# KelvinBei2



## Notations

### Traditional name

Kelvin function of the first kind

### Traditional notation

$\mathrm{bei}_\nu(z)$

### *Mathematica* StandardForm notation

```
KelvinBei[ν, z]
```

## Primary definition

03.17.02.0001.01

$$\mathrm{bei}_\nu(z) == -\frac{1}{2}\, i\, e^{-\frac{3}{4} i \pi \nu}\, z^\nu \left(\sqrt[4]{-1}\; z\right)^{-\nu} \left(e^{\frac{3 i \pi \nu}{2}}\, I_\nu\!\left(\sqrt[4]{-1}\; z\right) - J_\nu\!\left(\sqrt[4]{-1}\; z\right)\right)$$

## Specific values

### Specialized values

#### For fixed $\nu$

03.17.03.0001.01

$\mathrm{bei}_\nu(0) = 0 \;/;\; \nu \in \mathbb{N} \lor \mathrm{Re}(\nu) > 0$

03.17.03.0002.01

$\mathrm{bei}_\nu(0) = \tilde{\infty} \;/;\; \mathrm{Re}(\nu) < 0$

03.17.03.0003.01

$\mathrm{bei}_\nu(0) = ¿ \;/;\; \mathrm{Re}(\nu) = 0 \land \nu \neq 0$

#### For fixed $z$

### Explicit rational $\nu$

03.17.03.0004.01

$\mathrm{bei}_0(z) = \mathrm{bei}(z)$

03.17.03.0005.01

$$\mathrm{bei}_{-\frac{14}{3}}(z) = -\frac{1}{162\sqrt[3]{2}\,3^{5/6}\,z^{14/3}} \left(144\sqrt{3}\left(-9\,i\,z^2-110\right)\mathrm{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3}+144\sqrt{3}\left(9\,i\,z^2-110\right)\mathrm{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3}+\right.$$
$$\sqrt[6]{3}\left(-243\,z^4+12\,960\,i\,z^2+42\,240\right)\mathrm{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)-$$
$$3\sqrt[6]{3}\left(81\,z^4+4320\,i\,z^2-14\,080\right)\mathrm{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\left(1296\,i\,z^2\,((1+i)\,z)^{4/3}+15\,840\,((1+i)\,z)^{4/3}\right)$$
$$\mathrm{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\left(15\,840\,((1+i)\,z)^{4/3}-1296\,i\,z^2\,((1+i)\,z)^{4/3}\right)\mathrm{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\left.3^{2/3}\left(81\,z^4-4320\,i\,z^2-14\,080\right)\mathrm{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+3^{2/3}\left(81\,z^4+4320\,i\,z^2-14\,080\right)\mathrm{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.17.03.0006.01

$$\mathrm{bei}_{-\frac{9}{2}}(z) = \frac{(-1)^{3/8}}{2\sqrt{\pi}\,z^{9/2}}\left(\sqrt{2}\,i\left(z^4+45\,i\,z^2-105\right)\cos\left(\frac{(1+i)\,z}{\sqrt{2}}\right)-\right.$$
$$\left.(1-i)\left(z^4-45\,i\,z^2-105\right)\cosh\left(\frac{(1+i)\,z}{\sqrt{2}}\right)-5\,z\left((1+i)\left(2\,z^2+21\,i\right)\sin\left(\frac{(1+i)\,z}{\sqrt{2}}\right)+\sqrt{2}\left(2\,i\,z^2+21\right)\sinh\left(\frac{(1+i)\,z}{\sqrt{2}}\right)\right)\right)$$

03.17.03.0007.01

$$\mathrm{bei}_{-\frac{13}{3}}(z) =$$
$$-\frac{1}{54\,2^{2/3}\,3^{5/6}\,z^{13/3}}\left(\sqrt[4]{-1}\left(\sqrt{3}\left(81\,z^4-3024\,i\,z^2-4480\right)\mathrm{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\sqrt{3}\left(-81\,i\,z^4+3024\,z^2+4480\,i\right)\right.\right.$$
$$\mathrm{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\left(6720\sqrt[6]{3}\,((1+i)\,z)^{2/3}+756\sqrt[6]{3}\,i\,z^2\,((1+i)\,z)^{2/3}\right)\mathrm{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\left(756\sqrt[6]{3}\,z^2\,((1+i)\,z)^{2/3}+6720\sqrt[6]{3}\,i\,((1+i)\,z)^{2/3}\right)\mathrm{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\left(81\,z^4-3024\,i\,z^2-4480\right)\mathrm{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\left(-81\,i\,z^4+3024\,z^2+4480\,i\right)\mathrm{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\left(2240\,3^{2/3}\,((1+i)\,z)^{2/3}+252\,3^{2/3}\,i\,z^2\,((1+i)\,z)^{2/3}\right)\mathrm{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\left.\left.\left(252\,3^{2/3}\,z^2\,((1+i)\,z)^{2/3}+2240\,3^{2/3}\,i\,((1+i)\,z)^{2/3}\right)\mathrm{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)$$

03.17.03.0008.01

$$\mathrm{bei}_{-\frac{11}{3}}(z) = -\frac{20\sqrt[4]{-1}\,2^{2/3}}{27\,3^{5/6}\,z^{11/3}}\left(\frac{1}{160}\sqrt{3}\,9\left(160\,i-9\,z^2\right)\mathrm{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3}+\right.$$
$$\frac{1}{160}\sqrt{3}\,9\left(9\,i\,z^2-160\right)\mathrm{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3}+\frac{1}{4}\sqrt[6]{3}\,3\left(9\,z^2-32\,i\right)\mathrm{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\frac{1}{4}\sqrt[6]{3}\,3\left(32-9\,i\,z^2\right)\mathrm{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\left(\frac{81}{160}\,z^2\,((1+i)\,z)^{4/3}-9\,i\,((1+i)\,z)^{4/3}\right)\mathrm{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\left(9\,((1+i)\,z)^{4/3}-\frac{81}{160}\,i\,z^2\,((1+i)\,z)^{4/3}\right)\mathrm{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+$$
$$\left.\frac{1}{4}\,3^{2/3}\left(32\,i-9\,z^2\right)\mathrm{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\frac{1}{4}\,3^{2/3}\left(9\,i\,z^2-32\right)\mathrm{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.17.03.0009.01

$$\operatorname{bei}_{-\frac{7}{2}}(z) = -\frac{(-1)^{5/8}}{2\sqrt{\pi}\, z^{7/2}} \left(3\sqrt{2}\left(2z^2+5i\right)\cos\left(\frac{(1+i)z}{\sqrt{2}}\right) + (3+3i)\left(2iz^2+5\right)\cosh\left(\frac{(1+i)z}{\sqrt{2}}\right) + (1+i)z\left(z^2+15i\right)\sin\left(\frac{(1+i)z}{\sqrt{2}}\right) + \sqrt{2}\, z\left(z^2-15i\right)\sinh\left(\frac{(1+i)z}{\sqrt{2}}\right)\right)$$

03.17.03.0010.01

$$\operatorname{bei}_{-\frac{10}{3}}(z) = \frac{1}{18\, 2^{2/3}\, 3^{5/6}\, z^{10/3}} \left(16\sqrt{3}\left(9iz^2+14\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + 16\sqrt{3}\left(14-9iz^2\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(-336\sqrt[6]{3}\,((1+i)z)^{2/3} - 27i\sqrt[6]{3}\,z^2((1+i)z)^{2/3}\right)\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(336\sqrt[6]{3}\,((1+i)z)^{2/3} - 27i\sqrt[6]{3}\,z^2((1+i)z)^{2/3}\right)\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(144iz^2+224\right)\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(224-144iz^2\right)\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(-112\, 3^{2/3}((1+i)z)^{2/3} - 9i\, 3^{2/3} z^2((1+i)z)^{2/3}\right)\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(112\, 3^{2/3}((1+i)z)^{2/3} - 9i\, 3^{2/3} z^2((1+i)z)^{2/3}\right)\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.17.03.0011.01

$$\operatorname{bei}_{-\frac{8}{3}}(z) = -\frac{5i((1+i)z)^{8/3}}{12\sqrt[3]{2}\, z^{8/3}} \left(\frac{2\sqrt[6]{3}}{((1+i)z)^{4/3}} \left(\sqrt{3}\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - \sqrt{3}\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - \operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right) + \frac{1}{15\, 3^{2/3} z^2((1+i)z)^{2/3}} \left(-3\left(9z^2-40i\right)\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) - 3\left(9z^2+40i\right)\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \sqrt{3}\left(\left(9z^2-40i\right)\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \left(9z^2+40i\right)\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)\right)\right)$$

03.17.03.0012.01

$$\operatorname{bei}_{-\frac{5}{2}}(z) = \frac{(-1)^{5/8}}{2\sqrt{\pi}\, z^{5/2}} \left((-1-i)\left(z^2+3i\right)\cos\left(\frac{(1+i)z}{\sqrt{2}}\right) + \sqrt{2}\left(-iz^2-3\right)\cosh\left(\frac{(1+i)z}{\sqrt{2}}\right) + 3\sqrt{2}\, z\sin\left(\frac{(1+i)z}{\sqrt{2}}\right) + (3+3i)z\sinh\left(\frac{(1+i)z}{\sqrt{2}}\right)\right)$$

03.17.03.0013.01

$$\operatorname{bei}_{-\frac{7}{3}}(z) = -\frac{(-1)^{3/4}}{6\, 2^{2/3}\, 3^{5/6}\, z^{7/3}} \left(-24\sqrt[6]{3}\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 24\sqrt[6]{3}\, i\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} - 8\, 3^{2/3}\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + 8\, 3^{2/3}\, i\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3} + \sqrt{3}\left(9iz^2+16\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + \sqrt{3}\left(9z^2+16i\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + 9iz^2\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + 16\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + 9z^2\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right) + 16i\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.17.03.0014.01

$$\operatorname{bei}_{-\frac{5}{3}}(z) = \frac{1}{6\,6^{5/6}\,z^{5/3}} \left(-9\,i\,\sqrt{3}\,z\,\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{(1+i)\,z} - 9\,\sqrt{3}\,z\,\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{(1+i)\,z} + 9\,i\,z\,\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{(1+i)\,z} + 9\,z\,\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\sqrt[3]{(1+i)\,z} + \sqrt[6]{3}\,(12+12\,i)\,\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \sqrt[6]{3}\,(12-12\,i)\,\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - (4+4\,i)\,3^{2/3}\,\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - (4-4\,i)\,3^{2/3}\,\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.17.03.0015.01

$$\operatorname{bei}_{-\frac{3}{2}}(z) = \frac{(-1)^{3/8}}{2\,\sqrt{\pi}\,z^{3/2}} \left(-\sqrt{2}\,\cosh\left(\sqrt[4]{-1}\,z\right) + (1+i)\cosh\left((-1)^{3/4}\,z\right) + (1+i)\,z\sinh\left(\sqrt[4]{-1}\,z\right) + \sqrt{2}\,z\sinh\left((-1)^{3/4}\,z\right)\right)$$

03.17.03.0016.01

$$\operatorname{bei}_{-\frac{3}{2}}(z) = -\sqrt{\frac{2}{\pi}}\,z^{-3/2}\left(\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{z}{\sqrt{2}}\right)\left(z\cosh\left(\frac{z}{\sqrt{2}}\right) - \sinh\left(\frac{z}{\sqrt{2}}\right)\right) + \cos\left(\frac{3\pi}{8}\right)\cos\left(\frac{z}{\sqrt{2}}\right)\left(\cosh\left(\frac{z}{\sqrt{2}}\right) + z\sinh\left(\frac{z}{\sqrt{2}}\right)\right)\right)$$

03.17.03.0017.01

$$\operatorname{bei}_{-\frac{4}{3}}(z) = \frac{i}{2\,2^{2/3}\,3^{5/6}\,z^{4/3}} \left(3\,\sqrt[6]{3}\,\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3} + 3\,\sqrt[6]{3}\,\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3} + 3^{2/3}\,\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3} + 3^{2/3}\,\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3} - 2\,\sqrt{3}\,\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 2\,\sqrt{3}\,\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - 2\,\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 2\,\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.17.03.0018.01

$$\operatorname{bei}_{-\frac{2}{3}}(z) = \frac{1}{2\,\sqrt[3]{2}\,3^{2/3}\,z^{2/3}} \left(3\,\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 3\,\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \sqrt{3}\left(\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)$$

03.17.03.0019.01

$$\operatorname{bei}_{-\frac{1}{2}}(z) = \frac{(-1)^{7/8}}{2\,\sqrt{\pi}\,\sqrt{z}} \left(\sqrt{2}\,\cos\left(\sqrt[4]{-1}\,z\right) + (1+i)\cosh\left(\sqrt[4]{-1}\,z\right)\right)$$

03.17.03.0020.01

$$\operatorname{bei}_{-\frac{1}{2}}(z) = \frac{1}{\sqrt{z}}\sqrt{\frac{2}{\pi}}\left(\cos\left(\frac{3\pi}{8}\right)\sinh\left(\frac{z}{\sqrt{2}}\right)\sin\left(\frac{z}{\sqrt{2}}\right) - \sin\left(\frac{3\pi}{8}\right)\cosh\left(\frac{z}{\sqrt{2}}\right)\cos\left(\frac{z}{\sqrt{2}}\right)\right)$$

03.17.03.0021.01

$$\operatorname{bei}_{-\frac{1}{3}}(z) = \frac{i-1}{4\,\sqrt[3]{z}}\sqrt[6]{\frac{3}{2}} \left(\sqrt{3}\,i\,\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \sqrt{3}\,\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + i\,\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.17.03.0022.01

$$\operatorname{bei}_{\frac{1}{3}}(z) = \frac{\sqrt[6]{3}\ \sqrt[3]{(1+i)\,z}}{2\,2^{5/6}\,z^{2/3}} \left(\sqrt{3}\ i\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \sqrt{3}\ \operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - i\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.17.03.0023.01

$$\operatorname{bei}_{\frac{1}{2}}(z) = -\frac{(-1)^{7/8}}{\sqrt{2\pi}\ \sqrt{z}}\left(\sin\left(\sqrt[4]{-1}\ z\right) - \sqrt[4]{-1}\ \sin\left((-1)^{3/4}\,z\right)\right)$$

03.17.03.0024.01

$$\operatorname{bei}_{\frac{1}{2}}(z) = \frac{1}{\sqrt{z}}\sqrt{\frac{2}{\pi}}\left(\cosh\left(\frac{z}{\sqrt{2}}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{z}{\sqrt{2}}\right) + \cos\left(\frac{3\pi}{8}\right)\cos\left(\frac{z}{\sqrt{2}}\right)\sinh\left(\frac{z}{\sqrt{2}}\right)\right)$$

03.17.03.0025.01

$$\operatorname{bei}_{\frac{2}{3}}(z) = \frac{z^{2/3}}{6^{2/3}\,((1+i)\,z)^{4/3}} \left(3\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 3\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \sqrt{3}\left(\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)$$

03.17.03.0026.01

$$\operatorname{bei}_{\frac{4}{3}}(z) = \frac{i\,z^{4/3}}{\sqrt[3]{2}\ 3^{5/6}\,((1+i)\,z)^{8/3}} \left(\sqrt[6]{3}\left(-3\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - 3\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \sqrt{3}\left(\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)((1+i)\,z)^{2/3} + 2\left(\sqrt{3}\ \operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \sqrt{3}\ \operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)$$

03.17.03.0027.01

$$\operatorname{bei}_{\frac{3}{2}}(z) = -\frac{(-1)^{7/8}}{2\sqrt{\pi}\ z^{3/2}}\left((1+i)\,z\cosh\left(\sqrt[4]{-1}\ z\right) + \sqrt{2}\ z\cosh\left((-1)^{3/4}\,z\right) - \sqrt{2}\ \sinh\left(\sqrt[4]{-1}\ z\right) + (1+i)\sinh\left((-1)^{3/4}\,z\right)\right)$$

03.17.03.0028.01

$$\operatorname{bei}_{\frac{3}{2}}(z) = \sqrt{\frac{2}{\pi}}\ z^{-3/2}\left(\sin\left(\frac{3\pi}{8}\right)\cos\left(\frac{z}{\sqrt{2}}\right)\left(z\cosh\left(\frac{z}{\sqrt{2}}\right) - \sinh\left(\frac{z}{\sqrt{2}}\right)\right) - \cos\left(\frac{3\pi}{8}\right)\sin\left(\frac{z}{\sqrt{2}}\right)\left(\cosh\left(\frac{z}{\sqrt{2}}\right) + z\sinh\left(\frac{z}{\sqrt{2}}\right)\right)\right)$$

03.17.03.0029.01

$$\operatorname{bei}_{\frac{5}{3}}(z) = \frac{\sqrt[4]{-1}\ z^{5/3}}{3\,2^{2/3}\,3^{5/6}\,((1+i)\,z)^{10/3}} \left(9\,((1+i)\,z)^{4/3}\left(\sqrt{3}\ \operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + i\left(\sqrt{3}\ \operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right) + \operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right) - 8\sqrt[6]{3}\left(3\,i\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 3\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \sqrt{3}\left(i\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)\right)$$

03.17.03.0030.01

$$\operatorname{bei}_{\frac{7}{3}}(z) = \frac{1}{6\sqrt[3]{2}\, 3^{5/6}\, z^{5/3}\, ((1+i)\, z)^{2/3}} \left(\sqrt[4]{-1}\left(24\sqrt[6]{3}\, \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} - 24\, i\sqrt[6]{3}\, \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} - 8\, 3^{2/3}\, \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + 8\, 3^{2/3}\, i\, \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + \sqrt{3}\left(-9\, i\, z^2 - 16\right) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - \sqrt{3}\left(9\, z^2 + 16\, i\right) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(9\, i\, z^2 + 16\right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(9\, z^2 + 16\, i\right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)\right)$$

03.17.03.0031.01

$$\operatorname{bei}_{\frac{5}{2}}(z) = \frac{(-1)^{5/8}}{2\sqrt{\pi}\, z^{5/2}} \left(3\sqrt{2}\, z \cos\left(\frac{(1+i)\, z}{\sqrt{2}}\right) - (3-3\, i)\, z \cosh\left(\frac{(1+i)\, z}{\sqrt{2}}\right) + (1+i)\left(z^2 + 3\, i\right) \sin\left(\frac{(1+i)\, z}{\sqrt{2}}\right) + \sqrt{2}\left(z^2 - 3\, i\right) \sinh\left(\frac{(1+i)\, z}{\sqrt{2}}\right)\right)$$

03.17.03.0032.01

$$\operatorname{bei}_{\frac{8}{3}}(z) = \frac{5\, i\sqrt[3]{2}\, z^{8/3}}{3\, ((1+i)\, z)^{8/3}} \left(\frac{2\sqrt[6]{3}}{((1+i)\, z)^{4/3}} \left(\sqrt{3}\, \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - \sqrt{3}\, \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right) + \frac{1}{15\, 3^{2/3}\, z^2\, ((1+i)\, z)^{2/3}} \left(-3\left(9\, z^2 - 40\, i\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - 3\left(9\, z^2 + 40\, i\right) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - \sqrt{3}\left(\left(9\, z^2 - 40\, i\right) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(9\, z^2 + 40\, i\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)\right)\right)$$

03.17.03.0033.01

$$\operatorname{bei}_{\frac{10}{3}}(z) = \frac{1}{18\sqrt[3]{2}\, 3^{5/6}\, z^{8/3}\, ((1+i)\, z)^{2/3}} \left(-16\sqrt{3}\left(9\, z^2 - 14\, i\right) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 16\sqrt{3}\left(9\, z^2 + 14\, i\right) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(27\sqrt[6]{3}\, z^2\, ((1+i)\, z)^{2/3} - 336\, i\sqrt[6]{3}\, ((1+i)\, z)^{2/3}\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(27\sqrt[6]{3}\, z^2\, ((1+i)\, z)^{2/3} + 336\sqrt[6]{3}\, i\, ((1+i)\, z)^{2/3}\right) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(144\, z^2 - 224\, i\right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(-144\, z^2 - 224\, i\right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(112\, i\, 3^{2/3}\, ((1+i)\, z)^{2/3} - 9\, 3^{2/3}\, z^2\, ((1+i)\, z)^{2/3}\right) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(-9\, 3^{2/3}\, z^2\, ((1+i)\, z)^{2/3} - 112\, 3^{2/3}\, i\, ((1+i)\, z)^{2/3}\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.17.03.0034.01

$$\operatorname{bei}_{\frac{7}{2}}(z) = \frac{(-1)^{5/8}}{2\sqrt{\pi}\, z^{7/2}} \left( (1+i)\, z\, (z^2 + 15\, i) \cos\left(\frac{(1+i)\, z}{\sqrt{2}}\right) + \sqrt{2}\, z\, (i\, z^2 + 15) \cosh\left(\frac{(1+i)\, z}{\sqrt{2}}\right) - 3\sqrt{2}\, (2\, z^2 + 5\, i) \sin\left(\frac{(1+i)\, z}{\sqrt{2}}\right) + (3+3\, i)\, (5\, i - 2\, z^2) \sinh\left(\frac{(1+i)\, z}{\sqrt{2}}\right)\right)$$

03.17.03.0035.01

$$\operatorname{bei}_{\frac{11}{3}}(z) = \frac{320\, (-1)^{3/4}\, \sqrt[3]{2}\, z^{11/3}}{27\, 3^{5/6}\, ((1+i)\, z)^{22/3}} \left( \frac{1}{160} \sqrt{3}\, 9\, (9\, z^2 - 160\, i) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} + \frac{1}{160} \sqrt{3}\, 9\, (160 - 9\, i\, z^2) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} + \frac{1}{4} \sqrt[6]{3}\, 3\, i\, (9\, i\, z^2 + 32) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \frac{1}{4} \sqrt[6]{3}\, 3\, i\, (9\, z^2 + 32\, i) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(\frac{81}{160}\, z^2\, ((1+i)\, z)^{4/3} - 9\, i\, ((1+i)\, z)^{4/3}\right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(9\, ((1+i)\, z)^{4/3} - \frac{81}{160}\, i\, z^2\, ((1+i)\, z)^{4/3}\right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \frac{1}{4}\, 3^{2/3}\, i\, (9\, i\, z^2 + 32) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \frac{1}{4}\, 3^{2/3}\, i\, (9\, z^2 + 32\, i) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.17.03.0036.01

$$\operatorname{bei}_{\frac{13}{3}}(z) = \frac{1}{54\, \sqrt[3]{2}\, 3^{5/6}\, z^{11/3}\, ((1+i)\, z)^{2/3}} \left( \sqrt[4]{-1} \left( \sqrt{3}\, (81\, i\, z^4 + 3024\, z^2 - 4480\, i) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \sqrt{3}\, (81\, z^4 + 3024\, i\, z^2 - 4480) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(6720\, i\, \sqrt[6]{3}\, ((1+i)\, z)^{2/3} - 756\, \sqrt[6]{3}\, z^2\, ((1+i)\, z)^{2/3}\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(756\, i\, \sqrt[6]{3}\, z^2\, ((1+i)\, z)^{2/3} - 6720\, \sqrt[6]{3}\, ((1+i)\, z)^{2/3}\right) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + (-81\, i\, z^4 - 3024\, z^2 + 4480\, i) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + (-81\, z^4 - 3024\, i\, z^2 + 4480) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(252\, 3^{2/3}\, z^2\, ((1+i)\, z)^{2/3} - 2240\, i\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(2240\, 3^{2/3}\, ((1+i)\, z)^{2/3} - 252\, i\, 3^{2/3}\, z^2\, ((1+i)\, z)^{2/3}\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)\right)$$

03.17.03.0037.01

$$\operatorname{bei}_{\frac{9}{2}}(z) = -\frac{(-1)^{5/8}}{2\sqrt{\pi}\, z^{9/2}} \left( 5\sqrt{2}\, z\, (2\, z^2 + 21\, i) \cos\left(\frac{(1+i)\, z}{\sqrt{2}}\right) + (5+5\, i)\, z\, (2\, i\, z^2 + 21) \cosh\left(\frac{(1+i)\, z}{\sqrt{2}}\right) + (1+i)\, (z^4 + 45\, i\, z^2 - 105) \sin\left(\frac{(1+i)\, z}{\sqrt{2}}\right) + \sqrt{2}\, (z^4 - 45\, i\, z^2 - 105) \sinh\left(\frac{(1+i)\, z}{\sqrt{2}}\right)\right)$$

03.17.03.0038.01

$$\operatorname{bei}_{\frac{14}{3}}(z) = -\frac{14\,080\,\sqrt[3]{2}\,z^{14/3}}{81\,3^{5/6}\,((1+i)\,z)^{28/3}} \left(\frac{1}{110}\sqrt{3}\,9\,i\,(110\,i - 9\,z^2)\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3} + \frac{1}{110}\sqrt{3}\,9\,i\,(9\,z^2 + 110\,i)\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{4/3} + \left(-\frac{1}{110}\,81\,i\,z^2\,((1+i)\,z)^{4/3} - 9\,((1+i)\,z)^{4/3}\right)\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \left(\frac{81}{110}\,i\,z^2\,((1+i)\,z)^{4/3} - 9\,((1+i)\,z)^{4/3}\right)\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \frac{3\,\sqrt[6]{3}\,(81\,z^4 - 4320\,i\,z^2 - 14\,080)}{1760}\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \frac{3\,\sqrt[6]{3}\,(81\,z^4 + 4320\,i\,z^2 - 14\,080)}{1760}\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \frac{3^{2/3}\,(81\,z^4 - 4320\,i\,z^2 - 14\,080)}{1760}\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \frac{3^{2/3}\,(81\,z^4 + 4320\,i\,z^2 - 14\,080)}{1760}\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

## Symbolic rational $\nu$

03.17.03.0039.01

$$\operatorname{bei}_\nu(z) = \frac{(-1)^{3/8}\,e^{-i\pi\nu}}{\sqrt{2\pi}\,\sqrt{z}} \left(\sum_{k=0}^{\left\lfloor\frac{1}{4}(2|\nu|-3)\right\rfloor} \frac{\left(2k+|\nu|+\frac{1}{2}\right)!\,\left(2\sqrt[4]{-1}\,z\right)^{-2k-1}}{(2k+1)!\,\left(-2k+|\nu|-\frac{3}{2}\right)!} \left(e^{\frac{1}{4}i\pi(4\nu+1)}\cos\left(\frac{1}{2}\pi\left(\frac{1}{2}-\nu\right) - \frac{1}{\sqrt[4]{-1}}\,z\right) + (-1)^k\cos\left(\frac{1}{2}\pi\left(\nu-\frac{1}{2}\right) - \sqrt[4]{-1}\,z\right)\right) + \sum_{k=0}^{\left\lfloor\frac{1}{4}(2|\nu|-1)\right\rfloor} \frac{\left(2k+|\nu|-\frac{1}{2}\right)!\,\left(2\sqrt[4]{-1}\,z\right)^{-2k}}{(2k)!\,\left(-2k+|\nu|-\frac{1}{2}\right)!} \left((-1)^{3/4}\,e^{i\pi\nu}\sin\left(\frac{1}{2}\pi\left(\frac{1}{2}-\nu\right) - \frac{1}{\sqrt[4]{-1}}\,z\right) - (-1)^k\sin\left(\frac{1}{2}\pi\left(\nu-\frac{1}{2}\right) - \sqrt[4]{-1}\,z\right)\right)\right) /;\ \nu - \frac{1}{2} \in \mathbb{Z}$$

03.17.03.0040.01

$$\operatorname{bei}_\nu(z) = -\frac{i\,e^{\frac{1}{4}(-3)\,i\,\pi\,\nu}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma\left(-\frac{1}{3}\right)}{2\,\Gamma(1-|\nu|)}\left(\frac{2^{|\nu|-1}\left(\sqrt[4]{-1}\,z\right)^{-|\nu|}}{3^{5/6}}\sum_{k=0}^{|\nu|-\frac{1}{3}}\frac{4^{-k}\left(i\,z^2\right)^k\left(-k+|\nu|-\frac{1}{3}\right)!}{k!\left(-2\,k+|\nu|-\frac{1}{3}\right)!\left(\frac{1}{3}\right)_k(1-|\nu|)_k}\right.$$
$$\left(i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)}\operatorname{sgn}(\nu)\left(\sqrt{3}\,\operatorname{sgn}(\nu)\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)-\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)+\right.$$
$$\left.(-1)^k\,e^{\frac{3\,i\,\pi\,\nu}{2}}\left(\sqrt{3}\,\operatorname{sgn}(\nu)\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)-\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)+$$
$$\frac{2^{|\nu|-\frac{5}{3}}\left(\sqrt[4]{-1}\,z\right)^{\frac{2}{3}-|\nu|}}{3^{2/3}}\sum_{k=0}^{|\nu|-\frac{4}{3}}\frac{4^{-k}\left(i\,z^2\right)^k\left(-k+|\nu|-\frac{4}{3}\right)!}{k!\left(-2\,k+|\nu|-\frac{4}{3}\right)!\left(\frac{4}{3}\right)_k(1-|\nu|)_k}$$
$$\left(-i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)}\operatorname{sgn}(\nu)\left(\sqrt{3}\,\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)-3\operatorname{sgn}(\nu)\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)+\right.$$
$$\left.\left.(-1)^k\,e^{\frac{3\,i\,\pi\,\nu}{2}}\left(\sqrt{3}\,\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)-3\operatorname{sgn}(\nu)\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)\right)\bigg/;\ |\nu|-\frac{1}{3}\in\mathbb{Z}$$

03.17.03.0041.01

$$\operatorname{bei}_\nu(z) = -\frac{i\,e^{-\frac{3\pi i\nu}{4}}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma\left(-\frac{2}{3}\right)\operatorname{sgn}(\nu)}{2\,\Gamma(1-|\nu|)}\left(2^{|\nu|-\frac{7}{3}}\sqrt[6]{3}\left(\sqrt[4]{-1}\,z\right)^{\frac{4}{3}-|\nu|}\sum_{k=0}^{|\nu|-\frac{5}{3}}\frac{4^{-k}\left(i\,z^2\right)^k\left(-k+|\nu|-\frac{5}{3}\right)!}{k!\left(-2\,k+|\nu|-\frac{5}{3}\right)!\left(\frac{5}{3}\right)_k(1-|\nu|)_k}\right.$$
$$\left(-i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)}\left(\sqrt{3}\,\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\operatorname{sgn}(\nu)\right)+\right.$$
$$\left.(-1)^k\,e^{\frac{3\,i\,\pi\,\nu}{2}}\left(\sqrt{3}\,\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\operatorname{sgn}(\nu)\right)\right)+$$
$$\frac{2^{|\nu|}\left(\sqrt[4]{-1}\,z\right)^{-|\nu|}}{3\,3^{2/3}}\sum_{k=0}^{|\nu|-\frac{2}{3}}\frac{4^{-k}\left(i\,z^2\right)^k\left(-k+|\nu|-\frac{2}{3}\right)!}{k!\left(-2\,k+|\nu|-\frac{2}{3}\right)!\left(\frac{2}{3}\right)_k(1-|\nu|)_k}$$
$$\left(i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)}\left(3\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\sqrt{3}\,\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\operatorname{sgn}(\nu)\right)-\right.$$
$$\left.\left.(-1)^k\,e^{\frac{3\,i\,\pi\,\nu}{2}}\left(3\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)+\sqrt{3}\,\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\operatorname{sgn}(\nu)\right)\right)\right)\bigg/;\ |\nu|-\frac{2}{3}\in\mathbb{Z}$$

## Values at fixed points

03.17.03.0042.01

$$\operatorname{bei}_0(0) = 0$$

## Values at infinities

03.17.03.0043.01

$$\lim_{x\to\infty}\operatorname{bei}_\nu(x) = \tilde{\infty}$$

# General characteristics

## Domain and analyticity

$\text{bei}_\nu(z)$ is an analytical function of $\nu$ and $z$, which is defined in $\mathbb{C}^2$.

03.17.04.0001.01

$$(\nu * z) \longrightarrow \text{bei}_\nu(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

## Symmetries and periodicities

### Parity

03.17.04.0002.01

$$\text{bei}_\nu(-z) = (-z)^\nu z^{-\nu} \text{bei}_\nu(z)$$

03.17.04.0003.01

$$\text{bei}_{-n}(z) = (-1)^n \text{bei}_n(z) /; n \in \mathbb{Z}$$

### Mirror symmetry

03.17.04.0004.01

$$\text{bei}_{\bar{\nu}}(\bar{z}) == \overline{\text{bei}_\nu(z)} /; z \notin (-\infty, 0)$$

### Periodicity

No periodicity

## Poles and essential singularities

### With respect to $z$

For fixed $\nu$, the function $\text{bei}_\nu(z)$ has an essential singularity at $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is a branch point for generic $\nu$.

03.17.04.0005.01

$$\mathcal{Sing}_z(\text{bei}_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$$

### With respect to $\nu$

For fixed $z$, the function $\text{bei}_\nu(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

03.17.04.0006.01

$$\mathcal{Sing}_\nu(\text{bei}_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$$

## Branch points

### With respect to $z$

For fixed noninteger $\nu$, the function $\text{bei}_\nu(z)$ has two branch points: $z == 0$, $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is an essential singularity.

03.17.04.0007.01

$$\mathcal{BP}_z(\text{bei}_\nu(z)) == \{0, \tilde{\infty}\} /; \nu \notin \mathbb{Z}$$

03.17.04.0008.01

$\mathcal{BP}_z(\text{bei}_\nu(z)) = \{\} \,/;\, \nu \in \mathbb{Z}$

03.17.04.0009.01

$\mathcal{R}_z(\text{bei}_\nu(z), 0) = \log \,/;\, \nu \notin \mathbb{Q}$

03.17.04.0010.01

$\mathcal{R}_z\left(\text{ber}_{i\,\frac{p}{q}}(z), 0\right) = q \,/;\, p \in \mathbb{Z} \land q - 1 \in \mathbb{N}^+ \land \gcd(p, q) = 1$

03.17.04.0011.01

$\mathcal{R}_z(\text{bei}_\nu(z), \tilde{\infty}) = \log \,/;\, \nu \notin \mathbb{Q}$

03.17.04.0012.01

$\mathcal{R}_z\left(\text{bei}_{\frac{p}{q}}(z), \tilde{\infty}\right) = q \,/;\, p \in \mathbb{Z} \land q - 1 \in \mathbb{N}^+ \land \gcd(p, q) = 1$

#### With respect to $\nu$

For fixed $z$, the function $\text{bei}_\nu(z)$ does not have branch points.

03.17.04.0013.01

$\mathcal{BP}_\nu(\text{bei}_\nu(z)) = \{\}$

### Branch cuts

#### With respect to $z$

When $\nu$ is an integer, $\text{bei}_\nu(z)$ is an entire function of $z$. For fixed noninteger $\nu$, it has one infinitely long branch cut. For fixed noninteger $\nu$, the function $\text{bei}_\nu(z)$ is a single-valued function on the $z$-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

03.17.04.0014.01

$\mathcal{BC}_z(\text{bei}_\nu(z)) = \{\{(-\infty, 0), -i\}\} \,/;\, \nu \notin \mathbb{Z}$

03.17.04.0015.01

$\mathcal{BC}_z(\text{bei}_\nu(z)) = \{\} \,/;\, \nu \in \mathbb{Z}$

03.17.04.0016.01

$\lim_{\epsilon \to +0} \text{bei}_\nu(x + i\,\epsilon) = \text{bei}_\nu(x) \,/;\, x \in \mathbb{R} \land x < 0$

03.17.04.0017.01

$\lim_{\epsilon \to +0} \text{bei}_\nu(x - i\,\epsilon) = e^{-2\pi i \nu}\, \text{bei}_\nu(x) \,/;\, x \in \mathbb{R} \land x < 0$

#### With respect to $\nu$

For fixed $z$, the function $\text{bei}_\nu(z)$ is an entire function of $\nu$ and does not have branch cuts.

03.17.04.0018.01

$\mathcal{BC}_\nu(\text{bei}_\nu(z)) = \{\}$

## Series representations

### Generalized power series

## Expansions at $\nu == \pm n$

03.17.06.0001.01

$$\operatorname{bei}_\nu(z) \propto \operatorname{bei}_n(z) + \left(2^{n-1}\, n!\, z^{-n} \sum_{k=0}^{n-1} \frac{(-1)^{n-k}\, 2^{-k}\, z^k}{(n-k)\, k!} \left(\cos\left(\frac{1}{4}(k-n)\pi\right) \operatorname{bei}_k(z) - \sin\left(\frac{1}{4}(k-n)\pi\right) \operatorname{ber}_k(z)\right) + \frac{\pi}{2} \operatorname{ber}_n(z) - \operatorname{kei}_n(z)\right)(\nu - n) + \ldots \;/;\; (\nu \to n) \land n \in \mathbb{N}$$

03.17.06.0002.01

$$\operatorname{bei}_\nu(z) \propto (-1)^n \operatorname{bei}_n(z) + \left(\frac{1}{2}(-1)^n \pi \operatorname{ber}_n(z) - (-1)^n \operatorname{kei}_n(z) - (-1)^n\, 2^{n-1}\, n!\, z^{-n} \sum_{k=0}^{n-1} \frac{(-1)^{n-k}\, 2^{-k}\, z^k}{(n-k)\, k!} \left(\cos\left(\frac{1}{4}(k-n)\pi\right) \operatorname{bei}_k(z) - \sin\left(\frac{1}{4}(k-n)\pi\right) \operatorname{ber}_k(z)\right)\right)(\nu + n) + \ldots \;/;\; (\nu \to -n) \land n \in \mathbb{N}$$

## Expansions at generic point $z == z_0$

03.17.06.0003.01

$$\operatorname{bei}_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(\operatorname{bei}_\nu(z_0) + \left(-\frac{\operatorname{bei}_{\nu-1}(z_0)}{\sqrt{2}} + \frac{\operatorname{ber}_{\nu-1}(z_0)}{\sqrt{2}} - \frac{\nu \operatorname{bei}_\nu(z_0)}{z_0}\right)(z-z_0) + \frac{\left(2\nu(\nu+1)\operatorname{bei}_\nu(z_0) + z_0\left(\sqrt{2}\,(\operatorname{bei}_{\nu-1}(z_0) - \operatorname{ber}_{\nu-1}(z_0)) + 2\operatorname{ber}_\nu(z_0)\, z_0\right)\right)}{4 z_0^2}(z-z_0)^2 + \ldots\right) /;\; (z \to z_0)$$

03.17.06.0004.01

$$\operatorname{bei}_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(\operatorname{bei}_\nu(z_0) + \left(-\frac{\operatorname{bei}_{\nu-1}(z_0)}{\sqrt{2}} + \frac{\operatorname{ber}_{\nu-1}(z_0)}{\sqrt{2}} - \frac{\nu \operatorname{bei}_\nu(z_0)}{z_0}\right)(z-z_0) + \frac{\left(2\nu(\nu+1)\operatorname{bei}_\nu(z_0) + z_0\left(\sqrt{2}\,(\operatorname{bei}_{\nu-1}(z_0) - \operatorname{ber}_{\nu-1}(z_0)) + 2\operatorname{ber}_\nu(z_0)\, z_0\right)\right)}{4 z_0^2}(z-z_0)^2 + O\left((z-z_0)^3\right)\right)$$

03.17.06.0005.01

$$\operatorname{bei}_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{\operatorname{bei}_\nu^{(0,k)}(z_0)\,(z-z_0)^k}{k!}$$

03.17.06.0006.01

$$\operatorname{bei}_\nu(z) = 2^{-2\nu-1}\, i\, e^{\frac{1}{4}(-3)\, i\pi\nu} \sqrt{\pi}\; z_0^\nu\, \Gamma(\nu+1) \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^k\, z_0^{-k}}{k!} \left({}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1-k+\nu}{2}, \frac{2-k+\nu}{2}, \nu+1; -\frac{i z_0^2}{4}\right) - e^{\frac{3 i\pi\nu}{2}}\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1-k+\nu}{2}, \frac{2-k+\nu}{2}, \nu+1; \frac{i z_0^2}{4}\right)\right)(z-z_0)^k$$

03.17.06.0007.01

$$\operatorname{bei}_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^{-\frac{3k}{2}-1} (i-1)^k}{k!} \left( \sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \binom{k}{2j} \left( \left(1+i^k\right) \operatorname{bei}_{4j-k+\nu}(z_0) - i \left(1-i^k\right) \operatorname{ber}_{4j-k+\nu}(z_0) \right) + \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \binom{k}{2j+1} \left( i \left(1-i^k\right) \operatorname{ber}_{4j-k+\nu+2}(z_0) - \left(1+i^k\right) \operatorname{bei}_{4j-k+\nu+2}(z_0) \right) \right) (z-z_0)^k$$

03.17.06.0008.01

$$\operatorname{bei}_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{bei}_\nu(z_0) \left(1 + O(z-z_0)\right)$$

## Expansions on branch cuts

03.17.06.0009.01

$$\operatorname{bei}_\nu(z) \propto e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left( \operatorname{bei}_\nu(x) + \left( -\frac{\operatorname{bei}_{\nu-1}(x)}{\sqrt{2}} + \frac{\operatorname{ber}_{\nu-1}(x)}{\sqrt{2}} - \frac{\nu \operatorname{bei}_\nu(x)}{x} \right)(z-x) + \frac{x\left(\sqrt{2}\, (\operatorname{bei}_{\nu-1}(x) - \operatorname{ber}_{\nu-1}(x)) + 2x \operatorname{ber}_\nu(x)\right) + 2\nu(\nu+1) \operatorname{bei}_\nu(x)}{4x^2} (z-x)^2 + \ldots \right) /; (z \to x) \land x \in \mathbb{R} \land x < 0$$

03.17.06.0010.01

$$\operatorname{bei}_\nu(z) \propto e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left( \operatorname{bei}_\nu(x) + \left( -\frac{\operatorname{bei}_{\nu-1}(x)}{\sqrt{2}} + \frac{\operatorname{ber}_{\nu-1}(x)}{\sqrt{2}} - \frac{\nu \operatorname{bei}_\nu(x)}{x} \right)(z-x) + \frac{x\left(\sqrt{2}\, (\operatorname{bei}_{\nu-1}(x) - \operatorname{ber}_{\nu-1}(x)) + 2x \operatorname{ber}_\nu(x)\right) + 2\nu(\nu+1) \operatorname{bei}_\nu(x)}{4x^2} (z-x)^2 + O\left((z-x)^3\right) \right) /; x \in \mathbb{R} \land x < 0$$

03.17.06.0011.01

$$\operatorname{bei}_\nu(z) = 2^{-2\nu-1}\, i\, e^{-\frac{3 i \pi \nu}{4}} \sqrt{\pi}\, x^\nu\, \Gamma(\nu+1)\, e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^k x^{-k}}{k!} \left( {}_2\tilde{F}_3\left( \frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1+\nu-k}{2}, \frac{2+\nu-k}{2}, \nu+1; -\frac{i x^2}{4} \right) - e^{\frac{3 i \pi \nu}{2}}\, {}_2\tilde{F}_3\left( \frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1+\nu-k}{2}, \frac{2+\nu-k}{2}, \nu+1; \frac{i x^2}{4} \right) \right) (z-x)^k /; x \in \mathbb{R} \land x < 0$$

03.17.06.0012.01

$$\operatorname{bei}_\nu(z) = e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^{-\frac{3k}{2}-1} (i-1)^k}{k!} \left( \sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \binom{k}{2j} \left( \left(1+i^k\right) \operatorname{bei}_{4j-k+\nu}(x) - i \left(1-i^k\right) \operatorname{ber}_{4j-k+\nu}(x) \right) + \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \binom{k}{2j+1} \left( i \left(1-i^k\right) \operatorname{ber}_{4j-k+\nu+2}(x) - \left(1+i^k\right) \operatorname{bei}_{4j-k+\nu+2}(x) \right) \right) (z-x)^k /; x \in \mathbb{R} \land x < 0$$

03.17.06.0013.01

$$\operatorname{bei}_\nu(z) \propto e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \operatorname{bei}_\nu(x) \left(1 + O(z-x)\right) /; x \in \mathbb{R} \land x < 0$$

**Expansions at $z == 0$**

## For the function itself

### General case

03.17.06.0014.01

$$\operatorname{bei}_\nu(z) \propto \frac{2^{-\nu} z^\nu \sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)} \left(1 - \frac{z^4}{32(\nu+1)(\nu+2)} + \frac{z^8}{6144(\nu+1)(\nu+2)(\nu+3)(\nu+4)} + \ldots\right) + \frac{2^{-\nu-2} z^{\nu+2} \cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)} \left(1 - \frac{z^4}{96(\nu+2)(\nu+3)} + \frac{z^8}{30\,720(\nu+2)(\nu+3)(\nu+4)(\nu+5)} + \ldots\right) /; (z \to 0) \bigwedge -\nu \notin \mathbb{N}^+$$

03.17.06.0015.01

$$\operatorname{bei}_\nu(z) \propto \frac{2^{-\nu} z^\nu \sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)} \left(1 - \frac{z^4}{32(\nu+1)(\nu+2)} + \frac{z^8}{6144(\nu+1)(\nu+2)(\nu+3)(\nu+4)} + O\left(z^{12}\right)\right) + \frac{2^{-\nu-2} z^{\nu+2} \cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)} \left(1 - \frac{z^4}{96(\nu+2)(\nu+3)} + \frac{z^8}{30\,720(\nu+2)(\nu+3)(\nu+4)(\nu+5)} + O\left(z^{12}\right)\right) /; -\nu \notin \mathbb{N}^+$$

03.17.06.0016.01

$$\operatorname{bei}_\nu(z) == \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\nu+1)\, k!} \sin\left(\frac{\pi}{4}(2k+3\nu)\right) \left(\frac{z}{2}\right)^{2k}$$

03.17.06.0017.01

$$\operatorname{bei}_\nu(z) = \frac{\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)} \left(\frac{z}{2}\right)^{\nu+2} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu}{2}+1\right)_k \left(\frac{\nu+3}{2}\right)_k \left(\frac{3}{2}\right)_k k!} + \frac{\sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^{\nu} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu+1}{2}\right)_k \left(\frac{\nu}{2}+1\right)_k \left(\frac{1}{2}\right)_k k!} /; -\nu \notin \mathbb{N}^+$$

03.17.06.0018.01

$$\operatorname{bei}_\nu(z) = \frac{\sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu {}_0F_3\left(;\frac{1}{2}, \frac{\nu+1}{2}, \frac{\nu}{2}+1; -\frac{z^4}{256}\right) + \frac{\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)} \left(\frac{z}{2}\right)^{\nu+2} {}_0F_3\left(;\frac{3}{2}, \frac{\nu}{2}+1, \frac{\nu+3}{2}; -\frac{z^4}{256}\right) /; -\nu \notin \mathbb{N}^+$$

03.17.06.0019.01

$$\operatorname{bei}_\nu(z) = 4^{-\nu} \pi \sin\left(\frac{3\pi\nu}{4}\right) z^\nu {}_0\tilde{F}_3\left(;\frac{1}{2}, \frac{\nu+1}{2}, \frac{\nu}{2}+1; -\frac{z^4}{256}\right) + 4^{-\nu-2} \pi \cos\left(\frac{3\pi\nu}{4}\right) z^{\nu+2} {}_0\tilde{F}_3\left(;\frac{3}{2}, \frac{\nu}{2}+1, \frac{\nu+3}{2}; -\frac{z^4}{256}\right)$$

03.17.06.0020.01

$$\operatorname{bei}_\nu(z) \propto \frac{2^{-\nu} z^\nu \sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)} \left(1 + O\left(z^2\right)\right) /; -\nu \notin \mathbb{N}^+$$

03.17.06.0021.01

$$\mathrm{bei}_\nu(z) \propto \begin{cases} \frac{(-1)^{\nu/4} 2^{\nu-2} z^{2-\nu}}{(1-\nu)!} \left(1 + O(z^2)\right) & \frac{\nu}{4} \in \mathbb{Z} \bigwedge \nu < 0 \\ \frac{(-1)^{\frac{\nu-1}{4}} 2^{\nu-\frac{1}{2}} z^{-\nu}}{(-\nu)!} \left(1 + O(z^2)\right) & \frac{\nu-1}{4} \in \mathbb{Z} \bigwedge \nu < 0 \\ \frac{(-1)^{\frac{\nu-2}{4}} 2^{\nu} z^{-\nu}}{(-\nu)!} \left(1 + O(z^2)\right) & \frac{\nu-2}{4} \in \mathbb{Z} \bigwedge \nu < 0 \\ \frac{(-1)^{\frac{\nu-3}{4}} 2^{\nu-\frac{1}{2}} z^{-\nu}}{(-\nu)!} \left(1 + O(z^2)\right) & \frac{\nu-3}{4} \in \mathbb{Z} \bigwedge \nu < 0 \\ \frac{2^{-\nu} z^{\nu} \sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)} \left(1 + O(z^2)\right) & \text{True} \end{cases}$$

03.17.06.0022.01

$$\mathrm{bei}_\nu(z) = F_\infty(z, \nu) \,/;\, \left(\left(F_n(z, \nu) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{n} \frac{\sin\left(\frac{1}{4}\pi(2k+3\nu)\right)}{\Gamma(k+\nu+1)\,k!} \left(\frac{z}{2}\right)^{2k} = \mathrm{bei}_\nu(z) - \right.\right.$$

$$\left.\left.(-i)^n 2^{-2n-\nu-3} e^{-\frac{3i\pi\nu}{4}} z^{2n+\nu+2} \left((-1)^n e^{\frac{3i\pi\nu}{2}} {}_1\tilde{F}_2\left(1; n+2, n+\nu+2; \frac{i z^2}{4}\right) + {}_1\tilde{F}_2\left(1; n+2, n+\nu+2; -\frac{i z^2}{4}\right)\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

## Special cases

03.17.06.0023.01

$$\mathrm{bei}_{-2n}(z) \propto \frac{i\, i^n 2^{-2n-1} (1-(-1)^n) z^{2n}}{(2n)!} \left(1 - \frac{z^4}{64(n+1)(2n+1)} + \frac{z^8}{24576(n+1)(n+2)(2n+1)(2n+3)} + O(z^{12})\right) +$$

$$\frac{i^n 2^{-2n-3} (1+(-1)^n) z^{2n+2}}{(2n+1)!} \left(1 - \frac{z^4}{192(n+1)(2n+3)} + \frac{z^8}{122880(n+1)(n+2)(2n+3)(2n+5)} + O(z^{12})\right) /;\, n \in \mathbb{N}$$

03.17.06.0024.01

$$\mathrm{bei}_{-2n-1}(z) \propto \frac{(-1)^{n+\left\lfloor\frac{n-1}{2}\right\rfloor} 2^{-2n-\frac{3}{2}} z^{2n+1}}{(2n+1)!} \left(1 - \frac{z^4}{64(n+1)(2n+3)} + \frac{z^8}{24576(n+1)(n+2)(2n+3)(2n+5)} + O(z^{12})\right) +$$

$$\frac{(-1)^{n+\left\lfloor\frac{n}{2}\right\rfloor} 2^{-2n-\frac{7}{2}} z^{2n+3}}{(2n+2)!} \left(1 - \frac{z^4}{192(n+2)(2n+3)} + \frac{z^8}{122880(n+2)(n+3)(2n+3)(2n+5)} + O(z^{12})\right) /;\, n \in \mathbb{N}$$

03.17.06.0025.01

$$\mathrm{bei}_\nu(z) == \sum_{k=0}^{\infty} \frac{\sin\left(\frac{1}{4}\pi(2k+\nu)\right)}{k!\,\Gamma(k-\nu+1)} \left(\frac{z}{2}\right)^{2k-\nu} /;\, -\nu \in \mathbb{N}^+$$

03.17.06.0026.01

$$\mathrm{bei}_\nu(z) = \sum_{k=0}^{\infty} \frac{\sin\left(\frac{1}{4}\pi(2k+2\nu+|\nu|)\right)}{\Gamma(k+|\nu|+1)\,k!} \left(\frac{z}{2}\right)^{2k+|\nu|} /;\, \nu \in \mathbb{Z}$$

03.17.06.0027.01

$$\operatorname{bei}_{-2n}(z) = \frac{i\, i^n\, 2^{-2n-1}\,(1-(-1)^n)\, z^{2n}}{(2n)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{1}{2}\right)_k \left(n+\frac{1}{2}\right)_k (n+1)_k\, k!} + \frac{i^n\, 2^{-2n-3}\,(1+(-1)^n)\, z^{2n+2}}{(2n+1)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{3}{2}\right)_k \left(n+\frac{3}{2}\right)_k (n+1)_k\, k!} \;/;\; n \in \mathbb{N}$$

03.17.06.0028.01

$$\operatorname{bei}_{-2n-1}(z) = \frac{(-1)^{n+\left\lfloor \frac{n-1}{2} \right\rfloor}\, 2^{-2n-\frac{3}{2}}\, z^{2n+1}}{(2n+1)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{1}{2}\right)_k \left(n+\frac{3}{2}\right)_k (n+1)_k\, k!} + \frac{(-1)^{n+\left\lfloor \frac{n}{2} \right\rfloor}\, 2^{-2n-\frac{7}{2}}\, z^{2n+3}}{(2n+2)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{3}{2}\right)_k \left(n+\frac{3}{2}\right)_k (n+2)_k\, k!} \;/;\; n \in \mathbb{N}$$

03.17.06.0029.01

$$\operatorname{bei}_{-2n}(z) = \frac{i\, i^n\, 2^{-2n-1}\,(1-(-1)^n)\, z^{2n}}{(2n)!}\, {}_0F_3\!\left(;\frac{1}{2}, n+\frac{1}{2}, n+1; -\frac{z^4}{256}\right) + \frac{i^n\, 2^{-2n-3}\,(1+(-1)^n)\, z^{2n+2}}{(2n+1)!}\, {}_0F_3\!\left(;\frac{3}{2}, n+1, n+\frac{3}{2}; -\frac{z^4}{256}\right) \;/;\; n \in \mathbb{N}$$

03.17.06.0030.01

$$\operatorname{bei}_{-2n-1}(z) = \frac{(-1)^{n+\left\lfloor \frac{n-1}{2} \right\rfloor}\, 2^{-2n-\frac{3}{2}}\, z^{2n+1}}{(2n+1)!}\, {}_0F_3\!\left(;\frac{1}{2}, n+\frac{3}{2}, n+1; -\frac{z^4}{256}\right) + \frac{(-1)^{n+\left\lfloor \frac{n}{2} \right\rfloor}\, 2^{-2n-\frac{7}{2}}\, z^{2n+3}}{(2n+2)!}\, {}_0F_3\!\left(;\frac{3}{2}, n+\frac{3}{2}, n+2; -\frac{z^4}{256}\right) \;/;\; n \in \mathbb{N}$$

03.17.06.0031.01

$$\operatorname{bei}_{-2n}(z) = (-1)^{\frac{n+1}{2}}\, 2^{-4n-1}\,(1-(-1)^n)\,\pi\, z^{2n}\, {}_0\tilde{F}_3\!\left(;\frac{1}{2}, n+\frac{1}{2}, n+1; -\frac{z^4}{256}\right) + (-1)^{n/2}\, 2^{-4n-5}\,(1+(-1)^n)\,\pi\, z^{2n+2}\, {}_0\tilde{F}_3\!\left(;\frac{3}{2}, n+1, n+\frac{3}{2}; -\frac{z^4}{256}\right) \;/;\; n \in \mathbb{N}$$

03.17.06.0032.01

$$\operatorname{bei}_{-2n-1}(z) = (-1)^{n+\left\lfloor \frac{n-1}{2} \right\rfloor}\, 2^{-4n-\frac{5}{2}}\,\pi\, z^{2n+1}\, {}_0\tilde{F}_3\!\left(;\frac{1}{2}, n+\frac{3}{2}, n+1; -\frac{z^4}{256}\right) + (-1)^{n+\left\lfloor \frac{n}{2} \right\rfloor}\, 2^{-4n-\frac{13}{2}}\,\pi\, z^{2n+3}\, {}_0\tilde{F}_3\!\left(;\frac{3}{2}, n+\frac{3}{2}, n+2; -\frac{z^4}{256}\right) \;/;\; n \in \mathbb{N}$$

### Asymptotic series expansions

#### Expansions inside Stokes sectors

## Expansions containing $z \to \infty$

In exponential form ||| In exponential form

03.17.06.0033.01

$$\operatorname{bei}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{z}} \left( e^{\frac{z}{\sqrt{2}}} \left( e^{-\frac{1}{8}(3 i \pi)-\frac{i z}{\sqrt{2}}-\frac{i \pi \nu}{2}} + e^{\frac{3 i \pi}{8}+\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}} \right) + e^{-\frac{z}{\sqrt{2}}} \left( e^{\frac{i \pi}{8}+\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}} - e^{-\frac{1}{8}(i \pi)-\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \right) + \right.$$

$$\frac{1-4\nu^2}{8 z} \left( e^{\frac{z}{\sqrt{2}}} \left( e^{-\frac{1}{8}(i \pi)-\frac{i z}{\sqrt{2}}-\frac{i \pi \nu}{2}} + e^{\frac{i \pi}{8}+\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}} \right) + e^{-\frac{z}{\sqrt{2}}} \left( -e^{\frac{3 i \pi}{8}+\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}} + e^{-\frac{1}{8}(3 i \pi)-\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \right) \right) +$$

$$\frac{i\left(16\nu^4-40\nu^2+9\right)}{128 z^2} \left( e^{\frac{z}{\sqrt{2}}} \left( e^{-\frac{1}{8}(3 i \pi)-\frac{i z}{\sqrt{2}}-\frac{i \pi \nu}{2}} - e^{\frac{3 i \pi}{8}+\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}} \right) + e^{-\frac{z}{\sqrt{2}}} \left( e^{\frac{i \pi}{8}+\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}} + e^{-\frac{1}{8}(i \pi)-\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \right) \right) -$$

$$\frac{i\left(64\nu^6-560\nu^4+1036\nu^2-225\right)}{3072 z^3} \left( e^{\frac{z}{\sqrt{2}}} \left( e^{-\frac{1}{8}(i \pi)-\frac{i z}{\sqrt{2}}-\frac{i \pi \nu}{2}} - e^{\frac{i \pi}{8}+\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}} \right) + \right.$$

$$\left.\left. e^{-\frac{z}{\sqrt{2}}} \left( -e^{\frac{3 i \pi}{8}+\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}} - e^{-\frac{1}{8}(3 i \pi)-\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \right) \right) + \ldots \right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.17.06.0034.01

$$\operatorname{bei}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{z}} \left( \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{4 z^2}\right)^k \right.$$

$$\left( e^{-\frac{z}{\sqrt{2}}} \left( -(-1)^k e^{-\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}-\frac{\pi i}{8}} + e^{\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}+\frac{\pi i}{8}} \right) + e^{\frac{z}{\sqrt{2}}} \left( e^{-\frac{i z}{\sqrt{2}}-\frac{i \pi \nu}{2}-\frac{3 \pi i}{8}} + (-1)^k e^{\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}+\frac{3 \pi i}{8}} \right) \right) +$$

$$\frac{1}{2 z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(\frac{i}{4 z^2}\right)^k \left( e^{\frac{z}{\sqrt{2}}} \left( e^{-\frac{i z}{\sqrt{2}}-\frac{i \pi \nu}{2}-\frac{\pi i}{8}} + (-1)^k e^{\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}+\frac{\pi i}{8}} \right) + \right.$$

$$\left.\left. e^{-\frac{z}{\sqrt{2}}} \left( (-1)^k e^{-\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}-\frac{3 \pi i}{8}} - e^{\frac{i z}{\sqrt{2}}+\frac{i \pi \nu}{2}+\frac{3 \pi i}{8}} \right) \right) + \ldots \right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.17.06.0035.01

$$\operatorname{bei}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{z}}\left(e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}+\frac{\pi i}{8}}\,{}_4F_1\!\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)-e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}-\frac{\pi i}{8}}\,{}_4F_1\!\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\right)+e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{iz}{\sqrt{2}}-\frac{i\pi\nu}{2}-\frac{3\pi i}{8}}\,{}_4F_1\!\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)+e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}+\frac{3\pi i}{8}}\,{}_4F_1\!\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\right)+\frac{1-4\nu^2}{8z}\left(e^{-\frac{z}{\sqrt{2}}}\left(e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}-\frac{3\pi i}{8}}\,{}_4F_1\!\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)-e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}+\frac{3\pi i}{8}}\,{}_4F_1\!\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)\right)+e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{iz}{\sqrt{2}}-\frac{i\pi\nu}{2}-\frac{\pi i}{8}}\,{}_4F_1\!\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)+e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}+\frac{\pi i}{8}}\,{}_4F_1\!\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)\right)\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.17.06.0036.01

$$\operatorname{bei}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{z}}\left(e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}+\frac{\pi i}{8}}\left(1+O\!\left(\frac{1}{z^2}\right)\right)-e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}-\frac{\pi i}{8}}\left(1+O\!\left(\frac{1}{z^2}\right)\right)\right)+e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{iz}{\sqrt{2}}-\frac{i\pi\nu}{2}-\frac{3\pi i}{8}}\left(1+O\!\left(\frac{1}{z^2}\right)\right)+e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}+\frac{3\pi i}{8}}\left(1+O\!\left(\frac{1}{z^2}\right)\right)\right)\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

## In trigonometric form ||| In trigonometric form

03.17.06.0037.01

$$\operatorname{bei}_\nu(z) \propto e^{i\pi\nu-\frac{z}{\sqrt{2}}}(-i)\cos\!\left(\frac{1}{8}\left(\pi(4\nu+3)-4\sqrt{2}\,z\right)\right)-e^{\frac{z}{\sqrt{2}}}\sin\!\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)+\frac{1-4\nu^2}{8z}\left(i\,e^{i\pi\nu-\frac{z}{\sqrt{2}}}\cos\!\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)-e^{\frac{z}{\sqrt{2}}}\cos\!\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)\right)+\frac{16\nu^4-40\nu^2+9}{128z^2}\left(i\,e^{i\pi\nu-\frac{z}{\sqrt{2}}}\sin\!\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+3)\right)\right)-e^{\frac{z}{\sqrt{2}}}\cos\!\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(1-4\nu)\right)\right)\right)-\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\left(-i\,e^{i\pi\nu-\frac{z}{\sqrt{2}}}\sin\!\left(\frac{1}{8}\left(\pi(4\nu+1)-4\sqrt{2}\,z\right)\right)-e^{\frac{z}{\sqrt{2}}}\sin\!\left(\frac{1}{8}\left(-4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)\right)+\ldots\;/;\;-\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.17.06.0038.01

$$\operatorname{bei}_\nu(z) \propto \frac{1}{\sqrt{2\pi z}} \left( -\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(-\frac{1}{4z^2}\right)^k \left( e^{i\pi\nu - \frac{z}{\sqrt{2}}} i \cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(\pi(4\nu+3) - 4\sqrt{2}\, z\right)\right) + (-1)^k e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{\pi k}{2} + \frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right)\right) + \frac{1}{2z} \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(-\frac{1}{4z^2}\right)^k \left( i(-1)^k e^{i\pi\nu - \frac{z}{\sqrt{2}}} \cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z - \pi(4\nu+1)\right)\right) - e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right)\right) + \ldots \right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.17.06.0039.01

$$\operatorname{bei}_\nu(z) \propto \frac{1}{\sqrt{2\pi z}} \left( -\left( e^{i\pi\nu - \frac{z}{\sqrt{2}}} i \cos\left(\frac{1}{8}\left(\pi(4\nu+3) - 4\sqrt{2}\, z\right)\right) + e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right)\right) {}_8F_3\left(\frac{1}{8}(1-2\nu), \frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(2\nu+1), \frac{1}{8}(2\nu+3), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7); \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{16}{z^4}\right) + \frac{1-4\nu^2}{8z} \left( i\, e^{i\pi\nu - \frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z - \pi(4\nu+1)\right)\right) - e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right)\right) {}_8F_3\left(\frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(2\nu+3), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9); \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; -\frac{16}{z^4}\right) - \frac{16\nu^4 - 40\nu^2 + 9}{128 z^2} \left( e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right) + e^{i\pi\nu - \frac{z}{\sqrt{2}}} i \sin\left(\frac{1}{8}\left(\pi(4\nu+3) - 4\sqrt{2}\, z\right)\right)\right) {}_8F_3\left(\frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(11-2\nu), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9), \frac{1}{8}(2\nu+11); \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; -\frac{16}{z^4}\right) + \frac{-64\nu^6 + 560\nu^4 - 1036\nu^2 + 225}{3072 z^3} \left( -i\, e^{i\pi\nu - \frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z - \pi(4\nu+1)\right)\right) - e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi(4\nu+1)\right)\right)\right) {}_8F_3\left(\frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(11-2\nu), \frac{1}{8}(13-2\nu), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9), \frac{1}{8}(2\nu+11), \frac{1}{8}(2\nu+13); \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -\frac{16}{z^4}\right)\right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.17.06.0040.01

$$\operatorname{bei}_\nu(z) \propto -\frac{1}{\sqrt{2\pi z}} \left( e^{i\pi\nu - \frac{z}{\sqrt{2}}} i \cos\left(\frac{1}{8}\left(\pi(4\nu+3) - 4\sqrt{2}\, z\right)\right) + e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(\pi(1-4\nu) - 4\sqrt{2}\, z\right)\right)\right) \left(1 + O\left(\frac{1}{z^4}\right)\right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

## Expansions containing $z \to -\infty$

## In exponential form ||| In exponential form

03.17.06.0041.01

$$\operatorname{bei}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(e^{\frac{z}{\sqrt{2}}}\left(-e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}}+e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}}+e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\right)+\right.$$
$$\frac{1-4\nu^2}{8z}\left(e^{-\frac{z}{\sqrt{2}}}\left(-e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}}-e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\right)+e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}-e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}}\right)\right)+$$
$$\frac{i\left(16\nu^4-40\nu^2+9\right)}{128z^2}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}}+e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}}-e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\right)\right)-$$
$$\frac{i\left(64\nu^6-560\nu^4+1036\nu^2-225\right)}{3072z^3}\left(e^{-\frac{z}{\sqrt{2}}}\left(-e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}}+e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\right)+\right.$$
$$\left.\left.e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}+e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}}\right)\right)+\ldots\right)/;\ \frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.17.06.0042.01

$$\operatorname{bei}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{4z^2}\right)^k\right.$$
$$\left(e^{\frac{z}{\sqrt{2}}}\left(-(-1)^k e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}-\frac{\pi i}{8}}+e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}+\frac{\pi i}{8}}\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}-\frac{3\pi i}{8}}+(-1)^k e^{-\frac{iz}{\sqrt{2}}+\frac{6i\pi\nu}{4}+\frac{3\pi i}{8}}\right)\right)+$$
$$\frac{1}{2z}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{4z^2}\right)^k\left(e^{-\frac{z}{\sqrt{2}}}\left(-e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}-\frac{\pi i}{8}}-e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}+\frac{\pi i}{8}}(-1)^k\right)+\right.$$
$$\left.\left.e^{\frac{z}{\sqrt{2}}}\left(-e^{\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}-\frac{3\pi i}{8}}(-1)^k+e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}+\frac{3\pi i}{8}}\right)\right)+\ldots\right)/;\ \frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)\bigwedge n\in\mathbb{N}$$

03.17.06.0043.01

$$\operatorname{bei}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}+\frac{\pi i}{8}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)-e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}-\frac{\pi i}{8}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}-\frac{3\pi i}{8}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)+e^{-\frac{iz}{\sqrt{2}}+\frac{6i\pi\nu}{4}+\frac{3\pi i}{8}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\right)+\frac{1-\nu^2}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}+\frac{3\pi i}{8}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)-e^{\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}-\frac{3\pi i}{8}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(-e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}-\frac{\pi i}{8}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)-e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}+\frac{\pi i}{8}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)\right)\right)\;/;\;\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.17.06.0044.01

$$\operatorname{bei}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}+\frac{\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)-e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}-\frac{\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}-\frac{3\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)+e^{-\frac{iz}{\sqrt{2}}+\frac{6i\pi\nu}{4}+\frac{3\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)+\frac{1-\nu^2}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}+\frac{3\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)-e^{\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}-\frac{3\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(-e^{\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}-\frac{\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)-e^{-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}+\frac{\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)\right)\right)\;/;\;\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

## In trigonometric form ||| In trigonometric form

03.17.06.0045.01

$$\mathrm{bei}_\nu(z) \propto -\frac{i\, e^{i\pi\nu}}{\sqrt{2\pi}\,\sqrt{-z}} \left( \left( e^{\frac{z}{\sqrt{2}}+i\pi\nu} \sin\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)\right) + \frac{1-4\nu^2}{8z}\left(e^{\frac{z}{\sqrt{2}}+i\pi\nu}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right) + e^{-\frac{z}{\sqrt{2}}}\, i\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)\right) + \frac{16\nu^4-40\nu^2+9}{128z^2}\left(e^{\frac{z}{\sqrt{2}}+i\pi\nu}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(1-4\nu)\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(-4\sqrt{2}\,z-\pi(1-4\nu)\right)\right)\right) + \frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\left(i\, e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(\pi(4\nu+1)-4\sqrt{2}\,z\right)\right) - e^{\frac{z}{\sqrt{2}}+i\pi\nu}\sin\left(\frac{1}{8}\left(-4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)\right) + \ldots\right) /; (z\to -\infty)$$

03.17.06.0046.01

$$\mathrm{bei}_\nu(z) \propto -\frac{i\, e^{i\pi\nu}}{\sqrt{2\pi}\,\sqrt{-z}} \left( \sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{1}{4z^2}\right)^k \left(e^{\frac{z}{\sqrt{2}}+i\pi\nu}\sin\left(\frac{\pi k}{2}+\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{\pi k}{2}+\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)\right) + \frac{1}{2z}\sum_{k=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{1}{4z^2}\right)^k \left(e^{-\frac{z}{\sqrt{2}}}\, i\cos\left(\frac{\pi k}{2}+\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right) + e^{\frac{z}{\sqrt{2}}+i\pi\nu}(-1)^k\cos\left(\frac{\pi k}{2}+\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)\right) + \ldots\right) /; (z\to -\infty)\land n\in\mathbb{N}$$

03.17.06.0047.01

$$\operatorname{bei}_\nu(z) \propto -\frac{i\, e^{i\pi\nu}}{\sqrt{2\pi}\,\sqrt{-z}} \left(\left(e^{\frac{z}{\sqrt{2}}+i\pi\nu} \sin\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)\right) {}_8F_3\left(\frac{1}{8}(1-2\nu), \frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(2\nu+1), \frac{1}{8}(2\nu+3), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7); \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{16}{z^4}\right) - \frac{1-4\nu^2}{8z}\left(e^{-\frac{z}{\sqrt{2}}}(-i)\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right) - e^{\frac{z}{\sqrt{2}}+i\pi\nu}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)\right) {}_8F_3\left(\frac{1}{8}(3-2\nu), \frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(2\nu+3), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9); \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; -\frac{16}{z^4}\right) + \frac{16\nu^4-40\nu^2+9}{128z^2}\left(e^{\frac{z}{\sqrt{2}}+i\pi\nu}\cos\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)\right) {}_8F_3\left(\frac{1}{8}(5-2\nu), \frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(11-2\nu), \frac{1}{8}(2\nu+5), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9), \frac{1}{8}(2\nu+11); \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; -\frac{16}{z^4}\right) + \frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\left(e^{\frac{z}{\sqrt{2}}+i\pi\nu}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)\right) {}_8F_3\left(\frac{1}{8}(7-2\nu), \frac{1}{8}(9-2\nu), \frac{1}{8}(11-2\nu), \frac{1}{8}(13-2\nu), \frac{1}{8}(2\nu+7), \frac{1}{8}(2\nu+9), \frac{1}{8}(2\nu+11), \frac{1}{8}(2\nu+13); \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -\frac{16}{z^4}\right)\right) /; (z \to -\infty)$$

03.17.06.0048.01

$$\operatorname{bei}_\nu(z) \propto -\frac{i\, e^{i\pi\nu}}{\sqrt{2\pi}\,\sqrt{-z}} \left(e^{\frac{z}{\sqrt{2}}+i\pi\nu} \sin\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)\right)\left(1+O\left(\frac{1}{z^4}\right)\right) /; (z \to -\infty)$$

### Expansions for any $z$ in exponential form

## Using exponential function with branch cut-free arguments

03.17.06.0049.01

$$\operatorname{ber}_{\nu}(z) \propto \frac{e^{-\frac{1}{4} i \pi \nu} z^{\nu}}{2 \sqrt{2 \pi} \sqrt[4]{-1}} \left( e^{-\frac{z}{\sqrt{2}}} \left( e^{\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left( \frac{\sqrt[4]{-1} \sqrt{i z^2} \cos(\pi \nu)}{z} - \sin(\pi \nu) \right) - e^{\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1} z\right)^{-\nu-\frac{1}{2}} \right) + \right.$$

$$e^{\frac{z}{\sqrt{2}}} \left( e^{-\frac{i z}{\sqrt{2}}} \left( \frac{(-1)^{3/4} \sqrt{-i z^2} \cos(\pi \nu)}{z} + \sin(\pi \nu) \right) \left(-\sqrt[4]{-1} z\right)^{-\nu-\frac{1}{2}} + e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \right) -$$

$$\frac{(-1)^{3/4} \left(1-4 \nu^2\right)}{8 z} \left( e^{-\frac{z}{\sqrt{2}}} \left( e^{\frac{i z}{\sqrt{2}}} i \left(-\sqrt[4]{-1} z\right)^{-\nu-\frac{1}{2}} + e^{\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left( \sin(\pi \nu) - \frac{\sqrt[4]{-1} \sqrt{i z^2} \cos(\pi \nu)}{z} \right) \right) + \right.$$

$$\left. e^{\frac{z}{\sqrt{2}}} \left( e^{-\frac{i z}{\sqrt{2}}} \left( i \sin(\pi \nu) - \frac{\sqrt[4]{-1} \sqrt{-i z^2} \cos(\pi \nu)}{z} \right) \left(-\sqrt[4]{-1} z\right)^{-\nu-\frac{1}{2}} + e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \right) \right) +$$

$$\frac{i \left(16 \nu^4-40 \nu^2+9\right)}{128 z^2} \left( e^{-\frac{z}{\sqrt{2}}} \left( -e^{\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1} z\right)^{-\nu-\frac{1}{2}} - e^{\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left( \frac{\sqrt[4]{-1} \sqrt{i z^2} \cos(\pi \nu)}{z} - \sin(\pi \nu) \right) \right) + \right.$$

$$\left. e^{\frac{z}{\sqrt{2}}} \left( e^{-\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1} z\right)^{-\nu-\frac{1}{2}} \left( \frac{(-1)^{3/4} \sqrt{-i z^2} \cos(\pi \nu)}{z} + \sin(\pi \nu) \right) - e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \right) \right) -$$

$$\frac{\sqrt[4]{-1} (2 \nu-5) (2 \nu-3) (2 \nu-1) (2 \nu+1) (2 \nu+3) (2 \nu+5)}{3072 z^3}$$

$$\left( e^{-\frac{z}{\sqrt{2}}} \left( i e^{\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1} z\right)^{-\nu-\frac{1}{2}} - e^{\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left( \sin(\pi \nu) - \frac{\sqrt[4]{-1} \sqrt{i z^2} \cos(\pi \nu)}{z} \right) \right) + e^{\frac{z}{\sqrt{2}}} \right.$$

$$\left. \left. \left( e^{-\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1} z\right)^{-\nu-\frac{1}{2}} \left( i \sin(\pi \nu) - \frac{\sqrt[4]{-1} \sqrt{-i z^2} \cos(\pi \nu)}{z} \right) - e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \right) \right) + \ldots \right) /; (|z| \to \infty)$$

03.17.06.0050.01

$$\begin{aligned}
\operatorname{bei}_\nu(z) \propto \; & \frac{1}{2\sqrt{2\pi}\,\sqrt[4]{-1}}\, e^{-\frac{i\pi\nu}{4}} z^\nu \\
& \Bigg(\Bigg(e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(\frac{\sqrt[4]{-1}\,\sqrt{i z^2}}{z}\cos(\pi\nu)-\sin(\pi\nu)\right)\left(\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{i}{z^2}\right)^k+ \right. \\
& \left. O\left(\frac{1}{z^{2\lfloor\frac{n}{2}\rfloor+2}}\right)\right)-e^{\frac{iz}{\sqrt{2}}}\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\left(\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\lfloor\frac{n}{2}\rfloor+2}}\right)\right)\Bigg)+e^{\frac{z}{\sqrt{2}}} \\
& \Bigg(e^{-\frac{iz}{\sqrt{2}}}\left(\frac{(-1)^{3/4}\sqrt{-i z^2}}{z}\cos(\pi\nu)+\sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\left(\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\lfloor\frac{n}{2}\rfloor+2}}\right)\right)+ \\
& e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}\frac{2^{-2k}\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\lfloor\frac{n}{2}\rfloor+2}}\right)\right)\Bigg)\Bigg)- \\
& \frac{(-1)^{3/4}}{z}\Bigg(e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{iz}{\sqrt{2}}}\,i\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}}\left(\sum_{k=0}^{\lfloor\frac{n-1}{2}\rfloor}\frac{2^{-2k-1}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\lfloor\frac{n-1}{2}\rfloor+2}}\right)\right)+ \\
& e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(\sin(\pi\nu)-\frac{\sqrt[4]{-1}\,\sqrt{i z^2}}{z}\cos(\pi\nu)\right)\left(\sum_{k=0}^{\lfloor\frac{n-1}{2}\rfloor}\frac{2^{-2k-1}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{i}{z^2}\right)^k+ \right. \\
& \left. O\left(\frac{1}{z^{2\lfloor\frac{n-1}{2}\rfloor+2}}\right)\right)\Bigg)+e^{\frac{z}{\sqrt{2}}}\Bigg(e^{-\frac{iz}{\sqrt{2}}}\left(i\sin(\pi\nu)-\frac{\sqrt[4]{-1}\,\sqrt{-i z^2}}{z}\cos(\pi\nu)\right)\left(-\sqrt[4]{-1}\,z\right)^{-\nu-\frac{1}{2}} \\
& \left(\sum_{k=0}^{\lfloor\frac{n-1}{2}\rfloor}\frac{2^{-2k-1}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\lfloor\frac{n-1}{2}\rfloor+2}}\right)\right)+e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \\
& \left(\sum_{k=0}^{\lfloor\frac{n-1}{2}\rfloor}\frac{2^{-2k-1}\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{i}{z^2}\right)^k+O\left(\frac{1}{z^{2\lfloor\frac{n-1}{2}\rfloor+2}}\right)\right)\Bigg)\Bigg)\Bigg) \;/;\; (|z|\to\infty)\land n\in\mathbb{N}
\end{aligned}$$

03.17.06.0051.01

$$\operatorname{bei}_\nu(z) \propto \frac{(1-i)\, e^{-\frac{1}{4} i \pi \nu} z^\nu}{4\sqrt{\pi}} \Bigg(\Bigg(e^{\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(e^{-\sqrt[4]{-1}\, z} \left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi\nu)}{z} - \sin(\pi\nu)\right) + e^{\sqrt[4]{-1}\, z}\right)$$

$$\left(\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(-\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\lfloor \frac{n}{2} \rfloor + 2}}\right)\right) - \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}}$$

$$\left(e^{(-1)^{3/4} z} - e^{-(-1)^{3/4} z} \left(\frac{(-1)^{3/4} \sqrt{-i z^2}\, \cos(\pi\nu)}{z} + \sin(\pi\nu)\right)\right) \left(\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\lfloor \frac{n}{2} \rfloor + 2}}\right)\right)\Bigg) +$$

$$\frac{(-1)^{3/4}}{2z} \left(e^{\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(e^{-\sqrt[4]{-1}\, z} \left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi\nu)}{z} - \sin(\pi\nu)\right) - e^{\sqrt[4]{-1}\, z}\right)\right.$$

$$\left(\sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(-\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\lfloor \frac{n-1}{2} \rfloor + 2}}\right)\right) -$$

$$\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(i\, e^{(-1)^{3/4} z} + e^{-(-1)^{3/4} z} \left(i \sin(\pi\nu) - \frac{\sqrt[4]{-1}\, \sqrt{-i z^2}\, \cos(\pi\nu)}{z}\right)\right)$$

$$\left.\left(\sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\lfloor \frac{n-1}{2} \rfloor + 2}}\right)\right)\right)\Bigg) \;/;\; (|z| \to \infty) \land n \in \mathbb{N}$$

03.17.06.0052.01

$$\operatorname{ber}_\nu(z) \propto \frac{1}{2\sqrt{2\pi}\,\sqrt[4]{-1}}\, e^{-\frac{i\pi\nu}{4}} z^\nu \Bigg(\Bigg(e^{\frac{z}{\sqrt{2}}}\Bigg(e^{-\frac{iz}{\sqrt{2}}}\left(\frac{(-1)^{3/4}\sqrt{-i z^2}}{z}\cos(\pi\nu)+\sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} {}_4F_1\!\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)+ e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} {}_4F_1\!\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\Bigg)+ e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(\frac{\sqrt[4]{-1}\sqrt{i z^2}}{z}\cos(\pi\nu)-\sin(\pi\nu)\right) {}_4F_1\!\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)- e^{\frac{iz}{\sqrt{2}}}\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} {}_4F_1\!\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)\Bigg)\Bigg)-\frac{(-1)^{3/4}\left(1-4\nu^2\right)}{8z} \Bigg(e^{\frac{z}{\sqrt{2}}}\Bigg(e^{-\frac{iz}{\sqrt{2}}}\left(i\sin(\pi\nu)-\frac{\sqrt[4]{-1}\sqrt{-i z^2}}{z}\cos(\pi\nu)\right)\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} {}_4F_1\!\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)+ e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} {}_4F_1\!\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\Bigg)+ e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{iz}{\sqrt{2}}}\, i\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} {}_4F_1\!\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)+ e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(\sin(\pi\nu)-\frac{\sqrt[4]{-1}\sqrt{i z^2}}{z}\cos(\pi\nu)\right) {}_4F_1\!\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\Bigg)\Bigg)\Bigg) /;\ (|z|\to\infty)$$

03.17.06.0053.01

$$\operatorname{ber}_\nu(z) \propto \frac{1}{2\sqrt{2\pi}\,\sqrt[4]{-1}}\, e^{-\frac{i\pi\nu}{4}} z^\nu \Bigg(\Bigg(e^{\frac{z}{\sqrt{2}}}\Bigg(e^{-\frac{iz}{\sqrt{2}}}\left(\frac{(-1)^{3/4}\sqrt{-i z^2}}{z}\cos(\pi\nu)+\sin(\pi\nu)\right)\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}}\left(1+O\!\left(\frac{1}{z^2}\right)\right)+ e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(1+O\!\left(\frac{1}{z^2}\right)\right)\Bigg)+ e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(\frac{\sqrt[4]{-1}\sqrt{i z^2}}{z}\cos(\pi\nu)-\sin(\pi\nu)\right)\left(1+O\!\left(\frac{1}{z^2}\right)\right)- e^{\frac{iz}{\sqrt{2}}}\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}}\left(1+O\!\left(\frac{1}{z^2}\right)\right)\Bigg)\Bigg)\Bigg) /;\ (|z|\to\infty)$$

03.17.06.0054.01

$$\operatorname{bei}_\nu(z) \propto \begin{cases} \frac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z - \frac{i\pi\nu}{2}} \left(i\, e^{\sqrt{2}\, z} - (-1)^{3/4} e^{2 i\pi\nu} - \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z + i\pi\nu} - e^{\sqrt{2}\, i z + i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & -\frac{\pi}{4} < \arg(z) \le \frac{\pi}{4} \\ \frac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z - \frac{i\pi\nu}{2}} \left(i\, e^{\sqrt{2}\, z} - (-1)^{3/4} e^{2 i\pi\nu} - e^{\sqrt{2}\, i z + i\pi\nu} + \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z + 3 i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & \frac{\pi}{4} < \arg(z) \le \frac{3\pi}{4} \\ \frac{\sqrt[8]{-1}\, e^{\frac{i\pi\nu}{2} - \sqrt[4]{-1}\, z} \left(-e^{i\sqrt{2}\, z} - (-1)^{3/4} e^{i\pi\nu} - i\, e^{\sqrt{2}\, z + i\pi\nu} + \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z + 2 i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & \arg(z) > \frac{3\pi}{4} \\ \frac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z - \frac{3 i\pi\nu}{2}} \left(e^{i\sqrt{2}\, z} - (-1)^{3/4} e^{3 i\pi\nu} + i\, e^{\sqrt{2}\, z + i\pi\nu} - \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z + 2 i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & -\frac{3\pi}{4} < \arg(z) \le -\frac{\pi}{4} \\ \frac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z - \frac{3 i\pi\nu}{2}} \left(e^{i\sqrt{2}\, z} + (-1)^{3/4} e^{i\pi\nu} + i\, e^{\sqrt{2}\, z + i\pi\nu} - \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z + 2 i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & \text{True} \end{cases} \quad /;\ (|z| \to \infty)$$

### Residue representations

03.17.06.0055.01

$$\operatorname{bei}_\nu(z) = \pi \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{z}{4}\right)^{-4s} \Gamma\left(s + \frac{\nu+2}{4}\right)}{\Gamma(s+\nu)\,\Gamma(1-s-\nu)\,\Gamma\left(\frac{\nu+2}{4} - s\right)\Gamma\left(-s + \frac{\nu}{4} + 1\right)}\, \Gamma\left(s + \frac{\nu}{4}\right)\right)\left(-j - \frac{\nu}{4}\right) + \pi \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{z}{4}\right)^{-4s} \Gamma\left(s + \frac{\nu}{4}\right)}{\Gamma(s+\nu)\,\Gamma(1-s-\nu)\,\Gamma\left(\frac{\nu+2}{4} - s\right)\Gamma\left(-s + \frac{\nu}{4} + 1\right)}\, \Gamma\left(s + \frac{\nu+2}{4}\right)\right)\left(-j - \frac{\nu+2}{4}\right)$$

## Integral representations

### On the real axis

#### Of the direct function

03.17.07.0001.01

$$\operatorname{bei}_\nu(z) = \frac{\left(\frac{z}{2}\right)^\nu}{\Gamma\left(\nu + \frac{1}{2}\right)\sqrt{\pi}} \int_0^\pi \left(\cos\left(\frac{z\cos(t)}{\sqrt{2}}\right)\cosh\left(\frac{z\cos(t)}{\sqrt{2}}\right)\sin\left(\frac{3\pi\nu}{4}\right) + \cos\left(\frac{3\pi\nu}{4}\right)\sin\left(\frac{z\cos(t)}{\sqrt{2}}\right)\sinh\left(\frac{z\cos(t)}{\sqrt{2}}\right)\right)\sin^{2\nu}(t)\, dt \;/;\; \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.17.07.0002.01

$$\operatorname{bei}_\nu(z) = \frac{2^{1-\nu} z^\nu}{\sqrt{\pi}\,\Gamma\left(\nu + \frac{1}{2}\right)} \int_0^1 \left(1 - t^2\right)^{\nu - \frac{1}{2}} \left(\cos\left(\frac{t z}{\sqrt{2}}\right)\cosh\left(\frac{t z}{\sqrt{2}}\right)\sin\left(\frac{3\pi\nu}{4}\right) + \cos\left(\frac{3\pi\nu}{4}\right)\sin\left(\frac{t z}{\sqrt{2}}\right)\sinh\left(\frac{t z}{\sqrt{2}}\right)\right) dt \;/;\; \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.17.07.0003.01

$$\operatorname{bei}_\nu(z) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\cos\left(\frac{z\sin(t)}{\sqrt{2}}\right)\cosh\left(\frac{z\sin(t)}{\sqrt{2}}\right)\sin\left(\frac{3\pi\nu}{4}\right) + \cos\left(\frac{3\pi\nu}{4}\right)\sin\left(\frac{z\sin(t)}{\sqrt{2}}\right)\sinh\left(\frac{z\sin(t)}{\sqrt{2}}\right)\right)\cos^{2\nu}(t)\, dt \;/;\; \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.17.07.0004.01

$$\operatorname{bei}_n(z) = \frac{1}{\pi}\int_0^{\pi} e^{-\frac{z\cos(t)}{\sqrt{2}}}\left(\cos\left(\frac{z\cos(t)}{\sqrt{2}}\right)\sin\left(\frac{n\pi}{2}\right)+\cos\left(\frac{n\pi}{2}\right)\sin\left(\frac{z\cos(t)}{\sqrt{2}}\right)\right)\cos(n\,t)\,dt\,/;\,n\in\mathbb{N}^+$$

03.17.07.0005.01

$$\operatorname{bei}_n(z) = \frac{1}{\pi}\int_0^{\pi}\sin\left(n\,t+\frac{z\sin(t)}{\sqrt{2}}\right)\sinh\left(\frac{z\sin(t)}{\sqrt{2}}\right)dt\,/;\,n\in\mathbb{Z}$$

03.17.07.0006.01

$$\operatorname{bei}_\nu(z) = \frac{1}{2\pi i}\left(\frac{z}{2}\right)^{\nu}\int_{\gamma-i\infty}^{i\infty+\gamma} e^{\frac{z^2}{4\sqrt{2}\,t}+t}\sin\left(\frac{3\pi\nu}{4}-\frac{z^2}{4\sqrt{2}\,t}\right)t^{-\nu-1}\,dt\,/;\,\gamma>0\bigwedge\operatorname{Re}(\nu)>0$$

### Contour integral representations

03.17.07.0007.01

$$\operatorname{bei}_\nu(z) = \frac{1}{2i}\int_{\mathcal{L}}\frac{\Gamma\left(s+\frac{\nu}{4}\right)\Gamma\left(s+\frac{\nu+2}{4}\right)}{\Gamma(s+\nu)\,\Gamma(1-s-\nu)\,\Gamma\left(\frac{\nu+2}{4}-s\right)\Gamma\left(1-s+\frac{\nu}{4}\right)}\left(\frac{z}{4}\right)^{-4s}ds$$

## Limit representations

03.17.09.0001.01

$$\operatorname{bei}_\nu(z) = i\,2^{-\nu-1}z^{\nu}\lim_{n\to\infty}\left(\frac{1}{n^{\nu}}\left(e^{-\frac{3i\pi\nu}{4}}P_n^{(\nu,b)}\left(\cos\left(\frac{(1+i)z}{\sqrt{2}\,n}\right)\right)-e^{\frac{3i\pi\nu}{4}}P_n^{(\nu,b)}\left(\cosh\left(\frac{(1+i)z}{\sqrt{2}\,n}\right)\right)\right)\right)$$

03.17.09.0002.01

$$\operatorname{bei}_\nu(z) = i\,2^{-\nu-1}z^{\nu}\lim_{n\to\infty}\left(\frac{1}{n^{\nu}}\left(e^{-\frac{3i\pi\nu}{4}}L_n^{\nu}\left(\frac{i\,z^2}{4n}\right)-e^{\frac{3i\pi\nu}{4}}L_n^{\nu}\left(-\frac{i\,z^2}{4n}\right)\right)\right)$$

03.17.09.0003.01

$$\operatorname{bei}_\nu(z) = \frac{1}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^{\nu}\lim_{a\to\infty}\left(\frac{\cos\left(\frac{3\pi\nu}{4}\right)z^2}{4(\nu+1)}\,{}_1F_3\left(a;\frac{3}{2},\frac{\nu+3}{2},\frac{\nu}{2}+1;-\frac{z^4}{256\,a}\right)+\sin\left(\frac{3\pi\nu}{4}\right){}_1F_3\left(a;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256\,a}\right)\right)$$

## Generating functions

03.17.11.0001.01

$$\sum_{k=-\infty}^{\infty}t^k\operatorname{bei}_k(x) = e^{-\frac{\left(t-\frac{1}{t}\right)x}{2\sqrt{2}}}\sin\left(\frac{\left(t-\frac{1}{t}\right)x}{2\sqrt{2}}\right)$$

## Differential equations

### Ordinary linear differential equations and wronskians

#### For the direct function itself

03.17.13.0001.01

$$w^{(4)}(z)\,z^4+2\,w^{(3)}(z)\,z^3-\left(2\,\nu^2+1\right)w''(z)\,z^2+\left(2\,\nu^2+1\right)w'(z)\,z+\left(z^4+\nu^4-4\,\nu^2\right)w(z)=0\;/;\;w(z)=\mathrm{ber}_\nu(z)\,c_1+\mathrm{bei}_\nu(z)\,c_2+\mathrm{ker}_\nu(z)\,c_3+\mathrm{kei}_\nu(z)\,c_4$$

03.17.13.0002.01

$$W_z(\mathrm{ber}_\nu(z),\,\mathrm{bei}_\nu(z),\,\mathrm{ker}_\nu(z),\,\mathrm{kei}_\nu(z))=-\frac{1}{z^2}$$

03.17.13.0003.01

$$g(z)^4\;g'(z)^3\,w^{(4)}(z)+2\,g(z)^3\left(g'(z)^2-3\,g(z)\,g''(z)\right)g'(z)^2\,w^{(3)}(z)\;-\\
g(z)^2\left(\left(2\,\nu^2+1\right)g'(z)^4+6\,g(z)\,g''(z)\,g'(z)^2+4\,g(z)^2\,g^{(3)}(z)\,g'(z)-15\,g(z)^2\,g''(z)^2\right)g'(z)\,w''(z)\;+\\
g(z)\;\left(\left(2\,\nu^2+1\right)g'(z)^6+\left(2\,\nu^2+1\right)g(z)\,g''(z)\,g'(z)^4-2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3\;+\\
g(z)^2\left(6\,g''(z)^2-g(z)\,g^{(4)}(z)\right)g'(z)^2+10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z)-15\,g(z)^3\,g''(z)^3\right)w'(z)\;+\\
\left(\nu^4-4\,\nu^2+g(z)^4\right)g'(z)^7\;w(z)\;=0\;/;\;w(z)=c_1\,\mathrm{ber}_\nu(g(z))\;+c_2\,\mathrm{bei}_\nu(g(z))\;+c_3\,\mathrm{ker}_\nu(g(z))\;+c_4\,\mathrm{kei}_\nu(g(z))$$

03.17.13.0004.01

$$W_z(\mathrm{ber}_\nu(g(z)),\,\mathrm{bei}_\nu(g(z)),\,\mathrm{ker}_\nu(g(z)),\,\mathrm{kei}_\nu(g\,(z)))=-\frac{g'(z)^6}{g(z)^2}$$

03.17.13.0005.01

$$g(z)^4\,g'(z)^3\;\;h(z)^4\,w^{(4)}(z)+2\,g(z)^3\,g'(z)^2\left(h(z)\left(g'(z)^2-3\,g(z)\,g''(z)\right)-2\,g(z)\,g'(z)\,h'(z)\right)h(z)^3\,w^{(3)}(z)\;+\\
g(z)^2\,g'(z)\left(-\left(\left(2\,\nu^2+1\right)g'(z)^4+6\,g(z)\,g''(z)\,g'(z)^2+4\,g(z)^2\,g^{(3)}(z)\,g'(z)-15\,g(z)^2\,g''(z)^2\right)h(z)^2\;-\\
6\,g(z)\,g'(z)\left(h'(z)\,g'(z)^2+g(z)\,h''(z)\,g'(z)-3\,g(z)\,h'(z)\,g''(z)\right)h(z)+12\,g(z)^2\,g'(z)^2\,h'(z)^2\right)h(z)^2\,w''(z)\;+\\
g(z)\left(\left(\left(\left(2\,\nu^2+1\right)g'(z)^6+\left(2\,\nu^2+1\right)g(z)\,g''(z)\,g'(z)^4-2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3+g(z)^2\left(6\,g''(z)^2-g(z)\,g^{(4)}(z)\right)g'(z)^2\;+\right.\right.\right.\\
10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z)-15\,g(z)^3\,g''(z)^3\right)h(z)^3+2\,g(z)\,g'(z)\left(\left(2\,\nu^2+1\right)h'(z)\,g'(z)^4-3\,g(z)\,h''(z)\,g'(z)^3\;-\\
2\,g(z)\left(g(z)\,h^{(3)}(z)-3\,h'(z)\,g''(z)\right)g'(z)^2+g(z)^2\left(9\,g''(z)\,h''(z)+4\,h'(z)\,g^{(3)}(z)\right)g'(z)-15\,g(z)^2\,h'(z)\,g''(z)^2\right)h(z)^2\;+\\
12\,g(z)^2\,g'(z)^2\,h'(z)\left(h'(z)\,g'(z)^2+2\,g(z)\,h''(z)\,g'(z)-3\,g(z)\,h'(z)\,g''(z)\right)h(z)-24\,g(z)^3\,g'(z)^3\,h'(z)^3\right)h(z)\,w'(z)\;+\\
\left(\left(\nu^4-4\,\nu^2+g(z)^4\right)h(z)^4\,g'(z)^7+g(z)^4\left(24\,h'(z)^4-36\,h(z)\,h''(z)\,h'(z)^2+8\,h(z)^2\,h^{(3)}(z)\,h'(z)+h(z)^2\left(6\,h''(z)^2-h(z)\,h^{(4)}(z)\right)\right)\right.\\
g'(z)^3-2\,g(z)^3\,h(z)\left(g'(z)^2-3\,g(z)\,g''(z)\right)\left(6\,h'(z)^3-6\,h(z)\,h''(z)\,h'(z)+h(z)^2\,h^{(3)}(z)\right)g'(z)^2\;+\\
g(z)^2\,h(z)^2\left(h(z)\,h''(z)-2\,h'(z)^2\right)\left(\left(2\,\nu^2+1\right)g'(z)^4+6\,g(z)\,g''(z)\,g'(z)^2+4\,g(z)^2\,g^{(3)}(z)\,g'(z)-15\,g(z)^2\,g''(z)^2\right)g'(z)\;-\\
g(z)\,h(z)^3\,h'(z)\left(\left(2\,\nu^2+1\right)g'(z)^6+\left(2\,\nu^2+1\right)g(z)\,g''(z)\,g'(z)^4-2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3\;+\\
\left.g(z)^2\left(6\,g''(z)^2-g(z)\,g^{(4)}(z)\right)g'(z)^2+10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z)-15\,g(z)^3\,g''(z)^3\right)\right)w(z)\;=0\;/;\\
w(z)=c_1\,h(z)\,\mathrm{ber}_\nu(g(z))\;+c_2\,h(z)\,\mathrm{bei}_\nu(g(z))\;+c_3\,h(z)\,\mathrm{ker}_\nu(g(z))\;+c_4\,h(z)\,\mathrm{kei}_\nu(g(z))$$

03.17.13.0006.01

$$W_z(h(z)\,\mathrm{ber}_\nu(g(z)),\,h(z)\,\mathrm{bei}_\nu(g(z)),\,h(z)\,\mathrm{ker}_\nu(g(z)),\,h(z)\,\mathrm{kei}_\nu(g\,(z)))=-\frac{h(z)^4\,g'(z)^6}{g(z)^2}$$

03.17.13.0007.01

$$z^4\,w^{(4)}(z)\;+(6-4\,r-4\,s)\;z^3\,w^{(3)}(z)+\left(7-2\left(\nu^2-2\right)r^2+12\,(s-1)\,r+6\,(s-2)\,s\right)z^2\,w''(z)\;+(2\,r+2\,s-1)\\
\left(2\,r^2\,\nu^2-2\,(s-1)\,s+r\,(2-4\,s)-1\right)z\,w'(z)\;+\left(\left(a^4\,z^{4\,r}+\nu^4-4\,\nu^2\right)r^4-4\,s\,\nu^2\,r^3-2\,s^2\left(\nu^2-2\right)r^2+4\,s^3\,r+s^4\right)w(z)=0\;/;\\
w(z)=\;c_1\,z^s\,\mathrm{ber}_\nu(a\,z^r)+\;c_2\,z^s\,\mathrm{bei}_\nu(a\,z^r)+c_3\,z^s\,\mathrm{ker}_\nu(a\,z^r)\;+c_4\,z^s\,\mathrm{kei}_\nu(a\,z^r)$$

03.17.13.0008.01

$$W_z(z^s\,\mathrm{ber}_\nu(a\,z^r),\,z^s\,\mathrm{bei}_\nu(a\,z^r),\,z^s\,\mathrm{ker}_\nu(a\,z^r),\,z^s\,\mathrm{kei}_\nu(a\,z^r))=-a^4\,r^6\,z^{4\,r+4\,s-6}$$

03.17.13.0009.01

$$w^{(4)}(z) - 4(\log(r) + \log(s))\, w^{(3)}(z) + 2\left(-\left(\nu^2 - 2\right)\log^2(r) + 6\log(s)\log(r) + 3\log^2(s)\right) w''(z) + 4(\log(r) + \log(s))\left(\nu^2\log^2(r) - 2\log(s)\log(r) - \log^2(s)\right) w'(z) + \left(\left(a^4 r^{4z} + \nu^4 - 4\nu^2\right)\log^4(r) - 4\nu^2\log(s)\log^3(r) - 2\left(\nu^2 - 2\right)\log^2(s)\log^2(r) + 4\log^3(s)\log(r) + \log^4(s)\right) w(z) = 0 \;/;\; w(z) = c_1 s^z \operatorname{ber}_\nu(a r^z) + c_2 s^z \operatorname{bei}_\nu(a r^z) + c_3 s^z \operatorname{ker}_\nu(a r^z) + c_4 s^z \operatorname{kei}_\nu(a r^z)$$

03.17.13.0010.01

$$W_z(s^z \operatorname{ber}_\nu(a r^z), s^z \operatorname{bei}_\nu(a r^z), s^z \operatorname{ker}_\nu(a r^z), s^z \operatorname{kei}_\nu(a r^z)) = -a^4 r^{4z} s^{4z} \log^6(r)$$

# Transformations

## Transformations and argument simplifications

### Argument involving basic arithmetic operations

03.17.16.0001.01

$$\operatorname{bei}_\nu(-z) = (-z)^\nu z^{-\nu} \operatorname{bei}_\nu(z)$$

03.17.16.0002.01

$$\operatorname{bei}_\nu(i z) = (i z)^\nu z^{-\nu}\left(\sin\left(\frac{3\pi\nu}{2}\right)\operatorname{ber}_\nu(z) - \cos\left(\frac{3\pi\nu}{2}\right)\operatorname{bei}_\nu(z)\right)$$

03.17.16.0003.01

$$\operatorname{bei}_\nu(-i z) = (-i z)^\nu z^{-\nu}\left(\sin\left(\frac{3\pi\nu}{2}\right)\operatorname{ber}_\nu(z) - \cos\left(\frac{3\pi\nu}{2}\right)\operatorname{bei}_\nu(z)\right)$$

03.17.16.0004.01

$$\operatorname{bei}_\nu\left(\frac{1}{\sqrt[4]{-1}} z\right) = -\left(\sqrt[4]{-1}\, z\right)^{-\nu}\left(-(-1)^{3/4} z\right)^\nu\left(\cos\left(\frac{3\pi\nu}{2}\right)\operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) - \sin\left(\frac{3\pi\nu}{2}\right)\operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right)\right)$$

03.17.16.0005.01

$$\operatorname{bei}_\nu\left((-1)^{-3/4} z\right) = \left((-1)^{-3/4} z\right)^\nu\left(\sqrt[4]{-1}\, z\right)^{-\nu}\operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right)$$

03.17.16.0006.01

$$\operatorname{bei}_\nu\left((-1)^{3/4} z\right) = -\left(\sqrt[4]{-1}\, z\right)^{-\nu}\left((-1)^{3/4} z\right)^\nu\left(\cos\left(\frac{3\pi\nu}{2}\right)\operatorname{bei}_\nu\left(\sqrt[4]{-1}\, z\right) - \sin\left(\frac{3\pi\nu}{2}\right)\operatorname{ber}_\nu\left(\sqrt[4]{-1}\, z\right)\right)$$

03.17.16.0007.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z^4}\right) = \frac{1}{2} z^{-\nu-2}\left(z^4\right)^{\nu/4}\left(2\left(\sqrt{z^4}\cos^2\left(\frac{3\pi\nu}{4}\right) + z^2\sin^2\left(\frac{3\pi\nu}{4}\right)\right)\operatorname{bei}_\nu(z) - \sin\left(\frac{3\pi\nu}{2}\right)\left(\sqrt{z^4} - z^2\right)\operatorname{ber}_\nu(z)\right)$$

03.17.16.0008.01

$$\operatorname{bei}_{-\nu}(z) = \cos(\pi\nu)\operatorname{bei}_\nu(z) - \sin(\pi\nu)\operatorname{ber}_\nu(z) + \frac{2\sin(\pi\nu)}{\pi}\operatorname{kei}_\nu(z)$$

## Addition formulas

03.17.16.0009.01

$$\operatorname{bei}_\nu(z_1 - z_2) = \sum_{k=-\infty}^{\infty}\left(\operatorname{bei}_{k+\nu}(z_1)\operatorname{ber}_k(z_2) + \operatorname{bei}_k(z_2)\operatorname{ber}_{k+\nu}(z_1)\right) \;/;\; \left|\frac{z_2}{z_1}\right| < 1 \bigvee \nu \in \mathbb{Z}$$

03.17.16.0010.01

$$\operatorname{bei}_\nu(z_1+z_2) = \sum_{k=-\infty}^{\infty} (\operatorname{bei}_k(z_2)\operatorname{ber}_{\nu-k}(z_1) + \operatorname{bei}_{\nu-k}(z_1)\operatorname{ber}_k(z_2)) \;/;\; \left|\frac{z_2}{z_1}\right| < 1 \bigvee \nu \in \mathbb{Z}$$

## Multiple arguments

03.17.16.0011.01

$$\operatorname{bei}_\nu(z_1 z_2) = z_1^\nu \sum_{k=0}^{\infty} \frac{(1-z_1^2)^k}{k!} \left(\frac{z_2}{2}\right)^k \left(\cos\left(\frac{3k\pi}{4}\right)\operatorname{bei}_{k+\nu}(z_2) + \operatorname{ber}_{k+\nu}(z_2)\sin\left(\frac{3k\pi}{4}\right)\right) \;/;\; \left|\frac{z_2}{z_1}\right| < 1 \bigvee \nu \in \mathbb{Z}$$

## Related transformations

### Involving $\operatorname{ber}_\nu(z)$

03.17.16.0012.01

$$\operatorname{bei}_\nu(z) - i\operatorname{ber}_\nu(z) = -\frac{i\, e^{\frac{3i\pi\nu}{4}} z^\nu}{\left(\sqrt[4]{-1}\, z\right)^\nu} I_\nu\left(\sqrt[4]{-1}\, z\right)$$

03.17.16.0013.01

$$\operatorname{bei}_\nu(z) + i\operatorname{ber}_\nu(z) = \frac{i\, e^{\frac{1}{4}(-3)i\pi\nu} z^\nu}{\left((-1)^{3/4} z\right)^\nu} I_\nu\left((-1)^{3/4} z\right)$$

# Identities

## Recurrence identities

### Consecutive neighbors

03.17.17.0001.01

$$\operatorname{bei}_\nu(z) = -\operatorname{bei}_{\nu+2}(z) - \frac{\sqrt{2}\,(\nu+1)}{z}(\operatorname{bei}_{\nu+1}(z) + \operatorname{ber}_{\nu+1}(z))$$

03.17.17.0002.01

$$\operatorname{bei}_\nu(z) = -\operatorname{bei}_{\nu-2}(z) - \frac{\sqrt{2}\,(\nu-1)}{z}(\operatorname{bei}_{\nu-1}(z) + \operatorname{ber}_{\nu-1}(z))$$

### Distant neighbors

## Increasing

03.17.17.0003.01

$$\operatorname{bei}_\nu(z) = (\nu+1)_{n-1}\left((n+\nu)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k (n-k)!\, 2^{n-2k} z^{2k-n}}{k!\,(n-2k)!\,(-n-\nu)_k (\nu+1)_k}\left(\cos\left(\frac{1}{4}(2k-3n)\pi\right)\operatorname{bei}_{n+\nu}(z)+\sin\left(\frac{1}{4}(2k-3n)\pi\right)\operatorname{ber}_{n+\nu}(z)\right)+\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{(-1)^k(-k+n-1)!\,2^{-2k+n-1} z^{2k-n+1}}{k!\,(-2k+n-1)!\,(-n-\nu+1)_k(\nu+1)_k}\left(\cos\left(\frac{1}{4}(2k-3n-1)\pi\right)\operatorname{bei}_{n+\nu+1}(z)+\sin\left(\frac{1}{4}(2k-3n-1)\pi\right)\operatorname{ber}_{n+\nu+1}(z)\right)\right)/;\ n\in\mathbb{N}$$

03.17.17.0004.01

$$\operatorname{bei}_\nu(z) = -\operatorname{bei}_{\nu+2}(z)+\frac{\sqrt{2}\,(\nu+1)\operatorname{bei}_{\nu+3}(z)}{z}+\frac{4(\nu+1)(\nu+2)\operatorname{ber}_{\nu+2}(z)}{z^2}+\frac{\sqrt{2}\,(\nu+1)\operatorname{ber}_{\nu+3}(z)}{z}$$

03.17.17.0005.01

$$\operatorname{bei}_\nu(z) = \frac{2\sqrt{2}\,(\nu+2)\left(z^2+2(\nu+1)(\nu+3)\right)\operatorname{bei}_{\nu+3}(z)}{z^3}+\operatorname{bei}_{\nu+4}(z)+\frac{2\sqrt{2}\,(\nu+2)\left(z^2-2(\nu+1)(\nu+3)\right)\operatorname{ber}_{\nu+3}(z)}{z^3}-\frac{4(\nu+1)(\nu+2)\operatorname{ber}_{\nu+4}(z)}{z^2}$$

03.17.17.0006.01

$$\operatorname{bei}_\nu(z) = \frac{z^4-16(\nu+1)(\nu+2)(\nu+3)(\nu+4)}{z^4}\operatorname{bei}_{\nu+4}(z)+\frac{2\sqrt{2}\,(\nu+2)\left(-z^2+2\nu^2+8\nu+6\right)}{z^3}\operatorname{ber}_{\nu+5}(z)-\frac{12(\nu+2)(\nu+3)}{z^2}\operatorname{ber}_{\nu+4}(z)-\frac{2\sqrt{2}\,(\nu+2)\left(z^2+2\nu^2+8\nu+6\right)}{z^3}\operatorname{bei}_{\nu+5}(z)$$

03.17.17.0007.01

$$\operatorname{bei}_\nu(z) = \frac{\sqrt{2}\,(\nu+3)\left(-3z^4-16\left(\nu^2+6\nu+8\right)z^2+16\left(\nu^4+12\nu^3+49\nu^2+78\nu+40\right)\right)}{z^5}\operatorname{bei}_{\nu+5}(z)+\frac{\sqrt{2}\,(\nu+3)\left(-3z^4+16\left(\nu^2+6\nu+8\right)z^2+16\left(\nu^4+12\nu^3+49\nu^2+78\nu+40\right)\right)}{z^5}\operatorname{ber}_{\nu+5}(z)+\frac{12(\nu+2)(\nu+3)}{z^2}\operatorname{ber}_{\nu+6}(z)-\frac{\left(z^4-16\left(\nu^4+10\nu^3+35\nu^2+50\nu+24\right)\right)}{z^4}\operatorname{bei}_{\nu+6}(z)$$

## Decreasing

03.17.17.0008.01

$$\operatorname{bei}_\nu(z) = (1-\nu)_{n-1}\left((n-\nu)\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \frac{(n-k)!\,(-1)^k\,2^{n-2k}\,z^{2k-n}}{k!\,(n-2k)!\,(1-\nu)_k\,(\nu-n)_k}\left(\cos\left(\frac{1}{4}(2k+n)\pi\right)\operatorname{bei}_{\nu-n}(z)+\sin\left(\frac{1}{4}(2k+n)\pi\right)\operatorname{ber}_{\nu-n}(z)\right) - \sum_{k=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor} \frac{(-k+n-1)!\,(-1)^k\,2^{-2k+n-1}\,z^{2k-n+1}}{k!\,(-2k+n-1)!\,(1-\nu)_k\,(-n+\nu+1)_k}\left(\cos\left(\frac{1}{4}(2k+n-1)\pi\right)\operatorname{bei}_{-n+\nu-1}(z)+\sin\left(\frac{1}{4}(2k+n-1)\pi\right)\operatorname{ber}_{-n+\nu-1}(z)\right)\right) /;\ n\in\mathbb{N}$$

03.17.17.0009.01

$$\operatorname{bei}_\nu(z) = \frac{\sqrt{2}\,(\nu-1)}{z}\operatorname{bei}_{\nu-3}(z) - \operatorname{bei}_{\nu-2}(z) + \frac{\sqrt{2}\,(\nu-1)}{z}\operatorname{ber}_{\nu-3}(z) + \frac{4(\nu-2)(\nu-1)}{z^2}\operatorname{ber}_{\nu-2}(z)$$

03.17.17.0010.01

$$\operatorname{bei}_\nu(z) = \operatorname{bei}_{\nu-4}(z) + \frac{2\sqrt{2}\,(\nu-2)\left(z^2+2\nu^2-8\nu+6\right)}{z^3}\operatorname{bei}_{\nu-3}(z) + \frac{2\sqrt{2}\,(\nu-2)\left(z^2-2\nu^2+8\nu-6\right)}{z^3}\operatorname{ber}_{\nu-3}(z) - \frac{4(\nu-2)(\nu-1)}{z^2}\operatorname{ber}_{\nu-4}(z)$$

03.17.17.0011.01

$$\operatorname{bei}_\nu(z) = -\frac{2\sqrt{2}\,(\nu-2)\left(z^2+2\nu^2-8\nu+6\right)}{z^3}\operatorname{bei}_{\nu-5}(z) + \frac{\left(z^4-16\left(\nu^4-10\nu^3+35\nu^2-50\nu+24\right)\right)}{z^4}\operatorname{bei}_{\nu-4}(z) - \frac{12(\nu-3)(\nu-2)}{z^2}\operatorname{ber}_{\nu-4}(z) - \frac{2\sqrt{2}\,(\nu-2)\left(z^2-2\nu^2+8\nu-6\right)}{z^3}\operatorname{ber}_{\nu-5}(z)$$

03.17.17.0012.01

$$\operatorname{bei}_\nu(z) = -\frac{\left(z^4-16\left(\nu^4-10\nu^3+35\nu^2-50\nu+24\right)\right)}{z^4}\operatorname{bei}_{\nu-6}(z) + \frac{\sqrt{2}\,(\nu-3)\left(-3z^4-16\left(\nu^2-6\nu+8\right)z^2+16\left(\nu^4-12\nu^3+49\nu^2-78\nu+40\right)\right)}{z^5}\operatorname{bei}_{\nu-5}(z) + \frac{12(\nu-3)(\nu-2)}{z^2}\operatorname{ber}_{\nu-6}(z) + \frac{\sqrt{2}\,(\nu-3)\left(-3z^4+16\left(\nu^2-6\nu+8\right)z^2+16\left(\nu^4-12\nu^3+49\nu^2-78\nu+40\right)\right)}{z^5}\operatorname{ber}_{\nu-5}(z)$$

## Functional identities

### Relations between contiguous functions

03.17.17.0013.01

$$\operatorname{bei}_\nu(z) = \frac{z}{2\sqrt{2}\,\nu}\left(-\operatorname{bei}_{\nu-1}(z) - \operatorname{bei}_{\nu+1}(z) + \operatorname{ber}_{\nu-1}(z) + \operatorname{ber}_{\nu+1}(z)\right)$$

# Differentiation

## Low-order differentiation

### With respect to $\nu$

03.17.20.0001.01

$$\mathrm{bei}_\nu^{(1,0)}(z) = -\left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{\sin\left(\frac{1}{4}\pi(2k+3\nu)\,\psi(k+\nu+1)\right)}{k!\,\Gamma(k+\nu+1)} \left(\frac{z}{2}\right)^{2k} + \frac{3\pi}{4}\mathrm{ber}_\nu(z) + \log\left(\frac{z}{2}\right)\mathrm{bei}_\nu(z)$$

03.17.20.0002.01

$$\mathrm{bei}_n^{(1,0)}(z) = 2^{n-1}\,n!\,z^{-n}\sum_{k=0}^{n-1}\frac{(-1)^{n-k}\,2^{-k}\,z^k}{(n-k)\,k!}\left(\cos\left(\frac{1}{4}(k-n)\pi\right)\mathrm{bei}_k(z) - \sin\left(\frac{1}{4}(k-n)\pi\right)\mathrm{ber}_k(z)\right) + \frac{1}{2}\pi\,\mathrm{ber}_n(z) - \mathrm{kei}_n(z) \;/;\; n \in \mathbb{N}$$

03.17.20.0003.01

$$\mathrm{bei}_{-n}^{(1,0)}(z) = -(-1)^n\,2^{n-1}\,n!\,z^{-n}\sum_{k=0}^{n-1}\frac{(-1)^{n-k}\,2^{-k}\,z^k}{(n-k)\,k!}\left(\cos\left(\frac{1}{4}(k-n)\pi\right)\mathrm{bei}_k(z) - \sin\left(\frac{1}{4}(k-n)\pi\right)\mathrm{ber}_k(z)\right) + \frac{1}{2}(-1)^n\pi\,\mathrm{ber}_n(z) - (-1)^n\,\mathrm{kei}_n(z) \;/;\; n \in \mathbb{N}$$

03.17.20.0004.01

$$\mathrm{bei}_{-n}^{(1,0)}(z) + (-1)^n\,\mathrm{bei}_n^{(1,0)}(z) = (-1)^n\left(\pi\,\mathrm{ber}_n(z) - 2\,\mathrm{kei}_n(z)\right) \;/;\; n \in \mathbb{N}$$

03.17.20.0005.01

$$\mathrm{bei}_{n+\frac{1}{2}}^{(1,0)}(z) = \frac{3}{4}\pi\,\mathrm{ber}_{n+\frac{1}{2}}(z) + \left(\log(z) - \log\left(\sqrt[4]{-1}\,z\right)\right)\mathrm{bei}_{n+\frac{1}{2}}(z) + \frac{\sqrt[8]{-1}\,2^{\frac{1}{2}-n}\,e^{\frac{3in\pi}{4}}\,z^{\frac{1}{2}-n}}{n!\,\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}2^{2k}\binom{n}{2k+1}(-2k+2n-1)!\,i^k$$
$$\left((-1)^{3/4}\,e^{\frac{in\pi}{2}}\left(\cosh\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{3}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right) - \sinh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right)\right) -\right.$$
$$\left.(-1)^k\left(\cos\left(\sqrt[4]{-1}\,z\right)\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{3}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right) + \sin\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right)\right)z^{2k} -$$
$$\frac{(-1)^{7/8}\,2^{-n-\frac{1}{2}}\,e^{\frac{3in\pi}{4}}\,z^{-n-\frac{1}{2}}}{n!\,\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}2^{2k}\binom{n}{2k}(2n-2k)!\,i^k$$
$$\left((-1)^{3/4}\,i^n\left(\cosh\left(\sqrt[4]{-1}\,z\right)\mathrm{Shi}\left(2\sqrt[4]{-1}\,z\right) - \left(\mathrm{Chi}\left(2\sqrt[4]{-1}\,z\right) - \psi^{(0)}\left(k+\frac{1}{2}\right) + \psi^{(0)}\left(k-n+\frac{1}{2}\right)\right)\sinh\left(\sqrt[4]{-1}\,z\right)\right) +\right.$$
$$\left.(-1)^k\left(\left(\mathrm{Ci}\left(2\sqrt[4]{-1}\,z\right) - \psi^{(0)}\left(k+\frac{1}{2}\right) + \psi^{(0)}\left(k-n+\frac{1}{2}\right)\right)\sin\left(\sqrt[4]{-1}\,z\right) - \cos\left(\sqrt[4]{-1}\,z\right)\mathrm{Si}\left(2\sqrt[4]{-1}\,z\right)\right)\right)z^{2k} \;/;\; n \in \mathbb{N}$$

03.17.20.0006.01

$$\operatorname{bei}^{(1,0)}_{-n-\frac{1}{2}}(z) = \frac{3}{4}\pi \operatorname{ber}_{-n-\frac{1}{2}}(z) + \left(\log(z) - \log\left(\sqrt[4]{-1}\, z\right)\right)\operatorname{bei}_{-n-\frac{1}{2}}(z) - \frac{(-1)^{7/8}\, 2^{-n-\frac{1}{2}}\, e^{-\frac{1}{4}(i n \pi)}\, z^{-n-\frac{1}{2}}}{\sqrt{\pi}\, n!} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} 2^{2k} \binom{n}{2k} (2n-2k)!\, i^k \left(e^{\frac{1}{4}(-3) i (2n+1)\pi} \left(\cosh\left(\sqrt[4]{-1}\, z\right)\operatorname{Chi}\left(2\sqrt[4]{-1}\, z\right) + \cosh\left(\sqrt[4]{-1}\, z\right)\left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) - \sinh\left(\sqrt[4]{-1}\, z\right)\operatorname{Shi}\left(2\sqrt[4]{-1}\, z\right)\right) - (-1)^k \left(\cos\left(\sqrt[4]{-1}\, z\right)\operatorname{Ci}\left(2\sqrt[4]{-1}\, z\right) + \cos\left(\sqrt[4]{-1}\, z\right)\left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) + \sin\left(\sqrt[4]{-1}\, z\right)\operatorname{Si}\left(2\sqrt[4]{-1}\, z\right)\right)\right) z^{2k} + \frac{\sqrt[8]{-1}\, 2^{\frac{1}{2}-n}\, e^{-\frac{1}{4}(i n \pi)}\, z^{\frac{1}{2}-n}}{\sqrt{\pi}\, n!} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} 2^{2k} \binom{n}{2k+1} (-2k+2n-1)!\, i^k \left(e^{\frac{1}{4}(-3) i (2n+1)\pi} \left(-\operatorname{Chi}\left(2\sqrt[4]{-1}\, z\right)\sinh\left(\sqrt[4]{-1}\, z\right) - \left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right)\sinh\left(\sqrt[4]{-1}\, z\right) + \cosh\left(\sqrt[4]{-1}\, z\right)\operatorname{Shi}\left(2\sqrt[4]{-1}\, z\right)\right) - (-1)^k \left(\operatorname{Ci}\left(2\sqrt[4]{-1}\, z\right)\sin\left(\sqrt[4]{-1}\, z\right) + \left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right)\sin\left(\sqrt[4]{-1}\, z\right) - \cos\left(\sqrt[4]{-1}\, z\right)\operatorname{Si}\left(2\sqrt[4]{-1}\, z\right)\right)\right) z^{2k} \;/;\; n \in \mathbb{N}$$

### With respect to $z$

03.17.20.0007.01

$$\frac{\partial \operatorname{bei}_\nu(z)}{\partial z} = \frac{1}{\sqrt{2}\, z}\left(-z \operatorname{bei}_{\nu-1}(z) - \sqrt{2}\, \nu \operatorname{bei}_\nu(z) + z \operatorname{ber}_{\nu-1}(z)\right)$$

03.17.20.0008.01

$$\frac{\partial \operatorname{bei}_\nu(z)}{\partial z} = \frac{1}{2\sqrt{2}}\left(-\operatorname{bei}_{\nu-1}(z) + \operatorname{bei}_{\nu+1}(z) + \operatorname{ber}_{\nu-1}(z) - \operatorname{ber}_{\nu+1}(z)\right)$$

03.17.20.0009.01

$$\frac{\partial (z^\nu \operatorname{bei}_\nu(z))}{\partial z} = \frac{z^\nu}{\sqrt{2}}\left(\operatorname{ber}_{\nu-1}(z) - \operatorname{bei}_{\nu-1}(z)\right)$$

03.17.20.0010.01

$$\frac{\partial (z^{-\nu} \operatorname{bei}_\nu(z))}{\partial z} = \frac{z^{-\nu}}{\sqrt{2}}\left(\operatorname{bei}_{\nu+1}(z) - \operatorname{ber}_{\nu+1}(z)\right)$$

03.17.20.0011.01

$$\frac{\partial^2 \operatorname{bei}_\nu(z)}{\partial z^2} = \frac{1}{4}\left(-\operatorname{ber}_{\nu-2}(z) + 2\operatorname{ber}_\nu(z) - \operatorname{ber}_{\nu+2}(z)\right)$$

03.17.20.0012.01

$$\frac{\partial^2 \operatorname{bei}_\nu(z)}{\partial z^2} = \frac{\operatorname{bei}_{\nu-1}(z)}{\sqrt{2}\, z} + \frac{(\nu(\nu+1))\operatorname{bei}_\nu(z)}{z^2} + \operatorname{ber}_\nu(z) - \frac{\operatorname{ber}_{\nu-1}(z)}{\sqrt{2}\, z}$$

## Symbolic differentiation

### With respect to $\nu$

03.17.20.0013.01

$$\text{bei}_\nu^{(m,0)}(z) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{z}{2}\right)^{2k} \frac{\partial^m \frac{\left(\frac{z}{2}\right)^\nu \sin\left(\frac{1}{4}\pi(2k+3\nu)\right)}{\Gamma(k+\nu+1)}}{\partial \nu^m} /; m \in \mathbb{N}$$

### With respect to *z*

03.17.20.0014.01

$$\frac{\partial^n \text{bei}_\nu(z)}{\partial z^n} = z^{-n} \sum_{m=0}^{n} (-1)^{m+n} \binom{n}{m} (-\nu)_{n-m} \sum_{k=0}^{m} \frac{(-1)^k 2^{2k-m} (-m)_{2(m-k)} (\nu)_k}{(m-k)!} \left( \text{bei}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \frac{(-1)^j (k-2j)!}{(2j)!\,(k-4j)!\,(-k-\nu+1)_{2j}\,(\nu)_{2j}} \left(\frac{z}{2}\right)^{4j} + \frac{z}{2\sqrt{2}} (\text{bei}_{\nu-1}(z) - \text{ber}_{\nu-1}(z)) \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{\left((-1)^j (-2j+k-1)!\right) \left(\frac{z}{2}\right)^{4j}}{(2j)!\,(-4j+k-1)!\,(-k-\nu+1)_{2j}\,(\nu)_{2j+1}} - \frac{1}{4} z^2 \text{ber}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{(-1)^j (-2j+k-1)!}{(2j+1)!\,(-4j+k-2)!\,(-k-\nu+1)_{2j+1}\,(\nu)_{2j+1}} \left(\frac{z}{2}\right)^{4j} - \frac{z^3}{8\sqrt{2}} (\text{bei}_{\nu-1}(z) + \text{ber}_{\nu-1}(z)) \sum_{j=0}^{\left\lfloor \frac{k-2}{2} \right\rfloor} \frac{\left((-1)^j (-2j+k-2)!\right) \left(\frac{z}{2}\right)^{4j}}{(2j+1)!\,(-4j+k-3)!\,(-k-\nu+1)_{2j+1}\,(\nu)_{2j+2}} \right) /; n \in \mathbb{N}$$

03.17.20.0015.01

$$\frac{\partial^n \text{bei}_\nu(z)}{\partial z^n} = 2^{n-2\nu-1}\, i\, e^{\frac{1}{4}(-3) i \pi \nu} \sqrt{\pi}\, z^{\nu-n}\, \Gamma(\nu+1) \left( {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-n+\nu+1), \frac{1}{2}(-n+\nu+2), \nu+1; -\frac{1}{4}\left(i z^2\right)\right) - e^{\frac{3 i \pi \nu}{2}}\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-n+\nu+1), \frac{1}{2}(-n+\nu+2), \nu+1; \frac{i z^2}{4}\right)\right) /; n \in \mathbb{N}$$

03.17.20.0016.01

$$\frac{\partial^n \text{bei}_\nu(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1} (i-1)^n \left( \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \binom{n}{2k} ((1+i^n)\, \text{bei}_{4k-n+\nu}(z) - i(1-i^n)\, \text{ber}_{4k-n+\nu}(z)) + \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n}{2k+1} (i(1-i^n)\, \text{ber}_{4k-n+\nu+2}(z) - (1+i^n)\, \text{bei}_{4k-n+\nu+2}(z)) \right) /; n \in \mathbb{N}$$

03.17.20.0017.01

$$\frac{\partial^n \text{bei}_\nu(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1} (i-1)^n \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \left( \frac{n+1}{2k+1} \binom{n}{2k} \left((1+i^n)\, \text{bei}_{4k-n+\nu}(z) + \left(-i + i^{n+1}\right) \text{ber}_{4k-n+\nu}(z)\right) + \frac{\sqrt{2}\,(1+i)(4k-n+\nu+1)}{z} \binom{n}{2k+1} \left(\left(1 - i^{n+1}\right) \text{bei}_{4k-n+\nu+1}(z) + (-i + i^n)\, \text{ber}_{4k-n+\nu+1}(z)\right) \right) /; n \in \mathbb{N}$$

03.17.20.0018.01

$$\frac{\partial^n \operatorname{bei}_\nu(z)}{\partial z^n} = \pi\, G_{5,9}^{2,4}\left(\frac{z}{4}, \frac{1}{4} \,\middle|\, \begin{matrix} -\frac{n}{4}, \frac{1-n}{4}, \frac{2-n}{4}, \frac{3-n}{4}, \frac{1}{4}(-n+4\nu) \\ \frac{1}{4}(-n+\nu+2), \frac{\nu-n}{4}, \frac{1}{4}(-n-\nu+2), \frac{1}{4}(-n-\nu), \frac{1}{4}(-n+4\nu), 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right) /; n \in \mathbb{Z} \land n \geq 3$$

### Fractional integro-differentiation

#### With respect to $z$

03.17.20.0019.01

$$\frac{\partial^\alpha \operatorname{bei}_\nu(z)}{\partial z^\alpha} = 2^{-\nu} z^{\nu-\alpha} \sum_{k=0}^{\infty} \frac{\sin\left(\frac{1}{4}\pi(2k+3\nu)\right)\Gamma(2k+\nu+1)}{\Gamma(k+\nu+1)\,\Gamma(2k-\alpha+\nu+1)\,k!}\left(\frac{z}{2}\right)^{2k}$$

03.17.20.0020.01

$$\frac{\partial^\alpha \operatorname{bei}_\nu(z)}{\partial z^\alpha} = \frac{2^{-\nu-1}\, i\, z^{\nu-\alpha}}{\Gamma(-\alpha+\nu+1)} \left(e^{-\frac{3 i \pi \nu}{4}} {}_2F_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1; \frac{\nu-\alpha+1}{2}, \frac{\nu-\alpha}{2}+1, \nu+1; -\frac{i z^2}{4}\right) - e^{\frac{3 i \pi \nu}{4}} {}_2F_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1; \frac{\nu-\alpha+1}{2}, \frac{\nu-\alpha}{2}+1, \nu+1; \frac{i z^2}{4}\right)\right)$$

## Integration

### Indefinite integration

03.17.21.0001.01

$$\int \operatorname{bei}_\nu(a z)\, dz = \frac{1}{4}\pi z\, G_{2,6}^{2,1}\left(\frac{a z}{4}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{3}{4}, \nu \\ \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{1}{4}, \frac{2-\nu}{4}, -\frac{\nu}{4}, \nu \end{matrix}\right)$$

### Definite integration

03.17.21.0002.01

$$\int_0^\infty t^{\alpha-1} e^{-p t} \operatorname{bei}_\nu(t)\, dt = \frac{1}{\Gamma(\nu+1)} 2^{-\nu-2} p^{-\alpha-\nu} \Gamma(\alpha+\nu) \left(\frac{(\alpha+\nu)(\alpha+\nu+1)\cos\left(\frac{3\pi\nu}{4}\right)}{p^2(\nu+1)} {}_4F_3\left(\frac{\alpha+\nu+2}{4}, \frac{\alpha+\nu+3}{4}, \frac{\alpha+\nu}{4}+1, \frac{\alpha+\nu+5}{4}; \frac{3}{2}, \frac{\nu}{2}+1, \frac{\nu+3}{2}; -\frac{1}{p^4}\right) + 4\sin\left(\frac{3\pi\nu}{4}\right) {}_4F_3\left(\frac{\alpha+\nu}{4}, \frac{\alpha+\nu+1}{4}, \frac{\alpha+\nu+2}{4}, \frac{\alpha+\nu+3}{4}; \frac{1}{2}, \frac{\nu+1}{2}, \frac{\nu}{2}+1; -\frac{1}{p^4}\right)\right) /; \operatorname{Re}(\alpha+\nu) > 0 \bigwedge \operatorname{Re}(p) > \frac{1}{\sqrt{2}}$$

## Integral transforms

### Laplace transforms

03.17.22.0001.01

$$\mathcal{L}_t[\text{bei}_\nu(t)](z) = 2^{-2(\nu+2)}\pi z^{-\nu-3}\Gamma(\nu+1)\left(16\sin\left(\frac{3\pi\nu}{4}\right)z^2\,{}_4\tilde{F}_3\left(\frac{\nu+1}{4},\frac{\nu+2}{4},\frac{\nu+3}{4},\frac{\nu+4}{4};\frac{1}{2},\frac{\nu+1}{2},\frac{\nu+2}{2};-\frac{1}{z^4}\right)+(\nu+1)(\nu+2)\cos\left(\frac{3\pi\nu}{4}\right){}_4\tilde{F}_3\left(\frac{\nu+3}{4},\frac{\nu+4}{4},\frac{\nu+5}{4},\frac{\nu+6}{4};\frac{3}{2},\frac{\nu+2}{2},\frac{\nu+3}{2};-\frac{1}{z^4}\right)\right)/;\ \text{Re}(\nu)>-1\bigwedge\text{Re}(z)>\frac{1}{\sqrt{2}}$$

## Mellin transforms

03.17.22.0002.01

$$\mathcal{M}_t\left[e^{-pt}\,\text{bei}_\nu(t)\right](z) = \frac{1}{\Gamma(\nu+1)}2^{-\nu-2}p^{-z-\nu}\Gamma(z+\nu)\left(\frac{(z+\nu)(z+\nu+1)\cos\left(\frac{3\pi\nu}{4}\right)}{p^2(\nu+1)}{}_4F_3\left(\frac{1}{4}(z+\nu+2),\frac{1}{4}(z+\nu+3),\frac{z+\nu}{4}+1,\frac{1}{4}(z+\nu+5);\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu+3}{2};-\frac{1}{p^4}\right)+4\sin\left(\frac{3\pi\nu}{4}\right){}_4F_3\left(\frac{z+\nu}{4},\frac{1}{4}(z+\nu+1),\frac{1}{4}(z+\nu+2),\frac{1}{4}(z+\nu+3);\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{1}{p^4}\right)\right)/;\ \text{Re}(z+\nu)>0\bigwedge\text{Re}(p)>\frac{1}{\sqrt{2}}$$

# Representations through more general functions

## Through hypergeometric functions

### Involving ${}_p\tilde{F}_q$

03.17.26.0001.01

$$\text{bei}_\nu(z) = 4^{-\nu}\pi\sin\left(\frac{3\pi\nu}{4}\right)z^\nu\,{}_0\tilde{F}_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)+2^{-2(\nu+2)}\pi\cos\left(\frac{3\pi\nu}{4}\right)z^{\nu+2}\,{}_0\tilde{F}_3\left(;\frac{3}{2},\frac{\nu+3}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)$$

### Involving ${}_pF_q$

03.17.26.0002.01

$$\text{bei}_\nu(z) = \frac{\cos\left(\frac{3\pi\nu}{4}\right)\left(\frac{z}{2}\right)^{\nu+2}}{\Gamma(\nu+2)}{}_0F_3\left(;\frac{3}{2},\frac{\nu+3}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)+\frac{\sin\left(\frac{3\pi\nu}{4}\right)\left(\frac{z}{2}\right)^{\nu}}{\Gamma(\nu+1)}{}_0F_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)/;\ -\nu\notin\mathbb{N}^+$$

## Through Meijer *G*

### Classical cases for the direct function itself

03.17.26.0003.01

$$\text{bei}_\nu(z) = \pi\,G_{1,5}^{2,0}\left(\frac{z^4}{256}\,\middle|\,\begin{matrix}\nu\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4},\nu\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

03.17.26.0004.01

$$\text{bei}_{-\nu}(z)+\text{bei}_\nu(z) = 2\pi\cos\left(\frac{\pi\nu}{2}\right)G_{3,7}^{4,0}\left(\frac{z^4}{256}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

### Classical cases for powers of bei

03.17.26.0005.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)^2 = \frac{1}{2}\pi^{3/2} G_{1,5}^{1,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) - \frac{\sqrt{\pi}}{2^{3/2}} G_{3,7}^{2,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, 2\nu+\frac{1}{2} \\ \frac{\nu}{2}, \frac{\nu+1}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, 2\nu+\frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0006.01

$$\operatorname{bei}_\nu(z)^2 = \frac{1}{2}\pi^{3/2} G_{1,5}^{1,0}\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) - \frac{\sqrt{\pi}}{2^{3/2}} G_{3,7}^{2,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, 2\nu+\frac{1}{2} \\ \frac{\nu}{2}, \frac{\nu+1}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, 2\nu+\frac{1}{2} \end{matrix}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

### Classical cases for products of bei

03.17.26.0007.01

$$\operatorname{bei}_{-\nu}(z)\operatorname{bei}_\nu(z) = \frac{1}{4}\sqrt{\pi}\left(e^{-\frac{1}{2}(3 i \pi \nu)} G_{1,5}^{2,0}\left(-\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right) + e^{\frac{3 i \pi \nu}{2}} G_{1,5}^{2,0}\left(-\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)\right) - \frac{\pi^{3/2}}{2\sqrt{2}} G_{3,7}^{1,2}\left(-\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1}{2} \\ 0, \frac{1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) /; -\frac{\pi}{2} < \arg(z) \le 0$$

### Classical cases involving powers of ber

03.17.26.0008.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)^2 + \operatorname{ber}_\nu\left(\sqrt[4]{z}\right)^2 = \pi^{3/2} G_{1,5}^{1,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0009.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)^2 - \operatorname{ber}_\nu\left(\sqrt[4]{z}\right)^2 = -\sqrt{\frac{\pi}{2}}\, G_{3,7}^{2,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, 2\nu+\frac{1}{2} \\ \frac{\nu}{2}, \frac{\nu+1}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, 2\nu+\frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0010.01

$$\operatorname{bei}_\nu(z)^2 + \operatorname{ber}_\nu(z)^2 = \pi^{3/2} G_{1,5}^{1,0}\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0011.01

$$\operatorname{bei}_\nu(z)^2 - \operatorname{ber}_\nu(z)^2 = -\sqrt{\frac{\pi}{2}}\, G_{3,7}^{2,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, 2\nu+\frac{1}{2} \\ \frac{\nu}{2}, \frac{\nu+1}{2}, 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, 2\nu+\frac{1}{2} \end{matrix}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

### Classical cases involving ber

03.17.26.0012.01

$$\operatorname{bei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{\nu}\left(\sqrt[4]{z}\right)=\frac{\sqrt{\pi}}{2\sqrt{2}}\,G_{3,7}^{2,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},2\nu\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0013.01

$$\operatorname{bei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{-\nu}\left(\sqrt[4]{z}\right)=\frac{1}{4}\,e^{-\frac{1}{2}(3i\pi\nu)}\,i\sqrt{\pi}\,G_{1,5}^{2,0}\left(-\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)-\frac{1}{4}\,e^{\frac{3i\pi\nu}{2}}\,i\sqrt{\pi}\,G_{1,5}^{2,0}\left(-\frac{z}{64}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+\frac{i\pi^{3/2}}{2\sqrt{2}}\,G_{3,7}^{1,2}\left(-\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},0\\ \frac{1}{2},0,0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\;/;\;-\pi<\arg(z)\leq 0$$

03.17.26.0014.01

$$\operatorname{bei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{\mu}\left(\sqrt[4]{z}\right)+\operatorname{bei}_{\mu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{\nu}\left(\sqrt[4]{z}\right)=-2^{3/2}\,\pi^{5/2}\,G_{6,10}^{2,3}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{2},0,\frac{1}{3},\frac{2}{3}\\ \frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{3},\frac{2}{3},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{2-\mu-\nu+2}{4},\frac{\nu-\mu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0015.01

$$\operatorname{bei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{-\nu}\left(\sqrt[4]{z}\right)+\operatorname{bei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{\nu}\left(\sqrt[4]{z}\right)=-2^{3/2}\,\pi^{5/2}\,G_{4,8}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{3},\frac{2}{3}\\ \frac{1}{2},0,\frac{1}{3},\frac{2}{3},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0016.01

$$\operatorname{ber}_{\mu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{\nu}\left(\sqrt[4]{z}\right)-\operatorname{bei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{bei}_{\mu}\left(\sqrt[4]{z}\right)=2^{3/2}\,\pi^{5/2}\,G_{6,10}^{2,3}\left(\frac{z}{16}\,\middle|\,\begin{matrix}0,\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\ \frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{6},\frac{5}{6},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{\nu-\mu}{4},\frac{2-\mu-\nu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0017.01

$$\operatorname{ber}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{\nu}\left(\sqrt[4]{z}\right)-\operatorname{bei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{bei}_{\nu}\left(\sqrt[4]{z}\right)=2^{3/2}\,\pi^{5/2}\,G_{5,9}^{2,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\ 0,\frac{1}{2},\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0018.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ber}_{\nu}(z)=\frac{\sqrt{\pi}}{2^{3/2}}\,G_{3,7}^{2,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},2\nu\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu\end{matrix}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0019.01

$$\operatorname{bei}_\nu(z)\operatorname{ber}_{-\nu}(z) = \frac{1}{4} e^{-\frac{1}{2}(3 i \pi \nu)} i \sqrt{\pi}\, G_{1,5}^{2,0}\left(-\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right) - \frac{1}{4} e^{\frac{3 i \pi \nu}{2}} i \sqrt{\pi}\, G_{1,5}^{2,0}\left(-\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) + \frac{i \pi^{3/2}}{2\sqrt{2}} G_{3,7}^{1,2}\left(-\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, 0 \\ \frac{1}{2}, 0, 0, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) /; -\frac{\pi}{2} < \arg(z) \leq 0$$

03.17.26.0020.01

$$\operatorname{bei}_\nu(z)\operatorname{ber}_\mu(z) + \operatorname{bei}_\mu(z)\operatorname{ber}_\nu(z) = -2^{3/2} \pi^{5/2} G_{6,10}^{2,3}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1}{2}, 0, \frac{1}{3}, \frac{2}{3} \\ \frac{\mu+\nu}{4}, \frac{\mu+\nu+2}{4}, \frac{1}{3}, \frac{2}{3}, -\frac{\mu+\nu}{4}, \frac{\mu-\nu}{4}, \frac{2-\mu-\nu}{4}, \frac{\nu-\mu}{4}, \frac{\mu-\nu+2}{4}, \frac{2-\mu+\nu}{4} \end{matrix}\right) /; -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0021.01

$$\operatorname{bei}_\nu(z)\operatorname{ber}_{-\nu}(z) + \operatorname{bei}_{-\nu}(z)\operatorname{ber}_\nu(z) = -2^{3/2} \pi^{5/2} G_{4,8}^{1,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1}{3}, \frac{2}{3} \\ \frac{1}{2}, 0, \frac{1}{3}, \frac{2}{3}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) /; -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0022.01

$$\operatorname{ber}_\mu(z)\operatorname{ber}_\nu(z) - \operatorname{bei}_\nu(z)\operatorname{bei}_\mu(z) = 2^{3/2} \pi^{5/2} G_{6,10}^{2,3}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} 0, \frac{1}{4}, \frac{3}{4}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6} \\ \frac{\mu+\nu}{4}, \frac{\mu+\nu+2}{4}, \frac{1}{6}, \frac{5}{6}, -\frac{\mu+\nu}{4}, \frac{\mu-\nu}{4}, \frac{\nu-\mu}{4}, \frac{2-\mu-\nu}{4}, \frac{\mu-\nu+2}{4}, \frac{2-\mu+\nu}{4} \end{matrix}\right) /; -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0023.01

$$\operatorname{ber}_{-\nu}(z)\operatorname{ber}_\nu(z) - \operatorname{bei}_{-\nu}(z)\operatorname{bei}_\nu(z) = 2^{3/2} \pi^{5/2} G_{5,9}^{2,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6} \\ 0, \frac{1}{2}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{1-\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) /; -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

### Classical cases involving kei

03.17.26.0024.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{matrix}\right) - \frac{1}{2^{7/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0025.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_{-\nu}\left(\sqrt[4]{z}\right) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) - \frac{1}{8\sqrt{2\pi}} G_{3,7}^{4,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1}{2}-\nu \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}-\nu, \frac{1-\nu}{2}, -\frac{\nu}{2} \end{matrix}\right)$$

03.17.26.0026.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z^4}{64} \middle| \begin{matrix} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{matrix}\right) - \frac{1}{2^{7/2}\sqrt{\pi}}\, G_{2,6}^{3,2}\left(\frac{z^4}{16} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right) /; -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0027.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_{-\nu}(z) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z^4}{64} \middle| \begin{matrix} \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) - \frac{1}{8\sqrt{2\pi}}\, G_{3,7}^{4,2}\left(\frac{z^4}{16} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1}{2}-\nu \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1-\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}-\nu \end{matrix}\right) /; -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

## Classical cases involving ker

03.17.26.0028.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z}{64} \middle| \begin{matrix} \frac{3\nu}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{3\nu}{2} \end{matrix}\right) - \frac{1}{2^{7/2}\sqrt{\pi}}\, G_{2,6}^{3,2}\left(\frac{z}{16} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, 0, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0029.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_{-\nu}\left(\sqrt[4]{z}\right) = \frac{1}{8}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{64} \middle| 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}\right) - \frac{1}{8\sqrt{2\pi}}\, G_{3,7}^{4,2}\left(\frac{z}{16} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4}, -\nu \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, -\nu \end{matrix}\right)$$

03.17.26.0030.01

$$\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z^4}{64} \middle| \begin{matrix} \frac{3\nu}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{3\nu}{2} \end{matrix}\right) - \frac{1}{2^{7/2}\sqrt{\pi}}\, G_{2,6}^{3,2}\left(\frac{z^4}{16} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, 0 \end{matrix}\right) /; -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0031.01

$$\operatorname{bei}_\nu(z)\operatorname{ker}_{-\nu}(z) = \frac{1}{8}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z^4}{64} \middle| 0, \frac{1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}\right) - \frac{1}{8\sqrt{2\pi}}\, G_{3,7}^{4,2}\left(\frac{z^4}{16} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4}, -\nu \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, -\nu \end{matrix}\right) /; -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

## Classical cases involving ber, ker and kei

03.17.26.0032.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right) + \operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right) = \frac{1}{4}\sqrt{\pi}\, G_{1,5}^{3,0}\left(\frac{z}{64} \middle| \begin{matrix} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0033.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right) - \operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right) = -\frac{1}{2^{5/2}\sqrt{\pi}}\, G_{2,6}^{3,2}\left(\frac{z}{16} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0034.01

$$\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)+\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=-\frac{1}{2^{5/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{array}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0035.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)-\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{array}{c}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0036.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{array}{c}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{array}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0037.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)-\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0038.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{array}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.17.26.0039.01

$$\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)-\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{array}{c}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{array}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

## Classical cases involving Bessel *J*

03.17.26.0040.01

$$J_\nu\left(\frac{1}{\sqrt[4]{-1}}z\right)\operatorname{bei}_\nu(z)=$$

$$-i\,2^{-\frac{3\nu}{2}-1}e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{1}{\sqrt[4]{-1}}z\right)^\nu\left(e^{\frac{3i\pi\nu}{2}}2^{\frac{3\nu}{2}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\left(i\,z^2\,\middle|\,\begin{array}{c}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{array}\right)-\right.$$

$$\left.G_{0,4}^{1,0}\left(-\frac{z^4}{64}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)\right)/;-\frac{\pi}{2}<\arg(z)\leq 0$$

03.17.26.0041.01

$$J_{-\nu}\left(\frac{1}{\sqrt[4]{-1}}\,z\right)\operatorname{bei}_\nu(z) = -i\sqrt{\frac{\pi}{2}}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{1}{\sqrt[4]{-1}}\,z\right)^{-\nu}$$
$$\left(e^{\frac{3i\pi\nu}{4}}\,G_{2,4}^{1,1}\left(i\,z^2\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\,\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},\,-\frac{\nu}{2},\,\frac{3\nu}{2},\,\frac{1}{4}(2\nu+1)\end{matrix}\right)-e^{-\frac{1}{4}(3i\pi\nu)}\,2^{\frac{1}{2}(3\nu-1)}\,G_{1,5}^{2,0}\left(-\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{2-\nu}{4}\\ \frac{\nu}{4},\,\frac{\nu+2}{4},\,\frac{3\nu}{4},\,-\frac{\nu}{4},\,\frac{2-\nu}{4}\end{matrix}\right)\right)/;\,-\frac{\pi}{2}<\arg(z)\le 0$$

## Classical cases involving Bessel *I*

03.17.26.0042.01

$$I_\nu\left(\sqrt[4]{-1}\,z\right)\operatorname{bei}_\nu(z) = -\frac{1}{2}(-1)^{3/4}\,e^{-\frac{1}{4}i\pi(3\nu+1)}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}$$
$$\left(e^{\frac{3i\pi\nu}{2}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\left(i\,z^2\,\middle|\,\begin{matrix}\frac{1}{2},\,\frac{1}{4}\\ \nu,\,0,\,\frac{1}{4},\,-\nu\end{matrix}\right)-G_{0,4}^{1,0}\left(-\frac{z^4}{64}\,\middle|\,\frac{\nu}{2},\,0,\,\frac{1}{2},\,-\frac{\nu}{2}\right)\right)/;\,-\frac{\pi}{2}<\arg(z)\le 0$$

03.17.26.0043.01

$$I_{-\nu}\left(\sqrt[4]{-1}\,z\right)\operatorname{bei}_\nu(z) = -i\sqrt{\frac{\pi}{2}}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(e^{\frac{3i\pi\nu}{4}}\,G_{2,4}^{1,1}\left(i\,z^2\,\middle|\,\begin{matrix}\frac{1}{2},\,\frac{1}{4}\\ 0,\,\frac{1}{4},\,\nu,\,-\nu\end{matrix}\right)-\frac{e^{\frac{1}{4}(-3)i\pi\nu}}{\sqrt{2}}\,G_{1,5}^{2,0}\left(-\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\,\frac{1}{2},\,\frac{1-\nu}{2},\,-\frac{\nu}{2},\,\frac{\nu}{2}\end{matrix}\right)\right)/;$$
$$-\frac{\pi}{2}<\arg(z)\le 0$$

03.17.26.0044.01

$$\left(I_\nu\left(\sqrt[4]{-1}\,z\right)-I_{-\nu}\left(\sqrt[4]{-1}\,z\right)\right)\operatorname{bei}_\nu(z) = \frac{1}{2}\left(-\left(i\sqrt{\pi}\,\sin(\pi\nu)\right)\right)z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}$$
$$\left(\frac{e^{-\frac{1}{4}(3i\pi\nu)}}{2\pi^2}\,G_{0,4}^{3,0}\left(-\frac{z^4}{64}\,\middle|\,0,\,\frac{1}{2},\,\frac{\nu}{2},\,-\frac{\nu}{2}\right)+\sqrt{2}\,e^{\frac{3i\pi\nu}{4}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{3,5}^{2,1}\left(i\,z^2\,\middle|\,\begin{matrix}\frac{1}{2},\,\frac{1}{4},\,\nu-\frac{1}{4}\\ 0,\,\nu,\,-\nu,\,\frac{1}{4},\,\nu-\frac{1}{4}\end{matrix}\right)\right)/;\,-\frac{\pi}{2}<\arg(z)\le 0$$

## Classical cases involving Bessel *K*

03.17.26.0045.01

$$K_\nu\left(\sqrt[4]{-1}\,z\right)\operatorname{bei}_\nu(z) = \frac{1}{4}\,i\,\pi^{3/2}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}$$
$$\left(\frac{e^{-\frac{1}{4}(3i\pi\nu)}}{2\pi^2}\,G_{0,4}^{3,0}\left(-\frac{z^4}{64}\,\middle|\,0,\,\frac{1}{2},\,\frac{\nu}{2},\,-\frac{\nu}{2}\right)+\sqrt{2}\,e^{\frac{3i\pi\nu}{4}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{3,5}^{2,1}\left(i\,z^2\,\middle|\,\begin{matrix}\frac{1}{2},\,\frac{1}{4},\,\nu-\frac{1}{4}\\ 0,\,\nu,\,-\nu,\,\frac{1}{4},\,\nu-\frac{1}{4}\end{matrix}\right)\right)/;\,-\frac{\pi}{2}<\arg(z)\le 0$$

## Classical cases involving ${}_0F_1$

03.17.26.0046.01

$${}_0F_1\left(;\nu+1;\,\frac{i\sqrt{z}}{4}\right)\operatorname{bei}_\nu\left(\sqrt[4]{z}\right) = -\frac{i\sqrt{\pi}}{2\sqrt{2}}\,\Gamma(\nu+1)\left(-\pi\,e^{-\frac{3i\pi\nu}{4}}\,2^{\frac{1-\nu}{2}}\,G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{\nu+2}{4}\\ \frac{\nu}{4},\,-\frac{\nu}{4},\,\frac{2-\nu}{4},\,-\frac{1}{4}(3\nu),\,\frac{\nu+2}{4}\end{matrix}\right)+\right.$$
$$\left.e^{\frac{3i\pi\nu}{4}}\,G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\,\frac{1-\nu}{4}\\ \frac{\nu}{4},\,-\frac{\nu}{4},\,\frac{\nu+2}{4},\,\frac{2-\nu}{4},\,\frac{1}{4}(2-3\nu),\,-\frac{1}{4}(3\nu)\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}\,i\,G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\,\frac{1-\nu}{4}\\ \frac{\nu+2}{4},\,-\frac{\nu}{4},\,\frac{\nu}{4},\,\frac{2-\nu}{4},\,\frac{1}{4}(2-3\nu),\,-\frac{1}{4}(3\nu)\end{matrix}\right)\right)$$

03.17.26.0047.01

$$ {}_0F_1\left(;1-\nu;\frac{i\sqrt{z}}{4}\right)\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)= i\sqrt{\pi}\,\Gamma(1-\nu)\left(2^{\frac{\nu}{2}-1}e^{-\frac{3i\pi\nu}{4}}\pi\left(G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{4}(3\nu+2)\\ \frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right)+i\tan\left(\frac{\pi\nu}{2}\right)G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{4}+1\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{3\nu}{4}+1\end{matrix}\right)\right)-\frac{e^{\frac{3i\pi\nu}{4}}}{2\sqrt{2}}\left(G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)+i\,G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)\right)\right) $$

03.17.26.0048.01

$$ {}_0F_1\left(;\nu+1;\frac{iz^2}{4}\right)\operatorname{bei}_\nu(z)=-\frac{i\sqrt{\pi}}{2\sqrt{2}}\Gamma(\nu+1)\left(-\pi e^{-\frac{1}{4}(3i\pi\nu)}2^{\frac{1-\nu}{2}}G_{1,5}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{\nu+2}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu),\frac{\nu+2}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}i\,G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu+2}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4} $$

03.17.26.0049.01

$$ {}_0F_1\left(;1-\nu;\frac{iz^2}{4}\right)\operatorname{bei}_\nu(z)=i\sqrt{\pi}\,\Gamma(1-\nu)\left(2^{\frac{\nu}{2}-1}e^{-\frac{3i\pi\nu}{4}}\pi\left(G_{1,5}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{4}(3\nu+2)\\ \frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right)+i\tan\left(\frac{\pi\nu}{2}\right)G_{1,5}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{4}+1\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{3\nu}{4}+1\end{matrix}\right)\right)-\frac{e^{\frac{3i\pi\nu}{4}}}{2\sqrt{2}}\left(G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)+i\,G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)\right)\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4} $$

03.17.26.0050.01

$$ {}_0F_1\left(;\nu+1;\frac{iz^2}{4}\right)\operatorname{bei}_\nu(z)= -i\,2^{-\frac{\nu}{2}-1}e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma(\nu+1)\left(e^{\frac{3i\pi\nu}{2}}2^{\frac{3\nu}{2}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\left(iz^2\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{matrix}\right)-G_{0,4}^{1,0}\left(-\frac{z^4}{64}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)\right)/;-\frac{\pi}{2}<\arg(z)\le 0 $$

03.17.26.0051.01

$$ {}_0F_1\left(;1-\nu;\frac{iz^2}{4}\right)\operatorname{bei}_\nu(z)=-2^{-\nu-\frac{1}{2}}i\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma(1-\nu)\left(e^{\frac{3i\pi\nu}{2}}G_{2,4}^{1,1}\left(iz^2\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{1}{4}(2\nu+1)\end{matrix}\right)-2^{\frac{1}{2}(3\nu-1)}G_{1,5}^{2,0}\left(-\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{2-\nu}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4}\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\le 0 $$

## Classical cases involving ${}_0\tilde{F}_1$

03.17.26.0052.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i\sqrt{z}}{4}\right)\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)= -\frac{i\sqrt{\pi}}{2\sqrt{2}}\left(-\pi\,e^{-\frac{3i\pi\nu}{4}}\,2^{\frac{1-\nu}{2}}\,G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{\nu+2}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu),\frac{\nu+2}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}\,G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)+ e^{\frac{3i\pi\nu}{4}}\,i\,G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu+2}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)\right) $$

03.17.26.0053.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i\sqrt{z}}{4}\right)\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)= i\sqrt{\pi}\left(2^{\frac{\nu}{2}-1}\,e^{-\frac{3i\pi\nu}{4}}\,\pi\left(G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{4}(3\nu+2)\\ \frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right)+i\tan\left(\frac{\pi\nu}{2}\right)G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{4}+1\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{3\nu}{4}+1\end{matrix}\right)\right)- \frac{e^{\frac{3i\pi\nu}{4}}}{2\sqrt{2}}\left(G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)+i\,G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)\right)\right) $$

03.17.26.0054.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{bei}_\nu(z)=-\frac{i\sqrt{\pi}}{2\sqrt{2}}\,\Gamma(\nu+1) \left(-\pi\,e^{-\frac{1}{4}(3i\pi\nu)}\,2^{\frac{1-\nu}{2}}\,G_{1,5}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{\nu+2}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu),\frac{\nu+2}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}\,G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)+ e^{\frac{3i\pi\nu}{4}}\,i\,G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu+2}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4} $$

03.17.26.0055.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{bei}_\nu(z)=i\sqrt{\pi}\,\Gamma(1-\nu) \left(2^{\frac{\nu}{2}-1}\,e^{-\frac{3i\pi\nu}{4}}\,\pi\left(G_{1,5}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{4}(3\nu+2)\\ \frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right)+i\tan\left(\frac{\pi\nu}{2}\right)G_{1,5}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{4}+1\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{3\nu}{4}+1\end{matrix}\right)\right)-\frac{e^{\frac{3i\pi\nu}{4}}}{2\sqrt{2}} \left(G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)+i\,G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)\right)\right)\;/;\;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4} $$

03.17.26.0056.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\mathrm{bei}_\nu(z) = $$

$$ -i\,2^{-\frac{\nu}{2}-1}\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(e^{\frac{3i\pi\nu}{2}}\,2^{\frac{3\nu}{2}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\left(i z^2\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{matrix}\right)-\right. $$

$$ \left.G_{0,4}^{1,0}\left(-\frac{z^4}{64}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)\right)\;/;\;-\frac{\pi}{2}<\arg(z)\le 0 $$

03.17.26.0057.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\mathrm{bei}_\nu(z) = -2^{-\nu-\frac{1}{2}}\,i\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu} $$

$$ \left(e^{\frac{3i\pi\nu}{2}}\,G_{2,4}^{1,1}\left(i z^2\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{1}{4}(2\nu+1)\end{matrix}\right)-2^{\frac{1}{2}(3\nu-1)}\,G_{1,5}^{2,0}\left(-\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{2-\nu}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4}\end{matrix}\right)\right)\;/;\;-\frac{\pi}{2}<\arg(z)\le 0 $$

## Generalized cases for the direct function itself

03.17.26.0058.01

$$ \mathrm{bei}_\nu(z) = \pi\,G_{1,5}^{2,0}\left(\frac{z}{4},\frac{1}{4}\,\middle|\,\begin{matrix}\nu\\ \frac{\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{2-\nu}{4},\nu\end{matrix}\right) $$

03.17.26.0059.01

$$ \mathrm{bei}_{-\nu}(z)+\mathrm{bei}_\nu(z) = 2\pi\cos\left(\frac{\pi\nu}{2}\right)G_{3,7}^{4,0}\left(\frac{z}{4},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\end{matrix}\right) $$

## Generalized cases for powers of bei

03.17.26.0060.01

$$ \mathrm{bei}_\nu(z)^2 = \frac{1}{2}\pi^{3/2}\,G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ \frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)-\frac{\sqrt{\pi}}{2^{3/2}}\,G_{3,7}^{2,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},2\nu+\frac{1}{2}\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu+\frac{1}{2}\end{matrix}\right) $$

Brychkov Yu.A. (2006)

## Generalized cases for products of bei

03.17.26.0061.01

$$ \mathrm{bei}_{-\nu}(z)\,\mathrm{bei}_\nu(z) = \frac{1}{4}\sqrt{\pi}\left(e^{-\frac{1}{2}(3i\pi\nu)}\,G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)+e^{\frac{3i\pi\nu}{2}}\,G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\right)- $$

$$ \frac{\pi^{3/2}}{2\sqrt{2}}\,G_{3,7}^{1,2}\left(\frac{1}{2}\sqrt[4]{-1}\,z,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{2}\\ 0,\frac{1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right) $$

## Generalized cases involving powers of ber

03.17.26.0062.01

$$ \mathrm{bei}_\nu(z)^2+\mathrm{ber}_\nu(z)^2 = \pi^{3/2}\,G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ \frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right) $$

Brychkov Yu.A. (2006)

03.17.26.0063.01

$$\operatorname{bei}_\nu(z)^2-\operatorname{ber}_\nu(z)^2=-\sqrt{\frac{\pi}{2}}\,G_{3,7}^{2,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},2\nu+\frac{1}{2}\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu+\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

## Generalized cases involving ber

03.17.26.0064.01

$$\operatorname{bei}_\nu(z)\operatorname{ber}_\nu(z)=\frac{\sqrt{\pi}}{2\sqrt{2}}\,G_{3,7}^{2,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},2\nu\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0065.01

$$\operatorname{bei}_\nu(z)\operatorname{ber}_{-\nu}(z)=\frac{1}{4}\,e^{-\frac{1}{2}(3i\pi\nu)}\,i\sqrt{\pi}\,G_{1,5}^{2,0}\!\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)-$$
$$\frac{1}{4}\,i\,e^{\frac{3i\pi\nu}{2}}\sqrt{\pi}\,G_{1,5}^{2,0}\!\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+\frac{i\,\pi^{3/2}}{2\sqrt{2}}\,G_{3,7}^{1,2}\!\left(\frac{1}{2}\sqrt[4]{-1}\,z,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},0\\ \frac{1}{2},0,0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

03.17.26.0066.01

$$\operatorname{bei}_\nu(z)\operatorname{ber}_\mu(z)+\operatorname{bei}_\mu(z)\operatorname{ber}_\nu(z)=-2^{3/2}\,\pi^{5/2}\,G_{6,10}^{2,3}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{2},0,\frac{1}{3},\frac{2}{3}\\ \frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{3},\frac{2}{3},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{2-\mu-\nu}{4},\frac{\nu-\mu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0067.01

$$\operatorname{bei}_\nu(z)\operatorname{ber}_{-\nu}(z)+\operatorname{bei}_{-\nu}(z)\operatorname{ber}_\nu(z)=-2^{3/2}\,\pi^{5/2}\,G_{4,8}^{1,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{3},\frac{2}{3}\\ \frac{1}{2},0,\frac{1}{3},\frac{2}{3},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0068.01

$$\operatorname{ber}_\mu(z)\operatorname{ber}_\nu(z)-\operatorname{bei}_\nu(z)\operatorname{bei}_\mu(z)=2^{3/2}\,\pi^{5/2}\,G_{6,10}^{2,3}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}0,\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\ \frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{6},\frac{5}{6},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{\nu-\mu}{4},\frac{2-\mu-\nu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0069.01

$$\operatorname{ber}_{-\nu}(z)\operatorname{ber}_\nu(z)-\operatorname{bei}_{-\nu}(z)\operatorname{bei}_\nu(z)=2^{3/2}\,\pi^{5/2}\,G_{5,9}^{2,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\ 0,\frac{1}{2},\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

## Generalized cases involving kei

03.17.26.0070.01

$$\operatorname{bei}_{\nu}(z)\operatorname{kei}_{\nu}(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{array}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0071.01

$$\operatorname{bei}_{\nu}(z)\operatorname{kei}_{-\nu}(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{\nu+1}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{array}\right)-\frac{1}{8\sqrt{2\pi}}\,G_{3,7}^{4,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},\frac{1}{2}-\nu\\0,\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{1}{2}-\nu\end{array}\right)$$

## Generalized cases involving ker

03.17.26.0072.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ker}_{\nu}(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{array}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},0,-\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0073.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ker}_{-\nu}(z)=\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right)-\frac{1}{8\sqrt{2\pi}}\,G_{3,7}^{4,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},-\nu\\0,\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},-\nu\end{array}\right)$$

## Generalized cases involving ber, ker and kei

03.17.26.0074.01

$$\operatorname{bei}_{\nu}(z)\operatorname{kei}_{\nu}(z)+\operatorname{ber}_{\nu}(z)\operatorname{ker}_{\nu}(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{array}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0075.01

$$\operatorname{bei}_{\nu}(z)\operatorname{kei}_{\nu}(z)-\operatorname{ber}_{\nu}(z)\operatorname{ker}_{\nu}(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0076.01

$$\operatorname{ber}_{\nu}(z)\operatorname{kei}_{\nu}(z)+\operatorname{bei}_{\nu}(z)\operatorname{ker}_{\nu}(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{array}\right)$$

Brychkov Yu.A. (2006)

03.17.26.0077.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ker}_{\nu}(z)-\operatorname{ber}_{\nu}(z)\operatorname{kei}_{\nu}(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

## Generalized cases involving Bessel *J*

03.17.26.0078.01

$$J_\nu\left(\frac{1}{\sqrt[4]{-1}}\,z\right)\mathrm{bei}_\nu(z) = -i\,2^{-\frac{3\nu}{2}-1}\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{1}{\sqrt[4]{-1}}\,z\right)^\nu \left(e^{\frac{3i\pi\nu}{2}}\,2^{\frac{3\nu}{2}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{matrix}\right)-G_{0,4}^{1,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)\right)$$

03.17.26.0079.01

$$J_{-\nu}\left(\frac{1}{\sqrt[4]{-1}}\,z\right)\mathrm{bei}_\nu(z) = -i\sqrt{\frac{\pi}{2}}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{1}{\sqrt[4]{-1}}\,z\right)^{-\nu} \left(e^{\frac{3i\pi\nu}{4}}\,G_{2,4}^{1,1}\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{1}{4}(2\nu+1)\end{matrix}\right)-e^{-\frac{3i\pi\nu}{4}}\,2^{\frac{3\nu-1}{2}}\,G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{2-\nu}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4}\end{matrix}\right)\right)$$

## Generalized cases involving Bessel *I*

03.17.26.0080.01

$$I_\nu\left(\sqrt[4]{-1}\,z\right)\mathrm{bei}_\nu(z) = -\frac{1}{2}(-1)^{3/4}\,e^{-\frac{1}{4}i\pi(3\nu+1)}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu} \left(e^{\frac{3i\pi\nu}{2}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4}\\ \nu,0,\frac{1}{4},-\nu\end{matrix}\right)-G_{0,4}^{1,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2}\right)\right)$$

03.17.26.0081.01

$$I_{-\nu}\left(\sqrt[4]{-1}\,z\right)\mathrm{bei}_\nu(z) = -i\sqrt{\frac{\pi}{2}}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(-\frac{e^{-\frac{3i\pi\nu}{4}}}{\sqrt{2}}\,G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}\,G_{2,4}^{1,1}\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4}\\ 0,\frac{1}{4},\nu,-\nu\end{matrix}\right)\right)$$

03.17.26.0082.01

$$\left(I_\nu\left(\sqrt[4]{-1}\,z\right)-I_{-\nu}\left(\sqrt[4]{-1}\,z\right)\right)\mathrm{bei}_\nu(z) = -\frac{i\sqrt{\pi}\,\sin(\pi\nu)}{2}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu} \left(\sqrt{2}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)e^{\frac{3i\pi\nu}{4}}\,G_{3,5}^{2,1}\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4},\nu-\frac{1}{4}\\ 0,\nu,-\nu,\frac{1}{4},\nu-\frac{1}{4}\end{matrix}\right)+\frac{e^{-\frac{3i\pi\nu}{4}}}{2\pi^2}\,G_{0,4}^{3,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)\right)$$

## Generalized cases involving Bessel *K*

03.17.26.0083.01

$$K_\nu\left(\sqrt[4]{-1}\,z\right)\mathrm{bei}_\nu(z) = i\,\frac{\pi^{3/2}}{4}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu} \left(\sqrt{2}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)e^{\frac{3i\pi\nu}{4}}\,G_{3,5}^{2,1}\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4},\nu-\frac{1}{4}\\ 0,\nu,-\nu,\frac{1}{4},\nu-\frac{1}{4}\end{matrix}\right)+\frac{e^{-\frac{3i\pi\nu}{4}}}{2\pi^2}\,G_{0,4}^{3,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)\right)$$

## Generalized cases involving ${}_0F_1$

03.17.26.0084.01

$$ {}_0F_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{bei}_\nu(z) = -\frac{i\sqrt{\pi}\,\Gamma(\nu+1)}{2\sqrt{2}}\left(-2^{\frac{1-\nu}{2}} e^{-\frac{3i\pi\nu}{4}}\pi\, G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac14 \middle| \begin{matrix} \frac{\nu+2}{4} \\ \frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac14(3\nu),\frac{\nu+2}{4}\end{matrix}\right) + e^{\frac{3i\pi\nu}{4}} G_{2,6}^{1,2}\left(\frac z2,\frac14\middle| \begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac\nu4,-\frac\nu4,\frac{\nu+2}{4},\frac{2-\nu}{4},\frac14(2-3\nu),-\frac14(3\nu)\end{matrix}\right) + e^{\frac{3i\pi\nu}{4}}\, i\, G_{2,6}^{1,2}\left(\frac z2,\frac14\middle|\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu+2}{4},-\frac\nu4,\frac\nu4,\frac{2-\nu}{4},\frac14(2-3\nu),-\frac14(3\nu)\end{matrix}\right)\right) $$

03.17.26.0085.01

$$ {}_0F_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{bei}_\nu(z) = i\sqrt{\pi}\,\Gamma(1-\nu)\left(2^{\frac\nu2-1}e^{-\frac{3i\pi\nu}{4}}\pi\left(G_{1,5}^{1,0}\left(\frac{z}{2\sqrt2},\frac14\middle|\begin{matrix}\frac14(3\nu+2)\\ \frac\nu4,\frac{3\nu}{4},\frac{\nu+2}{4},-\frac\nu4,\frac14(3\nu+2)\end{matrix}\right)+i\tan\left(\frac{\pi\nu}{2}\right)G_{1,5}^{1,0}\left(\frac{z}{2\sqrt2},\frac14\middle|\begin{matrix}\frac{3\nu}{4}+1\\ \frac{\nu+2}{4},\frac\nu4,\frac{3\nu}4,-\frac\nu4,\frac{3\nu}4+1\end{matrix}\right)\right)-\frac{e^{\frac{3i\pi\nu}{4}}}{2\sqrt2}\left(G_{2,6}^{1,2}\left(\frac z2,\frac14\middle|\begin{matrix}\frac{\nu+1}4,\frac{\nu+3}4\\ \frac\nu4,\frac{\nu+2}4,\frac14(3\nu+2),\frac{3\nu}4,\frac{2-\nu}4,-\frac\nu4\end{matrix}\right)+i\,G_{2,6}^{1,2}\left(\frac z2,\frac14\middle|\begin{matrix}\frac{\nu+1}4,\frac{\nu+3}4\\ \frac{\nu+2}4,\frac\nu4,\frac14(3\nu+2),\frac{3\nu}4,\frac{2-\nu}4,-\frac\nu4\end{matrix}\right)\right)\right) $$

03.17.26.0086.01

$$ {}_0F_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{bei}_\nu(z) = -i\,2^{-\frac\nu2-1}e^{\frac14(-3)i\pi\nu}\sqrt\pi\, z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma(\nu+1)\left(e^{\frac{3i\pi\nu}{2}}2^{\frac{3\nu}2}\csc\left(\pi\left(\nu+\frac34\right)\right)G_{2,4}^{1,1}\left(\sqrt[4]{-1}\,z,\frac12\middle|\begin{matrix}\frac{1-\nu}2,\frac14(1-2\nu)\\ \frac\nu2,-\frac\nu2,-\frac12(3\nu),\frac14(1-2\nu)\end{matrix}\right)-G_{0,4}^{1,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt2},\frac14\middle|\begin{matrix}\\ \frac\nu4,-\frac\nu4,\frac{2-\nu}4,-\frac14(3\nu)\end{matrix}\right)\right) $$

03.17.26.0087.01

$$ {}_0F_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{bei}_\nu(z) = -2^{-\nu-\frac12}\,i\,e^{\frac14(-3)i\pi\nu}\sqrt\pi\, z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma(1-\nu)\left(e^{\frac{3i\pi\nu}2}G_{2,4}^{1,1}\left(\sqrt[4]{-1}\,z,\frac12\middle|\begin{matrix}\frac{\nu+1}2,\frac14(2\nu+1)\\ \frac\nu2,-\frac\nu2,\frac{3\nu}2,\frac14(2\nu+1)\end{matrix}\right)-2^{\frac{3\nu-1}2}G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt2},\frac14\middle|\begin{matrix}\frac{2-\nu}4\\ \frac\nu4,\frac{\nu+2}4,\frac{3\nu}4,-\frac\nu4,\frac{2-\nu}4\end{matrix}\right)\right) $$

## Generalized cases involving ${}_0\tilde{F}_1$

03.17.26.0088.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{bei}_\nu(z) = -\frac{i\sqrt{\pi}}{2\sqrt{2}}\left(-2^{\frac{1-\nu}{2}} e^{-\frac{3i\pi\nu}{4}}\pi\, G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac14 \middle| \begin{matrix} \frac{\nu+2}{4} \\ \frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac14(3\nu),\frac{\nu+2}{4}\end{matrix}\right) + e^{\frac{3i\pi\nu}{4}} G_{2,6}^{1,2}\left(\frac z2,\frac14\middle| \begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac\nu4,-\frac\nu4,\frac{\nu+2}{4},\frac{2-\nu}{4},\frac14(2-3\nu),-\frac14(3\nu)\end{matrix}\right) + e^{\frac{3i\pi\nu}{4}}\, i\, G_{2,6}^{1,2}\left(\frac z2,\frac14\middle|\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu+2}{4},-\frac\nu4,\frac\nu4,\frac{2-\nu}{4},\frac14(2-3\nu),-\frac14(3\nu)\end{matrix}\right)\right) $$

03.17.26.0089.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\mathrm{bei}_\nu(z) = i\sqrt{\pi}\left(2^{\frac{\nu}{2}-1} e^{-\frac{3 i \pi \nu}{4}} \pi \left(G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4} \middle| \begin{matrix}\frac{1}{4}(3\nu+2)\\ \frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right) + i\tan\left(\frac{\pi\nu}{2}\right) G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\middle|\begin{matrix}\frac{3\nu}{4}+1\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{3\nu}{4}+1\end{matrix}\right)\right) - \frac{e^{\frac{3 i \pi \nu}{4}}}{2\sqrt{2}}\left(G_{2,6}^{1,2}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right) + i\, G_{2,6}^{1,2}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)\right)\right) $$

03.17.26.0090.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\mathrm{bei}_\nu(z) = -i\,2^{-\frac{\nu}{2}-1} e^{\frac{1}{4}(-3) i \pi \nu}\sqrt{\pi}\, z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} \left(e^{\frac{3 i \pi \nu}{2}}\, 2^{\frac{3\nu}{2}} \csc\left(\pi\left(\nu+\frac{3}{4}\right)\right) G_{2,4}^{1,1}\left(\sqrt[4]{-1}\, z,\frac{1}{2}\middle|\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{matrix}\right) - G_{0,4}^{1,0}\left(\frac{\sqrt[4]{-1}\, z}{2\sqrt{2}},\frac{1}{4}\middle| \frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)\right) $$

03.17.26.0091.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\mathrm{bei}_\nu(z) = -2^{-\nu-\frac{1}{2}}\, i\, e^{\frac{1}{4}(-3) i \pi \nu}\sqrt{\pi}\, z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} \left(e^{\frac{3 i \pi \nu}{2}} G_{2,4}^{1,1}\left(\sqrt[4]{-1}\, z,\frac{1}{2}\middle|\begin{matrix}\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{1}{4}(2\nu+1)\end{matrix}\right) - 2^{\frac{3\nu-1}{2}} G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\, z}{2\sqrt{2}},\frac{1}{4}\middle|\begin{matrix}\frac{2-\nu}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4}\end{matrix}\right)\right) $$

### Through other functions

03.17.26.0092.01

$$ \mathrm{bei}_\nu(z) = -\frac{(-1)^{5/8}\sqrt{z}}{2^{3/4}\sqrt{(1+i)\, z}}\left(i\, \boldsymbol{H}_{-\nu}\left(\sqrt[4]{-1}\, z\right) + e^{\frac{i \pi \nu}{2}} \boldsymbol{L}_{-\nu}\left(\sqrt[4]{-1}\, z\right)\right) /;\ \nu-\frac{1}{2}\in\mathbb{N} $$

## Representations through equivalent functions

### With related functions

03.17.27.0001.01

$$ \mathrm{bei}_\nu(z) = \csc(\pi\nu)\,\mathrm{ber}_{-\nu}(z) - \cot(\pi\nu)\,\mathrm{ber}_\nu(z) - \frac{2}{\pi}\mathrm{ker}_\nu(z) /;\ \nu\notin\mathbb{Z} $$

03.17.27.0002.01

$$ \mathrm{bei}_\nu(z) = \frac{1}{2} z^\nu \left(-z^4\right)^{-\frac{1}{4}(2+\nu)} \left(I_\nu\left(\sqrt[4]{-z^4}\right)\left(\cos\left(\frac{3\pi\nu}{4}\right) z^2 + \sqrt{-z^4}\,\sin\left(\frac{3\pi\nu}{4}\right)\right) + J_\nu\left(\sqrt[4]{-z^4}\right)\left(\sqrt{-z^4}\,\sin\left(\frac{3\pi\nu}{4}\right) - z^2\cos\left(\frac{3\pi\nu}{4}\right)\right)\right) $$

03.17.27.0003.01

$$ \mathrm{bei}_\nu(z) = -\frac{1}{2}\, i\, e^{-\frac{3}{4} i \pi \nu} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu}\left(e^{\frac{3 i \pi \nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) - J_\nu\left(\sqrt[4]{-1}\, z\right)\right) $$

03.17.27.0004.01

$$ \mathrm{bei}_\nu(z) = -\frac{1}{2}\, i \left(e^{-\frac{1}{2}(i\pi\nu)} I_\nu\left(\sqrt[4]{-1}\, z\right) - J_\nu\left(\sqrt[4]{-1}\, z\right)\right) /;\ \nu\in\mathbb{Z} $$

03.17.27.0005.01

$$\mathrm{bei}_\nu(z) = \begin{cases} \frac{1}{2} i e^{i\pi\nu} J_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{1}{2} i e^{\frac{5 i \pi \nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) & \frac{3\pi}{4} < \arg(z) \leq \pi \\ \frac{1}{2} i e^{-i\pi\nu} J_\nu\left(\sqrt[4]{-1}\, z\right) - \frac{1}{2} i e^{\frac{i \pi \nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) & \text{True} \end{cases}$$

03.17.27.0006.01

$$\mathrm{bei}_\nu(z) + i\, \mathrm{ber}_\nu(z) = i\, e^{-\frac{1}{4}(3 i \pi \nu)} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} J_\nu\left(\sqrt[4]{-1}\, z\right)$$

03.17.27.0007.01

$$\mathrm{bei}_\nu(z) + i\, \mathrm{ber}_\nu(z) = \begin{cases} i\, e^{i\pi\nu} J_\nu\left(\sqrt[4]{-1}\, z\right) & \frac{3\pi}{4} < \arg(z) \leq \pi \\ i\, e^{-i\pi\nu} J_\nu\left(\sqrt[4]{-1}\, z\right) & \text{True} \end{cases}$$

03.17.27.0008.01

$$\mathrm{bei}_\nu(z) - i\, \mathrm{ber}_\nu(z) = -i\, e^{\frac{3 i \pi \nu}{4}} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} I_\nu\left(\sqrt[4]{-1}\, z\right)$$

03.17.27.0009.01

$$\mathrm{bei}_\nu(z) - i\, \mathrm{ber}_\nu(z) = \begin{cases} -i\, e^{\frac{5 i \pi \nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) & \frac{3\pi}{4} < \arg(z) \leq \pi \\ -i\, e^{\frac{i \pi \nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) & \text{True} \end{cases}$$

## Theorems

## History

## Copyright