LegendreQ2General

Notations

Traditional name

Assosiated Legendre function of the second kind of type 2

Traditional notation

$Q_\nu^\mu(z)$

Mathematica StandardForm notation

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LegendreQ[ν, μ, 2, z]
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Primary definition

07.11.02.0001.01

$$Q_\nu^\mu(z) = \frac{\pi \csc(\mu \pi)}{2} \left(\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}} \cos(\mu \pi) \, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right) - \frac{\Gamma(\mu+\nu+1)}{\Gamma(-\mu+\nu+1)} \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; \mu+1; \frac{1-z}{2}\right)\right) /; \mu \notin \mathbb{Z}$$

The function LegendreQ[ν, μ, 2, z] is the analytic continuation of the function LegendreQ[ν, μ, z] from the unit disk $|z| < 1$ to the cut complex plane. Inside the unit disk they coincide.

For $\mu \in \mathbb{Z}$ the above definition becomes indeterminate, and the function LegendreQ[ν, μ, 2, z] is defined by taking a limit. Series expansions for this case can be found in the *Series representations* section.

07.11.02.0002.01

$$Q_\nu^\mu(z) = \lim_{\tilde{\mu} \to \mu} Q_\nu^{\tilde{\mu}}(z) /; \mu \in \mathbb{Z}$$

Specific values

Specialized values

For fixed ν, μ

07.11.03.0001.01

$$Q_\nu^\mu(0) = -\frac{2^{\mu-1} \pi^{3/2}}{\Gamma\left(\frac{1-\mu-\nu}{2}\right) \Gamma\left(\frac{\nu-\mu}{2}+1\right)} \tan\left(\frac{\pi(\mu+\nu)}{2}\right)$$

07.11.03.0002.01

$$Q_\nu^\mu(1) = ¿ /; \mu \notin \mathbb{Z}$$

07.11.03.0003.01

$$Q_\nu^\mu(1) = \tilde{\infty} \,/;\, \mu \in \mathbb{Z}$$

07.11.03.0004.01

$$Q_\nu^\mu(-1) = ¿ \,/;\, \mu \notin \mathbb{Z}$$

07.11.03.0005.01

$$Q_\nu^\mu(-1) = \tilde{\infty} \,/;\, \mu \in \mathbb{Z}$$

For fixed ν, z

07.11.03.0006.01

$$Q_\nu^0(z) = Q_\nu(z)$$

07.11.03.0007.01

$$Q_\nu^{\frac{1}{2}}(z) = -\sqrt{\frac{\pi}{2}}\, \sin\!\left(\left(\nu + \frac{1}{2}\right)\cos^{-1}(z)\right) \frac{1}{\left(1-z^2\right)^{1/4}}$$

07.11.03.0008.01

$$Q_\nu^{-n-\nu}(z) = \tilde{\infty} \,/;\, n \in \mathbb{N}^+$$

07.11.03.0009.01

$$Q_\nu^{-\nu}(z) = \frac{2^{-\nu-1}}{\Gamma(2\nu+1)} \left(1-z^2\right)^{\nu/2} \left(\cos(\pi\nu)\,\Gamma(-\nu)\,\Gamma(2\nu+1) - \Gamma(\nu+1)\, B_{\frac{1-z}{2}}(-\nu, -\nu)\right) /;\, \nu \notin \mathbb{Z}$$

07.11.03.0010.01

$$Q_\nu^{\nu}(z) = \frac{2^{-\nu-1}}{\Gamma(-\nu)} \left(1-z^2\right)^{\nu/2} \left(\pi\cot(\pi\nu)\; B_{\frac{1-z}{2}}(-\nu, -\nu) - 4^\nu \sqrt{\pi}\; \csc(\pi\nu)\,\Gamma(-\nu)\,\Gamma\!\left(\nu+\frac{1}{2}\right)\right) /;\, \nu \notin \mathbb{Z}$$

07.11.03.0011.01

$$Q_\nu^{\nu+1}(z) = \frac{2^\nu\,\pi\cot(\pi\nu)}{\Gamma(-\nu)} \left(1-z^2\right)^{-\frac{\nu+1}{2}}$$

For fixed μ, z

07.11.03.0012.01

$$Q_0^\mu(z) = \frac{\pi\csc(\pi\mu)}{2\,\Gamma(1-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left((z+1)^\mu\cos(\pi\mu) - (1-z)^\mu\right) /;\, \mu \notin \mathbb{Z}$$

07.11.03.0013.01

$$Q_0^m(z) = \frac{(m-1)!}{2} (1-z)^{m/2} (1+z)^{-m/2} \left((z-1)^{-m} (z+1)^m - 1\right) /;\, m \in \mathbb{N}^+$$

07.11.03.0014.01

$$Q_0^{-m}(z) = \tilde{\infty} \,/;\, m \in \mathbb{N}^+$$

07.11.03.0015.01

$$Q_1^\mu(z) = \frac{\pi\csc(\pi\mu)}{2\,\Gamma(2-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left((z+1)^\mu (z-\mu)\cos(\pi\mu) - (1-z)^\mu (z+\mu)\right)$$

07.11.03.0016.01

$$Q_2^\mu(z) = \frac{\pi\;\csc(\pi\mu)}{2\,\Gamma(3-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left((z+1)^\mu \left(3z^2 - 3\mu z + \mu^2 - 1\right)\cos(\pi\mu) - (1-z)^\mu \left(3z^2 + 3\mu z + \mu^2 - 1\right)\right)$$

07.11.03.0017.01

$$Q_3^{\mu}(z) = \frac{\pi \csc(\pi \mu)}{2\,\Gamma(4-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left((z+1)^{\mu} \left(15 z^3 - 15 \mu z^2 + \left(6 \mu^2 - 9\right) z - \mu^3 + 4 \mu\right) \cos(\pi \mu) - (1-z)^{\mu} \left(15 z^3 + 15 \mu z^2 + \left(6 \mu^2 - 9\right) z + \mu^3 - 4 \mu\right)\right)$$

07.11.03.0018.01

$$Q_4^{\mu}(z) = \frac{\pi \csc(\pi \mu)}{2\,\Gamma(5-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left((z+1)^{\mu} \left(105 z^4 - 105 \mu z^3 + 45 \left(\mu^2 - 2\right) z^2 + \left(55 \mu - 10 \mu^3\right) z + \mu^4 - 10 \mu^2 + 9\right) \cos(\pi \mu) - (1-z)^{\mu} \left(105 z^4 + 105 \mu z^3 + 45 \left(\mu^2 - 2\right) z^2 + 5 \mu \left(2 \mu^2 - 11\right) z + \mu^4 - 10 \mu^2 + 9\right)\right)$$

07.11.03.0019.01

$$Q_5^{\mu}(z) = \frac{\pi \csc(\pi \mu)}{2\,\Gamma(6-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left((z+1)^{\mu} \left(945 z^5 - 945 \mu z^4 + 210 \left(2 \mu^2 - 5\right) z^3 - 105 \mu \left(\mu^2 - 7\right) z^2 + 15 \left(\mu^4 - 13 \mu^2 + 15\right) z - \mu \left(\mu^4 - 20 \mu^2 + 64\right)\right) \cos(\pi \mu) - (1-z)^{\mu} \left(945 z^5 + 945 \mu z^4 + 210 \left(2 \mu^2 - 5\right) z^3 + 105 \mu \left(\mu^2 - 7\right) z^2 + 15 \left(\mu^4 - 13 \mu^2 + 15\right) z + \mu^5 - 20 \mu^3 + 64 \mu\right)\right)$$

07.11.03.0020.01

$$Q_6^{\mu}(z) = \frac{\pi \csc(\pi \mu)}{2\,\Gamma(7-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left((z+1)^{\mu} \left(10\,395 z^6 - 10\,395 \mu z^5 + 4725 \left(\mu^2 - 3\right) z^4 - 630 \mu \left(2 \mu^2 - 17\right) z^3 + 105 \left(2 \mu^4 - 32 \mu^2 + 45\right) z^2 - 21 \mu \left(\mu^4 - 25 \mu^2 + 99\right) z + (\mu-5)(\mu-3)(\mu-1)(\mu+1)(\mu+3)(\mu+5)\right) \cos(\pi \mu) - (1-z)^{\mu} \left(10\,395 z^6 + 10\,395 \mu z^5 + 4725 \left(\mu^2 - 3\right) z^4 + 630 \mu \left(2 \mu^2 - 17\right) z^3 + 105 \left(2 \mu^4 - 32 \mu^2 + 45\right) z^2 + 21 \mu \left(\mu^4 - 25 \mu^2 + 99\right) z + (\mu-5)(\mu-3)(\mu-1)(\mu+1)(\mu+3)(\mu+5)\right)\right)$$

07.11.03.0021.01

$$Q_7^{\mu}(z) = \frac{\pi \csc(\pi \mu)}{2\,\Gamma(8-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left((z+1)^{\mu} \left(135\,135 z^7 - 135\,135 \mu z^6 + 31\,185 \left(2 \mu^2 - 7\right) z^5 - 17\,325 \mu \left(\mu^2 - 10\right) z^4 + 1575 \left(2 \mu^4 - 38 \mu^2 + 63\right) z^3 - 189 \mu \left(2 \mu^4 - 60 \mu^2 + 283\right) z^2 + 7 \left(4 \mu^6 - 170 \mu^4 + 1516 \mu^2 - 1575\right) z - \mu^3 \left(\mu^2 - 28\right)^2 + 2304 \mu\right) \cos(\pi \mu) - (1-z)^{\mu} \left(135\,135 z^7 + 135\,135 \mu z^6 + 31\,185 \left(2 \mu^2 - 7\right) z^5 + 17\,325 \mu \left(\mu^2 - 10\right) z^4 + 1575 \left(2 \mu^4 - 38 \mu^2 + 63\right) z^3 + 189 \mu \left(2 \mu^4 - 60 \mu^2 + 283\right) z^2 + 7 \left(4 \mu^6 - 170 \mu^4 + 1516 \mu^2 - 1575\right) z + \mu^3 \left(\mu^2 - 28\right)^2 - 2304 \mu\right)\right)$$

07.11.03.0022.01

$$Q_8^{\mu}(z) = \frac{\pi \csc(\pi \mu)}{2\,\Gamma(9-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left((z+1)^{\mu} \left(2\,027\,025 z^8 - 2\,027\,025 \mu z^7 + 945\,945 \left(\mu^2 - 4\right) z^6 - 135\,135 \mu \left(2 \mu^2 - 23\right) z^5 + 51\,975 \left(\mu^4 - 22 \mu^2 + 42\right) z^4 - 3465 \mu \left(2 \mu^4 - 70 \mu^2 + 383\right) z^3 + 315 (\mu-6)(\mu+6) \left(2 \mu^4 - 28 \mu^2 + 35\right) z^2 - 9 \mu \left(4 \mu^6 - 266 \mu^4 + 4396 \mu^2 - 15\,159\right) z + (\mu-7)(\mu-5)(\mu-3)(\mu-1)(\mu+1)(\mu+3)(\mu+5)(\mu+7)\right) \cos(\pi \mu) - (1-z)^{\mu} \left(2\,027\,025 z^8 + 2\,027\,025 \mu z^7 + 945\,945 \left(\mu^2 - 4\right) z^6 + 135\,135 \mu \left(2 \mu^2 - 23\right) z^5 + 51\,975 \left(\mu^4 - 22 \mu^2 + 42\right) z^4 + 3465 \mu \left(2 \mu^4 - 70 \mu^2 + 383\right) z^3 + 315 (\mu-6)(\mu+6) \left(2 \mu^4 - 28 \mu^2 + 35\right) z^2 + 9 \mu \left(4 \mu^6 - 266 \mu^4 + 4396 \mu^2 - 15\,159\right) z + (\mu-7)(\mu-5)(\mu-3)(\mu-1)(\mu+1)(\mu+3)(\mu+5)(\mu+7)\right) \csc(\pi \mu)\right)$$

07.11.03.0023.01

$$Q_9^\mu(z) = \frac{\pi \csc(\pi\mu)}{2\,\Gamma(10-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \Big((z+1)^\mu \big(34\,459\,425\, z^9 - 34\,459\,425\, \mu\, z^8 + 8\,108\,100 \left(2\mu^2-9\right) z^7 - 4\,729\,725\, \mu \left(\mu^2-13\right) z^6 + 945\,945 \left(\mu^4 - 25\mu^2 + 54\right) z^5 - 135\,135\, \mu \left(\mu^4 - 40\mu^2 + 249\right) z^4 + 6930 \left(2\mu^6 - 115\mu^4 + 1373\mu^2 - 1890\right) z^3 - 495\,\mu \left(2\mu^6 - 154\mu^4 + 2933\mu^2 - 11\,601\right) z^2 + 45 \left(\mu^8 - 98\mu^6 + 2674\mu^4 - 20\,217\mu^2 + 19\,845\right) z - (\mu-8)(\mu-6)(\mu-4)(\mu-2)\,\mu\,(\mu+2)(\mu+4)(\mu+6)(\mu+8)\big) \cos(\pi\mu) - (1-z)^\mu \big(34\,459\,425\, z^9 + 34\,459\,425\, \mu\, z^8 + 8\,108\,100 \left(2\mu^2-9\right) z^7 + 4\,729\,725\, \mu \left(\mu^2-13\right) z^6 + 945\,945 \left(\mu^4 - 25\mu^2 + 54\right) z^5 + 135\,135\, \mu \left(\mu^4 - 40\mu^2 + 249\right) z^4 + 6930 \left(2\mu^6 - 115\mu^4 + 1373\mu^2 - 1890\right) z^3 + 495\,\mu \left(2\mu^6 - 154\mu^4 + 2933\mu^2 - 11\,601\right) z^2 + 45 \left(\mu^8 - 98\mu^6 + 2674\mu^4 - 20\,217\mu^2 + 19\,845\right) z + (\mu-8)(\mu-6)(\mu-4)(\mu-2)\,\mu\,(\mu+2)(\mu+4)(\mu+6)(\mu+8)\big)\Big)$$

07.11.03.0024.01

$$Q_{10}^\mu(z) = \frac{\pi \csc(\pi\mu)}{2\,\Gamma(11-\mu)} \left(1-z^2\right)^{-\frac{\mu}{2}} \Big((z+1)^\mu \big(654\,729\,075\, z^{10} - 654\,729\,075\, \mu\, z^9 + 310\,134\,825 \left(\mu^2-5\right) z^8 - 45\,945\,900\, \mu \left(2\mu^2-29\right) z^7 + 9\,459\,450 \left(2\mu^4 - 56\mu^2 + 135\right) z^6 - 2\,837\,835\, \mu \left(\mu^4 - 45\mu^2 + 314\right) z^5 + 315\,315 \left(\mu^6 - 65\mu^4 + 874\mu^2 - 1350\right) z^4 - 12\,870\, \mu \left((\mu-7)\,\mu^2\,(\mu+7)\left(2\mu^2-77\right) - 16\,830\right) z^3 + 1485 \left(\mu^8 - 112\mu^6 + 3479\mu^4 - 29\,828\mu^2 + 33\,075\right) z^2 - 55\,\mu \left(\mu^8 - 138\mu^6 + 5754\mu^4 - 78\,877\mu^2 + 251\,865\right) z + (\mu-9)(\mu-7)(\mu-5)(\mu-3)(\mu-1)(\mu+1)(\mu+3)(\mu+5)(\mu+7)(\mu+9)\big) \cos(\pi\mu) - (1-z)^\mu \big(654\,729\,075\, z^{10} + 654\,729\,075\, \mu\, z^9 + 310\,134\,825 \left(\mu^2-5\right) z^8 + 45\,945\,900\, \mu \left(2\mu^2-29\right) z^7 + 9\,459\,450 \left(2\mu^4 - 56\mu^2 + 135\right) z^6 + 2\,837\,835\, \mu \left(\mu^4 - 45\mu^2 + 314\right) z^5 + 315\,315 \left(\mu^6 - 65\mu^4 + 874\mu^2 - 1350\right) z^4 + 12\,870\, \mu \left((\mu-7)\,\mu^2\,(\mu+7)\left(2\mu^2-77\right) - 16\,830\right) z^3 + 1485 \left(\mu^8 - 112\mu^6 + 3479\mu^4 - 29\,828\mu^2 + 33\,075\right) z^2 + 55\,\mu \left(\mu^8 - 138\mu^6 + 5754\mu^4 - 78\,877\mu^2 + 251\,865\right) z + (\mu-9)(\mu-7)(\mu-5)(\mu-3)(\mu-1)(\mu+1)(\mu+3)(\mu+5)(\mu+7)(\mu+9)\big)\Big)$$

07.11.03.0025.01

$$Q_n^\mu(z) = \frac{\pi \csc(\mu\pi)}{2} \left(\frac{\cos(\mu\pi)}{\Gamma(1-\mu)} \frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}} \sum_{k=0}^{n} \frac{(-n)_k\,(n+1)_k}{(1-\mu)_k\,k!} \left(\frac{1-z}{2}\right)^k - \frac{(n-\mu+1)_{2\mu}}{\Gamma(\mu+1)} \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}} \sum_{k=0}^{n} \frac{(-n)_k\,(n+1)_k}{(\mu+1)_k\,k!} \left(\frac{1-z}{2}\right)^k \right) /;$$

$n \in \mathbb{N} \bigwedge \mu \notin \mathbb{N}$

07.11.03.0026.01

$$Q_{-n}^\mu(z) = \frac{\pi \csc(\mu\pi)}{2} \left(\frac{\cos(\mu\pi)}{\Gamma(1-\mu)} \frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}} \sum_{k=0}^{n-1} \frac{(n)_k\,(1-n)_k}{(1-\mu)_k\,k!} \left(\frac{1-z}{2}\right)^k - \frac{(1-n-\mu)_{2\mu}}{\Gamma(\mu+1)} \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}} \sum_{k=0}^{n-1} \frac{(n)_k\,(1-n)_k}{(\mu+1)_k\,k!} \left(\frac{1-z}{2}\right)^k \right) /;$$

$n \in \mathbb{N}^+ \bigwedge \mu \notin \mathbb{N}$

07.11.03.0027.01

$$Q_n^m(z) = 2^{-n-1} (-1)^{\frac{m+|m|}{2}} (z+1)^n (m+n)!\, n!\, \frac{(1-z)^{\frac{|m|}{2}}}{(1+z)^{\frac{|m|}{2}}} \left(\sum_{k=n-|m|+1}^{n} \frac{(-1)^{m+n-k-1} (k-n+|m|-1)!}{k!\,(k+|m|)!\,(n-k)!} \left(\frac{z-1}{z+1}\right)^k + \sum_{k=0}^{n-|m|} \frac{1}{k!\,(k+|m|)!\,(n-k)!\,(-k+n-|m|)!} (\log(1+z) - \log(1-z) - \psi(n-k+1) - \psi(n-|m|-k+1) + \psi(k+1) + \psi(k+|m|+1)) \left(\frac{z-1}{z+1}\right)^k + \sum_{k=0}^{|m|-1} \frac{(-1)^{m-k} (|m|-k-1)!}{k!\,(n-k)!\,(n+|m|-k)!} \left(\frac{z-1}{z+1}\right)^{k-|m|} \right) /;\ n \in \mathbb{N} \wedge m \in \mathbb{Z} \wedge |m| \leq n$$

07.11.03.0028.01

$$Q_n^m(z) = 2^{-n-1} (z+1)^n (m+n)!\, n!\, \frac{(1-z)^{\frac{m}{2}}}{(1+z)^{\frac{m}{2}}} \left(\sum_{k=0}^{m-1} \frac{(-1)^k (m-k-1)!}{k!\,(n-k)!\,(m+n-k)!} \left(\frac{z-1}{z+1}\right)^{k-m} - \sum_{k=0}^{n} \frac{(-1)^{n-k} (k+m-n-1)!}{k!\,(k+m)!\,(n-k)!} \left(\frac{z-1}{z+1}\right)^k \right) /;\ n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+ \bigwedge m > n$$

07.11.03.0029.01

$$Q_{-n}^m(z) = (m-n)!\, \frac{(1-z)^{\frac{m}{2}}}{(1+z)^{\frac{m}{2}}} \left((-1)^n (m+n-1)!\, \frac{(1+z)^{\frac{m}{2}}}{(z-1)^{\frac{m}{2}}} \mathrm{P}_{-n}^{-m}(z) + \frac{2^{n-1} (1+z)^{-n}}{(n-1)!} \sum_{k=0}^{m-n} \frac{(n+k-1)!\,(m-k-1)!}{k!\,(m-n-k)!} \left(\frac{z-1}{z+1}\right)^{k-m} \right) /;\ n \in \mathbb{N}^+ \bigwedge m \in \mathbb{N}^+ \bigwedge m \geq n$$

07.11.03.0030.01

$$Q_{-n-\mu}^{\mu}(z) = \tilde{\infty} /;\ n \in \mathbb{N}^+$$

For fixed *z*

07.11.03.0031.01

$$Q_0^0(z) = \frac{1}{2} (\log(1+z) - \log(1-z))$$

07.11.03.0032.01

$$Q_0^1(z) = -\frac{1}{\sqrt{1-z^2}}$$

07.11.03.0033.01

$$Q_0^2(z) = \frac{2z}{1-z^2}$$

07.11.03.0034.01

$$Q_0^3(z) = -\frac{2+6z^2}{\left(1-z^2\right)^{3/2}}$$

07.11.03.0035.01

$$Q_0^4(z) = \frac{24\left(z+z^3\right)}{\left(z^2-1\right)^2}$$

07.11.03.0036.01

$$Q_0^5(z) = -\frac{24\left(1+5z^2\left(2+z^2\right)\right)}{\left(1-z^2\right)^{5/2}}$$

07.11.03.0037.01

$$Q_0^6(z) = -\frac{240\,z\left(z^2+3\right)\left(3z^2+1\right)}{\left(z^2-1\right)^3}$$

07.11.03.0038.01

$$Q_0^7(z) = -\frac{720\left(7z^6+35z^4+21z^2+1\right)}{(1-z)^{7/2}\,(z+1)^{7/2}}$$

07.11.03.0039.01

$$Q_0^8(z) = \frac{40\,320\left(z^7+7z^5+7z^3+z\right)}{\left(z^2-1\right)^4}$$

07.11.03.0040.01

$$Q_0^9(z) = -\frac{40\,320\left(9z^8+84z^6+126z^4+36z^2+1\right)}{(1-z)^{9/2}\,(z+1)^{9/2}}$$

07.11.03.0041.01

$$Q_0^{10}(z) = -\frac{725\,760\,z\left(z^4+10z^2+5\right)\left(5z^4+10z^2+1\right)}{\left(z^2-1\right)^5}$$

07.11.03.0042.01

$$Q_1^0(z) = \frac{z}{2}\left(\log(z+1)-\log(1-z)\right)-1$$

07.11.03.0043.01

$$Q_1^1(z) = \frac{\left(z^2-1\right)\left(\log(z+1)-\log(1-z)\right)-2z}{2\sqrt{1-z^2}}$$

07.11.03.0044.01

$$Q_1^2(z) = \frac{2}{z^2-1}$$

07.11.03.0045.01

$$Q_1^3(z) = -\frac{8z}{\left(1-z^2\right)^{3/2}}$$

07.11.03.0046.01

$$Q_1^4(z) = \frac{8\left(1+5z^2\right)}{\left(z^2-1\right)^2}$$

07.11.03.0047.01

$$Q_1^5(z) = -\frac{48z\left(3+5z^2\right)}{\left(1-z^2\right)^{5/2}}$$

07.11.03.0048.01

$$Q_1^6(z) == \frac{48\left(3+42 z^2+35 z^4\right)}{\left(1-z^2\right)^3}$$

07.11.03.0049.01

$$Q_1^7(z) == -\frac{1920 z\left(3+7 z^2\left(2+z^2\right)\right)}{\left(1-z^2\right)^{7/2}}$$

07.11.03.0050.01

$$Q_1^8(z) == \frac{5760\left(1+3 z^2\left(9+7 z^2\left(3+z^2\right)\right)\right)}{\left(z^2-1\right)^4}$$

07.11.03.0051.01

$$Q_1^9(z) == -\frac{80\,640 z\left(5+45 z^2+63 z^4+15 z^6\right)}{\left(1-z^2\right)^{9/2}}$$

07.11.03.0052.01

$$Q_1^{10}(z) == \frac{80\,640\left(5+11 z^2\left(20+90 z^2+84 z^4+15 z^6\right)\right)}{\left(1-z^2\right)^5}$$

07.11.03.0053.01

$$Q_2^0(z) == \frac{1}{4}\left(\left(3 z^2-1\right)(\log(z+1)-\log(1-z))-6 z\right)$$

07.11.03.0054.01

$$Q_2^1(z) == \frac{4-6 z^2-3(z-1)(z+1) z(\log(1-z)-\log(z+1))}{2 \sqrt{1-z^2}}$$

07.11.03.0055.01

$$Q_2^2(z) == \frac{2 z\left(3 z^2-5\right)-3\left(z^2-1\right)^2(\log(z+1)-\log(1-z))}{2\left(z^2-1\right)}$$

07.11.03.0056.01

$$Q_2^3(z) == -\frac{8}{\left(1-z^2\right)^{3/2}}$$

07.11.03.0057.01

$$Q_2^4(z) == \frac{48 z}{\left(z^2-1\right)^2}$$

07.11.03.0058.01

$$Q_2^5(z) == -\frac{48\left(1+7 z^2\right)}{\left(1-z^2\right)^{5/2}}$$

07.11.03.0059.01

$$Q_2^6(z) == \frac{384 z\left(3+7 z^2\right)}{\left(1-z^2\right)^3}$$

07.11.03.0060.01

$$Q_2^7(z) = -\frac{1152\left(1+18 z^2+21 z^4\right)}{\left(1-z^2\right)^{7/2}}$$

07.11.03.0061.01

$$Q_2^8(z) = \frac{11\,520\,z\left(5+30 z^2+21 z^4\right)}{\left(z^2-1\right)^4}$$

07.11.03.0062.01

$$Q_2^9(z) = -\frac{11\,520\left(5+33 z^2\left(5+15 z^2+7 z^4\right)\right)}{\left(1-z^2\right)^{9/2}}$$

07.11.03.0063.01

$$Q_2^{10}(z) = \frac{967\,680\,z\left(5+55 z^2+99 z^4+33 z^6\right)}{\left(1-z^2\right)^5}$$

07.11.03.0064.01

$$Q_3^0(z) = \frac{1}{12}\left(8-30 z^2-3\left(5 z^2-3\right)(\log(1-z)-\log(z+1))\, z\right)$$

07.11.03.0065.01

$$Q_3^1(z) = \frac{-30 z^3+26 z+3\left(5 z^4-6 z^2+1\right)(\log(z+1)-\log(1-z))}{4\sqrt{1-z^2}}$$

07.11.03.0066.01

$$Q_3^2(z) = \frac{30 z^4-50 z^2+16+15\left(z^2-1\right)^2(\log(1-z)-\log(z+1))\, z}{2\left(z^2-1\right)}$$

07.11.03.0067.01

$$Q_3^3(z) = -\frac{30 z^5-80 z^3+66 z+15\left(1-z^2\right)^3(\log(z+1)-\log(1-z))}{2\left(1-z^2\right)^{3/2}}$$

07.11.03.0068.01

$$Q_3^4(z) = \frac{48}{\left(z^2-1\right)^2}$$

07.11.03.0069.01

$$Q_3^5(z) = -\frac{384\, z}{\left(1-z^2\right)^{5/2}}$$

07.11.03.0070.01

$$Q_3^6(z) = \frac{384\left(1+9 z^2\right)}{\left(1-z^2\right)^3}$$

07.11.03.0071.01

$$Q_3^7(z) = -\frac{11\,520\left(z+3 z^3\right)}{\left(1-z^2\right)^{7/2}}$$

07.11.03.0072.01

$$Q_3^8(z) == \frac{11\,520\left(1+22\,z^2+33\,z^4\right)}{\left(z^2-1\right)^4}$$

07.11.03.0073.01

$$Q_3^9(z) == -\frac{46\,080\,z\left(15+110\,z^2+99\,z^4\right)}{\left(1-z^2\right)^{9/2}}$$

07.11.03.0074.01

$$Q_3^{10}(z) == \frac{138\,240\left(5+195\,z^2+715\,z^4+429\,z^6\right)}{\left(1-z^2\right)^5}$$

07.11.03.0075.01

$$Q_4^0(z) == \frac{1}{48}\left(-210\,z^3+110\,z+3\left(35\,z^4-30\,z^2+3\right)(\log(z+1)-\log(1-z))\right)$$

07.11.03.0076.01

$$Q_4^1(z) == \frac{-210\,z^4+230\,z^2-15\,(z-1)\,(z+1)\left(7\,z^2-3\right)(\log(1-z)-\log(z+1))\,z-32}{12\sqrt{1-z^2}}$$

07.11.03.0077.01

$$Q_4^2(z) == \frac{2\,z\left(105\,z^4-190\,z^2+81\right)-15\left(z^2-1\right)^2\left(7\,z^2-1\right)(\log(z+1)-\log(1-z))}{4\left(z^2-1\right)}$$

07.11.03.0078.01

$$Q_4^3(z) == -\frac{210\,z^6-560\,z^4+462\,z^2-96+105\left(z^2-1\right)^3 z\,(\log(1-z)-\log(z+1))}{2\left(1-z^2\right)^{3/2}}$$

07.11.03.0079.01

$$Q_4^4(z) == \frac{-210\,z^7+770\,z^5-1022\,z^3+558\,z+105\left(z^2-1\right)^4(\log(z+1)-\log(1-z))}{2\left(z^2-1\right)^2}$$

07.11.03.0080.01

$$Q_4^5(z) == -\frac{384}{\left(1-z^2\right)^{5/2}}$$

07.11.03.0081.01

$$Q_4^6(z) == \frac{3840\,z}{\left(z^2-1\right)^3}$$

07.11.03.0082.01

$$Q_4^7(z) == -\frac{3840\left(1+11\,z^2\right)}{\left(1-z^2\right)^{7/2}}$$

07.11.03.0083.01

$$Q_4^8(z) == \frac{46\,080\,z\left(3+11\,z^2\right)}{\left(z^2-1\right)^4}$$

07.11.03.0084.01

$$Q_4^9(z) == -\frac{46\,080\left(3+78 z^2+143 z^4\right)}{\left(1-z^2\right)^{9/2}}$$

07.11.03.0085.01

$$Q_4^{10}(z) == \frac{645\,120\, z\left(15+130 z^2+143 z^4\right)}{\left(1-z^2\right)^5}$$

07.11.03.0086.01

$$Q_5^0(z) == \frac{1}{240}\left(-2\left(945 z^4-735 z^2+64\right)-15 z\left(63 z^4-70 z^2+15\right)(\log(1-z)-\log(z+1))\right)$$

07.11.03.0087.01

$$Q_5^1(z) == \frac{-630 z^5+840 z^3-226 z+15\left(21 z^6-35 z^4+15 z^2-1\right)(\log(z+1)-\log(1-z))}{16\sqrt{1-z^2}}$$

07.11.03.0088.01

$$Q_5^2(z) == \frac{14\left(45\left(z^2-2\right) z^2+49\right) z^2-64+105\left(z^2-1\right)^2\left(3 z^2-1\right) z\,(\log(1-z)-\log(z+1))}{4\left(z^2-1\right)}$$

07.11.03.0089.01

$$Q_5^3(z) == -\frac{2 z\left(945 z^6-2625 z^4+2359 z^2-663\right)-105\left(z^2-1\right)^3\left(9 z^2-1\right)(\log(z+1)-\log(1-z))}{4\left(1-z^2\right)^{3/2}}$$

07.11.03.0090.01

$$Q_5^4(z) == \frac{945 z\left(z^2-1\right)^4(\log(z+1)-\log(1-z))-6\left(3\left(7 z^2\left(15 z^4-55 z^2+73\right)-279\right) z^2+128\right)}{2\left(z^2-1\right)^2}$$

07.11.03.0091.01

$$Q_5^5(z) == -\frac{3\left(2 z\left(3\left(105 z^6-490 z^4+896 z^2-790\right) z^2+965\right)-315\left(z^2-1\right)^5(\log(z+1)-\log(1-z))\right)}{2\left(1-z^2\right)^{5/2}}$$

07.11.03.0092.01

$$Q_5^6(z) == \frac{3840}{\left(1-z^2\right)^3}$$

07.11.03.0093.01

$$Q_5^7(z) == -\frac{46\,080\, z}{\left(1-z^2\right)^{7/2}}$$

07.11.03.0094.01

$$Q_5^8(z) == \frac{46\,080\left(13 z^2+1\right)}{\left(z^2-1\right)^4}$$

07.11.03.0095.01

$$Q_5^9(z) == -\frac{645\,120\, z\left(3+13 z^2\right)}{\left(1-z^2\right)^{9/2}}$$

07.11.03.0096.01

$$Q_5^{10}(z) = \frac{1\,935\,360\,(1+30\,z^2+65\,z^4)}{(1-z^2)^5}$$

07.11.03.0097.01

$$Q_6^0(z) = \frac{1}{160}\left(5\left(21\,z^2\left(11\,z^4-15\,z^2+5\right)-5\right)(\log(z+1)-\log(1-z))-14\,z\left(165\,z^4-170\,z^2+33\right)\right)$$

07.11.03.0098.01

$$Q_6^1(z) = \frac{1}{80\sqrt{1-z^2}}\left(-42\left(165\,z^4-260\,z^2+103\right)z^2-105\,(z-1)\,(z+1)\left(33\,z^4-30\,z^2+5\right)(\log(1-z)-\log(z+1))\,z+256\right)$$

07.11.03.0099.01

$$Q_6^2(z) = \frac{2\,z\left(3465\,z^6-7665\,z^4+5103\,z^2-919\right)-105\left(z^2-1\right)^2\left(33\,z^4-18\,z^2+1\right)(\log(z+1)-\log(1-z))}{16\left(z^2-1\right)}$$

07.11.03.0100.01

$$Q_6^3(z) = -\frac{1}{4\left(1-z^2\right)^{3/2}}\left(315\,z\left(11\,z^2-3\right)(\log(1-z)-\log(z+1))\left(z^2-1\right)^3+6\,z^2\left(7\,z^2\left(165\,z^4-485\,z^2+483\right)-1221\right)+512\right)$$

07.11.03.0101.01

$$Q_6^4(z) = -\frac{1}{4\left(z^2-1\right)^2}\left(3\left(6930\,z^9-26\,040\,z^7+36\,036\,z^5-21\,480\,z^3+4490\,z-315\left(z^2-1\right)^4\left(11\,z^2-1\right)(\log(z+1)-\log(1-z))\right)\right)$$

07.11.03.0102.01

$$Q_6^5(z) = -\frac{1}{2\left(1-z^2\right)^{5/2}}\left(3\left(3465\,z\,(\log(1-z)-\log(z+1))\left(z^2-1\right)^5+22\,z^2\left(3\left(105\,z^6-490\,z^4+896\,z^2-790\right)z^2+965\right)-2560\right)\right)$$

07.11.03.0103.01

$$Q_6^6(z) = \frac{3}{2\left(z^2-1\right)^3}$$

$$\left(3465\log(1-z)\left(z^2-1\right)^6-3465\log(z+1)\left(z^2-1\right)^6+2\,z\left(11\,z^2\left(315\,z^8-1785\,z^6+4158\,z^4-5058\,z^2+3335\right)-11\,895\right)\right)$$

07.11.03.0104.01

$$Q_6^7(z) = -\frac{46\,080}{\left(1-z^2\right)^{7/2}}$$

07.11.03.0105.01

$$Q_6^8(z) = \frac{645\,120\,z}{\left(z^2-1\right)^4}$$

07.11.03.0106.01

$$Q_6^9(z) = -\frac{645\,120\left(1+15\,z^2\right)}{\left(1-z^2\right)^{9/2}}$$

07.11.03.0107.01

$$Q_6^{10}(z) = \frac{30\,965\,760\left(z+5\,z^3\right)}{\left(1-z^2\right)^5}$$

07.11.03.0108.01

$$Q_7^0(z) = \frac{1}{1120}\left(512 - 14\left(55\left(39 z^2 - 50\right) z^2 + 849\right) z^2 - 35\left(429 z^6 - 693 z^4 + 315 z^2 - 35\right)(\log(1-z) - \log(z+1))\, z\right)$$

07.11.03.0109.01

$$Q_7^1(z) = \frac{1}{160\sqrt{1-z^2}}$$
$$\left(-30\,030 z^7 + 54\,670 z^5 - 28\,546 z^3 + 3746 z - +35\left(429 z^8 - 924 z^6 + 630 z^4 - 140 z^2 + 5\right)(\log(z+1) - \log(1-z))\right)$$

07.11.03.0110.01

$$Q_7^2(z) = \frac{1}{80\left(z^2-1\right)}$$
$$\left(6\left(7 z^2\left(55\left(39 z^2 - 95\right) z^2 + 4119\right) - 7641\right) z^2 + 315\left(z^2-1\right)^2\left(143 z^4 - 110 z^2 + 15\right) z\,(\log(1-z) - \log(z+1)) + 2048\right)$$

07.11.03.0111.01

$$Q_7^3(z) = -\frac{1}{16\left(1-z^2\right)^{3/2}}$$
$$\left(2 z\left(3\left(7 z^2\left(2145 z^4 - 6710 z^2 + 7404\right) - 22\,950\right) z^2 + 9295\right) - 315\left(z^2-1\right)^3\left(143 z^4 - 66 z^2 + 3\right)(\log(z+1) - \log(1-z))\right)$$

07.11.03.0112.01

$$Q_7^4(z) = \frac{1}{4\left(z^2-1\right)^2}$$
$$\left(-3465 z\left(13 z^2 - 3\right)\left(z^2-1\right)^4(\log(1-z) - \log(z+1)) - 22 z^2\left(3\left(1365 z^6 - 5320 z^4 + 7798 z^2 - 5160\right) z^2 + 4175\right) + 5120\right)$$

07.11.03.0113.01

$$Q_7^5(z) = -\frac{3}{4\left(1-z^2\right)^{5/2}}\left(90\,090 z^{11} - 427\,350 z^9 + 801\,108 z^7 -\right.$$
$$\left.736\,956 z^5 + 328\,130 z^3 - 54\,510 z - 3465\left(z^2-1\right)^5\left(13 z^2 - 1\right)(\log(z+1) - \log(1-z))\right)$$

07.11.03.0114.01

$$Q_7^6(z) = \frac{3}{2\left(z^2-1\right)^3}$$
$$\left(45\,045 z\,(\log(1-z) - \log(z+1))\left(z^2-1\right)^6 + 2\left(13\left(11 z^2\left(315 z^8 - 1785 z^6 + 4158 z^4 - 5058 z^2 + 3335\right) - 11\,895\right) z^2 + 15\,360\right)\right)$$

07.11.03.0115.01

$$Q_7^7(z) = -\frac{3}{2\left(1-z^2\right)^{7/2}}\left(45\,045 \log(1-z)\left(z^2-1\right)^7 - 45\,045 \log(z+1)\left(z^2-1\right)^7 +\right.$$
$$\left.2 z\left(13\left(11 z^2\left(315 z^8 - 2100 z^6 + 5943 z^4 - 9216 z^2 + 8393\right) - 48\,580\right) z^2 + 169\,995\right)\right)$$

07.11.03.0116.01

$$Q_7^8(z) = \frac{645\,120}{\left(z^2-1\right)^4}$$

07.11.03.0117.01

$$Q_7^9(z) = -\frac{10\,321\,920 z}{\left(1-z^2\right)^{9/2}}$$

07.11.03.0118.01

$$Q_7^{10}(z) = \frac{10\,321\,920\,(17 z^2 + 1)}{(1 - z^2)^5}$$

07.11.03.0119.01

$$Q_8^0(z) = \frac{1}{8960}\left(-450\,450 z^7 + 690\,690 z^5 - 294\,910 z^3 + 30\,318 z + 35\,(6435 z^8 - 12\,012 z^6 + 6930 z^4 - 1260 z^2 + 35)(\log(z+1) - \log(1-z))\right)$$

07.11.03.0120.01

$$Q_8^1(z) = \frac{1}{1120\sqrt{1 - z^2}}\left(6\,(20\,901 - 385 z^2 (195 z^4 - 403 z^2 + 261))\, z^2 - 315\,(z-1)(z+1)\,(11 z^2 (65 z^4 - 91 z^2 + 35) - 35)(\log(1-z) - \log(z+1))\, z - 4096\right)$$

07.11.03.0121.01

$$Q_8^2(z) = \frac{1}{32\,(z^2 - 1)}\left(90\,090 z^9 - 240\,240 z^7 + 218\,988 z^5 - 76\,464 z^3 + 7562 z - 315\,(z^2 - 1)^2\,(143 z^6 - 143 z^4 + 33 z^2 - 1)(\log(z+1) - \log(1-z))\right)$$

07.11.03.0122.01

$$Q_8^3(z) = -\frac{1}{16\,(1 - z^2)^{3/2}}\left(3465 z\,(39 z^4 - 26 z^2 + 3)(\log(1-z) - \log(z+1))\,(z^2 - 1)^3 + 2\,(33 z^2 (4095 z^8 - 13\,650 z^6 + 16\,604 z^4 - 8718 z^2 + 1733) - 2048)\right)$$

07.11.03.0123.01

$$Q_8^4(z) = -\frac{1}{16\,(z^2 - 1)^2}\left(3\,(2 z\,(11 z^2 (3\,(35 z^2 (195 z^4 - 793 z^2 + 1238) - 31\,806)\, z^2 + 31\,615) - 37\,495) - 3465\,(z^2 - 1)^4\,(65 z^4 - 26 z^2 + 1)(\log(z+1) - \log(1-z))\right)\right)$$

07.11.03.0124.01

$$Q_8^5(z) = -\frac{1}{4\,(1 - z^2)^{5/2}}\left(3\,(45\,045 z\,(5 z^2 - 1)(\log(1-z) - \log(z+1))\,(z^2 - 1)^5 + 26 z^2 (11 z^2 (3\,(35 z^2 (15 z^4 - 73 z^2 + 142) - 4846)\, z^2 + 7195) - 17\,015) + 20\,480)\right)$$

07.11.03.0125.01

$$Q_8^6(z) = \frac{1}{4\,(z^2 - 1)^3}\left(3\,(2 z\,(13\,(33 z^2 ((35 z^2 (45 z^4 - 258 z^2 + 611) - 26\,676)\, z^2 + 18\,361) - 215\,110)\, z^2 + 385\,035) - 45\,045\,(z^2 - 1)^6\,(15 z^2 - 1)(\log(z+1) - \log(1-z)))\right)$$

07.11.03.0126.01

$$Q_8^7(z) = -\frac{45}{2\,(1 - z^2)^{7/2}}\left(45\,045 z\,(\log(1-z) - \log(z+1))\,(z^2 - 1)^7 + 2\,(z^2 (13\,(11 z^2 (315 z^8 - 2100 z^6 + 5943 z^4 - 9216 z^2 + 8393) - 48\,580)\, z^2 + 169\,995) - 14\,336)\right)$$

07.11.03.0127.01

$$Q_8^8(z) == -\frac{45}{2\left(z^2-1\right)^4}\left(45\,045\log(1-z)\left(z^2-1\right)^8-45\,045\log(z+1)\left(z^2-1\right)^8+\right.$$
$$\left.2\,z\left(z^2\left(13\left(11\,z^2\left(315\,z^8-2415\,z^6+8043\,z^4-15\,159\,z^2+17\,609\right)-140\,903\right)z^2+801\,535\right)-184\,331\right)\right)$$

07.11.03.0128.01

$$Q_8^9(z) == -\frac{10\,321\,920}{\left(1-z^2\right)^{9/2}}$$

07.11.03.0129.01

$$Q_8^{10}(z) == \frac{185\,794\,560\,z}{\left(1-z^2\right)^5}$$

07.11.03.0130.01

$$Q_9^0(z) == \frac{1}{80\,640}\left(-2\left(165\left(91\,z^2\left(255\,z^4-455\,z^2+249\right)-3867\right)z^2+16\,384\right)-\right.$$
$$\left.315\,z\left(11\left(13\,z^2\left(85\,z^4-180\,z^2+126\right)-420\right)z^2+315\right)(\log(1-z)-\log(z+1))\right)$$

07.11.03.0131.01

$$Q_9^1(z) == \frac{1}{1792\sqrt{1-z^2}}\left(315\,(z-1)\,(z+1)\left(11\left(221\,z^6-364\,z^4+182\,z^2-28\right)z^2+7\right)(\log(z+1)-\log(1-z))-\right.$$
$$\left.2\,z\left(33\left(91\,z^2\left(255\,z^4-590\,z^2+456\right)-11\,910\right)z^2+30\,563\right)\right)$$

07.11.03.0132.01

$$Q_9^2(z) == \frac{1}{224\left(z^2-1\right)}\left(3465\,z\left(221\,z^6-273\,z^4+91\,z^2-7\right)(\log(1-z)-\log(z+1))\left(z^2-1\right)^2+\right.$$
$$\left.2\left(11\,z^2\left(3\left(91\,z^2\left(255\,z^4-740\,z^2+766\right)-30\,180\right)z^2+14\,179\right)-4096\right)\right)$$

07.11.03.0133.01

$$Q_9^3(z) == -\frac{1}{32\left(1-z^2\right)^{3/2}}\left(2\,z\left(11\,z^2\left(3\left(91\,z^2\left(255\,z^4-905\,z^2+1206\right)-66\,678\right)z^2+52\,411\right)-45\,687\right)-\right.$$
$$\left.3465\left(z^2-1\right)^3\left(221\,z^6-195\,z^4+39\,z^2-1\right)(\log(z+1)-\log(1-z))\right)$$

07.11.03.0134.01

$$Q_9^4(z) == \frac{1}{16\left(z^2-1\right)^2}\left(3\left(-45\,045\,z\left(17\,z^4-10\,z^2+1\right)\left(z^2-1\right)^4(\log(1-z)-\log(z+1))-\right.\right.$$
$$\left.\left.2\left(13\left(11\,z^2\left(5355\,z^8-22\,785\,z^6+37\,926\,z^4-30\,714\,z^2+12\,079\right)-21\,111\right)z^2+8192\right)\right)\right)$$

07.11.03.0135.01

$$Q_9^5(z) == -\frac{3}{16\left(1-z^2\right)^{5/2}}\left(2\,z\left(13\left(11\,z^2\left(15\left(1785\,z^6-8960\,z^4+18\,193\,z^2-18\,904\right)z^2+155\,813\right)-453\,320\right)z^2+528\,395\right)-\right.$$
$$\left.45\,045\left(z^2-1\right)^5\left(85\,z^4-30\,z^2+1\right)(\log(z+1)-\log(1-z))\right)$$

07.11.03.0136.01

$$Q_9^6(z) = \frac{15}{4\left(z^2-1\right)^3}\left(45045\,z\left(17z^2-3\right)(\log(1-z)-\log(z+1))\left(z^2-1\right)^6 + 2\left(z^2\left(13\left(11z^2\left(5355z^8-31290z^6+76041z^4-98460z^2+71869\right)-312270\right)z^2+725025\right)-28672\right)\right)$$

07.11.03.0137.01

$$Q_9^7(z) = -\frac{45}{4\left(1-z^2\right)^{7/2}}\left(2z\left(z^2\left(13\left(11z^2\left(3\left(7z^2\left(255z^4-1715z^2+4911\right)-54205\right)z^2+151897\right)-918183\right)z^2+3521455\right)-413707\right) - 45045\left(z^2-1\right)^7\left(17z^2-1\right)(\log(z+1)-\log(1-z))\right)$$

07.11.03.0138.01

$$Q_9^8(z) = -\frac{45}{2\left(z^2-1\right)^4}\left(765765\,z(\log(1-z)-\log(z+1))\left(z^2-1\right)^8 + 2\left(17\left(z^2\left(13\left(11z^2\left(315z^8-2415z^6+8043z^4-15159z^2+17609\right)-140903\right)z^2+801535\right)-184331\right)z^2+229376\right)\right)$$

07.11.03.0139.01

$$Q_9^9(z) = -\frac{45}{2\left(1-z^2\right)^{9/2}}\left(765765\log(1-z)\left(z^2-1\right)^9 - 765765\log(z+1)\left(z^2-1\right)^9 + 2z\left(17\left(z^2\left(13\left(11z^2\left(3\left(105z^6-910z^4+3486z^2-7734\right)z^2+32768\right)-334602\right)z^2+2633274\right)-985866\right)z^2 + 3363003\right)\right)$$

07.11.03.0140.01

$$Q_9^{10}(z) = \frac{185794560}{\left(1-z^2\right)^5}$$

07.11.03.0141.01

$$Q_{10}^0(z) = \frac{1}{161280}\left(315\left(11z^2\left(13\left(323z^6-765z^4+630z^2-210\right)z^2+315\right)-63\right)(\log(z+1)-\log(1-z)) - 22z\left(39\left(7z^2\left(4845z^4-9860z^2+6594\right)-11220\right)z^2+27985\right)\right)$$

07.11.03.0142.01

$$Q_{10}^1(z) = \frac{1}{16128\sqrt{1-z^2}}\left(-22\left(39\left(33915z^6-86870z^4+78008z^2-28650\right)z^2+143995\right)z^2 - 3465(z-1)(z+1)\left(13\left(323z^6-612z^4+378z^2-84\right)z^2+63\right)(\log(1-z)-\log(z+1))\,z+65536\right)$$

07.11.03.0143.01

$$Q_{10}^2(z) = \frac{1}{1792\left(z^2-1\right)}\left(2z\left(11z^2\left(39\left(7z^2\left(85\left(57z^2-179\right)z^2+17634\right)-63762\right)z^2+537025\right)-368961\right) - 3465\left(z^2-1\right)^2\left(13\left(323z^6-476z^4+210z^2-28\right)z^2+7\right)(\log(z+1)-\log(1-z))\right)$$

07.11.03.0144.01

$$Q_{10}^3(z) = -\frac{1}{224\left(1-z^2\right)^{3/2}}\left(45045\,z\left(323z^6-357z^4+105z^2-7\right)(\log(1-z)-\log(z+1))\left(z^2-1\right)^3 + 2\left(13\left(11z^2\left(3\left(7z^2\left(85\left(57z^2-215\right)z^2+26514\right)-128106\right)z^2+124885\right)-172353\right)z^2+49152\right)\right)$$

07.11.03.0145.01

$$Q_{10}^{4}(z) = \frac{1}{32\left(z^2-1\right)^2}\left(45045\left(z^2-1\right)^4\left(323 z^6-255 z^4+45 z^2-1\right)(\log(z+1)-\log(1-z))-2 z\left(13\left(11 z^2\left(3\left(7 z^2\left(85\left(57 z^2-254\right) z^2+38279\right)-237852\right) z^2+324919\right)-748874\right) z^2+643083\right)\right)$$

07.11.03.0146.01

$$Q_{10}^{5}(z) = -\frac{3}{16\left(1-z^2\right)^{5/2}}\left(45045 z\left(323 z^4-170 z^2+15\right)\left(z^2-1\right)^5(\log(1-z)-\log(z+1))+2 z^2\left(13\left(11 z^2\left(3\left(7 z^2\left(85\left(57 z^2-296\right) z^2+53469\right)-414840\right) z^2+754915\right)-2609040\right) z^2+4485285\right)-229376\right)$$

07.11.03.0147.01

$$Q_{10}^{6}(z) = \frac{15}{16\left(z^2-1\right)^3}\left(45045\left(323 z^4-102 z^2+3\right)\log(1-z)\left(z^2-1\right)^6-45045\left(323 z^4-102 z^2+3\right)\log(z+1)\left(z^2-1\right)^6+2 z\left(z^2\left(13\left(11 z^2\left(3\left(7 z^2\left(85\left(57 z^2-341\right) z^2+72669\right)-687735\right) z^2+1605595\right)-7750869\right) z^2+22164765\right)-1699873\right)\right)$$

07.11.03.0148.01

$$Q_{10}^{7}(z) = -\frac{15}{4\left(1-z^2\right)^{7/2}}\left(765765 z\left(19 z^2-3\right)(\log(1-z)-\log(z+1))\left(z^2-1\right)^7+2\left(17\left(z^2\left(13\left(11 z^2\left(3\left(7 z^2\left(285 z^4-1945 z^2+5677\right)-64311\right) z^2+187115\right)-1199989\right) z^2+5124525\right)-782369\right) z^2+458752\right)\right)$$

07.11.03.0149.01

$$Q_{10}^{8}(z) = -\frac{45}{4\left(z^2-1\right)^4}\left(2 z\left(17\left(z^2\left(13\left(11 z^2\left(3\left(7 z^2\left(285 z^4-2200 z^2+7392\right)-98688\right) z^2+349730\right)-2870856\right) z^2+17060904\right)-4303824\right) z^2+7491771\right)-765765\left(z^2-1\right)^8\left(19 z^2-1\right)(\log(z+1)-\log(1-z))\right)$$

07.11.03.0150.01

$$Q_{10}^{9}(z) = -\frac{45}{2\left(1-z^2\right)^{9/2}}\left(14549535 z(\log(1-z)-\log(z+1))\left(z^2-1\right)^9+2\left(19 z^2\left(17\left(z^2\left(13\left(11 z^2\left(3\left(105 z^6-910 z^4+3486 z^2-7734\right) z^2+32768\right)-334602\right) z^2+2633274\right)-985866\right) z^2+3363003\right)-4128768\right)\right)$$

07.11.03.0151.01

$$Q_{10}^{10}(z) = \frac{45}{2\left(z^2-1\right)^5}\left(14549535\log(1-z)\left(z^2-1\right)^{10}-14549535\log(z+1)\left(z^2-1\right)^{10}+2 z\left(19 z^2\left(17\left(z^2\left(13\left(11 z^2\left(315 z^8-3045 z^6+13188 z^4-33660 z^2+55970\right)-695050\right) z^2+6983100\right)-3619140\right) z^2+20122725\right)-68025825\right)\right)$$

General characteristics

Domain and analyticity

$Q_\nu^\mu(z)$ is an analytical function of ν, μ and z which is defined over $\mathbb{C}^3$. For integer ν and noninteger μ, $Q_\nu^\mu(z)$ degenerates to a sum of two polynomial in z multiplied on functions $\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}$ and $\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}$ correspondently. For integers ν, $\frac{\mu}{2}$ /; $\mu > -\nu$, $Q_\nu^\mu(z)$ becomes the meromorthic function.

07.11.04.0001.01

$$(\nu * \mu * 2 * z) \longrightarrow Q_\nu^\mu(z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \{2\} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.11.04.0002.01

$$Q_{\bar{\nu}}^{\bar{\mu}}(\bar{z}) = \overline{Q_\nu^\mu(z)} \text{ /; } z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν, μ (except $\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu$), the function $Q_\nu^\mu(z)$ does not have poles and essential singularities.

07.11.04.0003.01

$$\mathcal{Sing}_z(Q_\nu^\mu(z)) = \{\} \text{ /; } \neg\left(\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu\right)$$

For integer ν and integer $\frac{\mu}{2}$ /; $\mu > 1 \wedge \mu > -\nu$, the function $Q_\nu^\mu(z)$ is a meromorthic function with poles at points $z = \pm 1$ of orders $\frac{\mu}{2}$ and at (maybe) $z = \tilde{\infty}$ (for $\nu < -1$ of order $-\nu - 1$).

07.11.04.0004.01

$$\mathcal{Sing}_z(Q_\nu^\mu(z)) = \left\{\left\{1, \frac{\mu}{2}\right\}, \left\{-1, \frac{\mu}{2}\right\}\right\} \text{ /; } \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu \bigwedge \nu \geq 0$$

07.11.04.0005.01

$$\mathcal{Sing}_z(Q_\nu^\mu(z)) = \left\{\left\{1, \frac{\mu}{2}\right\}, \left\{-1, \frac{\mu}{2}\right\}, \{\tilde{\infty}, -\nu - 1\}\right\} \text{ /; } \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu \bigwedge \nu < -1$$

With respect to μ

For fixed ν, z, the function $Q_\nu^\mu(z)$ has an infinite set of singular points:

a) $\mu = -\nu - k$ /; $k \in \mathbb{N}^+$, are the simple poles with residues $-\frac{\pi \csc(\pi \nu)}{2 (k-1)! \, \Gamma(k+2\nu+1)} P_\nu^{k+\nu}(z)$;

b) $\mu = \tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

07.11.04.0006.01

$$\mathcal{Sing}_\mu(Q_\nu^\mu(z)) = \left\{\left\{\{-\nu - k, 1\} \text{ /; } k \in \mathbb{N}^+\right\}, \{\tilde{\infty}, \infty\}\right\}$$

07.11.04.0007.01

$$\operatorname{res}_{\mu}\left(Q_{\nu}^{\mu}(z)\right)(-\nu-k) = -\frac{\pi \csc(\pi \nu)}{2 (k-1)! \, \Gamma(k+2\nu+1)} P_{\nu}^{k+\nu}(z) /; k \in \mathbb{N}^{+}$$

With respect to ν

For fixed μ, z, the function $Q_{\nu}^{\mu}(z)$ has an infinite set of singular points:

a) $\nu = -\mu - k /; k \in \mathbb{N}^{+}$, are the simple poles with residues $\frac{(-1)^{k} \pi \csc(\pi \mu)}{2 (k-1)! \, \Gamma(1-k-2\mu)} P_{-k-\mu}^{-\mu}(z)$;

b) $\nu = \tilde{\infty}$ is the point of accumulation of poles, which is an essential singular point.

07.11.04.0008.01

$$\mathcal{S}ing_{\nu}\left(Q_{\nu}^{\mu}(z)\right) = \left\{\{\{-\mu-k, 1\} /; k \in \mathbb{N}^{+}\}, \{\tilde{\infty}, \infty\}\right\}$$

07.11.04.0009.01

$$\operatorname{res}_{\nu}\left(Q_{\nu}^{\mu}(z)\right)(-\mu-k) = \frac{(-1)^{k} \pi \csc(\pi \mu)}{2 (k-1)! \, \Gamma(1-k-2\mu)} P_{-k-\mu}^{-\mu}(z) /; k \in \mathbb{N}^{+}$$

Branch points

With respect to z

For fixed ν, μ (except $\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^{+} \bigwedge \mu > -\nu$), the function $Q_{\nu}^{\mu}(z)$ has three singular branch points: $z = -1$, $z = 1$and $z = \tilde{\infty}$.

For integer ν and integer $\frac{\mu}{2} /; \mu > 1 \land \mu > -\nu$, the function $Q_{\nu}^{\mu}(z)$ does not have branch points.

07.11.04.0010.01

$$\mathcal{BP}_{z}\left(Q_{\nu}^{\mu}(z)\right) = \{-1, 1, \tilde{\infty}\} /; \neg\left(\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^{+} \bigwedge \mu > -\nu\right)$$

07.11.04.0011.01

$$\mathcal{BP}_{z}\left(Q_{\nu}^{\mu}(z)\right) = \{\} /; \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^{+} \bigwedge \mu > -\nu$$

07.11.04.0012.01

$$\mathcal{R}_{z}\left(Q_{\nu}^{\mu}(z), -1\right) = \log /; \nu \in \mathbb{Z} \bigwedge \neg\left(\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{Z} \bigwedge \frac{\mu}{2} > 0 \bigwedge \mu > -\nu\right)$$

07.11.04.0013.01

$$\mathcal{R}_{z}\left(Q_{\nu}^{\mu}(z), -1\right) = s /; \mu = \frac{2r}{s} \bigwedge \{r, s\} \in \mathbb{Z} \bigwedge s > 1 \bigwedge \gcd(r, s) = 1$$

07.11.04.0014.01

$$\mathcal{R}_{z}\left(Q_{\nu}^{\mu}(z), 1\right) = \log /; \nu \in \mathbb{Z} \bigwedge \neg\left(\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{Z} \bigwedge \frac{\mu}{2} > 0 \bigwedge \mu > -\nu\right)$$

07.11.04.0015.01

$$\mathcal{R}_{z}\left(Q_{\nu}^{\mu}(z), 1\right) = s /; \mu = \frac{2r}{s} \bigwedge \{r, s\} \in \mathbb{Z} \bigwedge s > 1 \bigwedge \gcd(r, s) = 1$$

07.11.04.0016.01

$$\mathcal{R}_{z}\left(Q_{\nu}^{\mu}(z), \tilde{\infty}\right) = \log /; 2\nu \in \mathbb{Z} \bigvee a \notin \mathbb{Q}$$

07.11.04.0017.01

$$\mathcal{R}_z\left(Q_\nu^\mu(z),\ \tilde\infty\right) = s\ /;\ \nu = \frac{r}{s} \bigwedge \{r,\ s\} \in \mathbb{Z} \bigwedge s > 1 \bigwedge \gcd(r,\ s) = 1$$

With respect to μ

For fixed ν, z, the function $Q_\nu^\mu(z)$ does not have branch points.

07.11.04.0018.01

$$\mathcal{BP}_\mu\left(Q_\nu^\mu(z)\right) = \{\}$$

With respect to ν

For fixed μ, z, the function $Q_\nu^\mu(z)$ does not have branch points.

07.11.04.0019.01

$$\mathcal{BP}_\nu\left(Q_\nu^\mu(z)\right) = \{\}$$

Branch cuts

With respect to z

For fixed ν, μ (except $\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu$), the function $Q_\nu^\mu(z)$ is a single-valued function on the z-plane cut along the intervals $(-\infty,\ -1)$ and $(1,\ \infty)$. The function $Q_\nu^\mu(z)$ is continuous from above on the interval $(-\infty,\ -1]$ and from below on the interval $[1,\ \infty)$.

For integer ν and integer $\frac{\mu}{2}$ /; $\mu > 1 \wedge \mu > -\nu$, the function $Q_\nu^\mu(z)$ is a meromorthic function and does not have branch cuts.

07.11.04.0020.01

$$\mathcal{BC}_z\left(Q_\nu^\mu(z)\right) = \{\{(-\infty,\ -1),\ -i\},\ \{(1,\ \infty),\ i\}\}\ /;\ \neg\left(\nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu\right)$$

07.11.04.0021.01

$$\mathcal{BC}_z\left(Q_\nu^\mu(z)\right) = \{\}\ /;\ \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{N}^+ \bigwedge \mu > -\nu$$

07.11.04.0022.01

$$\lim_{\epsilon\to+0} Q_\nu^\mu(x + i\,\epsilon) = Q_\nu^\mu(x)\ /;\ x < -1$$

07.11.04.0023.01

$$\lim_{\epsilon\to+0} Q_\nu^\mu(x - i\,\epsilon) = -\frac{1}{2}\, i\,\pi\,(3\cos(\pi\,\nu) + \cos(\pi\,(2\,\mu + \nu)))\,P_\nu^\mu(-x) + i\,\pi\cos(\pi\,\mu)\,P_\nu^\mu(x) + e^{-i\,\pi\,\mu}\,Q_\nu^\mu(x) + 2\,i\cos(\pi\,\mu)\,Q_\nu^\mu(-x)\sin(\pi\,(\mu + \nu))\ /;\ x < -1$$

07.11.04.0024.01

$$\lim_{\epsilon\to+0} Q_\nu^\mu(x + i\,\epsilon) = i\,\pi\cos(\mu\,\pi)\,P_\nu^\mu(x) + e^{-i\,\mu\,\pi}\,Q_\nu^\mu(x)\ /;\ x > 1$$

07.11.04.0025.01

$$\lim_{\epsilon\to+0} Q_\nu^\mu(x - i\,\epsilon) = Q_\nu^\mu(x)\ /;\ x > 1$$

With respect to μ

For fixed ν, z, the function $Q_\nu^\mu(z)$ does not have branch cuts.

07.11.04.0026.01

$$\mathcal{BC}_\mu\left(Q_\nu^\mu(z)\right) = \{\}$$

With respect to ν

For fixed μ, z, the function $Q_\nu^\mu(z)$ does not have branch cuts.

07.11.04.0027.01

$$\mathcal{BC}_\nu\left(Q_\nu^\mu(z)\right) = \{\}$$

Series representations

Generalized power series

Expansions at $z = 0$

07.11.06.0001.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\frac{\cos(\mu\pi)}{\Gamma(1-\mu)}\sum_{k=0}^\infty\sum_{m=0}^\infty\sum_{j=0}^\infty\frac{(-\nu)_k(\nu+1)_k\left(\frac{\mu}{2}-k\right)_m\left(-\frac{\mu}{2}\right)_j(-1)^j z^{j+m}}{(1-\mu)_k\,k!\,2^k\,m!\,j!} - \frac{(\nu-\mu+1)_{2\mu}}{\Gamma(\mu+1)}\sum_{k=0}^\infty\sum_{m=0}^\infty\sum_{j=0}^\infty\frac{(-\nu)_k(\nu+1)_k\left(-k-\frac{\mu}{2}\right)_m\left(\frac{\mu}{2}\right)_j(-1)^j z^{j+m}}{(\mu+1)_k\,k!\,2^k\,m!\,j!}\right)\,/;\,|z|<1\bigwedge\mu\notin\mathbb{Z}$$

07.11.06.0002.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\cos(\mu\pi)\left(\frac{\mu z}{2}\sum_{k=0}^\infty(k+1)!\left(\frac{\mu}{2}+1\right)_k z^k\,{}_1F_0\left(-\frac{\mu}{2};;-z\right){}_3\tilde{F}_2\left(-\nu,\nu+1,1-\frac{\mu}{2};k+2,1-\mu;-\frac{z}{2}\right)+\Gamma\left(1-\frac{\mu}{2}\right)\sum_{k=0}^\infty\left(-\frac{\mu}{2}\right)_k(-z)^k\,\tilde{F}_{2\times1\times0}^{3\times1\times0}\left(\begin{matrix}-\nu,\nu+1,1-\frac{\mu}{2};1;;\\1,1-\mu;1-\frac{\mu}{2};;\end{matrix}\,\frac{1}{2},-\frac{z}{2}\right)\right)-(\nu-\mu+1)_{2\mu}\left(-\frac{\mu z}{2}\sum_{k=0}^\infty(k+1)!\left(1-\frac{\mu}{2}\right)_k z^k\,{}_1F_0\left(\frac{\mu}{2};;-z\right){}_3\tilde{F}_2\left(-\nu,\nu+1,1+\frac{\mu}{2};k+2,1+\mu;-\frac{z}{2}\right)+\Gamma\left(1+\frac{\mu}{2}\right)\sum_{k=0}^\infty\left(\frac{\mu}{2}\right)_k(-z)^k\,\tilde{F}_{2\times1\times0}^{3\times1\times0}\left(\begin{matrix}-\nu,\nu+1,1+\frac{\mu}{2};1;;\\1,1+\mu;1+\frac{\mu}{2};;\end{matrix}\,\frac{1}{2},-\frac{z}{2}\right)\right)\right)\,/;\,\mu\notin\mathbb{Z}$$

07.11.06.0003.01

$$Q_\nu^\mu(z)\propto 2^{-\mu-1}\pi^{3/2}\left(\frac{4^\mu\cot(\pi\mu)}{\Gamma\left(\frac{1-\mu-\nu}{2}\right)\Gamma\left(\frac{\nu-\mu}{2}+1\right)}-\frac{\csc(\pi\mu)\,\Gamma(\mu+\nu+1)}{\Gamma\left(\frac{\mu-\nu+1}{2}\right)\Gamma(\nu-\mu+1)\,\Gamma\left(\frac{\mu+\nu}{2}+1\right)}\right)(1+O(z))\,/;\,(z\to 0)\bigwedge\mu\notin\mathbb{Z}$$

Expansions at $z = 1$

07.11.06.0004.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\frac{\cos(\mu\pi)}{\Gamma(1-\mu)}\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\left(1+\frac{\nu(1+\nu)}{2(1-\mu)}(z-1)-\frac{(1-\nu)\nu(1+\nu)(2+\nu)}{8(1-\mu)(2-\mu)}(z-1)^2+\ldots\right)-\frac{(\nu-\mu+1)_{2\mu}}{\Gamma(\mu+1)}\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\left(1+\frac{\nu(1+\nu)}{2(1+\mu)}(z-1)-\frac{(1-\nu)\nu(1+\nu)(2+\nu)}{8(1+\mu)(2+\mu)}(z-1)^2+\ldots\right)\right)\,/;\,\left|\frac{1-z}{2}\right|<1\bigwedge\mu\notin\mathbb{Z}$$

07.11.06.0005.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\frac{\cos(\mu\pi)}{\Gamma(1-\mu)}\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\sum_{k=0}^\infty\frac{(-\nu)_k(\nu+1)_k}{(1-\mu)_k\,k!}\left(\frac{1-z}{2}\right)^k-\frac{(\nu-\mu+1)_{2\mu}}{\Gamma(\mu+1)}\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\sum_{k=0}^\infty\frac{(-\nu)_k(\nu+1)_k}{(\mu+1)_k\,k!}\left(\frac{1-z}{2}\right)^k\right)\,/;\,\left|\frac{1-z}{2}\right|<1\bigwedge\mu\notin\mathbb{Z}$$

07.11.06.0006.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\cos(\mu\pi)\,{}_2\tilde{F}_1\left(-\nu,\nu+1;1-\mu;\frac{1-z}{2}\right)-(\nu-\mu+1)_{2\mu}\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\,{}_2\tilde{F}_1\left(-\nu,\nu+1;\mu+1;\frac{1-z}{2}\right)\right)\,/;\,\mu\notin\mathbb{Z}$$

07.11.06.0007.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\cos(\mu\pi)(1-z)^{-\mu/2}2^{\mu/2}\sum_{k=0}^\infty\sum_{j=0}^k\frac{(-1)^k\left(-\frac{\mu}{2}\right)_{k-j}(-\nu)_j(\nu+1)_j(z-1)^k}{\Gamma(j-\mu+1)(k-j)!\,j!\,2^k}-(\nu-\mu+1)_{2\mu}(1-z)^{\frac{\mu}{2}}2^{-\frac{\mu}{2}}\sum_{k=0}^\infty\sum_{j=0}^k\frac{(-1)^k\left(\frac{\mu}{2}\right)_{k-j}(-\nu)_j(\nu+1)_j(z-1)^k}{\Gamma(j+\mu+1)(k-j)!\,j!\,2^k}\right)\,/;\,\left|\frac{1-z}{2}\right|<1$$

07.11.06.0008.01

$$Q_\nu^\mu(z)\propto\frac{2^{-\frac{\mu}{2}-1}\Gamma(\mu+\nu+1)\Gamma(-\mu)}{\Gamma(\nu-\mu+1)}(1-z)^{\frac{\mu}{2}}(1+O(z-1))+2^{\frac{\mu}{2}-1}\cos(\mu\pi)\Gamma(\mu)\,(1-z)^{-\mu/2}(1+O(z-1))\,/;\,(z\to1)\bigwedge\mu\notin\mathbb{Z}$$

07.11.06.0009.01

$$Q_\nu^m(z) = \frac{1}{2}\left(\frac{(1+z)^{\frac{m}{2}}}{(1-z)^{\frac{m}{2}}}\sum_{k=0}^{m-1}\frac{(-1)^{m-k}(m-k-1)!\,(-\nu)_k(\nu+1)_k}{k!}\left(\frac{1-z}{2}\right)^k+\frac{(1-z^2)^{\frac{m}{2}}}{2^m}\sum_{k=0}^\infty\frac{(-\nu)_{k+m}(\nu+1)_{k+m}}{k!\,(k+m)!}\left(\frac{1}{2}(\log(1+z)-\log(1-z))+\psi(k+1)\right)\left(\frac{1-z}{2}\right)^k+\frac{(-1)^m\Gamma(m+\nu+1)}{\Gamma(\nu-m+1)}\frac{(1-z)^{\frac{m}{2}}}{(1+z)^{\frac{m}{2}}}\sum_{k=0}^\infty\frac{(-\nu)_k(\nu+1)_k}{k!\,(k+m)!}\left(\frac{1}{2}(\log(1+z)-\log(1-z))+\psi(k+m+1)-\psi(-m+\nu+1)-\psi(m+\nu+1)\right)\left(\frac{1-z}{2}\right)^k\right)\,/;\,\left|\frac{1-z}{2}\right|<1\bigwedge m\in\mathbb{N}\bigwedge(\nu\notin\mathbb{Z}\bigvee\nu\in\mathbb{Z}\land\nu\geq m)$$

07.11.06.0010.01

$$Q_n^m(z) = \frac{1}{2}\left(\frac{(1+z)^{\frac{m}{2}}}{(1-z)^{\frac{m}{2}}}\sum_{k=0}^{m-1}\frac{(-1)^{m-k}(m-k-1)!\,(-n)_k\,(n+1)_k}{k!}\left(\frac{1-z}{2}\right)^k - (-1)^n (m+n)!\,(m-n-1)!\,\frac{(1-z)^{\frac{m}{2}}}{(1+z)^{\frac{m}{2}}}\sum_{k=0}^{n}\frac{(-n)_k\,(n+1)_k}{k!\,(k+m)!}\left(\frac{1-z}{2}\right)^k\right) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge n \in \mathbb{N} \bigwedge m \in \mathbb{N} \bigwedge m > n$$

07.11.06.0011.01

$$Q_\nu^m(z) = \frac{1}{2}\left(\frac{1+z}{1-z}\right)^{-\frac{m}{2}}\left(\left(\frac{z+1}{z-1}\right)^m \sum_{k=0}^{m-1}\frac{(m-k-1)!\,(-\nu)_k\,(\nu+1)_k}{k!}\left(\frac{z-1}{2}\right)^k + \frac{(-1)^m\,\Gamma(m+\nu+1)}{m!\,\Gamma(\nu-m+1)}(\log(1+z)-\log(1-z)-\psi(\nu-m+1)-\psi(m+\nu+1))\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k}{(m+1)_k\,k!}\left(\frac{1-z}{2}\right)^k + \sum_{k=0}^{\infty}\frac{1}{k!\,(k+m)!}\left((-\nu)_{k+m}\,(\nu+1)_{k+m}\,\psi(k+1)\left(\frac{1+z}{2}\right)^m + \frac{(-1)^m\,\psi(k+m+1)\,\Gamma(m+\nu+1)\,(-\nu)_k\,(\nu+1)_k}{\Gamma(\nu-m+1)}\right)\left(\frac{1-z}{2}\right)^k\right) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge m \in \mathbb{N} \bigwedge \nu \notin \mathbb{Z}$$

07.11.06.0012.01

$$Q_\nu^m(z) \propto \frac{(-1)^m\,2^{-\frac{m}{2}-1}}{m!\,\Gamma(\nu-m+1)}(1-z)^{\frac{m}{2}}\left(2^m\,(m-1)!\,m!\,\Gamma(\nu-m+1)\,(1-z)^{-m} - \gamma\,\Gamma(m+\nu+1) + \Gamma(m+\nu+1)\,(\log(2)-\log(1-z)+\psi(m+1)-\psi(\nu-m+1)-\psi(m+\nu+1))\right)(1+O(z-1)) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge m \in \mathbb{N}^+$$

07.11.06.0013.01

$$Q_\nu^m(z) \propto (-1)^m\,2^{\frac{m}{2}-1}\,(m-1)!\,(1-z)^{-m/2}\,(1+O(z-1)) /; (z \to 1) \bigwedge m \in \mathbb{N}^+$$

07.11.06.0014.01

$$Q_\nu^0(z) = \frac{1}{2}(\log(1+z)-\log(1-z)-2\,\psi(\nu+1))\,{}_2F_1\!\left(-\nu,\,\nu+1;\,1;\,\frac{1-z}{2}\right) + \sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k\,\psi(k+1)}{k!^2}\left(\frac{1-z}{2}\right)^k /; \left|\frac{1-z}{2}\right| < 1$$

07.11.06.0015.01

$$Q_\nu^0(z) \propto \left(\frac{1}{2}(\log(2)-\log(1-z))-\psi(\nu+1)-\gamma\right)(1+O(z-1)) /; (z \to 1) \bigwedge \nu \notin \mathbb{Z}$$

07.11.06.0016.01

$$Q_n^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\frac{\cos(\mu\pi)}{\Gamma(1-\mu)}\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\sum_{k=0}^{n}\frac{(-n)_k\,(n+1)_k}{(1-\mu)_k\,k!}\left(\frac{1-z}{2}\right)^k - \frac{(n-\mu+1)_{2\mu}}{\Gamma(\mu+1)}\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\sum_{k=0}^{n}\frac{(-n)_k\,(n+1)_k}{(\mu+1)_k\,k!}\left(\frac{1-z}{2}\right)^k\right) /; n \in \mathbb{N}$$

07.11.06.0017.01

$$Q_{-n}^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\frac{\cos(\mu\pi)}{\Gamma(1-\mu)}\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\sum_{k=0}^{n-1}\frac{(n)_k\,(1-n)_k}{(1-\mu)_k\,k!}\left(\frac{1-z}{2}\right)^k - \frac{(1-n-\mu)_{2\mu}}{\Gamma(\mu+1)}\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\sum_{k=0}^{n-1}\frac{(n)_k\,(1-n)_k}{(\mu+1)_k\,k!}\left(\frac{1-z}{2}\right)^k\right) /; n \in \mathbb{N}^+$$

Expansions at $z == -1$

07.11.06.0018.01

$$Q_\nu^\mu(z) == -\frac{1}{2\pi}\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\left(\Gamma(-\mu)\,\Gamma(\mu-\nu)\,\Gamma(\mu+\nu+1)\sin(\pi(\mu-\nu))\right.$$
$$\cos(\pi(\mu+\nu))\,(1+z)^\mu(1-z)^{-\mu}\left(1-\frac{\nu(1+\nu)}{2(1+\mu)}(z+1)-\frac{(1-\nu)\,\nu\,(1+\nu)\,(2+\nu)}{8(1+\mu)(2+\mu)}(z+1)^2+\ldots\right)+$$
$$\left.\pi\cos(\pi\nu)\,\Gamma(\mu)\left(1-\frac{\nu(1+\nu)}{2(1-\mu)}(z+1)-\frac{(1-\nu)\,\nu\,(1+\nu)\,(2+\nu)}{8(1-\mu)(2-\mu)}(z+1)^2+\ldots\right)\right)/;\left|\frac{z+1}{2}\right|<1\bigwedge\mu\notin\mathbb{Z}$$

07.11.06.0019.01

$$Q_\nu^\mu(z) ==$$
$$-\frac{1}{2\pi}\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\left(\Gamma(-\mu)\,\Gamma(\mu-\nu)\,\Gamma(\mu+\nu+1)\sin(\pi(\mu-\nu))\cos(\pi(\mu+\nu))\,(1+z)^\mu(1-z)^{-\mu}\sum_{k=0}^\infty\frac{(-\nu)_k\,(\nu+1)_k}{(\mu+1)_k\,k!}\left(\frac{z+1}{2}\right)^k+\right.$$
$$\left.\pi\cos(\pi\nu)\,\Gamma(\mu)\sum_{k=0}^\infty\frac{(-\nu)_k\,(\nu+1)_k}{(1-\mu)_k\,k!}\left(\frac{z+1}{2}\right)^k\right)/;\left|\frac{z+1}{2}\right|<1\bigwedge\mu\notin\mathbb{Z}$$

07.11.06.0020.01

$$Q_\nu^\mu(z) == \frac{1}{2\pi}\sin(\pi(\mu-\nu))\sin(\pi\nu)\sin(\pi(\mu+\nu))\csc(\pi\mu)\,\Gamma(\mu+\nu+1)\left(1-z^2\right)^{-\mu/2}$$
$$\left(\Gamma(\mu)\,\Gamma(-\mu-\nu)\,(\cos(\pi\mu)\csc(\pi(\mu-\nu))+\csc(\pi\nu))\,(1-z)^\mu\,{}_2F_1\!\left(-\nu,\,\nu+1;\,1-\mu;\,\frac{z+1}{2}\right)-\right.$$
$$\left.\Gamma(-\mu)\,\Gamma(\mu-\nu)\,(\cos(\pi\mu)\csc(\pi\nu)-\csc(\pi(\mu+\nu)))\,(z+1)^\mu\,{}_2F_1\!\left(-\nu,\,\nu+1;\,\mu+1;\,\frac{z+1}{2}\right)\right)/;\mu\notin\mathbb{Z}$$

07.11.06.0021.01

$$Q_\nu^\mu(z) == \frac{\pi\csc(\mu\pi)}{2}\left(\cos(\mu\pi)\left(\frac{\Gamma(-\mu)\,(z+1)^{\mu/2}\,2^{-\frac{\mu}{2}}}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\sum_{k=0}^\infty\sum_{j=0}^k\frac{(-\nu)_j\,(\nu+1)_j\left(\frac{\mu}{2}\right)_{k-j}2^{-k}\,(z+1)^k}{(\mu+1)_j\,j!\,(k-j)!}-\right.\right.$$
$$\left.\frac{2^{\mu/2}\sin(\pi\nu)\,\Gamma(\mu)}{\pi}(z+1)^{-\frac{\mu}{2}}\sum_{k=0}^\infty\sum_{j=0}^k\frac{(-\mu-\nu)_j\,(\nu-\mu+1)_j\left(\frac{\mu}{2}\right)_{k-j}2^{-k}\,(z+1)^k}{(1-\mu)_j\,j!\,(k-j)!}\right)-$$
$$(\nu-\mu+1)_{2\mu}\left(\frac{\Gamma(\mu)\,(z+1)^{-\frac{\mu}{2}}\,2^{\mu/2}}{\Gamma(\mu-\nu)\,\Gamma(\mu+\nu+1)}\sum_{k=0}^\infty\sum_{j=0}^k\frac{(-\nu)_j\,(\nu+1)_j\left(-\frac{\mu}{2}\right)_{k-j}2^{-k}\,(z+1)^k}{(1-\mu)_j\,j!\,(k-j)!}-\right.$$
$$\left.\left.\frac{2^{-\frac{\mu}{2}}\sin(\pi\nu)\,\Gamma(-\mu)}{\pi}(z+1)^{\mu/2}\sum_{k=0}^\infty\sum_{j=0}^k\frac{(\mu-\nu)_j\,(\mu+\nu+1)_j\left(-\frac{\mu}{2}\right)_{k-j}2^{-k}\,(z+1)^k}{(\mu+1)_j\,j!\,(k-j)!}\right)\right)/;\left|\frac{z+1}{2}\right|<1\bigwedge\mu\notin\mathbb{Z}\bigwedge\nu\notin\mathbb{Z}$$

07.11.06.0022.01

$$Q_\nu^\mu(z)\propto-\frac{2^{-\frac{\mu}{2}-1}\cos(\pi(\mu+\nu))\,\Gamma(-\mu)\,\Gamma(\mu+\nu+1)}{\Gamma(\nu-\mu+1)}(z+1)^{\mu/2}(1+O(z+1))-2^{\frac{\mu}{2}-1}\cos(\pi\nu)\,\Gamma(\mu)\,(z+1)^{-\frac{\mu}{2}}(1+O(z+1))\;/;$$
$$(z\to-1)\bigwedge\mu\notin\mathbb{Z}\bigwedge\nu\notin\mathbb{Z}$$

07.11.06.0023.01

$$Q_\nu^m(z) = -\frac{\cos(\pi\nu)}{2\,m!}\left(1-z^2\right)^{-\frac{m}{2}}\left((1-z)^m\,m!\sum_{k=0}^{m-1}\frac{(m-k-1)!\,(-\nu)_k\,(\nu+1)_k}{k!}\left(-\frac{z+1}{2}\right)^k + 2^{-m}\left(1-z^2\right)^m\sum_{k=0}^{\infty}\frac{(m-\nu)_k\,(m+\nu+1)_k\,\psi(k+1)}{k!\,(m+1)_k}\left(\frac{z+1}{2}\right)^k + (z+1)^m\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k\,\psi(k+m+1)}{k!\,(m+1)_k}\left(\frac{z+1}{2}\right)^k - \psi(m+1)\,(z+1)^m\;{}_2F_1\left(-\nu,\,\nu+1;\,m+1;\,\frac{z+1}{2}\right)\right)\;/;\;\left|\frac{z+1}{2}\right|<1\bigwedge m\in\mathbb{N}^+$$

07.11.06.0024.01

$$Q_\nu^m(z) \propto -\cos(\pi\nu)\,2^{\frac{m}{2}-1}\,(m-1)!\,(z+1)^{-\frac{m}{2}}\,(1+O(z+1))\;/;\;(z\to-1)\bigwedge m\in\mathbb{N}^+$$

07.11.06.0025.01

$$Q_\nu^0(z) = \frac{1}{2}\left(\cos(\pi\nu)\,(-\pi\cot(\pi\nu)-\log(1-z)+\log(z+1)+2\,\psi(-\nu))-\pi\csc(\pi\nu)\right)\,{}_2F_1\left(-\nu,\,\nu+1;\,1;\,\frac{z+1}{2}\right) - \cos(\pi\nu)\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k\,\psi(k+1)}{k!^2}\left(\frac{z+1}{2}\right)^k\;/;\;\left|\frac{z+1}{2}\right|<1\bigwedge\nu\notin\mathbb{Z}$$

07.11.06.0026.01

$$Q_\nu^0(z) \propto \frac{1}{2}\left(\cos(\pi\nu)\log(z+1)-\pi\csc(\pi\nu)+\cos(\pi\nu)\,(2\,\gamma-\pi\cot(\pi\nu)-\log(2)+2\,\psi(-\nu))\right)(O(z+1)+1)\;/;\;(z\to-1)\bigwedge\nu\notin\mathbb{Z}$$

07.11.06.0027.01

$$Q_n^0(z) = (-1)^n\left(\frac{1}{2}\left(\log(1+z)-\log(1-z)\right)+\psi(n+1)\right){}_2F_1\left(-n,\,n+1;\,1;\,\frac{z+1}{2}\right)-(-1)^n\sum_{k=0}^{n}\frac{(k+n)!\,\psi(k+1)}{k!^2\,(n-k)!}\left(-\frac{z+1}{2}\right)^k\;/;\;n\in\mathbb{N}$$

07.11.06.0028.01

$$Q_n^0(z) \propto (-1)^n\left(\frac{1}{2}\log\left(\frac{z+1}{2}\right)+\psi(n+1)+\gamma\right)(1+O(z+1))\;/;\;(z\to-1)\land n\in\mathbb{N}$$

Expansions at $z = \infty$

07.11.06.0029.01

$$Q_\nu^\mu(z) = 2^{-\nu-2}\,e^{i\pi\mu}\,z^{-\mu-\nu-1}\left(1-z^2\right)^{\mu/2}\sqrt{\pi}\,\sec(\pi\nu)\left(\frac{2^{2\nu+1}\,i\,\Gamma(\mu-\nu)\sin(\pi(\mu-\nu))\,z^{2\nu+1}}{\Gamma\left(\frac{1}{2}-\nu\right)}\left(1+\frac{(\mu-\nu)(1+\mu-\nu)}{2\,(1-2\nu)\,z^2}+\frac{(\mu-\nu)(1+\mu-\nu)(2+\mu-\nu)(3+\mu-\nu)}{8\left(3-8\nu+4\nu^2\right)z^4}+\ldots\right)+\frac{\cos(\pi(\mu+\nu))+e^{i\pi(\nu-\mu)}}{\Gamma\left(\nu+\frac{3}{2}\right)}\Gamma(\mu+\nu+1)\left(1+\frac{(1+\mu+\nu)(2+\mu+\nu)}{2\,(3+2\nu)\,z^2}+\frac{(1+\mu+\nu)(2+\mu+\nu)(3+\mu+\nu)(4+\mu+\nu)}{8\,(3+2\nu)(5+2\nu)\,z^4}+\ldots\right)\right)\;/;\;|z|>1$$

07.11.06.0030.01

$$Q_\nu^\mu(z) = 2^{-\nu-2} e^{i\pi\mu} z^{-\mu-\nu-1} \sqrt{\pi} \sec(\pi\nu) \left(1-z^2\right)^{\mu/2} \left(2^{2\nu+1} i\, \Gamma(\mu-\nu) \sin(\pi(\mu-\nu)) z^{2\nu+1} \sum_{k=0}^{\infty} \frac{\left(\frac{\mu-\nu}{2}\right)_k \left(\frac{\mu-\nu+1}{2}\right)_k z^{-2k}}{\Gamma\left(k-\nu+\frac{1}{2}\right) k!} + \left(\cos(\pi(\mu+\nu)) + e^{i\pi(\nu-\mu)}\right) \Gamma(\mu+\nu+1) \sum_{k=0}^{\infty} \frac{\left(\frac{\mu+\nu+1}{2}\right)_k \left(\frac{\mu+\nu+2}{2}\right)_k z^{-2k}}{\Gamma\left(k+\nu+\frac{3}{2}\right) k!}\right) /; |z| > 1$$

07.11.06.0031.01

$$Q_\nu^\mu(z) = 2^{-\nu-2} e^{i\pi\mu} z^{-\mu-\nu-1} \sqrt{\pi} \sec(\pi\nu) \left(1-z^2\right)^{\mu/2} \left(2^{2\nu+1} i \sin(\pi(\mu-\nu))\, \Gamma(\mu-\nu) z^{2\nu+1} {}_2\tilde{F}_1\left(\frac{\mu-\nu}{2}, \frac{\mu-\nu+1}{2}; \frac{1}{2}-\nu; \frac{1}{z^2}\right) + \left(\cos(\pi(\mu+\nu)) + e^{i\pi(\nu-\mu)}\right) \Gamma(\mu+\nu+1)\, {}_2\tilde{F}_1\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1; \nu+\frac{3}{2}; \frac{1}{z^2}\right)\right) /; \mathrm{Im}(z) > 0 \bigvee (z \bigwedge (-\infty, -1) \bigvee z \in (0, \infty))$$

07.11.06.0032.01

$$Q_\nu^\mu(z) = \frac{2^{-\nu-2} \sqrt{\pi} \csc(\pi\mu)}{\Gamma(-\mu-\nu)} (z-1)^{-\nu-1} \left(1-z^2\right)^{-\mu/2} \left(\Gamma\left(-\nu-\frac{1}{2}\right) (\csc(\pi(\mu+\nu)) \sin(\pi(\mu-\nu)) (-z-1)^\mu + \cos(\pi\mu)(1+z)^\mu) \sum_{k=0}^{\infty} \frac{(\nu+1)_k (\mu+\nu+1)_k}{(2(\nu+1))_k\, k!} \left(\frac{2}{1-z}\right)^k - \frac{2^{2\nu+1} \Gamma\left(\nu+\frac{1}{2}\right) \Gamma(-\mu-\nu)}{\Gamma(\nu-\mu+1)} (z-1)^{2\nu+1} ((-z-1)^\mu - \cos(\pi\mu)(1+z)^\mu) \sum_{k=0}^{\infty} \frac{(-\nu)_k (\mu-\nu)_k}{(-2\nu)_k\, k!} \left(\frac{2}{1-z}\right)^k\right) /; \left|\frac{1-z}{2}\right| > 1 \bigwedge \mu \notin \mathbb{Z}$$

07.11.06.0033.01

$$Q_\nu^\mu(z) = \frac{2^{-\nu-2} \sqrt{\pi} \csc(\pi\mu)}{\Gamma(-\mu-\nu)} (z-1)^{-\nu-1} \left(1-z^2\right)^{-\mu/2} \left(\Gamma\left(-\nu-\frac{1}{2}\right) (\csc(\pi(\mu+\nu)) \sin(\pi(\mu-\nu)) (-z-1)^\mu + \cos(\pi\mu)(1+z)^\mu)\, {}_2F_1\left(\nu+1, \mu+\nu+1; 2(\nu+1); \frac{2}{1-z}\right) - \frac{2^{2\nu+1} \Gamma\left(\nu+\frac{1}{2}\right) \Gamma(-\mu-\nu)}{\Gamma(\nu-\mu+1)} (z-1)^{2\nu+1} ((-z-1)^\mu - \cos(\pi\mu)(1+z)^\mu)\, {}_2F_1\left(-\nu, \mu-\nu; -2\nu; \frac{2}{1-z}\right)\right) /; z \notin (-1, 1) \wedge \mu \notin \mathbb{Z}$$

07.11.06.0034.01

$$Q_\nu^\mu(z) \propto \frac{2^{-\nu-2} \sqrt{\pi} \csc(\pi\mu)}{\Gamma(-\mu-\nu)} z^{-\nu-1} \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}} \left(\Gamma\left(-\nu-\frac{1}{2}\right) \left(\cos(\pi\mu) \frac{(1+z)^\mu}{(1-z)^\mu} + \csc(\pi(\mu+\nu)) \sin(\pi(\mu-\nu))\right) \left(1+O\left(\frac{1}{z}\right)\right) - \frac{2^{2\nu+1} \Gamma\left(\nu+\frac{1}{2}\right) \Gamma(-\mu-\nu)}{\Gamma(\nu-\mu+1)} z^{2\nu+1} \left(1-\cos(\pi\mu) \frac{(1+z)^\mu}{(1-z)^\mu}\right) \left(1+O\left(\frac{1}{z}\right)\right)\right) /; (|z| \to \infty) \bigwedge \mu \notin \mathbb{Z}$$

07.11.06.0035.01

$$Q_\nu^m(z) = -\frac{(-1)^m 2^{-\nu-2} \pi \csc(2\pi\nu)}{\Gamma(\nu+1)\Gamma(\nu-m+1)} (z-1)^\nu \frac{(1-z)^{m/2}}{(1+z)^{m/2}} (\log(1+z) - \log(1-z) - 2\psi(-m-\nu) - 2\psi(\nu-m+1))$$
$$\,_2\tilde{F}_1\left(-m-\nu, -\nu; -2\nu; \frac{2}{1-z}\right) + \frac{(-1)^m 2^{\nu-1} \pi \csc(2\pi\nu)}{\Gamma(-m-\nu)\Gamma(-\nu)} (z-1)^{-\nu-1} \frac{(1-z)^{m/2}}{(1+z)^{m/2}}$$
$$(\log(1+z) - \log(1-z) + 4\pi\cot(\pi\nu) - 2\psi(m-\nu) - 2\psi(m+\nu+1)) \,_2\tilde{F}_1\left(\nu+1, \nu-m+1; 2\nu+2; \frac{2}{1-z}\right) -$$
$$\frac{2^{-\nu-2}\pi\csc(2\pi\nu)}{\Gamma(\nu+1)\Gamma(\nu-m+1)} (z-1)^\nu \frac{(1+z)^{m/2}}{(1-z)^{m/2}} (\log(1+z) - \log(1-z) - 2\pi\cot(\pi\nu)) \,_2\tilde{F}_1\left(m-\nu, -\nu; -2\nu; \frac{2}{1-z}\right) +$$
$$\frac{2^{\nu-1}\pi\csc(2\pi\nu)}{\Gamma(-m-\nu)\Gamma(-\nu)} (z-1)^{-\nu-1} \frac{(1+z)^{m/2}}{(1-z)^{m/2}} (2\pi\cot(\pi\nu) + \log(1+z) - \log(1-z)) \,_2\tilde{F}_1\left(\nu+1, m+\nu+1; 2\nu+2; \frac{2}{1-z}\right) +$$
$$\frac{2^{\nu-1}\Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\nu-m+1)} (z-1)^\nu \frac{(1-z)^{m/2}}{(1+z)^{m/2}}$$
$$\sum_{k=0}^{\infty} \frac{(-\nu)_k}{k!\,(-2\nu)_k} \left((m-\nu)_k \psi(k+m-\nu) \left(\frac{1+z}{1-z}\right)^m + (-1)^m (-m-\nu)_k \psi(k-m-\nu)\right) \left(\frac{2}{1-z}\right)^k +$$
$$\frac{2^{-\nu-2}\Gamma\left(-\nu-\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(-m-\nu)} (z-1)^{-\nu-1} \frac{(1-z)^{m/2}}{(1+z)^{m/2}} \sum_{k=0}^{\infty} \frac{(\nu+1)_k}{k!\,(2\nu+2)_k}$$
$$\left((m+\nu+1)_k \psi(k+m+\nu+1) \left(\frac{1+z}{1-z}\right)^m + (-1)^m (\nu-m+1)_k \psi(k-m+\nu+1)\right) \left(\frac{2}{1-z}\right)^k /; \left|\frac{1-z}{2}\right| > 1 \bigwedge m \in \mathbb{N}$$

07.11.06.0036.01

$$Q_\nu^m(z) = \frac{2^{-\nu-2}}{\sqrt{\pi}\,\Gamma(-m-\nu)\Gamma(\nu-m+1)} (z-1)^{-\nu-1} \frac{(1+z)^{m/2}}{(1-z)^{m/2}}$$
$$\left(2^{2\nu+1}\Gamma(-m-\nu)\Gamma\left(\nu+\frac{1}{2}\right)(\log(1+z) - \log(-z-1)) (z-1)^{2\nu+1} \sum_{k=0}^{\infty} \frac{(-\nu)_k (m-\nu)_k}{(-2\nu)_k\, k!} \left(\frac{2}{1-z}\right)^k + \Gamma\left(-\nu-\frac{1}{2}\right)\right.$$
$$\left.\Gamma(\nu-m+1)(2\pi\cot(\pi\nu) + \log(1+z) - \log(-z-1)) \sum_{k=0}^{\infty} \frac{(\nu+1)_k (m+\nu+1)_k}{(2\nu+2)_k\, k!} \left(\frac{2}{1-z}\right)^k\right) /; \left|\frac{1-z}{2}\right| > 1 \bigwedge m \in \mathbb{N}$$

07.11.06.0037.01

$$Q_\nu^m(z) = \frac{2^{-\nu-2}}{\sqrt{\pi}\,\Gamma(-m-\nu)\Gamma(\nu-m+1)} (z-1)^{-\nu-1} \frac{(1+z)^{m/2}}{(1-z)^{m/2}}$$
$$\left(2^{2\nu+1}\Gamma(-m-\nu)\Gamma\left(\nu+\frac{1}{2}\right)(\log(1+z) - \log(-z-1)) (z-1)^{2\nu+1} \,_2F_1\left(-\nu, m-\nu; -2\nu; \frac{2}{1-z}\right) + \Gamma\left(-\nu-\frac{1}{2}\right)\Gamma(\nu-m+1)\right.$$
$$\left.(2\pi\cot(\pi\nu) + \log(1+z) - \log(-z-1)) \,_2F_1\left(\nu+1, m+\nu+1; 2\nu+2; \frac{2}{1-z}\right)\right) /; z \notin (-1, 1) \bigwedge m \in \mathbb{N}$$

07.11.06.0038.01

$$Q_\nu^m(z) \propto \frac{2^{-\nu-2}}{\sqrt{\pi}\,\Gamma(-m-\nu)\,\Gamma(\nu-m+1)}\, z^{-\nu-1}\, \frac{(1+z)^{m/2}}{(1-z)^{m/2}} \left(2^{2\nu+1}\,\Gamma(-m-\nu)\,\Gamma\left(\nu+\frac{1}{2}\right)(\log(1+z)-\log(1-z))\, z^{2\nu+1}\left(1+O\left(\frac{1}{z}\right)\right) + \Gamma\left(-\nu-\frac{1}{2}\right)\Gamma(\nu-m+1)\,(2\pi\cot(\pi\nu)+\log(1+z)-\log(1-z))\left(1+O\left(\frac{1}{z}\right)\right)\right) /;\ (|z|\to\infty) \land m\in\mathbb{N}$$

07.11.06.0039.01

$$Q_\nu^{-m}(z) = -\frac{2^{-\nu-2}\sin(\pi\nu)}{\pi^{3/2}}\,(z-1)^{-\nu-1}\,\frac{(1+z)^{m/2}}{(1-z)^{m/2}} \left(2^{2\nu+1}\,\Gamma(-m-\nu)\,\Gamma\left(\nu+\frac{1}{2}\right)(\log(1+z)-\log(-z-1))\,(z-1)^{2\nu+1}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(m-\nu)_k}{(-2\nu)_k\,k!}\left(\frac{2}{1-z}\right)^k + \Gamma\left(-\nu-\frac{1}{2}\right)\Gamma(\nu-m+1)\,(2\pi\cot(\pi\nu)+\log(1+z)-\log(-z-1))\sum_{k=0}^{\infty}\frac{(\nu+1)_k\,(m+\nu+1)_k}{(2\nu+2)_k\,k!}\left(\frac{2}{1-z}\right)^k\right) /;\ \left|\frac{1-z}{2}\right|>1 \bigwedge m\in\mathbb{N}^+$$

07.11.06.0040.01

$$Q_\nu^{-m}(z) = -\frac{2^{-\nu-2}\sin(\pi\nu)}{\pi^{3/2}}\,(z-1)^{-\nu-1}\,\frac{(1+z)^{m/2}}{(1-z)^{m/2}} \left(2^{2\nu+1}\,\Gamma(-m-\nu)\,\Gamma\left(\nu+\frac{1}{2}\right)(\log(1+z)-\log(-z-1))\,(z-1)^{2\nu+1}\,{}_2F_1\left(-\nu,\, m-\nu;\, -2\nu;\, \frac{2}{1-z}\right) + \Gamma\left(-\nu-\frac{1}{2}\right)\Gamma(\nu-m+1)\,(2\pi\cot(\pi\nu)+\log(1+z)-\log(-z-1))\,{}_2F_1\left(\nu+1,\, m+\nu+1;\, 2\nu+2;\, \frac{2}{1-z}\right)\right) /;\ z\notin(-1,\,1) \bigwedge m\in\mathbb{N}^+$$

07.11.06.0041.01

$$Q_\nu^{-m}(z) \propto -\frac{2^{-\nu-2}\sin(\pi\nu)}{\pi^{3/2}}\, z^{-\nu-1}\,\frac{(1+z)^{m/2}}{(1-z)^{m/2}} \left(2^{2\nu+1}\,\Gamma(-m-\nu)\,\Gamma\left(\nu+\frac{1}{2}\right)(\log(1+z)-\log(1-z))\, z^{2\nu+1}\left(1+O\left(\frac{1}{z}\right)\right) + \Gamma\left(-\nu-\frac{1}{2}\right)\Gamma(\nu-m+1)\,(2\pi\cot(\pi\nu)+\log(1+z)-\log(1-z))\left(1+O\left(\frac{1}{z}\right)\right)\right) /;\ (|z|\to\infty) \land m\in\mathbb{N}^+$$

Integral representations

On the real axis

Of the direct function

07.11.07.0001.01

$$Q_\nu^\mu(z) = \frac{\Gamma(\nu+1)}{2\,\Gamma(\nu-\mu+1)}\int_0^\infty\left(e^{-\frac{\mu\pi i}{2}}\left(z+\sqrt{z^2-1}\,\cosh(t)\right)^{-\nu-1} + e^{\frac{\mu\pi i}{2}}\left(z-\sqrt{z^2-1}\,\cosh(t)\right)^{-\nu-1}\right)\cosh(\mu t)\,dt /;$$
$$-1<z<1 \bigwedge \mathrm{Re}(\mu+\nu)>-1 \bigwedge \mathrm{Re}(\nu-\mu)>-1$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

07.11.13.0001.01

$$\left(1-z^2\right) w''(z)-2\,z\,w'(z)+\left(\nu\,(\nu+1)-\frac{\mu^2}{1-z^2}\right) w(z)=0\;/;\; w(z)=c_1\,P_\nu^\mu(z)+c_2\,Q_\nu^\mu(z)$$

07.11.13.0002.02

$$W_z\left(P_\nu^\mu(z),\,Q_\nu^\mu(z)\right)=\frac{\Gamma(\mu+\nu+1)}{\left(1-z^2\right)\Gamma(-\mu+\nu+1)}$$

07.11.13.0003.01

$$g'(z)\,w''(z)-\left(\frac{2\,g(z)\,g'(z)^2}{1-g(z)^2}+g''(z)\right)w'(z)-\frac{\left(\mu^2-\nu\,(\nu+1)\left(1-g(z)^2\right)\right)g'(z)^3}{\left(1-g(z)^2\right)^2}\,w(z)=0\;/;\; w(z)=c_1\,P_\nu^\mu(g(z))+c_2\,Q_\nu^\mu(g(z))$$

07.11.13.0004.01

$$W_z\left(P_\nu^\mu(g(z)),\,Q_\nu^\mu(g(z))\right)=\frac{\Gamma(\mu+\nu+1)\,g'(z)}{\left(1-g(z)^2\right)\Gamma(1-\mu+\nu)}$$

07.11.13.0005.01

$$g'(z)\,h(z)^2\,w''(z)-\left(\left(\frac{2\,g(z)\,g'(z)^2}{1-g(z)^2}+g''(z)\right)h(z)^2+2\,g'(z)\,h'(z)\,h(z)\right)w'(z)+\left(-\frac{\mu^2-\nu\,(\nu+1)\left(1-g(z)^2\right)}{\left(1-g(z)^2\right)^2}\,h(z)^2\,g'(z)^3+2\,h'(z)^2\,g'(z)+h(z)\left(h'(z)\left(\frac{2\,g(z)\,g'(z)^2}{1-g(z)^2}+g''(z)\right)-g'(z)\,h''(z)\right)\right)w(z)=0\;/;\; w(z)=c_1\,h(z)\,P_\nu^\mu(g(z))+c_2\,h(z)\,Q_\nu^\mu(g(z))$$

07.11.13.0006.01

$$W_z\left(h(z)\,P_\nu^\mu(g(z)),\,h(z)\,Q_\nu^\mu(g(z))\right)=\frac{\Gamma(\mu+\nu+1)\,h(z)^2\,g'(z)}{\left(1-g(z)^2\right)\Gamma(1-\mu+\nu)}$$

07.11.13.0007.01

$$z^2\,w''(z)-z\left(2\,s+\frac{r\left(a^2\,z^{2r}+1\right)}{1-a^2\,z^{2r}}-1\right)w'(z)+\left(-\frac{a^2\,r^2\left(\mu^2+\left(a^2\,z^{2r}-1\right)\nu\,(\nu+1)\right)z^{2r}}{\left(1-a^2\,z^{2r}\right)^2}+s^2+\frac{r\,s\left(a^2\,z^{2r}+1\right)}{1-a^2\,z^{2r}}\right)w(z)=0\;/;\; w(z)=c_1\,z^s\,P_\nu^\mu(a\,z^r)+c_2\,z^s\,Q_\nu^\mu(a\,z^r)$$

07.11.13.0008.01

$$W_z\left(z^s\,P_\nu^\mu(a\,z^r),\,z^s\,Q_\nu^\mu(a\,z^r)\right)=\frac{a\,r\,z^{r+2s-1}\,\Gamma(\mu+\nu+1)}{\left(1-a^2\,z^{2r}\right)\Gamma(-\mu+\nu+1)}$$

07.11.13.0009.01

$$w''(z)-\frac{a^2\,(\log(r)-2\log(s))\,r^{2z}+\log(r)+2\log(s)}{1-a^2\,r^{2z}}\,w'(z)+\left(-\frac{a^2\left(\mu^2-\left(1-a^2\,r^{2z}\right)\nu\,(\nu+1)\right)\log^2(r)\,r^{2z}}{\left(1-a^2\,r^{2z}\right)^2}+\log^2(s)+\frac{\left(a^2\,r^{2z}+1\right)\log(r)\log(s)}{1-a^2\,r^{2z}}\right)w(z)=0\;/;\; w(z)=c_1\,s^z\,P_\nu^\mu\left(a\,r^z\right)+c_2\,s^z\,Q_\nu^\mu(a\,r^z)$$

07.11.13.0010.01

$$W_z\left(s^z\,P_\nu^\mu(a\,r^z),\,s^z\,Q_\nu^\mu(a\,r^z)\right)=\frac{a\,r^z\,s^{2z}\,\Gamma(\mu+\nu+1)\log(r)}{\left(1-a^2\,r^{2z}\right)\Gamma(-\mu+\nu+1)}$$

Identities

Recurrence identities

Consecutive neighbors

07.11.17.0001.01

$$Q_\nu^\mu(z) = \frac{(2\nu+3)z}{\mu+\nu+1} Q_{\nu+1}^\mu(z) + \frac{\mu-\nu-2}{\mu+\nu+1} Q_{\nu+2}^\mu(z)$$

07.11.17.0002.01

$$Q_\nu^\mu(z) = \frac{(2\nu-1)z}{\nu-\mu} Q_{\nu-1}^\mu(z) - \frac{\mu+\nu-1}{\nu-\mu} Q_{\nu-2}^\mu(z)$$

07.11.17.0003.01

$$Q_\nu^\mu(z) = \frac{2(\mu+1)z}{(\mu(\mu+1)-\nu(1+\nu))\sqrt{1-z^2}} Q_\nu^{\mu+1}(z) + \frac{1}{\mu(\mu+1)-\nu(\nu+1)} Q_\nu^{\mu+2}(z)$$

07.11.17.0004.01

$$Q_\nu^\mu(z) = \frac{2(1-\mu)z}{\sqrt{1-z^2}} Q_\nu^{\mu-1}(z) + ((\mu-1)(\mu-2)-\nu(\nu+1)) Q_\nu^{\mu-2}(z)$$

Distant neighbors

07.11.17.0017.01

$$Q_\nu^\mu(z) = C_n(\nu,\mu,z) Q_{\nu+n}^\mu(z) + \frac{\mu-\nu-n-1}{n+\mu+\nu} C_{n-1}(\nu,\mu,z) Q_{\nu+n+1}^\mu(z) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{(2\nu+3)z}{\mu+\nu+1} \bigwedge C_n(\nu,\mu,z) = \frac{z(2n+2\nu+1)}{n+\mu+\nu} C_{n-1}(\nu,\mu,z) + \frac{\mu-\nu-n}{n+\mu+\nu-1} C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.11.17.0018.01

$$Q_\nu^\mu(z) = C_n(\nu,\mu,z) Q_{\nu-n}^\mu(z) - \frac{\mu+\nu-n}{\nu-\mu-n+1} C_{n-1}(\nu,\mu,z) Q_{\nu-n-1}^\mu(z) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{(2\nu-1)z}{\nu-\mu} \bigwedge C_n(\nu,\mu,z) = \frac{z(2n-2\nu-1)}{n+\mu-\nu-1} C_{n-1}(\nu,\mu,z) - \frac{\mu+\nu-n+1}{\nu-\mu-n+2} C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.11.17.0019.01

$$Q_\nu^\mu(z) = C_n(\nu,\mu,z) Q_\nu^{\mu+n}(z) + \frac{1}{(n+\mu-1)(n+\mu)-\nu(\nu+1)} C_{n-1}(\nu,\mu,z) Q_\nu^{\mu+n+1}(z) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{2(\mu+1)z}{(\mu(\mu+1)-\nu(1+\nu))\sqrt{1-z^2}} \bigwedge C_n(\nu,\mu,z) = \frac{2z(n+\mu)}{\sqrt{1-z^2}\,((n-1+\mu)(n+\mu)-\nu(1+\nu))} C_{n-1}(\nu,\mu,z) + \frac{1}{(n-2+\mu)(n-1+\mu)-\nu(1+\nu)} C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.11.17.0020.01

$$Q_\nu^\mu(z) = C_n(\nu,\mu,z) Q_\nu^{\mu-n}(z) + ((\mu-n-1)(\mu-n)-\nu(\nu+1)) C_{n-1}(\nu,\mu,z) Q_\nu^{\mu-n-1}(z) /; C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{2(1-\mu)z}{\sqrt{1-z^2}} \bigwedge C_n(\nu,\mu,z) = \frac{2z(n-\mu)}{\sqrt{1-z^2}} C_{n-1}(\nu,\mu,z) + ((\mu-n)(\mu-n+1)-\nu(\nu+1)) C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

07.11.17.0005.01

$$(\mu+\nu)\,Q_{\nu-1}^{\mu}(z)+(\nu-\mu+1)\,Q_{\nu+1}^{\mu}(z)=(2\,\nu+1)\,z\,Q_{\nu}^{\mu}(z)$$

07.11.17.0006.01

$$Q_{\nu}^{\mu}(z)=\frac{(\mu+\nu)\,Q_{\nu-1}^{\mu}(z)+(\nu-\mu+1)\,Q_{\nu+1}^{\mu}(z)}{(2\,\nu+1)\,z}$$

07.11.17.0007.01

$$(\mu\,(\mu-1)-\nu\,(\nu+1))\,Q_{\nu}^{\mu-1}(z)-Q_{\nu}^{\mu+1}(z)=\frac{2\,\mu\,z}{\sqrt{1-z^2}}\,Q_{\nu}^{\mu}(z)$$

07.11.17.0008.01

$$Q_{\nu}^{\mu}(z)=\frac{\sqrt{1-z^2}}{2\,z\,\mu}\left((\mu\,(\mu-1)-\nu\,(\nu+1))\,Q_{\nu}^{\mu-1}(z)-Q_{\nu}^{\mu+1}(z)\right)$$

07.11.17.0021.01

$$z\,(\mu+\nu+1)\,Q_{\nu}^{\mu}(z)+\sqrt{1-z}\,\sqrt{z+1}\,Q_{\nu}^{\mu+1}(z)-(-\mu+\nu+1)\,Q_{\nu+1}^{\mu}(z)=0$$

Pavlyk O. (2006)

07.11.17.0022.01

$$Q_{\nu}^{\mu+1}(z)-z\,Q_{\nu+1}^{\mu+1}(z)-\sqrt{1-z}\,\sqrt{z+1}\,(-\mu+\nu+1)\,Q_{\nu+1}^{\mu}(z)=0$$

Pavlyk O. (2006)

Relations of special kind

07.11.17.0009.01

$$Q_{-\nu-1}^{\mu}(z)=Q_{\nu}^{\mu}(z)+\cos(\pi\,\nu)\,\Gamma(\mu-\nu)\,\Gamma(\mu+\nu+1)\,P_{\nu}^{-\mu}(z)$$

07.11.17.0015.01

$$Q_{-\nu-1}^{\mu}(z)=\csc(\pi\,(\mu-\nu))\left(\pi\cos(\mu\,\pi)\cos(\nu\,\pi)\,P_{\nu}^{\mu}(z)-\sin((\mu+\nu)\,\pi)\,Q_{\nu}^{\mu}(z)\right)$$

07.11.17.0010.01

$$Q_{-\nu-1}^{m}(z)=Q_{\nu}^{m}(z)-\pi\cot(\nu\,\pi)\,P_{\nu}^{m}(z)\,/;\,m\in\mathbb{Z}\bigwedge\nu\notin\mathbb{Z}$$

07.11.17.0011.01

$$Q_{\nu}^{-\mu}(z)=\frac{\Gamma(\nu-\mu+1)}{2\,\Gamma(\mu+\nu+1)}\left(2\cos(\pi\,\mu)\,Q_{\nu}^{\mu}(z)+\pi\,\sin(\pi\,\mu)\,P_{\nu}^{\mu}(z)\right)$$

07.11.17.0012.01

$$Q_{\nu}^{-m}(z)=\frac{(-1)^m\,(\nu-m)!}{(\nu+m)!}\,Q_{\nu}^{m}(z)\,/;\,m\in\mathbb{Z}$$

07.11.17.0013.01

$$Q_{\nu}^{\mu}(-z)=\frac{\pi\csc(\pi\,\mu)}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\,Q_{\nu}^{-\mu}(z)+\csc(\pi\,\mu)\sin(\pi\,\nu)\,Q_{\nu}^{\mu}(z)\,/;\,\mu\notin\mathbb{Z}\bigwedge|z|<1$$

07.11.17.0016.01

$$Q_\nu^\mu(-z) = -\cos((\mu+\nu)\pi)\, Q_\nu^\mu(z) - \frac{\pi}{2}\sin((\mu+\nu)\pi)\, P_\nu^\mu(z)$$

07.11.17.0014.01

$$Q_n^m(-z) = (-1)^{m+n+1}\, Q_n^m(z) \,/;\, m \in \mathbb{N} \land n \in \mathbb{N} \land |z| < 1$$

Differentiation

Low-order differentiation

With respect to ν

07.11.20.0001.01

$$\frac{\partial Q_\nu^\mu(z)}{\partial \nu} = \left(\pi\cot(\pi\nu) - \csc^2(\pi\mu)\,(\psi(\nu-\mu+1) - \psi(\mu+\nu+1))\right) Q_\nu^\mu(z) + \cot(\pi\mu)\csc(\pi\mu)\,(\nu-\mu+1)_{2\mu}\,(\psi(\nu-\mu+1)-\psi(\mu+\nu+1))\, Q_\nu^{-\mu}(z) + \frac{\pi\csc(\mu\pi)}{2}\left((\nu-\mu+1)_{2\mu}\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k\,(\psi(k-\nu)-\psi(k+\nu+1))}{\Gamma(k+\mu+1)\,k!}\left(\frac{1-z}{2}\right)^k - \cos(\mu\pi)\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k\,(\psi(k-\nu)-\psi(k+\nu+1))}{\Gamma(k-\mu+1)\,k!}\left(\frac{1-z}{2}\right)^k\right) /;\, \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z} \bigwedge \nu \notin \mathbb{Z}$$

07.11.20.0002.01

$$\frac{\partial Q_\nu^\mu(z)}{\partial \nu} = \frac{\pi\csc(\mu\pi)}{2}\left(\cos(\mu\pi)\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\sum_{k=0}^{\infty}\frac{1}{\Gamma(k-\mu+1)\,k!}\left(\frac{1-z}{2}\right)^k\sum_{j=1}^{k}S_k^{(j)}\,\nu^j\sum_{r=1}^{k}(-1)^r\,S_k^{(r)}\left(\frac{j}{\nu}+\frac{r}{\nu+1}\right)(\nu+1)^r - (\nu-\mu+1)_{2\mu}\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\sum_{k=0}^{\infty}\frac{1}{\Gamma(k+\mu+1)\,k!}\left(\frac{1-z}{2}\right)^k\sum_{j=1}^{k}S_k^{(j)}\,\nu^j\sum_{r=1}^{k}(-1)^r\,S_k^{(r)}\left(\frac{j}{\nu}+\frac{r}{\nu+1}\right)(\nu+1)^r - (\nu-\mu+1)_{2\mu}\,(\psi(\mu+\nu+1)-\psi(\nu-\mu+1))\,P_\nu^{-\mu}(z)\right) /;\, \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.11.20.0003.01

$$\frac{\partial Q_\nu^\mu(z)}{\partial \nu} = \frac{2\nu+1}{4\,\Gamma(\nu-\mu+1)}\frac{(1-z)^{\mu/2+1}}{(1+z)^{\mu/2}}\left(\cos(\mu\pi)\,\Gamma(\mu-1)\,\Gamma(\nu-\mu+1)\frac{(1+z)^{\mu}}{(1-z)^{\mu}}\,F_{2\times0\times2}^{2\times1\times3}\left(\begin{matrix}1-\nu,\,\nu+2;\,1;\,1,\,-\nu,\,\nu+1; \\ 2,\,2-\mu;;\,\nu+2,\,1-\nu;\end{matrix}\,\frac{1-z}{2},\,\frac{1-z}{2}\right) + \Gamma(-\mu-1)\,\Gamma(\mu+\nu+1)\,F_{2\times0\times2}^{2\times1\times3}\left(\begin{matrix}1-\nu,\,\nu+2;\,1;\,1,\,-\nu,\,\nu+1; \\ 2,\,2+\mu;;\,\nu+2,\,1-\nu;\end{matrix}\,\frac{1-z}{2},\,\frac{1-z}{2}\right)\right) + \frac{\pi\csc(\mu\pi)}{2}(\nu-\mu+1)_{2\mu}\,(\psi(\nu-\mu+1)-\psi(\mu+\nu+1))\,P_\nu^{-\mu}(z) \,/;\, \mu \notin \mathbb{Z}$$

07.11.20.0004.01

$$\frac{\partial^2 Q_\nu^\mu(z)}{\partial \nu^2} = \frac{\pi \csc(\mu \pi)}{2} \Bigg(\cos(\mu \pi) \frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{k! \, \Gamma(k-\mu+1)} \Big(\psi(k-\nu)^2 - 2 (\pi \cot(\pi \nu) + \psi(k+\nu+1)) \psi(k-\nu) + \psi(k+\nu+1)^2 + 2 \pi \cot(\pi \nu) \psi(k+\nu+1) + \psi^{(1)}(k-\nu) + \psi^{(1)}(k+\nu+1) \Big) \left(\frac{1-z}{2}\right)^k - (\nu-\mu+1)_{2\mu} \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{k! \, \Gamma(k+\mu+1)} \Big(\psi(k-\nu)^2 - 2 (\pi \cot(\pi \nu) + \psi(k+\nu+1) - \psi(\nu-\mu+1) + \psi(\mu+\nu+1)) \psi(k-\nu) + \psi(k+\nu+1)^2 + 2 \psi(k+\nu+1) (\pi \cot(\pi \nu) - \psi(\nu-\mu+1) + \psi(\mu+\nu+1)) + \psi^{(1)}(k-\nu) + \psi^{(1)}(k+\nu+1) \Big) \left(\frac{1-z}{2}\right)^k - \pi^2 \cos(\pi \mu) P_\nu^\mu(z) - (\nu-\mu+1)_{2\mu} \Big((\psi(\nu-\mu+1) - \psi(\mu+\nu+1)) (\psi(\nu-\mu+1) - \psi(\mu+\nu+1) - 2 \pi \cot(\pi \nu)) - \pi^2 - \psi^{(1)}(\nu-\mu+1) + \psi^{(1)}(\mu+\nu+1) \Big) P_\nu^{-\mu}(z) \Bigg) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.11.20.0005.01

$$\frac{\partial^2 Q_\nu^\mu(z)}{\partial \nu^2} = \frac{\pi \csc(\mu \pi)}{2} \Bigg(\cos(\mu \pi) \frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k-\mu+1) \, k!} \left(\frac{1-z}{2}\right)^k \sum_{i=1}^{k} \nu^{i-2} S_k^{(i)} \sum_{r=1}^{k} (-1)^r (\nu+1)^{r-2} \Big((r-1) r \nu^2 + i^2 (\nu+1)^2 + ((2r-1)\nu - 1) i (\nu+1) \Big) S_k^{(r)} - (\nu-\mu+1)_{2\mu} \Bigg(\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\mu+1) \, k!} \left(\frac{1-z}{2}\right)^k \sum_{j=1}^{k} S_k^{(j)} \nu^j \sum_{r=1}^{k} (-1)^r S_k^{(r)} (\nu+1)^r \left(\frac{(r-1) r \nu^2 + j^2 (\nu+1)^2 + j (\nu+1) ((2r-1)\nu - 1)}{\nu^2 (\nu+1)^2} + 2 \left(\frac{j}{\nu} + \frac{r}{\nu+1} \right) (\psi(\mu+\nu+1) - \psi(\nu-\mu+1)) \right) + \Big((\psi(\nu-\mu+1) - \psi(\mu+\nu+1))^2 - \psi^{(1)}(\nu-\mu+1) + \psi^{(1)}(\mu+\nu+1) \Big) P_\nu^{-\mu}(z) \Bigg) \Bigg) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

With respect to μ

07.11.20.0006.01

$$\frac{\partial Q_\nu^\mu(z)}{\partial \mu} = \frac{\pi \csc(\pi \mu)}{2} \Bigg((\nu-\mu+1)_{2\mu} \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k \psi(k+\mu+1)}{\Gamma(k+\mu+1) \, k!} \left(\frac{1-z}{2}\right)^k + \cos(\pi \mu) \frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k \psi(k-\mu+1)}{\Gamma(k-\mu+1) \, k!} \left(\frac{1-z}{2}\right)^k - \left(\pi \sin(\pi \mu) - \frac{\cos(\pi \mu)}{2} (\log(1+z) - \log(1-z)) \right) P_\nu^\mu(z) - 2 \cos(\pi \mu) Q_\nu^\mu(z) + (\nu-\mu+1)_{2\mu} \left(\frac{1}{2} (\log(1+z) - \log(1-z)) - \psi(\nu-\mu+1) - \psi(\mu+\nu+1) \right) P_\nu^{-\mu}(z) \Bigg) /; \left|\frac{1-z}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.11.20.0007.01

$$\frac{\partial Q_\nu^\mu(z)}{\partial \mu} = \frac{\pi\,\nu\,(\nu+1)\,(z-1)}{4}\left(\frac{\cot(\mu\,\pi)}{(1-\mu)\,\Gamma(2-\mu)}\,\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\,F_{2\times0\times1}^{2\times1\times2}\left(\begin{matrix}1-\nu,\,\nu+2;\,1;\,1,\,1-\mu;\\ 2,\,2-\mu;;\,2-\mu;\end{matrix}\,\frac{1-z}{2},\,\frac{1-z}{2}\right)+\frac{\csc(\mu\,\pi)\,\Gamma(\mu+\nu+1)}{(\mu+1)\,\Gamma(\mu+2)\,\Gamma(-\mu+\nu+1)}\,\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\,F_{2\times0\times1}^{2\times1\times2}\left(\begin{matrix}1-\nu,\,\nu+2;\,1;\,1,\,\mu+1;\\ 2,\,2+\mu;;\,\mu+2;\end{matrix}\,\frac{1-z}{2},\,\frac{1-z}{2}\right)\right)+\frac{\pi}{4}\left(\cot(\mu\,\pi)\,(\log(1+z)-\log(1-z)+2\,\psi(1-\mu))-2\,\pi\csc^2(\mu\,\pi)\right)P_\nu^\mu(z)+\frac{\pi\csc(\mu\,\pi)\,\Gamma(\mu+\nu+1)}{4\,\Gamma(\nu-\mu+1)}\,(\log(1+z)-\log(1-z)+2\,(\pi\cot(\mu\,\pi)+\psi(\mu+1)-\psi(\nu-\mu+1)-\psi(\mu+\nu+1)))\,P_\nu^{-\mu}(z)\,/;\,\mu\notin\mathbb{Z}$$

07.11.20.0008.01

$$\frac{\partial^2 Q_\nu^\mu(z)}{\partial \mu^2} = \frac{\pi\csc^2(\pi\,\mu)}{4}\,\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k}{k!\,\Gamma(k-\mu+1)}\left(((\log(1+z)-\log(1-z))\sin(2\,\pi\,\mu)-4\,\pi)\,\psi(k-\mu+1)+\sin(2\,\pi\,\mu)\left(\psi(k-\mu+1)^2-\psi^{(1)}(k-\mu+1)\right)\right)\left(\frac{1-z}{2}\right)^k-\frac{\pi\csc(\pi\,\mu)}{2}\,(\nu-\mu+1)_{2\,\mu}\,\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k}{k!\,\Gamma(k+\mu+1)}\left((\log(1+z)-\log(1-z)+2\,\pi\cot(\pi\,\mu)-2\,\psi(-\mu+\nu+1)-2\,\psi(\mu+\nu+1))\,\psi(k+\mu+1)+\psi(k+\mu+1)^2-\psi^{(1)}(k+\mu+1)\right)\left(\frac{1-z}{2}\right)^k+\frac{\pi}{2}\left((\nu-\mu+1)_{2\,\mu}\csc^2(\pi\,\mu)\left(\sin(\pi\,\mu)\left(-\frac{1}{4}\,(\log(1+z)-\log(1-z))^2+\pi^2+(\log(1+z)-\log(1-z)-\psi(\nu-\mu+1)-\psi(\mu+\nu+1))\,(\psi(\nu-\mu+1)+\psi(\mu+\nu+1))+\psi^{(1)}(\nu-\mu+1)-\psi^{(1)}(\mu+\nu+1)\right)-\pi\cos(\pi\,\mu)\,(\log(1+z)-\log(1-z)-2\,\psi(\nu-\mu+1)-2\,\psi(\mu+\nu+1))-2\,\pi^2\csc(\pi\,\mu)\right)P_\nu^{-\mu}(z)+\left(\cot(\pi\,\mu)\left(2\,\pi^2\csc^2(\pi\,\mu)+\frac{1}{4}\,(\log(1+z)-\log(1-z))^2\right)-\pi\csc^2(\pi\,\mu)\,(\log(1+z)-\log(1-z))\right)P_\nu^\mu(z)\right)/;\,\left|\frac{1-z}{2}\right|<1\bigwedge\mu\notin\mathbb{Z}\bigwedge m\in\mathbb{N}^+$$

With respect to z

07.11.20.0009.01

$$\frac{\partial Q_\nu^\mu(z)}{\partial z} = \frac{1}{z^2-1}\left(z\,\nu\,Q_\nu^\mu(z)-(\mu+\nu)\,Q_{\nu-1}^\mu(z)\right)$$

07.11.20.0010.01

$$\frac{\partial^2 Q_\nu^\mu(z)}{\partial z^2} = \frac{2\,z\,(\mu+\nu)\,Q_{\nu-1}^\mu(z)+\left(\mu^2+\left((\nu-1)\,z^2-\nu-1\right)\nu\right)Q_\nu^\mu(z)}{\left(z^2-1\right)^2}$$

Symbolic differentiation

With respect to ν

07.11.20.0011.02

$$\frac{\partial^m Q_\nu^\mu(z)}{\partial \nu^m} = \frac{\pi \csc(\mu\pi)}{2}\Bigg(\cos(\mu\pi)\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}} \sum_{k=0}^{\infty}\frac{1}{\Gamma(k-\mu+1)\,k!}\left(\frac{1-z}{2}\right)^k \sum_{j=0}^{m}\binom{m}{j}\sum_{i=1}^{k} S_k^{(i)}\,(i-j+1)_j\,\nu^{i-j}\sum_{r=1}^{k}(-1)^r S_k^{(r)}\,(j-m+r+1)_{m-j}\,(\nu+1)^{j-m+r} - \frac{\partial^m (\nu-\mu+1)_{2\mu}}{\partial \nu^m} P_\nu^{-\mu}(z) - \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\sum_{p=1}^{m}\binom{m}{p}\frac{\partial^{m-p}(\nu-\mu+1)_{2\mu}}{\partial \nu^{m-p}}\sum_{k=0}^{\infty}\frac{1}{\Gamma(k+\mu+1)\,k!}\left(\frac{1-z}{2}\right)^k \sum_{j=0}^{p}\binom{p}{j}\sum_{i=1}^{k}S_k^{(i)}\,(i-j+1)_j\,\nu^{i-j}\sum_{r=1}^{k}(-1)^r S_k^{(r)}\,(j-p+r+1)_{p-j}\,(\nu+1)^{j-p+r}\Bigg)\;/;\;\left|\frac{1-z}{2}\right|<1\bigwedge \mu\notin\mathbb{Z}\bigwedge m\in\mathbb{N}$$

With respect to z

07.11.20.0012.02

$$\frac{\partial^n Q_\nu^\mu(z)}{\partial z^n} = \frac{(-1)^n \pi\csc(\mu\pi)}{2}\Bigg(\cos(\mu\pi)\,\Gamma\!\left(\frac{\mu}{2}+1\right)\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2+n}}\sum_{j=0}^{n}\binom{n}{j}\,{}_2\tilde{F}_1\!\left(-j,\frac{\mu}{2};\frac{\mu}{2}-j+1;\frac{z+1}{z-1}\right){}_3\tilde{F}_2\!\left(1,-\nu,\nu+1;j-n+1,1-\mu;\frac{1-z}{2}\right)\left(\frac{z-1}{z+1}\right)^j - (\nu-\mu+1)_{2\mu}\,\Gamma\!\left(1-\frac{\mu}{2}\right)\frac{(1-z)^{\mu/2-n}}{(1+z)^{\mu/2}}\sum_{j=0}^{n}\binom{n}{j}\,{}_2\tilde{F}_1\!\left(-j,-\frac{\mu}{2};1-j-\frac{\mu}{2};\frac{z+1}{z-1}\right){}_3\tilde{F}_2\!\left(1,-\nu,\nu+1;j-n+1,\mu+1;\frac{1-z}{2}\right)\left(\frac{z-1}{z+1}\right)^j\Bigg)\;/;\;n\in\mathbb{N}\bigwedge\mu\notin\mathbb{Z}$$

07.11.20.0014.01

$$\frac{\partial^m Q_\nu^\mu(z)}{\partial z^m} = \frac{\Gamma\!\left(1-\frac{\mu}{2}\right)\Gamma(\mu+\nu+1)}{\Gamma(1-\mu+\nu)}\sum_{k=0}^{m}\sum_{j=0}^{k}\frac{(-1)^j\,2^{2j-k}\,k!\binom{m}{k}\Gamma(1-k+m-\mu+\nu)}{(k-j)!\,(2j-k)!\,\Gamma\!\left(1-j-\frac{\mu}{2}\right)\Gamma(k-m+\mu+\nu+1)}\,z^{2j-k}\left(1-z^2\right)^{\frac{1}{2}(-2j+k-m)}\,Q_\nu^{k-m+\mu}(z)\;/;\;m\in\mathbb{N}$$

07.11.20.0015.01

$$\frac{\partial^m Q_\nu^\mu(z)}{\partial z^m} = \sqrt{\pi}\sum_{k=0}^{m}(-1)^{m-k}\,z^{-k}\left(1-z^2\right)^{\frac{k-m}{2}}\binom{m}{k}(-\mu-\nu)_{m-k}\,{}_3\tilde{F}_2\!\left(1,-k,\frac{\mu}{2};\frac{1-k}{2},1-\frac{k}{2};\frac{z^2}{z^2-1}\right)(1-\mu+\nu)_{m-k}\,Q_\nu^{k-m+\mu}(z)\;/;\;m\in\mathbb{N}$$

Fractional integro-differentiation

With respect to z

07.11.20.0013.01

$$\frac{\partial^\alpha Q_\nu^\mu(z)}{\partial z^\alpha} = \frac{\pi\csc(\mu\pi)}{2}\left(\frac{\mu\cos(\mu\pi)}{2} z^{1-\alpha}\sum_{k=0}^{\infty}(k+1)!\left(\frac{\mu}{2}+1\right)_k z^k\,{}_{1\times0\times2}^{1\times1\times3}\tilde{F}\left(\begin{matrix}k+2;-\frac{\mu}{2};-\nu,\nu+1,1-\frac{\mu}{2};\\ k-\alpha+2;;k+2,1-\mu;\end{matrix}-z,-\frac{z}{2}\right)+\right.$$
$$\Gamma\left(1-\frac{\mu}{2}\right)z^{-\alpha}\sum_{k=0}^{\infty}\left(-\frac{\mu}{2}\right)_k(-z)^k\,{}_{2\times1\times1}^{3\times1\times1}\tilde{F}\left(\begin{matrix}-\nu,\nu+1,1-\frac{\mu}{2};1;k+1;\\ 1,1-\mu;1-\frac{\mu}{2};k-\alpha+1;\end{matrix}\frac{1}{2},-\frac{z}{2}\right)-$$
$$(\nu-\mu+1)_{2\mu}\left(\Gamma\left(\frac{\mu}{2}+1\right)z^{-\alpha}\sum_{k=0}^{\infty}\left(\frac{\mu}{2}\right)_k(-z)^k\,{}_{2\times1\times1}^{3\times1\times1}\tilde{F}\left(\begin{matrix}-\nu,\nu+1,\frac{\mu}{2}+1;1;k+1;\\ 1,\mu+1;\frac{\mu}{2}+1;k-\alpha+1;\end{matrix}\frac{1}{2},-\frac{z}{2}\right)-\right.$$
$$\left.\left.\frac{\mu}{2}z^{1-\alpha}\sum_{k=0}^{\infty}(k+1)!\left(1-\frac{\mu}{2}\right)_k z^k\,{}_{1\times0\times2}^{1\times1\times3}\tilde{F}\left(\begin{matrix}k+2;\frac{\mu}{2};-\nu,\nu+1,\frac{\mu}{2}+1;\\ k-\alpha+2;;k+2,\mu+1;\end{matrix}-z,-\frac{z}{2}\right)\right)\right)/;\mu\notin\mathbb{Z}$$

Integration

Indefinite integration

Involving only one direct function

07.11.21.0001.01

$$\int Q_\nu^\mu(z)\,dz = \frac{\pi\csc(\mu\pi)}{2}\left(\frac{\mu\cos(\mu\pi)}{2}z^2\sum_{k=0}^{\infty}(k+1)!\left(\frac{\mu}{2}+1\right)_k z^k z^k\,{}_{1\times0\times2}^{1\times1\times3}\tilde{F}\left(\begin{matrix}k+2;-\frac{\mu}{2};-\nu,\nu+1,1-\frac{\mu}{2};\\ k+3;;k+2,1-\mu;\end{matrix}-z,-\frac{z}{2}\right)+\right.$$
$$\Gamma\left(1-\frac{\mu}{2}\right)z\sum_{k=0}^{\infty}\left(-\frac{\mu}{2}\right)_k(-z)^k\,{}_{2\times1\times1}^{3\times1\times1}\tilde{F}\left(\begin{matrix}-\nu,\nu+1,1-\frac{\mu}{2};1;k+1;\\ 1,1-\mu;1-\frac{\mu}{2};k+2;\end{matrix}\frac{1}{2},-\frac{z}{2}\right)+$$
$$(\nu-\mu+1)_{2\mu}\left(\frac{\mu}{2}z^2\sum_{k=0}^{\infty}(k+1)!\left(1-\frac{\mu}{2}\right)_k z^k\,{}_{1\times0\times2}^{1\times1\times3}\tilde{F}\left(\begin{matrix}k+2;\frac{\mu}{2};-\nu,\nu+1,1+\frac{\mu}{2};\\ k+3;;k+2,1+\mu;\end{matrix}-z,-\frac{z}{2}\right)-\right.$$
$$\left.\left.z\,\Gamma\left(\frac{\mu}{2}+1\right)\sum_{k=0}^{\infty}\left(\frac{\mu}{2}\right)_k(-z)^k\,{}_{2\times1\times1}^{3\times1\times1}\tilde{F}\left(\begin{matrix}-\nu,\nu+1,1+\frac{\mu}{2};1;k+1;\\ 1,1+\mu;1+\frac{\mu}{2};k+2;\end{matrix}\frac{1}{2},-\frac{z}{2}\right)\right)\right)/;\mu\notin\mathbb{Z}$$

Operations

Limit operation

07.11.25.0001.01

$$\lim_{\nu\to\infty}\nu^{-\mu}Q_\nu^\mu\left(\cos\left(\frac{z}{\nu}\right)\right) = -\frac{\pi}{2}Y_{-\mu}(z)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_2\tilde{F}_1$

07.11.26.0001.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\cos(\mu\pi)\,{}_2\tilde{F}_1\!\left(-\nu,\,\nu+1;\,1-\mu;\,\frac{1-z}{2}\right) - \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}(\nu-\mu+1)_{2\mu}\,{}_2\tilde{F}_1\!\left(-\nu,\,\nu+1;\,\mu+1;\,\frac{1-z}{2}\right)\right)/;\ \mu\notin\mathbb{Z}$$

07.11.26.0028.01

$$Q_\nu^\mu(z) = \frac{1}{2}\sin(\pi(\mu-\nu))\sin(\pi\nu)\sin(\pi(\mu+\nu))\csc^2(\pi\mu)\,\Gamma(\mu+\nu+1)\left(1-z^2\right)^{-\frac{\mu}{2}}\Bigg((\cos(\pi\mu)\csc(\pi(\mu-\nu))+\csc(\pi\nu))\,\Gamma(-\mu-\nu)\,(1-z)^\mu\,{}_2\tilde{F}_1\!\left(-\nu,\,\nu+1;\,1-\mu;\,\frac{z+1}{2}\right) + (\cos(\pi\mu)\csc(\pi\nu)-\csc(\pi(\mu+\nu)))\,\Gamma(\mu-\nu)\,(z+1)^\mu\,{}_2\tilde{F}_1\!\left(-\nu,\,\nu+1;\,\mu+1;\,\frac{z+1}{2}\right)\Bigg)/;\ \mu\notin\mathbb{Z}$$

07.11.26.0002.01

$$Q_\nu^\mu(z) = \frac{2^{-\nu-1}\pi\csc(\pi\mu)}{\Gamma(\nu-\mu+1)}(1+z)^{\nu-\mu/2}(1-z)^{\mu/2}\left(\cos(\pi\mu)\,\Gamma(\nu-\mu+1)\,\frac{(1+z)^\mu}{(1-z)^\mu}\,{}_2\tilde{F}_1\!\left(-\nu,\,-\mu-\nu;\,1-\mu;\,\frac{z-1}{z+1}\right) - \Gamma(\mu+\nu+1)\,{}_2\tilde{F}_1\!\left(-\nu,\,\mu-\nu;\,\mu+1;\,\frac{z-1}{z+1}\right)\right)/;\ \mu\notin\mathbb{Z}\bigwedge z\notin(-\infty,\,-1)$$

07.11.26.0029.01

$$Q_\nu^\mu(z) = \frac{2^{-\nu-2}\sqrt{\pi}\,\csc(\pi\mu)\,(z-1)^{-\nu-1}}{\Gamma(-\mu-\nu)\left(1-z^2\right)^{\mu/2}}\Bigg(\Gamma\!\left(-\nu-\frac{1}{2}\right)\Gamma(2\nu+2)\,(\csc(\pi(\mu+\nu))\sin(\pi(\mu-\nu))(-z-1)^\mu+(z+1)^\mu\cos(\pi\mu))\,{}_2\tilde{F}_1\!\left(\nu+1,\,\mu+\nu+1;\,2(\nu+1);\,\frac{2}{1-z}\right) - \frac{2^{2\nu+1}}{\Gamma(-\mu+\nu+1)}\Gamma\!\left(\nu+\frac{1}{2}\right)\Gamma(-\mu-\nu)\,\Gamma(-2\nu)\,(z-1)^{2\nu+1}((-z-1)^\mu-\cos(\pi\mu)(z+1)^\mu)\,{}_2\tilde{F}_1\!\left(-\nu,\,\mu-\nu;\,-2\nu;\,\frac{2}{1-z}\right)\Bigg)/;\ \mu\notin\mathbb{Z}\bigwedge z\notin(-1,\,1)$$

07.11.26.0030.01

$$Q_\nu^\mu(z) = 2^\mu\pi\left(1-z^2\right)^{-\frac{\mu}{2}}\left(\frac{\cos\!\left(\frac{1}{2}(\mu+\nu)\pi\right)\Gamma\!\left(\frac{\mu+\nu}{2}+1\right)z}{2\,\Gamma\!\left(\frac{1}{2}(-\mu+\nu+1)\right)}\,{}_2\tilde{F}_1\!\left(\frac{1}{2}(-\mu-\nu+1),\,\frac{\nu-\mu}{2}+1;\,\frac{3}{2};\,z^2\right) - \frac{\sin\!\left(\frac{1}{2}(\mu+\nu)\pi\right)\Gamma\!\left(\frac{1}{2}(\mu+\nu+1)\right)}{2\,\Gamma\!\left(\frac{1}{2}(-\mu+\nu+2)\right)}\,{}_2\tilde{F}_1\!\left(\frac{1}{2}(-\mu-\nu),\,\frac{1}{2}(-\mu+\nu+1);\,\frac{1}{2};\,z^2\right)\right)$$

07.11.26.0031.01

$$Q_\nu^\mu(z) = \frac{1}{z\,\Gamma(-\mu-\nu)} \left(2^{-\nu-3}\sqrt{\pi}\,\left(-z^2\right)^{\frac{1}{2}(\mu-\nu-1)} \left(1-z^2\right)^{-\frac{\mu}{2}} \sec(\pi\,\nu) \left(1-\frac{1}{z^2}\right)^{\mu} \sin(\pi\,(\mu-\nu))\right.$$
$$\left(4^{\nu+1}\,z\,\Gamma(-\mu-\nu)\,\Gamma(\mu-\nu)\left(z\cos\left(\frac{1}{2}\pi\,(\mu+\nu)\right)-\sqrt{-z^2}\,\sin\left(\frac{1}{2}\pi\,(\mu+\nu)\right)\right)\left(-z^2\right)^{\nu}\,{}_2\tilde{F}_1\left(\frac{\mu-\nu}{2},\,\frac{1}{2}(\mu-\nu+1);\,\frac{1}{2}-\nu;\,\frac{1}{z^2}\right)+\right.$$
$$\pi\left(z\sec\left(\frac{1}{2}\pi\,(\mu-\nu)\right)\tan\left(\frac{1}{2}\pi\,(\mu+\nu)\right)-\sqrt{-z^2}\,\cot\left(\frac{1}{2}\pi\,(\mu+\nu)\right)\csc\left(\frac{1}{2}\pi\,(\mu-\nu)\right)\right)$$
$$\left.\left.{}_2\tilde{F}_1\left(\frac{1}{2}(\mu+\nu+1),\,\frac{1}{2}(\mu+\nu+2);\,\nu+\frac{3}{2};\,\frac{1}{z^2}\right)\right)\right)\,/;\,z\notin(-1,\,1)$$

Involving $_2F_1$

07.11.26.0004.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\,\pi)}{2}$$
$$\left(\frac{\cos(\mu\,\pi)}{\Gamma(1-\mu)}\,\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\,{}_2F_1\left(-\nu,\,\nu+1;\,1-\mu;\,\frac{1-z}{2}\right)-\frac{(\nu-\mu+1)_{2\mu}}{\Gamma(\mu+1)}\,\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\,{}_2F_1\left(-\nu,\,\nu+1;\,\mu+1;\,\frac{1-z}{2}\right)\right)\,/;\,\mu\notin\mathbb{Z}$$

07.11.26.0005.01

$$Q_\nu^\mu(z) = \frac{\sin(\pi\,(\mu-\nu))\sin(\pi\,\nu)\sin(\pi\,(\mu+\nu))\csc(\pi\,\mu)}{2\,\pi}\,\Gamma(\mu+\nu+1)\left(1-z^2\right)^{-\mu/2}$$
$$\left(\Gamma(\mu)\,\Gamma(-\mu-\nu)\,(1-z)^{\mu}\,(\cos(\pi\,\mu)\csc(\pi\,(\mu-\nu))+\csc(\pi\,\nu))\,{}_2F_1\left(-\nu,\,\nu+1;\,1-\mu;\,\frac{z+1}{2}\right)-\right.$$
$$\left.\Gamma(-\mu)\,\Gamma(\mu-\nu)\,(1+z)^{\mu}\,(\cos(\pi\,\mu)\csc(\pi\,\nu)-\csc(\pi\,(\mu+\nu)))\,{}_2F_1\left(-\nu,\,\nu+1;\,\mu+1;\,\frac{z+1}{2}\right)\right)\,/;\,\mu\notin\mathbb{Z}$$

07.11.26.0032.01

$$Q_\nu^\mu(z) = \frac{2^{-\nu-1}}{\mu\,\Gamma(\nu-\mu+1)}\,(z+1)^{\nu-\frac{\mu}{2}}\,(1-z)^{\mu/2}\left(\cos(\pi\,\mu)\,\Gamma(\mu+1)\,\Gamma(-\mu+\nu+1)\,\frac{(z+1)^{\mu}}{(1-z)^{\mu}}\,{}_2F_1\left(-\nu,\,-\mu-\nu;\,1-\mu;\,\frac{z-1}{z+1}\right)-\right.$$
$$\left.\Gamma(\mu+\nu+1)\,\Gamma(1-\mu)\,{}_2F_1\left(-\nu,\,\mu-\nu;\,\mu+1;\,\frac{z-1}{z+1}\right)\right)\,/;\,\mu\notin\mathbb{Z}\bigwedge z\notin(-\infty,\,-1)$$

07.11.26.0006.01

$$Q_\nu^\mu(z) = \frac{2^{-\nu-2}\sqrt{\pi}\,\csc(\pi\,\mu)}{\Gamma(-\mu-\nu)}\,\frac{(z-1)^{-\nu-1}}{\left(1-z^2\right)^{\mu/2}}$$
$$\left(\Gamma\left(-\nu-\frac{1}{2}\right)(\csc(\pi\,(\mu+\nu))\sin(\pi\,(\mu-\nu))\,(-z-1)^{\mu}+\cos(\pi\,\mu)\,(1+z)^{\mu})\,{}_2F_1\left(\nu+1,\,\mu+\nu+1;\,2\,(\nu+1);\,\frac{2}{1-z}\right)-\right.$$
$$\left.\frac{2^{2\,\nu+1}\,\Gamma\left(\nu+\frac{1}{2}\right)\Gamma(-\mu-\nu)}{\Gamma(\nu-\mu+1)}\,(z-1)^{2\,\nu+1}\,((-z-1)^{\mu}-\cos(\pi\,\mu)\,(1+z)^{\mu})\,{}_2F_1\left(-\nu,\,\mu-\nu;\,-2\,\nu;\,\frac{2}{1-z}\right)\right)\,/;\,\mu\notin\mathbb{Z}\bigwedge z\notin(-1,\,1)$$

07.11.26.0007.01

$$Q_\nu^m(z) = \frac{2^{-\nu-2}}{\sqrt{\pi}\,\Gamma(-m-\nu)\,\Gamma(\nu-m+1)} (z-1)^{-\nu-1} \frac{(1+z)^{m/2}}{(1-z)^{m/2}} \left(2^{2\nu+1}\,\Gamma(-m-\nu)\,\Gamma\left(\nu+\frac{1}{2}\right)(\log(1+z)-\log(-z-1))\,(z-1)^{2\nu+1}\,{}_2F_1\left(-\nu,\, m-\nu;\, -2\nu;\, \frac{2}{1-z}\right) + \Gamma\left(-\nu-\frac{1}{2}\right)\Gamma(\nu-m+1)\right.$$
$$\left.(2\pi\cot(\pi\nu)-\log(-z-1)+\log(1+z))\,{}_2F_1\left(\nu+1,\, m+\nu+1;\, 2\nu+2;\, \frac{2}{1-z}\right)\right) /;\ m\in\mathbb{N} \bigwedge z\notin(-1,\,1)$$

07.11.26.0008.01

$$Q_\nu^{-m}(z) = -\frac{2^{-\nu-2}\sin(\pi\nu)}{\pi^{3/2}} (z-1)^{-\nu-1} \frac{(1+z)^{m/2}}{(1-z)^{m/2}} \left(2^{2\nu+1}\,\Gamma(-m-\nu)\,\Gamma\left(\nu+\frac{1}{2}\right)(\log(1+z)-\log(-z-1))\,(z-1)^{2\nu+1}\,{}_2F_1\left(-\nu,\, m-\nu;\, -2\nu;\, \frac{2}{1-z}\right) + \Gamma\left(-\nu-\frac{1}{2}\right)\Gamma(\nu-m+1)\right.$$
$$\left.(2\pi\cot(\pi\nu)-\log(-z-1)+\log(1+z))\,{}_2F_1\left(\nu+1,\, m+\nu+1;\, 2\nu+2;\, \frac{2}{1-z}\right)\right) /;\ m\in\mathbb{N}^+ \bigwedge z\notin(-1,\,1)$$

07.11.26.0033.01

$$Q_\nu^\mu(z) = 2^\mu\sqrt{\pi}\,\left(1-z^2\right)^{-\frac{\mu}{2}} \left(\frac{\cos\left(\frac{1}{2}(\mu+\nu)\pi\right)\Gamma\left(\frac{\mu+\nu}{2}+1\right)z}{\Gamma\left(\frac{1}{2}(-\mu+\nu+1)\right)}\,{}_2F_1\left(\frac{1}{2}(-\mu-\nu+1),\, \frac{\nu-\mu}{2}+1;\, \frac{3}{2};\, z^2\right) - \right.$$
$$\left.\frac{\sin\left(\frac{1}{2}(\mu+\nu)\pi\right)\Gamma\left(\frac{1}{2}(\mu+\nu+1)\right)}{2\,\Gamma\left(\frac{1}{2}(-\mu+\nu+2)\right)}\,{}_2F_1\left(\frac{1}{2}(-\mu-\nu),\, \frac{1}{2}(-\mu+\nu+1);\, \frac{1}{2};\, z^2\right)\right)$$

07.11.26.0034.01

$$Q_\nu^\mu(z) = \frac{2^{-\nu-2}\sin(\pi(\mu-\nu))}{z\sqrt{\pi}\,(2\nu+1)\,\Gamma(-\mu-\nu)} \left(-z^2\right)^{\frac{1}{2}(\mu-\nu-1)} \left(1-z^2\right)^{-\frac{\mu}{2}} \left(1-\frac{1}{z^2}\right)^\mu$$
$$\left(4^{\nu+1}\,z\,\Gamma(-\mu-\nu)\,\Gamma(\mu-\nu)\,\Gamma\left(\nu+\frac{3}{2}\right)\left(z\cos\left(\frac{1}{2}\pi(\mu+\nu)\right)-\sqrt{-z^2}\,\sin\left(\frac{1}{2}\pi(\mu+\nu)\right)\right)\left(-z^2\right)^\nu\right.$$
$${}_2F_1\left(\frac{\mu-\nu}{2},\, \frac{1}{2}(\mu-\nu+1);\, \frac{1}{2}-\nu;\, \frac{1}{z^2}\right) + \pi\,\Gamma\left(\frac{1}{2}-\nu\right)\left(z\sec\left(\frac{1}{2}\pi(\mu-\nu)\right)\tan\left(\frac{1}{2}\pi(\mu+\nu)\right) - \right.$$
$$\left.\left.\sqrt{-z^2}\,\cot\left(\frac{1}{2}\pi(\mu+\nu)\right)\csc\left(\frac{1}{2}\pi(\mu-\nu)\right)\right){}_2F_1\left(\frac{1}{2}(\mu+\nu+1),\, \frac{1}{2}(\mu+\nu+2);\, \nu+\frac{3}{2};\, \frac{1}{z^2}\right)\right) /;\ z\notin(-1,\,1)$$

Through Meijer *G*

Classical cases for the direct function itself

07.11.26.0009.01

$$Q_\nu^\mu(z) = -\frac{\sin(\pi\nu)\csc(\pi\mu)}{2}\left(\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\cos(\pi\mu)\,G_{2,2}^{1,2}\left(\frac{z-1}{2}\,\middle|\,\begin{matrix}\nu+1,\,-\nu\\ 0,\,\mu\end{matrix}\right) - \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}(\nu-\mu+1)_{2\mu}\,G_{2,2}^{1,2}\left(\frac{z-1}{2}\,\middle|\,\begin{matrix}\nu+1,\,-\nu\\ 0,\,-\mu\end{matrix}\right)\right) /;$$
$$\nu\notin\mathbb{Z}\bigwedge\mu\notin\mathbb{Z}$$

07.11.26.0010.01

$$Q_\nu^\mu(2z+1) = -\frac{\sin(\pi\nu)\csc(\pi\mu)}{2}\left(\frac{(1+z)^{\mu/2}}{(-z)^{\mu/2}}\cos(\pi\mu)\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\nu+1,\,-\nu\\ 0,\,\mu\end{matrix}\right) - \frac{(-z)^{\mu/2}}{(1+z)^{\mu/2}}(\nu-\mu+1)_{2\mu}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\nu+1,\,-\nu\\ 0,\,-\mu\end{matrix}\right)\right) /;$$
$$\nu\notin\mathbb{Z}\bigwedge\mu\notin\mathbb{Z}$$

Classical cases involving algebraic functions

07.11.26.0011.01

$$(z+1)^{\nu}\,Q_{\nu}^{\mu}\left(\frac{1-z}{1+z}\right)=\frac{\pi\csc(\pi\mu)}{2\,\Gamma(-\nu)\,\Gamma(-\mu-\nu)}\left(\cos(\pi\mu)\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{\mu}{2}+\nu+1,\,1-\frac{\mu}{2}+\nu\\-\frac{\mu}{2},\,\frac{\mu}{2}\end{matrix}\right)+\frac{\sin(\pi(\mu-\nu))}{\sin(\pi(\mu+\nu))}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{\mu}{2}+\nu+1,\,1-\frac{\mu}{2}+\nu\\\frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)\right)/;\ \nu\notin\mathbb{Z}\land\mu\notin\mathbb{Z}\land z\notin(-\infty,0)$$

07.11.26.0012.01

$$(z+1)^{\nu}\,Q_{\nu}^{\mu}\left(\frac{z-1}{z+1}\right)=\frac{\pi\csc(\pi\mu)}{2\,\Gamma(-\nu)\,\Gamma(-\mu-\nu)}\left(\cos(\pi\mu)\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{\mu}{2}+\nu+1,\,1-\frac{\mu}{2}+\nu\\\frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)+\frac{\sin(\pi(\mu-\nu))}{\sin(\pi(\mu+\nu))}\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}1-\frac{\mu}{2}+\nu,\,\frac{\mu}{2}+\nu+1\\\frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)\right)/;\ \nu\notin\mathbb{Z}\land\mu\notin\mathbb{Z}\land z\notin(-1,0)$$

07.11.26.0013.01

$$(z+1)^{-\nu-1}\,Q_{\nu}^{\mu}\left(\frac{1-z}{1+z}\right)=\frac{\pi\csc(\pi\mu)}{2\,\Gamma(\nu+1)\,\Gamma(1-\mu+\nu)}\left(\cos(\pi\mu)\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{\mu}{2}-\nu,\,-\frac{\mu}{2}-\nu\\-\frac{\mu}{2},\,\frac{\mu}{2}\end{matrix}\right)-G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{\mu}{2}-\nu,\,-\frac{\mu}{2}-\nu\\\frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)\right)/;$$
$$\nu\notin\mathbb{Z}\land\mu\notin\mathbb{Z}\land z\notin(-\infty,0)$$

07.11.26.0014.01

$$(z+1)^{-\nu-1}\,Q_{\nu}^{\mu}\left(\frac{z-1}{z+1}\right)=\frac{\pi\csc(\pi\mu)}{2\,\Gamma(\nu+1)\,\Gamma(1-\mu+\nu)}\left(\cos(\pi\mu)\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{\mu}{2}-\nu,\,-\frac{\mu}{2}-\nu\\\frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)-G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}-\frac{\mu}{2}-\nu,\,\frac{\mu}{2}-\nu\\\frac{\mu}{2},\,-\frac{\mu}{2}\end{matrix}\right)\right)/;$$
$$\nu\notin\mathbb{Z}\land\mu\notin\mathbb{Z}\land z\notin(-1,0)$$

07.11.26.0015.01

$$(z+1)^{\nu/2}\,Q_{\nu}^{\mu}\left(\frac{1}{\sqrt{z+1}}\right)=-\frac{\Gamma(\mu+\nu+1)}{2^{\nu+2}\sqrt{\pi}}\left(\cos(\pi(\mu+\nu))\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\,\frac{\nu+1}{2}\\-\frac{\mu}{2},\,\frac{\mu}{2}\end{matrix}\right)+G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\,\frac{\nu+1}{2},\,\nu-\frac{\mu}{2}\\\frac{\mu}{2},\,-\frac{\mu}{2},\,\nu-\frac{\mu}{2}\end{matrix}\right)\right)/;$$
$$\nu\notin\mathbb{Z}\land\mu\notin\mathbb{Z}\land z\notin(-\infty,0)$$

07.11.26.0016.01

$$(z+1)^{\nu/2}\,Q_{\nu}^{\mu}\left(\sqrt{\frac{z}{z+1}}\right)=-\frac{\Gamma(\mu+\nu+1)}{2^{\nu+2}\sqrt{\pi}}\left(\cos(\pi(\mu+\nu))\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{\mu+\nu}{2}+1,\,\frac{\nu-\mu}{2}+1\\0,\,\frac{1}{2}\end{matrix}\right)+G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{\nu-\mu}{2}+1,\,\frac{\mu+\nu}{2}+1,\,\frac{\mu-\nu}{2}+1\\0,\,\frac{1}{2},\,\frac{\mu-\nu}{2}+1\end{matrix}\right)\right)/;$$
$$\nu\notin\mathbb{Z}\land\mu\notin\mathbb{Z}\land z\notin(-\infty,0)$$

07.11.26.0017.01

$$(z+1)^{-\frac{\nu+1}{2}}\,Q_{\nu}^{\mu}\left(\frac{1}{\sqrt{z+1}}\right)=\frac{\sqrt{\pi}}{2^{1-\nu}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}-\frac{\nu}{2},\,\frac{1-\nu}{2},\,\frac{\mu+1}{2}\\\frac{\mu}{2},\,-\frac{\mu}{2},\,\frac{\mu+1}{2}\end{matrix}\right)/;\ \nu\notin\mathbb{Z}\land\mu\notin\mathbb{Z}\land z\notin(-\infty,-1)$$

07.11.26.0018.01

$$(z+1)^{-\frac{\nu+1}{2}}\,Q_{\nu}^{\mu}\left(\sqrt{\frac{z}{z+1}}\right)=\frac{2^{\nu-1}\sqrt{\pi}}{\Gamma(1-\mu+\nu)}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{1+\mu-\nu}{2},\,\frac{1-\mu-\nu}{2},\,-\frac{\mu+\nu}{2}\\0,\,\frac{1}{2},\,-\frac{\mu+\nu}{2}\end{matrix}\right)/;\ \nu\notin\mathbb{Z}\land\mu\notin\mathbb{Z}\land z\notin(-\infty,0)$$

Classical cases involving unit step θ

07.11.26.0019.01

$$\frac{\theta(1-|z|)}{\sqrt{1-z}}\left(Q_\nu^\mu\left(-\sqrt{1-z}\right)+Q_\nu^\mu\left(\sqrt{1-z}\right)\right) = -\frac{2^\mu \pi^2 \tan\left(\frac{1}{2}(\mu+\nu)\pi\right)}{\Gamma\left(\frac{1-\mu-\nu}{2}\right)\Gamma\left(\frac{\nu-\mu}{2}+1\right)} G_{2,2}^{2,0}\left(z \,\middle|\, \begin{matrix} -\frac{\nu}{2}, \frac{\nu+1}{2} \\ -\frac{\mu}{2}, \frac{\mu}{2} \end{matrix}\right) /; \nu \notin \mathbb{Z} \land \mu \notin \mathbb{Z} \land z \notin (-1, 0)$$

07.11.26.0020.01

$$\frac{\theta(|z|-1)}{\sqrt{z-1}}\left(Q_\nu^\mu\left(-\sqrt{\frac{z-1}{z}}\right)+Q_\nu^\mu\left(\sqrt{\frac{z-1}{z}}\right)\right) = -\frac{2^\mu \pi^2 \tan\left(\frac{1}{2}(\mu+\nu)\pi\right)}{\Gamma\left(\frac{1-\mu-\nu}{2}\right)\Gamma\left(\frac{\nu-\mu}{2}+1\right)} G_{2,2}^{0,2}\left(z \,\middle|\, \begin{matrix} \frac{\mu+1}{2}, \frac{1-\mu}{2} \\ \frac{\nu+1}{2}, -\frac{\nu}{2} \end{matrix}\right) /; \nu \notin \mathbb{Z} \land \mu \notin \mathbb{Z} \land z \notin (-\infty, -1)$$

Classical cases involving sgn

07.11.26.0021.01

$$\frac{(\operatorname{sgn}(|z|-1)(z-1))^{\nu+\frac{1}{2}}}{\sqrt{\sqrt{z}+1}} Q_\nu^{\nu+\frac{1}{2}}\left(\frac{2\sqrt[4]{z}}{\sqrt{z}+1}\right) =$$

$$2^{-\nu-\frac{3}{2}}\Gamma(\nu+1)\Gamma\left(2\nu+\frac{3}{2}\right)\sec(\pi\nu)\left(\cos\left(\pi\left(\nu+\frac{1}{4}\right)\right)G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu+1, \nu+\frac{5}{4} \\ \frac{1}{4}, 0 \end{matrix}\right)+\sin\left(\pi\left(\nu+\frac{1}{4}\right)\right)G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \nu+\frac{5}{4}, \nu+1 \\ 0, \frac{1}{4} \end{matrix}\right)\right) -$$

$$\frac{2^{-\nu-\frac{5}{2}}\tan(\pi\nu)}{\Gamma\left(-2\nu-\frac{1}{2}\right)\Gamma(-\nu)} G_{2,2}^{2,2}\left(z \,\middle|\, \begin{matrix} \nu+1, \nu+\frac{5}{4} \\ 0, \frac{1}{4} \end{matrix}\right) /; z \notin (-1, 0)$$

Generalized cases involving algebraic functions

07.11.26.0022.01

$$\left(z^2+1\right)^{\nu/2} Q_\nu^\mu\left(\frac{z}{\sqrt{z^2+1}}\right) = -\frac{\Gamma(\mu+\nu+1)}{2^{\nu+2}\sqrt{\pi}}$$

$$\left(\cos(\pi(\mu+\nu))\, G_{2,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\mu+\nu}{2}+1, \frac{\nu-\mu}{2}+1 \\ 0, \frac{1}{2} \end{matrix}\right)+G_{3,3}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\nu-\mu}{2}+1, \frac{\mu+\nu}{2}+1, \frac{\mu-\nu}{2}+1 \\ 0, \frac{1}{2}, \frac{\mu-\nu}{2}+1 \end{matrix}\right)\right) /; \nu \notin \mathbb{Z} \land \mu \notin \mathbb{Z} \land \operatorname{Re}(z) > 0$$

07.11.26.0023.01

$$\left(z^2+1\right)^{-\frac{\nu+1}{2}} Q_\nu^\mu\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{2^{\nu-1}\sqrt{\pi}}{\Gamma(1-\mu+\nu)} G_{3,3}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1+\mu-\nu}{2}, \frac{1-\mu-\nu}{2}, -\frac{\mu+\nu}{2} \\ 0, \frac{1}{2}, -\frac{\mu+\nu}{2} \end{matrix}\right) /; \nu \notin \mathbb{Z} \land \mu \notin \mathbb{Z}$$

Generalized cases involving unit step θ

07.11.26.0024.01

$$\frac{\theta(|z|-1)}{\sqrt{z^2-1}}\left(Q_\nu^\mu\left(\frac{\sqrt{z^2-1}}{z}\right)+Q_\nu^\mu\left(-\frac{\sqrt{z^2-1}}{z}\right)\right) = -\frac{2^\mu \pi^2 \tan\left(\frac{1}{2}(\mu+\nu)\pi\right)}{\Gamma\left(\frac{1-\mu-\nu}{2}\right)\Gamma\left(\frac{\nu-\mu}{2}+1\right)} G_{2,2}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\mu+1}{2}, \frac{1-\mu}{2} \\ \frac{\nu+1}{2}, -\frac{\nu}{2} \end{matrix}\right) /; \nu \notin \mathbb{Z} \land \mu \notin \mathbb{Z} \land \operatorname{Re}(z) > 0$$

Generalized cases involving sgn

07.11.26.0025.01

$$\frac{\left(\operatorname{sgn}(|z|-1)\left(z^2-1\right)\right)^{\nu+\frac{1}{2}}}{\sqrt{z+1}}\, Q_\nu^{\nu+\frac{1}{2}}\left(\frac{2\sqrt{z}}{z+1}\right) = 2^{-\nu-\frac{3}{2}}\,\Gamma(\nu+1)\,\Gamma\left(2\nu+\frac{3}{2}\right)\sec(\pi\nu)\left(\cos\left(\pi\left(\nu+\frac{1}{4}\right)\right) G_{2,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \nu+1, \nu+\frac{5}{4} \\ \frac{1}{4}, 0 \end{matrix}\right) + \sin\left(\pi\left(\nu+\frac{1}{4}\right)\right) G_{2,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \nu+\frac{5}{4}, \nu+1 \\ 0, \frac{1}{4} \end{matrix}\right)\right) - \frac{2^{-\nu-\frac{5}{2}}\tan(\pi\nu)}{\Gamma\left(-2\nu-\frac{1}{2}\right)\Gamma(-\nu)}\, G_{2,2}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \nu+1, \nu+\frac{5}{4} \\ 0, \frac{1}{4} \end{matrix}\right) /;\ \operatorname{Re}(z) > 0$$

Through other functions

Involving some hypergeometric-type functions

07.11.26.0026.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)\,\Gamma(\nu+1)}{2\,\Gamma(\nu-\mu+1)}\left(\cos(\mu\pi)\,\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\, P_\nu^{(-\mu,\mu)}(z) - \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\, P_\nu^{(\mu,-\mu)}(z)\right) /;\ \mu \notin \mathbb{Z}$$

07.11.26.0027.01

$$Q_\nu^\mu(z) = \frac{2^{-\mu-1}\sin(\pi(\mu-\nu))}{\sqrt{\pi}}\,\frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}} \left(\cot(\pi\mu)\,\Gamma(\mu+\nu+1)\,\Gamma\left(\frac{1}{2}-\mu\right)\Gamma(\mu-\nu)\,(1-z)^{-\mu}\, C_{\mu+\nu}^{\frac{1}{2}-\mu}(z) + 2^{2\mu}\,\pi\,\csc(\pi\mu)\csc(\pi(\mu+\nu))\,\Gamma\left(\mu+\frac{1}{2}\right)(1+z)^\mu\, C_{-\mu-\nu-1}^{\mu+\frac{1}{2}}(z)\right)$$

Involving spheroidal functions

07.11.26.0035.01

$$Q_\nu^\mu(z) = QS_{\nu,\mu}(0, z)$$

Representations through equivalent functions

With related functions

07.11.27.0001.01

$$Q_\nu^\mu(z) = \frac{\pi\csc(\mu\pi)}{2}\left(\cos(\mu\pi)\,P_\nu^\mu(z) - (\nu-\mu+1)_{2\mu}\,P_\nu^{-\mu}(z)\right) /;\ \mu \notin \mathbb{Z}$$

07.11.27.0002.01

$$Q_\nu^\mu(z) = e^{-i\pi\mu}\,\frac{(-1-z)^{\mu/2}}{(1+z)^{\mu/2}}\,\mathfrak{Q}_\nu^\mu(z) + \frac{\pi}{2}(1+z)^{-\mu/2}(-1-z)^{-\mu/2}\left((1+z)^\mu\cot(\pi\mu) - (-z-1)^\mu\csc(\pi\mu)\right)\mathfrak{P}_\nu^\mu(z) /;\ \mu \notin \mathbb{Z}$$

07.11.27.0003.01

$$Q_\nu^\mu(z) = e^{-i\pi\mu}\,\frac{(-1-z)^{\mu/2}}{(1+z)^{\mu/2}}\,\mathfrak{Q}_\nu^\mu(z) + \frac{\pi}{2}(1+z)^{-\mu}\left((1+z)^\mu\cot(\pi\mu) - (-z-1)^\mu\csc(\pi\mu)\right)P_\nu^\mu(z) /;\ \mu \notin \mathbb{Z}$$

07.11.27.0004.01

$$Q_n^m(z) = -i^{2\left\lfloor\frac{m-1}{2}\right\rfloor}\left(\frac{\sqrt{-1-z}}{\sqrt{1+z}}\right)^{2\left\lfloor\frac{m-1}{2}\right\rfloor+2-m}\mathfrak{Q}_n^m(z) /;\ n \in \mathbb{N} \land m \in \mathbb{N}$$

07.11.27.0005.01

$$Q_\nu^\mu(z) = \frac{\csc(\pi\mu)}{2}\left(\cos(\pi(\mu+\nu))\sin(\pi(\mu-\nu))\,\Gamma(\mu-\nu)\,\Gamma(\mu+\nu+1)\,P_\nu^{-\mu}(-z) - \pi\cos(\pi\nu)\,P_\nu^\mu(-z)\right) /;$$
$$\mu \notin \mathbb{Z} \bigwedge z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

07.11.27.0006.01

$$Q_\nu^\mu(x) = \frac{1}{2} e^{-\pi i \mu}\left(e^{-\frac{\pi i \mu}{2}} \lim_{z\to x+i 0} \mathfrak{Q}_\nu^\mu(z) + e^{\frac{\pi i \mu}{2}} \lim_{z\to x-i 0} \mathfrak{Q}_\nu^\mu(z)\right) /; -1 < x < 1$$

07.11.27.0007.01

$$Q_\nu^m(z) = (-1)^m \left(1-z^2\right)^{m/2} \frac{\partial^m Q_\nu(z)}{\partial z^m} /; m \in \mathbb{N}$$

07.11.27.0008.01

$$Q_\nu^0(z) = \mathfrak{Q}_\nu^0(z) + \frac{1}{2}(\log(1+z) - \log(-z-1))\,P_\nu(z)$$

Theorems

The solution of Laplace equation

The solution of the Laplace equation using the method of separation of variables in toroidal coordinates

$$x = \frac{c\sinh(\eta)\cos(\vartheta)}{\cosh(\eta)-\cos(\varphi)}\ y = \frac{c\sinh(\eta)\sin(\vartheta)}{\cosh(\eta)-\cos(\varphi)}\ z = \frac{c\sin(\varphi)}{\cosh(\eta)-\cos(\varphi)}$$

is a linear combination of functions of the form

$$\sqrt{2(\cosh(\eta)-\cos(\varphi))}\ (c_1\cos(\varphi) + c_2\sin(\varphi)) \times$$
$$(c_3\cos(\vartheta) + c_4\sin(\vartheta))\left(c_5 P_{n-\frac{1}{2}}^{|m|}(\cosh(\eta)) + c_6 Q_{n-\frac{1}{2}}^{|m|}(\cosh(\eta))\right) /;\ m \in \mathbb{Z},\ n \in \mathbb{N}.$$

History

– E. Heine (1842)
– F. Neumann (1842)
– L. Schläfli (1881) used complex μ, ν
– R. Olbricht (1888)

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