu-\mu+1)\,\mathfrak{Q}_{\nu+1}^\mu(z) = (2\nu+1)\,z\,\mathfrak{Q}_\nu^\mu(z)$$

07.12.17.0006.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{(\mu+\nu)\,\mathfrak{Q}_{\nu-1}^\mu(z) + (\nu-\mu+1)\,\mathfrak{Q}_{\nu+1}^\mu(z)}{(2\nu+1)\,z}$$

07.12.17.0007.01

$$\mathfrak{Q}_{\nu}^{\mu+1}(z)+(\mu(\mu-1)-\nu(\nu+1))\,\mathfrak{Q}_{\nu}^{\mu-1}(z)=-\frac{2\mu z}{\sqrt{z-1}\,\sqrt{z+1}}\,\mathfrak{Q}_{\nu}^{\mu}(z)$$

07.12.17.0008.01

$$\mathfrak{Q}_{\nu}^{\mu}(z)=-\frac{\sqrt{z-1}\,\sqrt{z+1}}{2\mu z}\left((\mu(\mu-1)-\nu(\nu+1))\,\mathfrak{Q}_{\nu}^{\mu-1}(z)+\mathfrak{Q}_{\nu}^{\mu+1}(z)\right)$$

07.12.17.0017.01

$$z(\mu+\nu+1)\,Q_{\nu}^{\mu}(z)+\sqrt{z-1}\,\sqrt{z+1}\,Q_{\nu}^{\mu+1}(z)-(-\mu+\nu+1)\,Q_{\nu+1}^{\mu}(z)=0$$

Pavlyk O. (2006)

07.12.17.0018.01

$$Q_{\nu}^{\mu+1}(z)-z\,Q_{\nu+1}^{\mu+1}(z)+\sqrt{z-1}\,\sqrt{z+1}\,(-\mu+\nu+1)\,Q_{\nu+1}^{\mu}(z)=0$$

Pavlyk O. (2006)

Relations of special kind

07.12.17.0009.01

$$\mathfrak{Q}_{-\nu-1}^{\mu}(z)=\csc(\pi(\mu-\nu))\left(\pi\,e^{i\mu\pi}\cos(\nu\pi)\,\mathfrak{P}_{\nu}^{\mu}(z)-\sin((\mu+\nu)\pi)\,\mathfrak{Q}_{\nu}^{\mu}(z)\right)$$

07.12.17.0010.01

$$\mathfrak{Q}_{-\nu-1}^{m}(z)=\mathfrak{Q}_{\nu}^{m}(z)-\pi\cot(\nu\pi)\,\mathfrak{P}_{\nu}^{m}(z)\;/;\;m\in\mathbb{Z}\land\nu\notin\mathbb{Z}$$

07.12.17.0011.01

$$\mathfrak{Q}_{\nu}^{-\mu}(z)=e^{-2i\pi\mu}\,\frac{\Gamma(\nu-\mu+1)}{\Gamma(\nu+\mu+1)}\,\mathfrak{Q}_{\nu}^{\mu}(z)$$

07.12.17.0012.01

$$\mathfrak{Q}_{\nu}^{\mu}(-z)=-\exp\left(-\frac{\pi\nu\sqrt{-z^2}}{z}\right)\mathfrak{Q}_{\nu}^{\mu}(z)\;/;\;z\notin(-1,\,1)$$

Differentiation

Low-order differentiation

With respect to ν

07.12.20.0001.01

$$\frac{\partial\mathfrak{Q}_{\nu}^{\mu}(z)}{\partial\nu}=\frac{\pi\csc(\pi\mu)}{2}\,e^{i\pi\mu}\left((\nu-\mu+1)_{2\mu}\,\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k\,(\psi(k-\nu)-\psi(k+\nu+1))}{\Gamma(k+\mu+1)\,k!}\left(\frac{1-z}{2}\right)^k-\right.$$
$$\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k\,(\psi(k-\nu)-\psi(k+\nu+1))}{\Gamma(k-\mu+1)\,k!}\left(\frac{1-z}{2}\right)^k-$$
$$\left.(\nu-\mu+1)_{2\mu}\,(\pi\cot(\pi\nu)-\psi(\nu-\mu+1)+\psi(\mu+\nu+1))\,\mathfrak{P}_{\nu}^{-\mu}(z)+\pi\cot(\pi\nu)\,\mathfrak{P}_{\nu}^{\mu}(z)\right)/;\;\left|\frac{1-z}{2}\right|<1\bigwedge\mu\notin\mathbb{Z}\bigwedge\nu\notin\mathbb{Z}$$

07.12.20.0002.01

$$\frac{\partial \mathbb{Q}_\nu^\mu(z)}{\partial \nu} = \frac{\pi \csc(\pi \mu)}{2} e^{\pi i \mu} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k-\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k \sum_{i=1}^{k} S_k^{(i)} \nu^i \sum_{r=1}^{k} (-1)^r S_k^{(r)} \left(\frac{i}{\nu} + \frac{r}{\nu+1}\right) (\nu+1)^r - (\nu-\mu+1)_{2\mu} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k \sum_{i=1}^{k} S_k^{(i)} \nu^i \sum_{r=1}^{k} (-1)^r S_k^{(r)} \left(\frac{i}{\nu} + \frac{r}{\nu+1}\right) (\nu+1)^r - (\nu-\mu+1)_{2\mu} (\psi(\mu+\nu+1) - \psi(\nu-\mu+1))\, \mathbb{P}_\nu^{-\mu}(z) \right) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.12.20.0003.01

$$\frac{\partial \mathbb{Q}_\nu^\mu(z)}{\partial \nu} = \frac{\pi}{2} e^{i \mu \pi} \csc(\mu \pi) (\nu-\mu+1)_{2\mu} (\psi(\nu-\mu+1) - \psi(\mu+\nu+1))\, \mathbb{P}_\nu^{-\mu}(z) - \frac{e^{i \mu \pi} (2\nu+1)(z-1)}{4\,\Gamma(\nu-\mu+1)} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \left(\Gamma(\mu-1)\, \Gamma(\nu-\mu+1) \frac{(z+1)^\mu}{(z-1)^\mu} F_{2\times 0\times 2}^{2\times 1\times 3}\left(\begin{matrix} 1-\nu, \nu+2; 1; 1, -\nu, \nu+1; \\ 2, 2-\mu;; \nu+2, 1-\nu; \end{matrix} \frac{1-z}{2}, \frac{1-z}{2} \right) + \Gamma(-\mu-1)\, \Gamma(\mu+\nu+1) F_{2\times 0\times 2}^{2\times 1\times 3}\left(\begin{matrix} 1-\nu, \nu+2; 1; 1, -\nu, \nu+1; \\ 2, 2+\mu;; \nu+2, 1-\nu; \end{matrix} \frac{1-z}{2}, \frac{1-z}{2} \right) \right) /; \mu \notin \mathbb{Z}$$

07.12.20.0004.01

$$\frac{\partial^2 \mathbb{Q}_\nu^\mu(z)}{\partial \nu^2} = \frac{\pi \csc(\pi \mu)}{2} e^{\pi i \mu} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{k!\, \Gamma(k-\mu+1)} \left(\psi(k-\nu)^2 - 2(\pi \cot(\pi \nu) + \psi(k+\nu+1))\, \psi(k-\nu) + \psi(k+\nu+1)^2 + 2\pi \cot(\pi \nu)\, \psi(k+\nu+1) + \psi^{(1)}(k-\nu) + \psi^{(1)}(k+\nu+1) \right) \left(\frac{1-z}{2}\right)^k - (\nu-\mu+1)_{2\mu} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{k!\, \Gamma(k+\mu+1)} \left(\psi(k-\nu)^2 - 2(\pi \cot(\pi \nu) + \psi(k+\nu+1) - \psi(\nu-\mu+1) + \psi(\mu+\nu+1))\, \psi(k-\nu) + \psi(k+\nu+1)^2 + 2\psi(k+\nu+1)(\pi \cot(\pi \nu) - \psi(\nu-\mu+1) + \psi(\mu+\nu+1)) + \psi^{(1)}(k-\nu) + \psi^{(1)}(k+\nu+1) \right) \left(\frac{1-z}{2}\right)^k \right) - \frac{\pi \csc(\pi \mu)}{2} e^{i \pi \mu} \left(\pi^2\, \mathbb{P}_\nu^\mu(z) + (\nu-\mu+1)_{2\mu} \left(\psi(\nu-\mu+1)^2 - 2(\pi \cot(\pi \nu) + \psi(\mu+\nu+1))\, \psi(\nu-\mu+1) - \pi^2 + \psi(\mu+\nu+1)^2 + 2\pi \cot(\pi \nu)\, \psi^{(0)}(\mu+\nu+1) - \psi^{(1)}(\nu-\mu+1) + \psi^{(1)}(\mu+\nu+1) \right) \mathbb{P}_\nu^{-\mu}(z) \right) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.12.20.0005.01

$$\frac{\partial^2 \mathfrak{Q}_\nu^\mu(z)}{\partial \nu^2} = \frac{\pi \csc(\mu\pi)}{2} e^{\pi i \mu} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k-\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k \sum_{i=1}^{k} \nu^{i-2} S_k^{(i)} \sum_{r=1}^{k} (-1)^r (\nu+1)^{r-2} \left((r-1)\, r\, \nu^2 + i^2 (\nu+1)^2 + ((2r-1)\nu - 1)\, i\, (\nu+1)\right) S_k^{(r)} - (\nu-\mu+1)_{2\mu} \left(\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k \sum_{j=1}^{k} S_k^{(j)} \nu^j \sum_{r=1}^{k} (-1)^r S_k^{(r)} (\nu+1)^r \left(\frac{(r-1)\, r\, \nu^2 + j^2 (\nu+1)^2 + j(\nu+1)((2r-1)\nu - 1)}{\nu^2 (\nu+1)^2} + 2\left(\frac{j}{\nu} + \frac{r}{\nu+1}\right)(\psi(\mu+\nu+1) - \psi(\nu-\mu+1)) \right) + \left((\psi(\nu-\mu+1) - \psi(\mu+\nu+1))^2 - \psi^{(1)}(\nu-\mu+1) + \psi^{(1)}(\mu+\nu+1)\right) \mathfrak{P}_\nu^{-\mu}(z) \right) \right) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

With respect to μ

07.12.20.0006.01

$$\frac{\partial \mathfrak{Q}_\nu^\mu(z)}{\partial \mu} = \frac{\pi \csc(\pi\mu)}{2} e^{\pi i \mu} \left((\nu-\mu+1)_{2\mu} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k \psi(k+\mu+1)}{\Gamma(k+\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k + \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k \psi(k-\mu+1)}{\Gamma(k-\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k - (\nu-\mu+1)_{2\mu} \left(\psi(\nu-\mu+1) + \psi(\mu+\nu+1) + i\pi - \pi\cot(\pi\mu) - \frac{1}{2}(\log(z+1) - \log(z-1))\right) \mathfrak{P}_\nu^{-\mu}(z) + \left(i\pi - \pi\cot(\pi\mu) + \frac{1}{2}(\log(z+1) - \log(z-1))\right) \mathfrak{P}_\nu^{\mu}(z) \right) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.12.20.0007.01

$$\frac{\partial \mathfrak{Q}_\nu^\mu(z)}{\partial \mu} = \frac{1}{4}\left(e^{i\mu\pi} \pi\, \nu\, (\nu+1) \csc(\mu\pi)\right) \frac{(z-1)^{\mu/2+1}}{(z+1)^{\mu/2}} \left(\frac{1}{(1-\mu)\,\Gamma(2-\mu)} \frac{(z+1)^\mu}{(z-1)^\mu} F_{2\times0\times1}^{2\times1\times2}\left(\begin{matrix} 1-\nu,\ \nu+2;\ 1;\ 1,\ 1-\mu; \\ 2,\ 2-\mu;;\ 2-\mu; \end{matrix} \frac{1-z}{2}, \frac{1-z}{2} \right) + \frac{\Gamma(\mu+\nu+1)}{(\mu+1)\,\Gamma(\mu+2)\,\Gamma(\nu-\mu+1)} F_{2\times0\times1}^{2\times1\times2}\left(\begin{matrix} 1-\nu,\ \nu+2;\ 1;\ 1,\ \mu+1; \\ 2,\ 2+\mu;;\ \mu+2; \end{matrix} \frac{1-z}{2}, \frac{1-z}{2} \right) \right) - \frac{e^{i\mu\pi} \pi \csc(\mu\pi)}{2} \left(\pi\cot(\mu\pi) - i\pi - \psi(1-\mu) - \frac{1}{2}(\log(z+1) - \log(z-1))\right) \mathfrak{P}_\nu^\mu(z) + \frac{e^{i\mu\pi} \pi \csc(\mu\pi)}{2} (\nu-\mu+1)_{2\mu} \left(\frac{1}{2}(\log(z+1) - \log(z-1)) + \pi\cot(\mu\pi) - i\pi + \psi(\mu+1) - \psi(\nu-\mu+1) - \psi(\mu+\nu+1)\right) \mathfrak{P}_\nu^{-\mu}(z) /; \mu \notin \mathbb{Z}$$

07.12.20.0008.01

$$\frac{\partial^2 \mathfrak{Q}_\nu^\mu(z)}{\partial \mu^2} = \frac{\pi \csc(\pi\mu)}{2} e^{i\pi\mu} \left(\left(\pi^2 \left(\cot^2(\pi\mu) + \csc^2(\pi\mu) \right) - \left(\pi - \frac{i}{2} (\log(z+1) - \log(z-1)) \right)^2 - \pi \cot(\pi\mu) (2 i \pi + (\log(z+1) - \log(z-1))) \right) \mathfrak{P}_\nu^\mu(z) - (\nu-\mu+1)_{2\mu} \left(\frac{1}{4} (\log(z+1) - \log(z-1))^2 + \pi (\cot(\pi\mu) - i) (2\pi\cot(\pi\mu) + (\log(z+1) - \log(z-1))) - (2\pi\cot(\pi\mu) - 2 i \pi - \psi(\nu-\mu+1) - \psi(\mu+\nu+1) + (\log(z+1) - \log(z-1))) (\psi(\nu-\mu+1) + \psi(\mu+\nu+1)) - \psi^{(1)}(\nu-\mu+1) + \psi^{(1)}(\mu+\nu+1) \right) \mathfrak{P}_\nu^{-\mu}(z) \right) - e^{i\pi\mu} \pi (\nu-\mu+1)_{2\mu} \csc(\pi\mu) \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{k!\,\Gamma(k+\mu+1)} \left(\psi(k+\mu+1) \left(\frac{1}{2} (\log(z+1) - \log(z-1)) - i\pi + \pi\cot(\pi\mu) - \psi(\nu-\mu+1) - \psi(\mu+\nu+1) \right) + \frac{1}{2} \left(\psi(k+\mu+1)^2 - \psi^{(1)}(k+\mu+1) \right) \right) \left(\frac{1-z}{2} \right)^k + e^{i\pi\mu} \pi \csc(\pi\mu) \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{k!\,\Gamma(k-\mu+1)} \left(\left(\frac{1}{2} (\log(z+1) - \log(z-1)) - \pi\cot(\pi\mu) + i\pi \right) \psi(k-\mu+1) + \frac{1}{2} \left(\psi(k-\mu+1)^2 - \psi^{(1)}(k-\mu+1) \right) \right) \left(\frac{1-z}{2} \right)^k /; \left| \frac{1-z}{2} \right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

With respect to z

07.12.20.0009.01

$$\frac{\partial \mathfrak{Q}_\nu^\mu(z)}{\partial z} = \frac{1}{z^2-1} \left(z\,\nu\, \mathfrak{Q}_\nu^\mu(z) - (\mu+\nu)\, \mathfrak{Q}_{\nu-1}^\mu(z) \right)$$

07.12.20.0010.01

$$\frac{\partial^2 \mathfrak{Q}_\nu^\mu(z)}{\partial z^2} = \frac{2 z (\mu+\nu) \mathfrak{Q}_{\nu-1}^\mu(z) + \left(\mu^2 + \left((\nu-1) z^2 - \nu - 1 \right) \nu \right) \mathfrak{Q}_\nu^\mu(z)}{\left(z^2 - 1 \right)^2}$$

Symbolic differentiation

With respect to ν

07.12.20.0011.02

$$\frac{\partial^m \mathfrak{Q}_\nu^\mu(z)}{\partial \nu^m} = \frac{\pi \csc(\mu\pi)}{2} e^{\pi i \mu} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k-\mu+1)\,k!} \left(\frac{1-z}{2} \right)^k \sum_{j=0}^{m} \binom{m}{j} \sum_{i=1}^{k} S_k^{(i)} (i-j+1)_j\, \nu^{i-j} \sum_{r=1}^{k} (-1)^r S_k^{(r)} (j-m+r+1)_{m-j} (\nu+1)^{j-m+r} - \frac{\partial^m (\nu-\mu+1)_{2\mu}}{\partial \nu^m} \mathfrak{P}_\nu^{-\mu}(z) - \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} \sum_{p=1}^{m} \binom{m}{p} \frac{\partial^{m-p} (\nu-\mu+1)_{2\mu}}{\partial \nu^{m-p}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\mu+1)\,k!} \left(\frac{1-z}{2} \right)^k \sum_{j=0}^{p} \binom{p}{j} \sum_{i=1}^{k} S_k^{(i)} (i-j+1)_j\, \nu^{i-j} \sum_{r=1}^{k} (-1)^r S_k^{(r)} (j-p+r+1)_{p-j} (\nu+1)^{j-p+r} \right) /; \left| \frac{1-z}{2} \right| < 1 \bigwedge \mu \notin \mathbb{Z} \bigwedge m \in \mathbb{N}$$

With respect to z

07.12.20.0012.02

$$\frac{\partial^n \mathbb{Q}_\nu^\mu(z)}{\partial z^n} = \frac{\pi \csc(\mu \pi)}{2} e^{\pi i \mu}\left(\Gamma\left(\frac{\mu}{2}+1\right) \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2+n}} \sum_{j=0}^{n} \binom{n}{j} {}_2\tilde{F}_1\left(-j, \frac{\mu}{2}; \frac{\mu}{2}-j+1; \frac{z+1}{z-1}\right) {}_3\tilde{F}_2\left(1, -\nu, \nu+1; j-n+1, 1-\mu; \frac{1-z}{2}\right)\left(\frac{z-1}{z+1}\right)^j - (\nu-\mu+1)_{2\mu}\, \Gamma\left(1-\frac{\mu}{2}\right) \frac{(z-1)^{\mu/2-n}}{(z+1)^{\mu/2}} \sum_{j=0}^{n} \binom{n}{j} {}_2\tilde{F}_1\left(-j, -\frac{\mu}{2}; 1-j-\frac{\mu}{2}; \frac{z+1}{z-1}\right) {}_3\tilde{F}_2\left(1, -\nu, \nu+1; j-n+1, \mu+1; \frac{1-z}{2}\right)\left(\frac{z-1}{z+1}\right)^j\right) /; n \in \mathbb{N} \bigwedge \mu \notin \mathbb{Z}$$

07.12.20.0014.01

$$\frac{\partial^m \mathbb{Q}_\nu^\mu(z)}{\partial z^m} = \frac{\Gamma\left(1-\frac{\mu}{2}\right)\Gamma(\mu+\nu+1)}{\Gamma(1-\mu+\nu)} \sum_{k=0}^{m}\sum_{j=0}^{k} \frac{2^{2j-k}\binom{m}{k} k!\, \Gamma(1-k+m-\mu+\nu)}{(k-j)!\,(2j-k)!\,\Gamma\left(1-j-\frac{\mu}{2}\right)\Gamma(k-m+\mu+\nu+1)} z^{2j-k} (z-1)^{\frac{1}{2}(-2j+k-m)} (z+1)^{\frac{1}{2}(-2j+k-m)} \mathbb{Q}_\nu^{k-m+\mu}(z) /; m \in \mathbb{N}$$

07.12.20.0015.01

$$\frac{\partial^m \mathbb{Q}_\nu^\mu(z)}{\partial z^m} = \sqrt{\pi} \sum_{k=0}^{m} (-1)^{m-k} (z-1)^{\frac{k-m}{2}} z^{-k} (z+1)^{\frac{k-m}{2}} \binom{m}{k} (-\mu-\nu)_{m-k}\, {}_3\tilde{F}_2\left(1, -k, \frac{\mu}{2}; \frac{1-k}{2}, 1-\frac{k}{2}; \frac{z^2}{z^2-1}\right)(1-\mu+\nu)_{m-k}\, \mathbb{Q}_\nu^{k-m+\mu}(z) /; m \in \mathbb{N}$$

Fractional integro-differentiation

With respect to z

07.12.20.0013.01

$$\frac{\partial^\alpha \mathbb{Q}_\nu^\mu(z)}{\partial z^\alpha} = \frac{\pi \csc(\mu \pi)}{2} e^{\pi i \mu}\left(\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}}\left(\frac{\mu}{2} z^{1-\alpha} \sum_{k=0}^{\infty} (k+1)!\left(\frac{\mu}{2}+1\right)_k z^k \tilde{F}_{1\times 0\times 2}^{1\times 1\times 3}\left(\begin{matrix} k+2; -\frac{\mu}{2}; -\nu, \nu+1, 1-\frac{\mu}{2}; \\ k-\alpha+2;; k+2, 1-\mu; \end{matrix} -z, -\frac{z}{2}\right) + \Gamma\left(1-\frac{\mu}{2}\right) z^{-\alpha} \sum_{k=0}^{\infty}\left(-\frac{\mu}{2}\right)_k (-z)^k \tilde{F}_{2\times 1\times 1}^{3\times 1\times 1}\left(\begin{matrix} -\nu, \nu+1, 1-\frac{\mu}{2}; 1; k+1; \\ 1, 1-\mu; 1-\frac{\mu}{2}; k-\alpha+1; \end{matrix} \frac{1}{2}, -\frac{z}{2}\right)\right) - (\nu-\mu+1)_{2\mu} \frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}}\left(z^{-\alpha}\, \Gamma\left(\frac{\mu}{2}+1\right) \sum_{k=0}^{\infty}\left(\frac{\mu}{2}\right)_k (-z)^k \tilde{F}_{2\times 1\times 1}^{3\times 1\times 1}\left(\begin{matrix} -\nu, \nu+1, \frac{\mu}{2}+1; 1; k+1; \\ 1, \mu+1; \frac{\mu}{2}+1; k-\alpha+1; \end{matrix} \frac{1}{2}, -\frac{z}{2}\right) - \frac{\mu}{2} z^{1-\alpha} \sum_{k=0}^{\infty} (k+1)!\left(1-\frac{\mu}{2}\right)_k z^k \tilde{F}_{1\times 0\times 2}^{1\times 1\times 3}\left(\begin{matrix} k+2; \frac{\mu}{2}; -\nu, \nu+1, \frac{\mu}{2}+1; \\ k-\alpha+2;; k+2, \mu+1; \end{matrix} -z, -\frac{z}{2}\right)\right)\right) /; \mu \notin \mathbb{Z}$$

Integration

Indefinite integration

Involving only one direct function

07.12.21.0001.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{\pi\csc(\pi\mu)}{2}\, e^{\pi i \mu}\left(\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}}\left(\frac{\mu z^2}{2}\left(\sum_{k=0}^{\infty}(k+1)!\left(\frac{\mu}{2}+1\right)_k z^k\, {}_{1\times 0\times 2}^{1\times 1\times 3}\tilde{F}\left(\begin{matrix} k+2; -\frac{\mu}{2}; -\nu, \nu+1, 1-\frac{\mu}{2}; \\ k+3;; k+2, 1-\mu; \end{matrix} -z, -\frac{z}{2}\right)\right)+\Gamma\left(1-\frac{\mu}{2}\right)z\right.\right.$$
$$\left.\left.\left(\sum_{k=0}^{\infty}\left(-\frac{\mu}{2}\right)_k (-z)^k\, {}_{2\times 1\times 1}^{3\times 1\times 1}\tilde{F}\left(\begin{matrix} -\nu, \nu+1, 1-\frac{\mu}{2}; 1; k+1; \\ 1, 1-\mu; 1-\frac{\mu}{2}; k+2; \end{matrix} \frac{1}{2}, -\frac{z}{2}\right)\right)\right)\right) -$$
$$(\nu-\mu+1)_{2\mu}\frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}}\left(z\,\Gamma\left(\frac{\mu}{2}+1\right)\sum_{k=0}^{\infty}\left(\frac{\mu}{2}\right)_k (-z)^k\, {}_{2\times 1\times 1}^{3\times 1\times 1}\tilde{F}\left(\begin{matrix} -\nu, \nu+1, 1+\frac{\mu}{2}; 1; k+1; \\ 1, 1+\mu; 1+\frac{\mu}{2}; k+2; \end{matrix} \frac{1}{2}, -\frac{z}{2}\right) -\right.$$
$$\left.\left.\frac{\mu z^2}{2}\sum_{k=0}^{\infty}(k+1)!\left(1-\frac{\mu}{2}\right)_k z^k\, {}_{1\times 0\times 2}^{1\times 1\times 3}\tilde{F}\left(\begin{matrix} k+2; \frac{\mu}{2}; -\nu, \nu+1, 1+\frac{\mu}{2}; \\ k+3;; k+2, 1+\mu; \end{matrix} -z, -\frac{z}{2}\right)\right)\right)$$

Summation

Infinite summation

07.12.23.0001.01

$$\sum_{n=0}^{\infty} w^n\, \mathfrak{Q}_n^0(z) = \frac{1}{\sqrt{w^2-2zw+1}}\log\left(\frac{z-w+\sqrt{w^2-2zw+1}}{\sqrt{z^2-1}}\right) /; |z|>1 \land |w|<1$$

Operations

Limit operation

07.12.25.0001.01

$$\lim_{\nu\to\infty} \nu^{-\mu}\, \mathfrak{Q}_\nu^\mu\left(\cosh\left(\frac{z}{\nu}\right)\right) = e^{\mu\pi i} K_\mu(z)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_2\tilde{F}_1$

07.12.26.0001.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\, e^{\pi i\mu}\left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right) - \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}(\nu-\mu+1)_{2\mu}\, {}_2\tilde{F}_1\left(-\nu, \nu+1; \mu+1; \frac{1-z}{2}\right)\right) /;$$
$$\mu \notin \mathbb{Z}$$

07.12.26.0081.01

$$\mathfrak{Q}_\nu^\mu(z) == -\frac{\pi}{2}\csc^2(\mu\pi)\,e^{\pi i\mu}\left(\left(\sin(\nu\pi)\frac{(1-z)^\mu}{(z-1)^\mu}+\sin((\mu-\nu)\pi)\right)\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\,{}_2\tilde{F}_1\left(-\nu,\,\nu+1;\,1-\mu;\,\frac{z+1}{2}\right)+\frac{\pi}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\left(1-\csc((\mu+\nu)\pi)\sin(\nu\pi)\frac{(z-1)^\mu}{(1-z)^\mu}\right)\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\,{}_2\tilde{F}_1\left(-\nu,\,\nu+1;\,\mu+1;\,\frac{z+1}{2}\right)\right)\,/;\,\mu\notin\mathbb{Z}$$

07.12.26.0002.01

$$\mathfrak{Q}_\nu^\mu(z) == 2^{-\nu-1}\,e^{i\pi\mu}\,\pi\,(z+1)^{\nu-\frac{\mu}{2}}\,(z-1)^{\mu/2}\csc(\pi\mu)\left(\frac{(z+1)^\mu}{(z-1)^\mu}\,{}_2\tilde{F}_1\left(-\nu,\,-\mu-\nu;\,1-\mu;\,\frac{z-1}{z+1}\right)-(\nu-\mu+1)_{2\mu}\,{}_2\tilde{F}_1\left(-\nu,\,\mu-\nu;\,\mu+1;\,\frac{z-1}{z+1}\right)\right)\,/;\,\mu\notin\mathbb{Z}\land z\notin(-\infty,\,-1)$$

07.12.26.0082.01

$$\mathfrak{Q}_\nu^\mu(z) == 2^\nu\,e^{i\pi\mu}\,\Gamma(\mu+\nu+1)\,\Gamma(\nu+1)\,(z-1)^{-\nu-1}\left(\frac{z+1}{z-1}\right)^{\mu/2}\,{}_2\tilde{F}_1\left(\nu+1,\,\mu+\nu+1;\,2(\nu+1);\,\frac{2}{1-z}\right)\,/;\,z\notin(-1,\,1)$$

07.12.26.0083.01

$$\mathfrak{Q}_\nu^\mu(z) == 2^{\mu-1}\,e^{i\pi\mu}\,\pi\,(z-1)^{\mu/2}\left(-\frac{1}{z^2}\right)^{-\frac{1}{2}(\mu+\nu)}z^{1-\mu-\nu}\,(z+1)^{\mu/2}\left(\frac{\Gamma\left(\frac{1}{2}(\mu+\nu+2)\right)}{\Gamma\left(\frac{1}{2}(-\mu+\nu+1)\right)}\,{}_2\tilde{F}_1\left(\frac{1}{2}(\mu-\nu+1),\,\frac{\mu+\nu}{2}+1;\,\frac{3}{2};\,z^2\right)-\sqrt{-\frac{1}{z^2}}\,\frac{\Gamma\left(\frac{1}{2}(\mu+\nu+1)\right)}{\Gamma\left(\frac{1}{2}(-\mu+\nu+2)\right)}\,{}_2\tilde{F}_1\left(\frac{\mu-\nu}{2},\,\frac{1}{2}(\mu+\nu+1);\,\frac{1}{2};\,z^2\right)\right)\,/;\,z\notin(-\infty,\,0)\land z\notin(1,\,\infty)$$

07.12.26.0003.01

$$\mathfrak{Q}_\nu^\mu(z) == 2^{-\nu-1}\,e^{i\pi\mu}\,\sqrt{\pi}\,z^{-\mu-\nu-1}\,\Gamma(\mu+\nu+1)\,(z-1)^{\mu/2}\,(z+1)^{\mu/2}\,{}_2\tilde{F}_1\left(\frac{\mu+\nu+1}{2},\,\frac{\mu+\nu}{2}+1;\,\nu+\frac{3}{2};\,\frac{1}{z^2}\right)\,/;\,z\notin(-1,\,0)$$

07.12.26.0084.01

$$\mathfrak{Q}_\nu^\mu(z) == e^{\frac{1}{2}i\pi(\mu-\nu)}\,2^{\mu-1}\,\pi\,(z+1)^{\mu/2}\,(z-1)^{\mu/2}\left(\frac{z\,\Gamma\left(\frac{\mu+\nu}{2}+1\right)}{\Gamma\left(\frac{1-\mu+\nu}{2}\right)}\,{}_2\tilde{F}_1\left(\frac{\mu-\nu+1}{2},\,\frac{\mu+\nu+2}{2};\,\frac{3}{2};\,z^2\right)-\frac{i\,\Gamma\left(\frac{\mu+\nu+1}{2}\right)}{\Gamma\left(\frac{\nu-\mu}{2}+1\right)}\,{}_2\tilde{F}_1\left(\frac{\mu-\nu}{2},\,\frac{\mu+\nu+1}{2};\,\frac{1}{2};\,z^2\right)\right)\,/;\,z\notin(1,\,\infty)$$

Involving ${}_2F_1$

07.12.26.0004.01

$$\mathfrak{Q}_\nu^\mu(z) == \frac{\pi\csc(\mu\pi)}{2}\,e^{\pi i\mu}\left(\frac{1}{\Gamma(1-\mu)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\,{}_2F_1\left(-\nu,\,\nu+1;\,1-\mu;\,\frac{1-z}{2}\right)-\frac{(\nu-\mu+1)_{2\mu}}{\Gamma(\mu+1)}\,\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\,{}_2F_1\left(-\nu,\,\nu+1;\,\mu+1;\,\frac{1-z}{2}\right)\right)\,/;\,\mu\notin\mathbb{Z}$$

07.12.26.0005.01

$$\mathfrak{Q}_\nu^\mu(z) == -\frac{\pi}{2}\csc^2(\mu\pi)\,e^{\pi i\mu}\left(\left(\sin(\nu\pi)\frac{(1-z)^\mu}{(z-1)^\mu}+\sin((\mu-\nu)\pi)\right)\frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\,\frac{1}{\Gamma(1-\mu)}\,{}_2F_1\left(-\nu,\,\nu+1;\,1-\mu;\,\frac{z+1}{2}\right)+\frac{\pi}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\left(1-\csc((\mu+\nu)\pi)\sin(\nu\pi)\frac{(z-1)^\mu}{(1-z)^\mu}\right)\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\,\frac{1}{\Gamma(1+\mu)}\,{}_2F_1\left(-\nu,\,\nu+1;\,\mu+1;\,\frac{z+1}{2}\right)\right)\,/;\,\mu\notin\mathbb{Z}$$

07.12.26.0085.01

$$\mathfrak{Q}_\nu^\mu(z) = 2^{-\nu-1} e^{i\pi\mu} (z-1)^{\mu/2} (z+1)^{\nu-\frac{\mu}{2}} \left(\Gamma(\mu) \frac{(z+1)^\mu}{(z-1)^\mu} \, {}_2F_1\!\left(-\nu, -\mu-\nu; 1-\mu; \frac{z-1}{z+1}\right) + \Gamma(-\mu)\,(\nu-\mu+1)_{2\mu} \, {}_2F_1\!\left(-\nu, \mu-\nu; \mu+1; \frac{z-1}{z+1}\right)\right) /; \mu \notin \mathbb{Z} \land z \notin (-\infty, -1)$$

07.12.26.0006.01

$$\mathfrak{Q}_\nu^\mu(z) = -\frac{1}{\sqrt{\pi}} \left(\left(2^{-\nu-1} e^{i\pi\mu}\right) \cos(\pi\nu)\, \Gamma\!\left(-\nu-\frac{1}{2}\right) \Gamma(\mu+\nu+1)\,(z-1)^{-\nu-1} \left(\frac{z+1}{z-1}\right)^{\mu/2} {}_2F_1\!\left(\nu+1, \mu+\nu+1; 2(\nu+1); \frac{2}{1-z}\right)\right) /; z \notin (-1, 1)$$

07.12.26.0086.01

$$\mathfrak{Q}_\nu^\mu(z) = 2^{\mu-1} e^{i\pi\mu} \sqrt{\pi}\, (z-1)^{\mu/2} \left(-\frac{1}{z^2}\right)^{-\frac{1}{2}(\mu+\nu)} z^{1-\mu-\nu} (z+1)^{\mu/2} \left(\frac{2\,\Gamma\!\left(\frac{1}{2}(\mu+\nu+2)\right)}{\Gamma\!\left(\frac{1}{2}(-\mu+\nu+1)\right)} \, {}_2F_1\!\left(\frac{1}{2}(\mu-\nu+1), \frac{\mu+\nu}{2}+1; \frac{3}{2}; z^2\right) - \sqrt{-\frac{1}{z^2}} \, \frac{\Gamma\!\left(\frac{1}{2}(\mu+\nu+1)\right)}{\Gamma\!\left(\frac{1}{2}(-\mu+\nu+2)\right)} \, {}_2F_1\!\left(\frac{\mu-\nu}{2}, \frac{1}{2}(\mu+\nu+1); \frac{1}{2}; z^2\right)\right) /; z \notin (-\infty, 0) \bigwedge z \notin (1, \infty)$$

07.12.26.0087.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{2^{-\nu-1} e^{i\pi\mu} \sqrt{\pi}}{\Gamma\!\left(\nu+\frac{3}{2}\right)} \Gamma(\mu+\nu+1)\, z^{-\mu-\nu-1} (z-1)^{\mu/2} (z+1)^{\mu/2} \, {}_2F_1\!\left(\frac{1}{2}(\mu+\nu+1), \frac{\mu+\nu}{2}+1; \nu+\frac{3}{2}; \frac{1}{z^2}\right) /; z \notin (-1, 0)$$

07.12.26.0088.01

$$\mathfrak{Q}_\nu^\mu(z) = e^{\frac{1}{2} i\pi(\mu-\nu)} 2^{\mu-1} \sqrt{\pi}\, (z+1)^{\mu/2} (z-1)^{\mu/2} \left(\frac{2z\,\Gamma\!\left(\frac{\mu+\nu}{2}+1\right)}{\Gamma\!\left(\frac{1-\mu+\nu}{2}\right)} \, {}_2F_1\!\left(\frac{\mu-\nu+1}{2}, \frac{\mu+\nu+2}{2}; \frac{3}{2}; z^2\right) - \frac{i\,\Gamma\!\left(\frac{\mu+\nu+1}{2}\right)}{\Gamma\!\left(\frac{\nu-\mu}{2}+1\right)} \, {}_2F_1\!\left(\frac{\mu-\nu}{2}, \frac{\mu+\nu+1}{2}; \frac{1}{2}; z^2\right)\right) /; z \notin (1, \infty)$$

Through Meijer *G*

Classical cases for the direct function itself

07.12.26.0007.01

$$\mathfrak{Q}_\nu^\mu(z) = -\frac{\sin(\pi\nu)\csc(\pi\mu)}{2} e^{\pi i\mu} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} G_{2,2}^{1,2}\!\left(\frac{z-1}{2} \,\middle|\, \begin{matrix}\nu+1, -\nu \\ 0, \mu\end{matrix}\right) - \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}} (\nu-\mu+1)_{2\mu}\, G_{2,2}^{1,2}\!\left(\frac{z-1}{2} \,\middle|\, \begin{matrix}\nu+1, -\nu \\ 0, -\mu\end{matrix}\right)\right) /; \nu \notin \mathbb{Z} \bigwedge \mu \notin \mathbb{Z}$$

07.12.26.0008.01

$$\mathfrak{Q}_\nu^\mu(z) = -\frac{\sin(\pi\nu)\csc(\pi\mu)}{2} e^{\pi i\mu} \left(\frac{(z+1)^{\mu/2}}{z^{\mu/2}} G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix}\nu+1, -\nu \\ 0, \mu\end{matrix}\right) - \frac{z^{\mu/2}}{(z+1)^{\mu/2}} (\nu-\mu+1)_{2\mu}\, G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix}\nu+1, -\nu \\ 0, -\mu\end{matrix}\right)\right) /; \nu \notin \mathbb{Z} \bigwedge \mu \notin \mathbb{Z}$$

Classical cases involving algebraic functions

07.12.26.0009.01

$$(z+1)^{\mu/2} \mathfrak{Q}_\nu^\mu(2z+1) = \frac{e^{\pi\mu i}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(1-\mu+\nu)} G_{2,2}^{2,1}\!\left(z \,\middle|\, \begin{matrix}\frac{\mu}{2}-\nu, \frac{\mu}{2}+\nu+1 \\ \frac{\mu}{2}, -\frac{\mu}{2}\end{matrix}\right) /; z \notin (-1, 0)$$

07.12.26.0010.01

$$(z+1)^{-\frac{\mu}{2}}\, \mathfrak{Q}_\nu^\mu(2z+1) = \frac{1}{2} e^{\pi\mu i}\, G_{2,2}^{2,1}\!\left(z \,\middle|\, \begin{matrix} -\frac{\mu}{2}-\nu,\, 1-\frac{\mu}{2}+\nu \\ \frac{\mu}{2},\, -\frac{\mu}{2} \end{matrix}\right) /;\, z \notin (-1,\,0)$$

07.12.26.0011.01

$$(z+1)^{\mu/2}\, \mathfrak{Q}_\nu^\mu\!\left(1+\frac{2}{z}\right) = \frac{e^{\pi\mu i}\, \Gamma(\mu+\nu+1)}{2\,\Gamma(1-\mu+\nu)}\, G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix} 1,\, \mu+1 \\ \nu+1,\, -\nu \end{matrix}\right) /;\, z \notin (-1,\,0)$$

07.12.26.0012.01

$$(z+1)^{-\frac{\mu}{2}}\, \mathfrak{Q}_\nu^\mu\!\left(1+\frac{2}{z}\right) = \frac{1}{2} e^{\pi\mu i}\, G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix} 1-\mu,\, 1 \\ \nu+1,\, -\nu \end{matrix}\right) /;\, z \notin (-1,\,0)$$

07.12.26.0013.01

$$\mathfrak{Q}_\nu^\mu\!\left(\sqrt{z+1}\right) = \frac{2^{\mu-1}\, e^{\pi i\mu}\, \Gamma\!\left(\frac{\mu+\nu}{2}+1\right)}{\Gamma\!\left(\frac{1+\nu-\mu}{2}\right)}\, G_{2,2}^{2,1}\!\left(z \,\middle|\, \begin{matrix} \frac{1-\nu}{2},\, \frac{\nu}{2}+1 \\ \frac{\mu}{2},\, -\frac{\mu}{2} \end{matrix}\right) /;\, z \notin (-1,\,0)$$

07.12.26.0014.01

$$\mathfrak{Q}_\nu^\mu\!\left(\sqrt{\frac{z+1}{z}}\right) = \frac{2^{\mu-1}\, e^{\pi i\mu}\, \Gamma\!\left(\frac{\mu+\nu}{2}+1\right)}{\Gamma\!\left(\frac{1+\nu-\mu}{2}\right)}\, G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix} 1-\frac{\mu}{2},\, \frac{\mu}{2}+1 \\ \frac{\nu+1}{2},\, -\frac{\nu}{2} \end{matrix}\right) /;\, z \notin (-1,\,0)$$

07.12.26.0015.01

$$\frac{1}{\sqrt{z+1}}\, \mathfrak{Q}_\nu^\mu\!\left(\sqrt{z+1}\right) = \frac{2^{\mu-1}\, e^{\pi i\mu}\, \Gamma\!\left(\frac{1}{2}(\mu+\nu+1)\right)}{\Gamma\!\left(\frac{\nu-\mu}{2}+1\right)}\, G_{2,2}^{2,1}\!\left(z \,\middle|\, \begin{matrix} -\frac{\nu}{2},\, \frac{\nu+1}{2} \\ \frac{\mu}{2},\, -\frac{\mu}{2} \end{matrix}\right) /;\, z \notin (-1,\,0)$$

07.12.26.0016.01

$$\frac{1}{\sqrt{z+1}}\, \mathfrak{Q}_\nu^\mu\!\left(\sqrt{\frac{z+1}{z}}\right) = \frac{2^{\mu-1}\, e^{\pi i\mu}\, \Gamma\!\left(\frac{1}{2}(\mu+\nu+1)\right)}{\Gamma\!\left(\frac{\nu-\mu}{2}+1\right)}\, G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix} \frac{1-\mu}{2},\, \frac{\mu+1}{2} \\ \frac{\nu+1}{2},\, -\frac{\nu}{2} \end{matrix}\right) /;\, z \notin (-1,\,0)$$

07.12.26.0017.01

$$(z+1)^{\nu/2}\, \mathfrak{Q}_\nu^\mu\!\left(\frac{z+2}{2\sqrt{z+1}}\right) = \frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\sqrt{\pi}}\, G_{2,2}^{2,1}\!\left(z \,\middle|\, \begin{matrix} \frac{1}{2},\, \nu+1 \\ \mu,\, -\mu \end{matrix}\right) /;\, \mathrm{Re}(z) > 0$$

07.12.26.0018.01

$$(z+1)^{\nu/2}\, \mathfrak{Q}_\nu^\mu\!\left(\frac{2z+1}{2\sqrt{z}\,\sqrt{z+1}}\right) = \frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\sqrt{\pi}}\, G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix} 1-\mu+\frac{\nu}{2},\, \mu+\frac{\nu}{2}+1 \\ \frac{1+\nu}{2},\, -\frac{\nu}{2} \end{matrix}\right) /;\, \mathrm{Re}(z) > 0$$

07.12.26.0019.01

$$(z+1)^{-\frac{\nu+1}{2}}\, \mathfrak{Q}_\nu^\mu\!\left(\frac{z+2}{2\sqrt{z+1}}\right) = \frac{e^{\pi i\mu}\sqrt{\pi}}{\Gamma(1-\mu+\nu)}\, G_{2,2}^{2,1}\!\left(z \,\middle|\, \begin{matrix} -\nu,\, \frac{1}{2} \\ \mu,\, -\mu \end{matrix}\right) /;\, \mathrm{Re}(z) > 0$$

07.12.26.0020.01

$$(z+1)^{-\frac{\nu+1}{2}}\, \mathfrak{Q}_\nu^\mu\!\left(\frac{2z+1}{2\sqrt{z}\,\sqrt{z+1}}\right) = \frac{e^{i\pi\mu}\sqrt{\pi}}{\Gamma(1-\mu+\nu)}\, G_{2,2}^{1,2}\!\left(z \,\middle|\, \begin{matrix} \frac{1-\nu}{2}-\mu,\, \mu+\frac{1-\nu}{2} \\ \frac{\nu+1}{2},\, -\frac{\nu}{2} \end{matrix}\right) /;\, \mathrm{Re}(z) > 0$$

Classical cases involving unit step θ

07.12.26.0021.01

$$\theta(1-|z|)\,(1-z)^{\nu}\,\mathfrak{Q}_{\nu}^{\mu}\!\left(\frac{1+z}{1-z}\right)=\frac{1}{2}\,e^{\pi\mu i}\,\Gamma(\nu+1)\,\Gamma(\mu+\nu+1)\,G_{2,2}^{2,0}\!\left(z\;\middle|\;\begin{matrix}\frac{\mu}{2}+\nu+1,\,1-\frac{\mu}{2}+\nu\\ \frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)$$

07.12.26.0022.01

$$\theta(|z|-1)\,(z-1)^{\nu}\,\mathfrak{Q}_{\nu}^{\mu}\!\left(\frac{z+1}{z-1}\right)=\frac{1}{2}\,e^{\pi\mu i}\,\Gamma(\nu+1)\,\Gamma(\mu+\nu+1)\,G_{2,2}^{0,2}\!\left(z\;\middle|\;\begin{matrix}-\frac{\mu}{2}+\nu+1,\,\frac{\mu}{2}+\nu+1\\ -\frac{\mu}{2},\,\frac{\mu}{2}\end{matrix}\right)$$

07.12.26.0023.01

$$\theta(1-|z|)\,(1-z)^{\nu/2}\,\mathfrak{Q}_{\nu}^{\mu}\!\left(\frac{1}{\sqrt{1-z}}\right)=\frac{e^{\pi i\mu}\sqrt{\pi}\;\Gamma(\mu+\nu+1)}{2^{\nu+1}}\,G_{2,2}^{2,0}\!\left(z\;\middle|\;\begin{matrix}\frac{\nu}{2}+1,\,\frac{\nu+1}{2}\\ \frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)$$

07.12.26.0024.01

$$\theta(|z|-1)\,(z-1)^{\nu/2}\,\mathfrak{Q}_{\nu}^{\mu}\!\left(\sqrt{\frac{z}{z-1}}\right)=\frac{e^{\pi i\mu}\sqrt{\pi}\;\Gamma(\mu+\nu+1)}{2^{\nu+1}}\,G_{2,2}^{0,2}\!\left(z\;\middle|\;\begin{matrix}\frac{\nu-\mu}{2}+1,\,\frac{\mu+\nu}{2}+1\\ 0,\,\frac{1}{2}\end{matrix}\right)$$

07.12.26.0025.01

$$\theta(1-|z|)\,(1-z)^{\nu/2}\,\mathfrak{Q}_{\nu}^{\mu}\!\left(\frac{2-z}{2\sqrt{1-z}}\right)=e^{\pi i\mu}\sqrt{\pi}\;\Gamma(\mu+\nu+1)\,G_{2,2}^{2,0}\!\left(z\;\middle|\;\begin{matrix}\frac{1}{2},\,\nu+1\\ \mu,\,-\mu\end{matrix}\right)/;\ \mathrm{Re}(z)>0$$

07.12.26.0026.01

$$\theta(|z|-1)\,(z-1)^{\nu/2}\,\mathfrak{Q}_{\nu}^{\mu}\!\left(\frac{2z-1}{2\sqrt{z}\,\sqrt{z-1}}\right)=e^{\pi i\mu}\sqrt{\pi}\;\Gamma(\mu+\nu+1)\,G_{2,2}^{0,2}\!\left(z\;\middle|\;\begin{matrix}1-\mu+\frac{\nu}{2},\,\mu+\frac{\nu}{2}+1\\ \frac{\nu+1}{2},\,-\frac{\nu}{2}\end{matrix}\right)/;\ \mathrm{Re}(z)>0$$

Classical cases involving sgn

07.12.26.0027.01

$$(\mathrm{sgn}(|z|-1)\,(z-1))^{\nu}\,\mathfrak{Q}_{\nu}^{\mu}\!\left(\frac{z+1}{\mathrm{sgn}(|z|-1)\,(z-1)}\right)=$$
$$\frac{1}{2}\,e^{\pi\mu i}\,\Gamma(\nu+1)\,\Gamma(\mu+\nu+1)\left(G_{2,2}^{2,0}\!\left(z\;\middle|\;\begin{matrix}\frac{\mu}{2}+\nu+1,\,1-\frac{\mu}{2}+\nu\\ \frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)+G_{2,2}^{0,2}\!\left(z\;\middle|\;\begin{matrix}\frac{\mu}{2}+\nu+1,\,1-\frac{\mu}{2}+\nu\\ \frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)\right)$$

07.12.26.0028.01

$$(\mathrm{sgn}(|z|-1)\,(z-1))^{\nu}\,\mathfrak{Q}_{\nu}^{\mu}\!\left(\frac{z+1}{\mathrm{sgn}(|z|-1)\,(z-1)}\right)=$$
$$\frac{1}{2}\,e^{\pi\mu i}\,\Gamma(\nu+1)\,\Gamma(\mu+\nu+1)\left(G_{2,2}^{2,0}\!\left(z\;\middle|\;\begin{matrix}\frac{\mu}{2}+\nu+1,\,1-\frac{\mu}{2}+\nu\\ \frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)+G_{2,2}^{0,2}\!\left(z\;\middle|\;\begin{matrix}\frac{\mu}{2}+\nu+1,\,1-\frac{\mu}{2}+\nu\\ \frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)\right)/;\ z\notin(-1,\,0)$$

07.12.26.0029.01

$$(\mathrm{sgn}(1-|z|)\,(1-z))^{-\mu}\,\mathfrak{Q}_{\nu}^{\mu}\!\left(\frac{z+1}{2\sqrt{z}}\right)=e^{\pi i\mu}\sqrt{\pi}\;\Gamma\!\left(\frac{1}{2}-\mu\right)G_{2,2}^{1,1}\!\left(z\;\middle|\;\begin{matrix}\frac{1-\nu}{2}-\mu,\,-\mu+\frac{\nu}{2}+1\\ \frac{\nu+1}{2},\,-\frac{\nu}{2}\end{matrix}\right)/;\ z\notin(-1,\,0)$$

Classical cases for powers of Legendre Q

07.12.26.0030.01

$$\mathfrak{Q}_{\nu}^{\mu}\!\left(\sqrt{z+1}\right)^{2}=\frac{\sqrt{\pi}\;e^{2\pi i\mu}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\!\left(z\;\middle|\;\begin{matrix}-\nu,\,\frac{1}{2},\,\nu+1\\ 0,\,\mu,\,-\mu\end{matrix}\right)/;\ z\notin(-1,\,0)$$

07.12.26.0031.01

$$\mathfrak{Q}_\nu^\mu\left(\sqrt{\frac{z+1}{z}}\right)^2 = \frac{\sqrt{\pi}\, e^{2\pi i\mu}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(1-\mu+\nu)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} 1,\, 1-\mu,\, \mu+1 \\ \nu+1,\, \frac{1}{2},\, -\nu \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

Classical cases involving products of Legendre *Q*

07.12.26.0032.01

$$\mathfrak{Q}_{-\nu-1}^\mu\left(\sqrt{z+1}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{z+1}\right) = \frac{e^{2\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)\,\Gamma(\mu-\nu)}{2\sqrt{\pi}} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{2},\, -\nu,\, \nu+1 \\ 0,\, \mu,\, -\mu \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

07.12.26.0033.01

$$\mathfrak{Q}_{-\nu-1}^\mu\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{\frac{z+1}{z}}\right) = \frac{e^{2\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)\,\Gamma(\mu-\nu)}{2\sqrt{\pi}} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} 1,\, 1-\mu,\, \mu+1 \\ \frac{1}{2},\, -\nu,\, \nu+1 \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

07.12.26.0034.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{Q}_{\nu+1}^\mu\left(\sqrt{z+1}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{z+1}\right) = \frac{e^{2\pi i\mu}\sqrt{\pi}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(-\mu+\nu+2)} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} -\nu-1,\, \frac{1}{2},\, \nu+1 \\ 0,\, \mu,\, -\mu \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

07.12.26.0035.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{Q}_{\nu+1}^\mu\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{\frac{z+1}{z}}\right) = \frac{e^{2\pi i\mu}\sqrt{\pi}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(2-\mu+\nu)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} \frac{1}{2},\, \frac{1}{2}-\mu,\, \mu+\frac{1}{2} \\ \nu+\frac{3}{2},\, 0,\, -\nu-\frac{1}{2} \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

07.12.26.0036.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{Q}_\nu^\mu\left(\sqrt{z+1}\right)\mathfrak{Q}_\nu^{\mu+1}\left(\sqrt{z+1}\right) = -\frac{e^{2\pi i\mu}\sqrt{\pi}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(1-\mu+\nu)} G_{3,3}^{3,1}\left(z \,\middle|\, \begin{matrix} -\nu-\frac{1}{2},\, 0,\, \nu+\frac{1}{2} \\ -\frac{1}{2},\, -\mu-\frac{1}{2},\, \mu+\frac{1}{2} \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

07.12.26.0037.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{Q}_\nu^\mu\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{Q}_\nu^{\mu+1}\left(\sqrt{\frac{z+1}{z}}\right) = -\frac{e^{2\pi i\mu}\sqrt{\pi}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(1-\mu+\nu)} G_{3,3}^{1,3}\left(z \,\middle|\, \begin{matrix} 1,\, \mu+1,\, -\mu \\ \nu+1,\, \frac{1}{2},\, -\nu \end{matrix}\right) /;\, z \notin (-1,\, 0)$$

07.12.26.0038.01

$$\mathfrak{Q}_\nu^\mu\left(\sqrt{z+1}\right)\mathfrak{Q}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right) = \frac{e^{\pi i\left(\mu+\nu+\frac{1}{2}\right)}\,\Gamma(\mu+\nu+1)}{2\sqrt{2}} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{3}{4},\, \frac{1}{4}-\nu,\, \nu+\frac{5}{4} \\ \frac{1}{4},\, \mu+\frac{1}{4},\, \frac{1}{4}-\mu \end{matrix}\right) /;\, z \notin (-\infty,\, -1)$$

07.12.26.0039.01

$$\mathfrak{Q}_\nu^\mu\left(\sqrt{z+1}\right)\mathfrak{Q}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right) = \frac{e^{\pi i\left(\mu-\nu+\frac{1}{2}\right)}\,\pi\csc((\mu+\nu)\,\pi)}{2\sqrt{2}\,\Gamma(-\mu+\nu+1)} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{3}{4},\, \frac{1}{4}-\nu,\, \nu+\frac{5}{4} \\ \frac{1}{4},\, \frac{1}{4}-\mu,\, \mu+\frac{1}{4} \end{matrix}\right) /;\, z \notin (-\infty,\, -1)$$

Classical cases involving Legendre *P*

07.12.26.0040.01

$$\mathfrak{P}_\nu^\mu\left(\sqrt{z+1}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{z+1}\right) = \frac{e^{\pi i\mu}\,\Gamma(\mu+\nu+1)}{2\sqrt{\pi}\,\Gamma(1-\mu+\nu)} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2},\, -\nu,\, \nu+1 \\ 0,\, -\mu,\, \mu \end{matrix}\right) /;\, z \notin (-\infty,\, -1)$$

07.12.26.0041.01

$$\mathfrak{P}_\nu^\mu\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{\frac{z+1}{z}}\right) = \frac{e^{\pi i\mu}\,\Gamma(\mu+\nu+1)}{2\sqrt{\pi}\,\Gamma(1-\mu+\nu)} G_{3,3}^{2,2}\left(z \,\middle|\, \begin{matrix} 1,\, \mu+1,\, 1-\mu \\ \frac{1}{2},\, \nu+1,\, -\nu \end{matrix}\right) /;\, z \notin (-\infty,\, -1)$$

07.12.26.0042.01

$$\mathfrak{P}_{\nu}^{-\mu}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{2\sqrt{\pi}}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\,-\nu,\,\nu+1\\0,\,\mu,\,-\mu\end{matrix}\right)/;\,z\notin(-\infty,-1)$$

07.12.26.0043.01

$$\mathfrak{P}_{\nu}^{-\mu}\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{2\sqrt{\pi}}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}1,\,1-\mu,\,\mu+1\\\frac{1}{2},\,\nu+1,\,-\nu\end{matrix}\right)/;\,z\notin(-\infty,-1)$$

07.12.26.0044.01

$$\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{4}-\nu,\,\nu+\frac{5}{4},\,\frac{3}{4}\\\frac{1}{4},\,\mu+\frac{1}{4},\,\frac{1}{4}-\mu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.12.26.0045.01

$$\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}-\mu,\,\mu+\frac{3}{4}\\\nu+\frac{3}{4},\,-\nu-\frac{1}{4},\,\frac{1}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.12.26.0046.01

$$\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\pi\sqrt{2}}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,\nu+\frac{5}{4},\,\frac{1}{4}-\nu\\\frac{1}{4},\,\mu+\frac{1}{4},\,\frac{1}{4}-\mu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.12.26.0047.01

$$\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\pi\sqrt{2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}-\mu,\,\mu+\frac{3}{4}\\\frac{1}{4},\,-\nu-\frac{1}{4},\,\nu+\frac{3}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.12.26.0048.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\mu-\frac{1}{2}}^{-\nu-\frac{3}{2}}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(2-\mu+\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}-\nu-\frac{3}{4},\,\frac{3}{4},\,\nu+\frac{5}{4}\\\frac{1}{4},\,\mu+\frac{1}{4},\,\frac{1}{4}-\mu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.12.26.0049.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\mu-\frac{1}{2}}^{-\nu-\frac{3}{2}}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(2-\mu+\nu)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{1}{4},\,\frac{1}{4}-\mu,\,\mu+\frac{1}{4}\\\nu+\frac{5}{4},\,-\frac{1}{4},\,-\nu-\frac{3}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.12.26.0050.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\mu-\frac{1}{2}}^{\frac{1}{2}-\nu}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{4}-\nu,\,\frac{3}{4},\,\nu+\frac{1}{4}\\\frac{1}{4},\,\mu+\frac{1}{4},\,\frac{1}{4}-\mu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.12.26.0051.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\mu-\frac{1}{2}}^{\frac{1}{2}-\nu}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{1}{4},\,\frac{1}{4}-\mu,\,\mu+\frac{1}{4}\\\nu+\frac{1}{4},\,-\frac{1}{4},\,\frac{1}{4}-\nu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.12.26.0052.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\mu+\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}-\nu-\frac{1}{4},\,\frac{1}{4},\,\nu+\frac{3}{4}\\-\frac{1}{4},\,\mu+\frac{3}{4},\,-\mu-\frac{1}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.12.26.0053.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\mu+\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,-\mu-\frac{1}{4},\,\mu+\frac{3}{4}\\\nu+\frac{3}{4},\,\frac{1}{4},\,-\nu-\frac{1}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

Generalized cases involving algebraic functions

07.12.26.0054.01

$$\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{2^{\mu-1}\,e^{\pi i \mu}\,\Gamma\left(\frac{\mu+\nu}{2}+1\right)}{\Gamma\left(\frac{1-\mu+\nu}{2}\right)}\,G_{2,2}^{1,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{\mu}{2},\,\frac{\mu}{2}+1\\ \frac{\nu+1}{2},\,-\frac{\nu}{2}\end{matrix}\right) /;\, z\notin(-\infty,\,0)$$

07.12.26.0055.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{2^{\mu-1}\,e^{\pi i \mu}\,\Gamma\left(\frac{\mu+\nu+1}{2}\right)}{\Gamma\left(\frac{\nu-\mu}{2}+1\right)}\,G_{2,2}^{1,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\mu}{2},\,\frac{\mu+1}{2}\\ \frac{\nu+1}{2},\,-\frac{\nu}{2}\end{matrix}\right) /;\, z\notin(-\infty,\,0)$$

07.12.26.0056.01

$$\left(z^2+1\right)^{\nu/2}\,\mathfrak{Q}_\nu^\mu\left(\frac{2z^2+1}{2z\sqrt{z^2+1}}\right) = \frac{e^{\pi i \mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\sqrt{\pi}}\,G_{2,2}^{1,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1-\mu+\frac{\nu}{2},\,\mu+\frac{\nu}{2}+1\\ \frac{\nu+1}{2},\,-\frac{\nu}{2}\end{matrix}\right) /;\, \mathrm{Re}(z)>0$$

07.12.26.0057.01

$$\left(z^2+1\right)^{-\frac{\nu+1}{2}}\,\mathfrak{Q}_\nu^\mu\left(\frac{2z^2+1}{2z\sqrt{z^2+1}}\right) = \frac{e^{i\pi\mu}\sqrt{\pi}}{\Gamma(1-\mu+\nu)}\,G_{2,2}^{1,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}-\mu,\,\mu+\frac{1-\nu}{2}\\ \frac{\nu+1}{2},\,-\frac{\nu}{2}\end{matrix}\right) /;\, \mathrm{Re}(z)>0$$

Generalized cases involving unit step θ

07.12.26.0058.01

$$\theta(|z|-1)\left(z^2-1\right)^{\nu/2}\,\mathfrak{Q}_\nu^\mu\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{e^{\pi i \mu}\sqrt{\pi}\;\Gamma(\mu+\nu+1)}{2^{\nu+1}}\,G_{2,2}^{0,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu-\mu}{2}+1,\,\frac{\mu+\nu}{2}+1\\ 0,\,\frac{1}{2}\end{matrix}\right) /;\, \mathrm{Re}(z)>0$$

07.12.26.0059.01

$$\theta(|z|-1)\left(z^2-1\right)^{\nu/2}\,\mathfrak{Q}_\nu^\mu\left(\frac{2z^2-1}{2z\sqrt{z^2-1}}\right) = e^{\pi i \mu}\sqrt{\pi}\;\Gamma(\mu+\nu+1)\,G_{2,2}^{0,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}-\mu+\frac{\nu}{2}+1,\,\mu+\frac{\nu}{2}+1\\ \frac{\nu+1}{2},\,-\frac{\nu}{2}\end{matrix}\right) /;\, z>0$$

Generalized cases involving sgn

07.12.26.0060.01

$$\left(\mathrm{sgn}(1-|z|)\left(1-z^2\right)\right)^{-\mu}\,Q_\nu^\mu\left(\frac{z^2+1}{2z}\right) = e^{\pi i \mu}\sqrt{\pi}\;\Gamma\left(\frac{1}{2}-\mu\right)G_{2,2}^{1,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}-\mu,\,1-\mu+\frac{\nu}{2}\\ \frac{\nu+1}{2},\,-\frac{\nu}{2}\end{matrix}\right) /;\, \mathrm{Re}(z)>0$$

Generalized cases for powers of Legendre Q

07.12.26.0061.01

$$\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right)^2 = \frac{\sqrt{\pi}\;e^{2\pi i\mu}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1,\,1-\mu,\,\mu+1\\ \nu+1,\,\frac{1}{2},\,-\nu\end{matrix}\right) /;\, z\notin(-\infty,\,0)$$

Generalized cases involving products of Legendre Q

07.12.26.0062.01

$$\mathfrak{Q}_{-\nu-1}^\mu\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{2\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)\,\Gamma(\mu-\nu)}{2\sqrt{\pi}}\,G_{3,3}^{1,3}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1,\,1-\mu,\,\mu+1\\ \frac{1}{2},\,-\nu,\,\nu+1\end{matrix}\right)$$

07.12.26.0063.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{Q}_{\nu+1}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{2\pi i\mu}\sqrt{\pi}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(2-\mu+\nu)}\,G_{3,3}^{1,3}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\,\frac{1}{2}-\mu,\,\mu+\frac{1}{2}\\ \nu+\frac{3}{2},\,0,\,-\nu-\frac{1}{2}\end{matrix}\right)\,/;\,z\notin(-\infty,0)$$

07.12.26.0064.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{Q}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_{\nu}^{\mu+1}\left(\frac{\sqrt{z^2+1}}{z}\right) = -\frac{e^{2\pi i\mu}\sqrt{\pi}\,\Gamma(\mu+\nu+1)}{2\,\Gamma(-\mu+\nu+1)}\,G_{3,3}^{1,3}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1,\,\mu+1,\,-\mu\\ \nu+1,\,\frac{1}{2},\,-\nu\end{matrix}\right)\,/;\,z\notin(-\infty,0)$$

07.12.26.0065.01

$$\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z^2+1}\right)\mathfrak{Q}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\sqrt{1+\frac{1}{z^2}}\right) = \frac{e^{\pi i\left(\mu+\nu+\frac{1}{2}\right)}\,\Gamma(\mu+\nu+1)}{2\sqrt{2}}\,G_{3,3}^{2,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{1}{4}-\nu,\,\nu+\frac{5}{4}\\ \frac{1}{4},\,\mu+\frac{1}{4},\,\frac{1}{4}-\mu\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.12.26.0066.01

$$\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z^2+1}\right)\mathfrak{Q}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{1+\frac{1}{z^2}}\right) = \frac{e^{\pi i\left(\mu-\nu+\frac{1}{2}\right)}\,\pi\csc((\mu+\nu)\pi)}{2\sqrt{2}\,\Gamma(-\mu+\nu+1)}\,G_{3,3}^{2,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{1}{4}-\nu,\,\nu+\frac{5}{4}\\ \frac{1}{4},\,\frac{1}{4}-\mu,\,\mu+\frac{1}{4}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

Generalized cases involving Legendre *P*

07.12.26.0067.01

$$\mathfrak{P}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i\mu}\,\Gamma(\mu+\nu+1)}{2\sqrt{\pi}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{2,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1,\,\mu+1,\,1-\mu\\ \frac{1}{2},\,\nu+1,\,-\nu\end{matrix}\right)$$

07.12.26.0068.01

$$\mathfrak{P}_{\nu}^{-\mu}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i\mu}}{2\sqrt{\pi}}\,G_{3,3}^{2,2}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}1,\,1-\mu,\,\mu+1\\ \frac{1}{2},\,\nu+1,\,-\nu\end{matrix}\right)$$

07.12.26.0069.01

$$\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z^2+1}\right) = \frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}-\nu,\,\nu+\frac{5}{4},\,\frac{3}{4}\\ \frac{1}{4},\,\mu+\frac{1}{4},\,\frac{1}{4}-\mu\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.12.26.0070.01

$$\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{z^2+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}-\mu,\,\mu+\frac{3}{4}\\ \nu+\frac{3}{4},\,-\nu-\frac{1}{4},\,\frac{1}{4}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.12.26.0071.01

$$\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z^2+1}\right) = \frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\pi\sqrt{2}}\,G_{3,3}^{3,1}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\,\nu+\frac{5}{4},\,\frac{1}{4}-\nu\\ \frac{1}{4},\,\mu+\frac{1}{4},\,\frac{1}{4}-\mu\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.12.26.0072.01

$$\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\sqrt{z^2+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\pi\sqrt{2}}\,G_{3,3}^{1,3}\left(z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}-\mu,\,\mu+\frac{3}{4}\\ \frac{1}{4},\,-\nu-\frac{1}{4},\,\nu+\frac{3}{4}\end{matrix}\right)\,/;\,\mathrm{Re}(z)>0$$

07.12.26.0073.01

$$\frac{1}{\sqrt{z^2+1}}\, \mathfrak{P}_{\mu-\frac{1}{2}}^{-\nu-\frac{3}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right) \mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z^2+1}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(2-\mu+\nu)}\, G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} -\nu-\frac{3}{4}, \frac{3}{4}, \nu+\frac{5}{4} \\ \frac{1}{4}, \mu+\frac{1}{4}, \frac{1}{4}-\mu \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.12.26.0074.01

$$\frac{1}{\sqrt{z^2+1}}\, \mathfrak{P}_{\mu-\frac{1}{2}}^{-\nu-\frac{3}{2}}\left(\sqrt{z^2+1}\right) \mathfrak{Q}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(2-\mu+\nu)}\, G_{3,3}^{1,3}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{1}{4}, \frac{1}{4}-\mu, \mu+\frac{1}{4} \\ \nu+\frac{5}{4}, -\frac{1}{4}, -\nu-\frac{3}{4} \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.12.26.0075.01

$$\frac{1}{\sqrt{z^2+1}}\, \mathfrak{P}_{\mu-\frac{1}{2}}^{\frac{1}{2}-\nu}\left(\frac{\sqrt{z^2+1}}{z}\right) \mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z^2+1}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\, G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{1}{4}-\nu, \frac{3}{4}, \nu+\frac{1}{4} \\ \frac{1}{4}, \mu+\frac{1}{4}, \frac{1}{4}-\mu \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.12.26.0076.01

$$\frac{1}{\sqrt{z^2+1}}\, \mathfrak{P}_{\mu-\frac{1}{2}}^{\frac{1}{2}-\nu}\left(\sqrt{z^2+1}\right) \mathfrak{Q}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\, G_{3,3}^{1,3}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{1}{4}, \frac{1}{4}-\mu, \mu+\frac{1}{4} \\ \nu+\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}-\nu \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.12.26.0077.01

$$\frac{1}{\sqrt{z^2+1}}\, \mathfrak{P}_{\mu+\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right) \mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z^2+1}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(1-\mu+\nu)}\, G_{3,3}^{3,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} -\nu-\frac{1}{4}, \frac{1}{4}, \nu+\frac{3}{4} \\ -\frac{1}{4}, \mu+\frac{3}{4}, -\mu-\frac{1}{4} \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.12.26.0078.01

$$\frac{1}{\sqrt{z^2+1}}\, \mathfrak{P}_{\mu+\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{z^2+1}\right) \mathfrak{Q}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(1-\mu+\nu)}\, G_{3,3}^{1,3}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{3}{4}, -\mu-\frac{1}{4}, \mu+\frac{3}{4} \\ \nu+\frac{3}{4}, \frac{1}{4}, -\nu-\frac{1}{4} \end{array}\right) /; \operatorname{Re}(z) > 0$$

Through other functions

Involving some hypergeometric-type functions

07.12.26.0079.01

$$\mathfrak{Q}_{\nu}^{\mu}(z) = \frac{e^{i\pi\mu}\,\pi\csc(\pi\mu)\,\Gamma(\nu+1)}{2\,\Gamma(\nu-\mu+1)} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\, P_{\nu}^{(-\mu,\mu)}(z) - \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\, P_{\nu}^{(\mu,-\mu)}(z)\right) /; \mu \notin \mathbb{Z}$$

07.12.26.0080.01

$$\mathfrak{Q}_{\nu}^{\mu}(z) = 2^{-1-\mu}\operatorname{Csc}[\pi\mu]\, e^{i\pi\mu}\sqrt{\pi}\left(\frac{\Gamma(\mu+\nu+1)\,\Gamma\left(\frac{1}{2}-\mu\right)}{\Gamma(\nu-\mu+1)}\,(z-1)^{-\mu/2}\,(z+1)^{-\mu/2}\, C_{\mu+\nu}^{\frac{1}{2}-\mu}(z) + \right.$$

$$\left. 4^{\mu}\,\csc(\pi(\mu+\nu))\sin(\pi(\mu-\nu))\,\Gamma\left(\mu+\frac{1}{2}\right)(z-1)^{\mu/2}\,(z+1)^{\mu/2}\, C_{-\mu-\nu-1}^{\mu+\frac{1}{2}}(z)\right) /; \mu \notin \mathbb{Z}$$

Involving spheroidal functions

07.12.26.0089.01

$$Q_{\nu}^{\mu}(z) = \frac{e^{i\mu\pi}\,(z-1)^{\mu/2}}{(1-z)^{\mu/2}}\left(QS_{\nu,\mu}(0,z) - \frac{1}{2}\,\pi\csc(\mu\pi)\left(\cos(\mu\pi) - \frac{(1-z)^{\mu}}{(z-1)^{\mu}}\right) PS_{\nu,\mu}(0,z)\right) /; \mu \notin \mathbb{Z}$$

07.12.26.0090.01

$$Q_\nu^m(z) = \frac{(-1)^m (z-1)^{m/2}}{(1-z)^{m/2}} \left(QS_{\nu,m}(0, z) - \frac{\pi \sqrt{z-1}}{2\sqrt{1-z}} PS_{\nu,m}(0, z) \right) /; m \in \mathbb{Z}$$

Representations through equivalent functions

With related functions

07.12.27.0001.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{\pi \csc(\mu\pi)}{2} e^{\pi i \mu} \left(\mathfrak{P}_\nu^\mu(z) - (\nu-\mu+1)_{2\mu} \mathfrak{P}_\nu^{-\mu}(z) \right) /; \mu \notin \mathbb{Z}$$

07.12.27.0010.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{\pi}{2} \csc(\mu\pi) e^{i\mu\pi} \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} \left(P_\nu^\mu(z) - (\nu-\mu+1)_{2\mu} \frac{(z-1)^\mu}{(1-z)^\mu} P_\nu^{-\mu}(z) \right) /; \mu \notin \mathbb{Z}$$

07.12.27.0011.01

$$\mathfrak{Q}_\nu^\mu(z) = -\frac{\pi}{2} \csc^2(\pi\mu) e^{\pi i \mu} \left(\left(\sin(\nu\pi) \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} + \sin(\pi(\mu-\nu)) \frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}} \right) \frac{(-z-1)^{\mu/2}}{(1+z)^{\mu/2}} \mathfrak{P}_\nu^\mu(-z) + \right.$$
$$\left. \frac{\pi}{\Gamma(-\mu-\nu)\Gamma(\nu-\mu+1)} \left(\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} - \csc((\mu+\nu)\pi) \sin(\nu\pi) \frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}} \right) \frac{(z+1)^{\mu/2}}{(-z-1)^{\mu/2}} \mathfrak{P}_\nu^{-\mu}(-z) \right) /; \mu \notin \mathbb{Z}$$

07.12.27.0012.01

$$\mathfrak{Q}_\nu^\mu(z) = -\frac{\pi}{2} \csc^2(\pi\mu) e^{\pi i \mu} \left(\left(\sin(\nu\pi) \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} + \sin(\pi(\mu-\nu)) \frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}} \right) P_\nu^\mu(-z) + \right.$$
$$\left. \frac{\pi}{\Gamma(-\mu-\nu)\Gamma(\nu-\mu+1)} \left(\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} - \csc((\mu+\nu)\pi) \sin(\nu\pi) \frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}} \right) P_\nu^{-\mu}(-z) \right) /; \mu \notin \mathbb{Z}$$

07.12.27.0002.01

$$\mathfrak{Q}_\nu^\mu(z) = e^{i\mu\pi} \frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}} \left(Q_\nu^\mu(z) - \frac{\pi}{2} \csc(\mu\pi) \left(\cos(\mu\pi) - \frac{(1-z)^\mu}{(z-1)^\mu} \right) P_\nu^\mu(z) \right) /; \mu \notin \text{Integers} /; \mu \notin \mathbb{Z}$$

07.12.27.0013.01

$$\mathfrak{Q}_\nu^m(z) = (-1)^m \frac{(z-1)^{m/2}}{(1-z)^{m/2}} \left(Q_\nu^m(z) - \frac{\pi}{2} \frac{\sqrt{z-1}}{\sqrt{1-z}} P_\nu^m(z) \right) /; m \in \mathbb{Z}$$

07.12.27.0003.01

$$\mathfrak{Q}_\nu^\mu(z) = e^{i\mu\pi} \left(\frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}} Q_\nu^\mu(z) - \frac{\pi}{2} \csc(\mu\pi) \left(\frac{(z-1)^\mu}{(1-z)^\mu} \cos(\mu\pi) - 1 \right) \mathfrak{P}_\nu^\mu(z) \right) /; \mu \notin \mathbb{Z}$$

07.12.27.0014.01

$$\mathfrak{Q}_\nu^m(z) = (-1)^m \frac{(z-1)^{m/2}}{(1-z)^{m/2}} Q_\nu^m(z) - \frac{\pi}{2} \frac{\sqrt{z-1}}{\sqrt{1-z}} \mathfrak{P}_\nu^m(z) /; m \in \mathbb{Z}$$

07.12.27.0005.01

$$\mathfrak{Q}_\nu^\mu(z) = \sqrt{\frac{\pi}{2}} \frac{\Gamma(\mu+\nu+1) e^{\pi i \mu}}{\sqrt[4]{z-1}\sqrt[4]{z+1}} \mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\frac{z}{\sqrt{z-1}\sqrt{z+1}} \right) /; \text{Re}(z) > 0 \land z \notin (0, 1)$$

07.12.27.0006.01

$$\mathfrak{Q}_\nu^\mu(z) = \sqrt{\frac{\pi}{2}}\,\Gamma(\mu+\nu+1)\,e^{\pi i \mu}\,\frac{1}{\sqrt[4]{z^2-1}}\,\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\frac{z}{\sqrt{z^2-1}}\right) /;\ \mathrm{Re}(z) > 0 \bigwedge z \notin (0,1) \bigvee -i\,z \in (0,\infty)$$

07.12.27.0007.01

$$\mathfrak{Q}_\nu^\mu(\cosh(z)) = \sqrt{\frac{\pi}{2}}\,\Gamma(\mu+\nu+1)\,e^{\pi i \mu}\,\frac{1}{\sinh^{\frac{1}{2}}(z)}\,\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}(\coth(z)) /;\ \mathrm{Re}(z) > 0$$

07.12.27.0008.01

$$\mathfrak{Q}_\nu^m(z) = (z-1)^{m/2}\,(z+1)^{m/2}\left(\frac{\partial^m Q_\nu(z)}{\partial z^m} - \frac{\pi}{2}\,\frac{\sqrt{z-1}}{\sqrt{1-z}}\,\frac{\partial^m P_\nu(z)}{\partial z^m}\right) /;\ m \in \mathbb{N}$$

07.12.27.0015.01

$$\mathfrak{Q}_\nu^m(z) = (z-1)^{m/2}\,(z+1)^{m/2}\,\frac{\partial^m Q_\nu(z)}{\partial z^m} - \frac{\pi}{2}\,\frac{\sqrt{z-1}}{\sqrt{1-z}}\,\mathfrak{P}_\nu^\mu(z) /;\ m \in \mathbb{N}$$

07.12.27.0009.01

$$\mathfrak{Q}_\nu^\mu(z) = \frac{e^{\pi i \mu}}{\Gamma(-\mu)}\left(z^2-1\right)^{\mu/2}\int_z^\infty \mathfrak{Q}_\nu^0(z)\,(t-z)^{-\mu-1}\,dt /;\ \mathrm{Re}(\mu) < 0 \bigwedge \mathrm{Re}(\mu+\nu) > -1 \bigwedge z \notin (-\infty, 1)$$

07.12.27.0016.01

$$\mathfrak{Q}_\nu^m(z) = \frac{\Gamma(\nu+m+1)}{\Gamma(\nu-m+1)}\,\frac{(z-1)^{m/2}}{(1-z)^{m/2}}\left(Q_\nu^{-m}(z) - \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}}\,P_\nu^{-m}(z)\right) /;\ m \in \mathbb{N}$$

Copyright