JacobiDC

Notations

Traditional name

Jacobi elliptic function dc

Traditional notation

$\mathrm{dc}(z \mid m)$

Mathematica StandardForm notation

`JacobiDC[z, m]`

Primary definition

09.28.02.0001.01

$$\mathrm{dc}(z \mid m) == \frac{\mathrm{dn}(z \mid m)}{\mathrm{cn}(z \mid m)}$$

Specific values

Specialized values

For fixed *z*

Case *m* = 0

09.28.03.0001.01

$$\mathrm{dc}(z \mid 0) == \sec(z)$$

09.28.03.0002.01

$$\mathrm{dc}\left(z + \frac{\pi}{2} \,\middle|\, 0\right) == -\csc(z)$$

09.28.03.0025.01

$$\mathrm{dc}\left(z + \frac{\pi k}{2} \,\middle|\, 0\right) == \sec\left(z + \frac{\pi k}{2}\right) /; k \in \mathbb{Z}$$

Case *m* = 1

09.28.03.0003.01

$$\mathrm{dc}(z \mid 1) == 1$$

For fixed *m*

Values at quarter-period points in the fundamental period parallelogram

09.28.03.0004.01

$$\operatorname{dc}(0 \mid m) = 1$$

09.28.03.0005.01

$$\operatorname{dc}(K(m) \mid m) = \tilde{\infty}$$

09.28.03.0006.01

$$\operatorname{dc}(2\,K(m) \mid m) = -1$$

09.28.03.0007.01

$$\operatorname{dc}(3\,K(m) \mid m) = \tilde{\infty}$$

09.28.03.0008.01

$$\operatorname{dc}(4\,K(m) \mid m) = 1$$

09.28.03.0009.01

$$\operatorname{dc}(i\,K(1-m) \mid m) = \sqrt{m}$$

09.28.03.0010.01

$$\operatorname{dc}(2\,i\,K(1-m) \mid m) = 1$$

09.28.03.0011.01

$$\operatorname{dc}(3\,i\,K(1-m) \mid m) = \sqrt{m}$$

09.28.03.0012.01

$$\operatorname{dc}(4\,i\,K(1-m) \mid m) = 1$$

09.28.03.0013.01

$$\operatorname{dc}(K(m) + i\,K(1-m) \mid m) = 0$$

09.28.03.0014.01

$$\operatorname{dc}(2\,K(m) + i\,K(1-m) \mid m) = -\sqrt{m}$$

09.28.03.0015.01

$$\operatorname{dc}(3\,K(m) + i\,K(1-m) \mid m) = -\sqrt{m}$$

09.28.03.0016.01

$$\operatorname{dc}(4\,K(m) + i\,K(1-m) \mid m) = \sqrt{m}$$

09.28.03.0017.01

$$\operatorname{dc}(K(m) + 2\,i\,K(1-m) \mid m) = \tilde{\infty}$$

09.28.03.0018.01

$$\operatorname{dc}(2\,K(m) + 2\,i\,K(1-m) \mid m) = -1$$

09.28.03.0019.01

$$\operatorname{dc}(3\,K(m) + 2\,i\,K(1-m) \mid m) = \tilde{\infty}$$

09.28.03.0020.01

$$\operatorname{dc}(4\,K(m) + 2\,i\,K(1-m) \mid m) = 1$$

09.28.03.0021.01

$$\operatorname{dc}((2\,r+1)\,K(m) + 2\,i\,s\,K(1-m) \mid m) = \tilde{\infty} \;/;\; \{r, s\} \in \mathbb{Z}$$

Values at half-quarter-period points

09.28.03.0022.01

$$\mathrm{dc}\left(\frac{K(m)}{2} \middle| m\right) = \sqrt{1+\sqrt{1-m}}$$

09.28.03.0023.01

$$\mathrm{dc}\left(\frac{i\,K(1-m)}{2} \middle| m\right) = \sqrt[4]{m}$$

09.28.03.0024.01

$$\mathrm{dc}\left(\frac{K(m)}{2} + \frac{i\,K(1-m)}{2} \middle| m\right) = \left(\sqrt[4]{m}\left(\sqrt{1+\sqrt{1-m}} - i\sqrt{1-\sqrt{1-m}}\right)\right)\frac{1+i}{2}$$

General characteristics

Domain and analyticity

$\mathrm{dc}(z \mid m)$ is a meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.28.04.0001.01

$$(z * m) \longrightarrow \mathrm{dc}(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\mathrm{dc}(z \mid m)$ is an even function with respect to z.

09.28.04.0002.01

$$\mathrm{dc}(-z \mid m) = \mathrm{dc}(z \mid m)$$

Mirror symmetry

09.28.04.0003.01

$$\mathrm{dc}(\bar{z} \mid \bar{m}) = \overline{\mathrm{dc}(z \mid m)}$$

Periodicity

$\mathrm{dc}(z \mid m)$ is a doubly periodic function with respect to z with periods $2\,i\,K(1-m)$ and $4\,K(m)$.

09.28.04.0004.01

$$\mathrm{dc}(z + 2\,K(m) \mid m) = -\mathrm{dc}(z \mid m)$$

09.28.04.0005.01

$$\mathrm{dc}(z + 4\,K(m) \mid m) = \mathrm{dc}(z \mid m)$$

09.28.04.0006.01

$$\mathrm{dc}(z + 2\,i\,K(1-m) \mid m) = \mathrm{dc}(z \mid m)$$

09.28.04.0007.01

$$\mathrm{dc}(z + 2\,K(m) + 2\,i\,K(1-m) \mid m) = -\mathrm{dc}(z \mid m)$$

09.28.04.0008.01

$$\mathrm{dc}(z + 2\,i\,s\,K(1-m) + 2\,r\,K(m) \mid m) = (-1)^r\,\mathrm{dc}(z \mid m) \;/;\; \{r, s\} \in \mathbb{Z}$$

Poles and essential singularities

With respect to *z*

For fixed m, the function $\operatorname{dc}(z \mid m)$ has an infinite set of singular points:

a) $z = (2r+1)K(m) + 2isK(1-m)$, $\{r, s\} \in \mathbb{Z}$, are the simple poles with residues $(-1)^{r-1}$;

b) $z == \tilde{\infty}$ is an essential singular point.

09.28.04.0009.01

$$\mathcal{Sing}_z(\operatorname{dc}(z \mid m)) == \{\{\{2 s i K(1-m) + (2r+1)K(m), 1\} /; \{r, s\} \in \mathbb{Z}\}, \{\tilde{\infty}, \infty\}\}$$

09.28.04.0010.01

$$\operatorname{res}_z(\operatorname{dc}(z \mid m))(2 s i K(1-m) + (2r+1)K(m)) == (-1)^{r-1} /; \{r, s\} \in \mathbb{Z}$$

Branch points

With respect to *m*

For fixed z, the function $\operatorname{dc}(z \mid m)$ is a meromorphic function in m that has no branch points.

09.28.04.0013.01

$$\mathcal{BP}_m(\operatorname{dc}(z \mid m)) == \{\}$$

P. Walker

With respect to *z*

For fixed m, the function $\operatorname{dc}(z \mid m)$ does not have branch points.

09.28.04.0011.01

$$\mathcal{BP}_z(\operatorname{dc}(z \mid m)) == \{\}$$

Branch cuts

With respect to *m*

For fixed z, the function $\operatorname{cd}(z \mid m)$ is a meromorphic function in m that has no branch cuts.

09.28.04.0014.01

$$\mathcal{BC}_m(\operatorname{dc}(z \mid m)) == \{\}$$

P. Walker

With respect to *z*

For fixed m, the function $\operatorname{dc}(z \mid m)$ does not have branch cuts.

09.28.04.0012.01

$$\mathcal{BC}_z(\operatorname{dc}(z \mid m)) == \{\}$$

Series representations

Generalized power series

Expansions at $z = 0$

09.28.06.0005.01

$$\operatorname{dc}(z \mid m) \propto 1 + \frac{1}{2}(1-m)z^2 + \frac{1}{24}\left(5 - 6m + m^2\right)z^4 + \ldots \;/;\; (z \to 0)$$

09.28.06.0001.02

$$\operatorname{dc}(z \mid m) \propto 1 + \frac{1}{2}(1-m)z^2 + \frac{1}{24}\left(5 - 6m + m^2\right)z^4 + \frac{1}{720}\left(61 - 107m + 47m^2 - m^3\right)z^6 + \frac{\left(1385 - 3116m + 2142m^2 - 412m^3 + m^4\right)z^8}{40320} + \frac{1}{3628800}\left(\left(50521 - 138933m + 130250m^2 - 45530m^3 + 3693m^4 - m^5\right)z^{10}\right) + O\left(z^{12}\right)$$

09.28.06.0006.01

$$\operatorname{dc}(z \mid m) = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(j+1)(-1)^{k-j}\operatorname{dn}_{k-j}(m)}{(2k-2j)!}\sum_{r=0}^{j}\frac{(-1)^r}{r+1}\binom{j}{r}q_{r,j}\,z^{2k} \;/;\; q_{j,0} = 1 \bigwedge q_{j,k} = \frac{1}{k}\sum_{i=1}^{k}\frac{(j\,i + i - k)(-1)^i \operatorname{cn}_i(m)\,q_{j,k-i}}{(2i)!} \bigwedge$$
$$k \in \mathbb{N}^+ \bigwedge \operatorname{sn}_0(m) = 1 \bigwedge \operatorname{sn}_n(m) = \sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2n}{2j}\operatorname{cn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n} \bigwedge \operatorname{cn}_0(m) = 1 \bigwedge$$
$$\operatorname{cn}_n(m) = \sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n+1} \bigwedge \operatorname{dn}_0(m) = 1 \bigwedge \operatorname{dn}_n(m) = m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{cn}_k(m)\,\delta_{j+k-n+1}$$

09.28.06.0007.01

$$\operatorname{dc}(z \mid m) \propto 1 + O\left(z^2\right)$$

Expansions at $z = (2r+1)K(m) + 2 i s K(1-m)$

09.28.06.0008.01

$$\operatorname{dc}(z \mid m) \propto (-1)^{r-1}\left(\frac{1}{z - z_0} + \frac{1}{6}(m+1)(z - z_0) + \frac{1}{360}\left(7m^2 - 22m + 7\right)(z - z_0)^3 + \ldots\right) /;$$
$$(z \to z_0) \land z_0 = (2r+1)K(m) + 2 i s K(1-m) \land r \in \mathbb{Z} \land s \in \mathbb{Z}$$

09.28.06.0009.01

$$\operatorname{dc}(z \mid m) = (-1)^{r-1}\sum_{k=0}^{\infty}(k+1)\sum_{r=0}^{k}\frac{(-1)^r}{r+1}\binom{k}{r}p_{r,k}\,(z - z_0)^{2k-1} \;/;$$
$$z_0 = (2r+1)K(m) + 2 i s K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z} \bigwedge p_{j,0} = 1 \bigwedge p_{j,k} = \frac{1}{k}\sum_{i=1}^{k}\frac{(j\,i + i - k)(-1)^i \operatorname{sn}_i(m)\,p_{j,k-i}}{(2i+1)!} \bigwedge$$
$$k \in \mathbb{N}^+ \bigwedge \operatorname{sn}_0(m) = 1 \bigwedge \operatorname{sn}_n(m) = \sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2n}{2j}\operatorname{cn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n} \bigwedge \operatorname{cn}_0(m) = 1 \bigwedge$$
$$\operatorname{cn}_n(m) = \sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{dn}_k(m)\,\delta_{j+k-n+1} \bigwedge \operatorname{dn}_0(m) = 1 \bigwedge \operatorname{dn}_n(m) = m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2n-1}{2j+1}\operatorname{sn}_j(m)\operatorname{cn}_k(m)\,\delta_{j+k-n+1}$$

09.28.06.0010.01

$$\operatorname{dc}(z \mid m) \propto \frac{(-1)^{r-1}}{z - z_0}\left(1 + O\left((z - z_0)^2\right)\right) /;\; z_0 = (2r+1)K(m) + 2 i s K(1-m) \land r \in \mathbb{Z} \land s \in \mathbb{Z}$$

Expansions at $m == 0$

09.28.06.0011.01

$$\text{dc}(z \mid m) \propto \sec(z) - \frac{1}{4} (z + \cos(z)\sin(z)) \tan(z) \sec(z)\, m + \frac{1}{512} \left(-8\cos(2z) z^2 + 24 z^2 + 4\sin(2z) z + 4\sin(4z) z + 5\cos(4z) - 5\right) \sec^3(z)\, m^2 + \ldots /; (m \to 0)$$

09.28.06.0012.01

$$\text{dc}(z \mid m) \propto \sec(z) - \frac{1}{4} (z + \cos(z)\sin(z)) \tan(z) \sec(z)\, m + \frac{1}{512} \left(-8\cos(2z) z^2 + 24 z^2 + 4\sin(2z) z + 4\sin(4z) z + 5\cos(4z) - 5\right) \sec^3(z)\, m^2 +$$
$$\frac{1}{49152} \Big(3\left(160 z^2 - 83\right)\cos(z) - 3\left(88 z^2 - 43\right)\cos(3z) - 3\left(8 z^2 - 41\right)\cos(5z) - 3\cos(7z) - 32 z\left(23 z^2 - 6\right)\sin(z) + 8 z\left(4 z^2 + 39\right)\sin(3z) + 120 z \sin(5z)\Big) \sec^4(z)\, m^3 - \frac{1}{1572864}$$
$$\Big(-3680 z^4 - 2760 z^2 + 4\left(1976 z^2 - 1527\right)\sin(2z) z - 4\left(56 z^2 + 1227\right)\sin(4z) z + 4\left(8 z^2 - 297\right)\sin(6z) z + 36\sin(8z) z + \left(2432 z^4 + 312 z^2 + 1167\right)\cos(2z) - 2\left(16 z^4 - 1716 z^2 + 1287\right)\cos(4z) + 3\left(120 z^2 - 389\right)\cos(6z) + 60\cos(8z) + 2514\Big)$$
$$\sec^5(z)\, m^4 + \frac{1}{251658240} \Big(840\left(476 z^4 + 87 z^2 - 411\right)\cos(z) - 30\left(6288 z^4 + 10116 z^2 - 2755\right)\cos(3z) + 5\left(736 z^4 - 44904 z^2 + 41811\right)\cos(5z) + 10\left(16 z^4 - 1848 z^2 + 5835\right)\cos(7z) + 30\left(36 z^2 - 161\right)\cos(9z) + 15\cos(11z) - 16 z\left(13456 z^4 + 28940 z^2 - 23745\right)\sin(z) + 24 z\left(1264 z^4 - 18980 z^2 + 27975\right)\sin(3z) - 8 z\left(16 z^4 - 540 z^2 - 44145\right)\sin(5z) - 20 z\left(160 z^2 - 2859\right)\sin(7z) - 4500 z \sin(9z)\Big) \sec^6(z)\, m^5 +$$
$$\frac{1}{24159191040} \Big(3014144 z^6 + 17125920 z^4 - 2772000 z^2 - 24\left(485296 z^4 + 1407720 z^2 - 1672185\right)\sin(2z) z + 60\left(31808 z^4 - 276960 z^2 + 638391\right)\sin(4z) z - 180\left(32 z^4 - 408 z^2 - 80953\right)\sin(6z) z + 24\left(16 z^4 - 2760 z^2 + 62415\right)\sin(8z) z + 180\left(72 z^2 - 1381\right)\sin(10z) z + 900\sin(12z) z - 2\left(1349504 z^6 - 4620960 z^4 + 7884360 z^2 + 4624785\right)\cos(2z) + 4\left(46208 z^6 - 1922880 z^4 - 5262840 z^2 + 2644965\right)\cos(4z) - \left(256 z^6 - 204480 z^4 + 8508960 z^2 - 9467235\right)\cos(6z) + 30\left(400 z^4 - 11904 z^2 + 58617\right)\cos(8z) + 45\left(2160 z^2 - 4837\right)\cos(10z) + 1620\cos(12z) - 12339990\Big) \sec^7(z)\, m^6 +$$
$$\frac{1}{5411658792960} \Big(63\left(9614336 z^6 + 67529280 z^4 - 48535200 z^2 - 46854555\right)\cos(z) - 140\left(3269504 z^6 - 1174224 z^4 + 27202662 z^2 - 992079\right)\cos(3z) + 70\left(478976 z^6 - 10396272 z^4 - 37828116 z^2 + 25073235\right)\cos(5z) - 35\left(1792 z^6 - 490272 z^4 + 20050128 z^2 - 27450513\right)\cos(7z) - 7\left(256 z^6 + 84960 z^4 - 1830960 z^2 - 16994475\right) \cos(9z) - 630\left(432 z^4 - 22276 z^2 + 35213\right)\cos(11z) - 2520\left(25 z^2 - 109\right)\cos(13z) - 315\cos(15z) - 8 z\left(33244544 z^6 + 169790880 z^4 + 373682400 z^2 - 460109475\right)\sin(z) + 12 z\left(5176064 z^6 - 94597440 z^4 - 370032320 z^2 + 594245715\right)\sin(3z) - 16 z\left(139456 z^6 - 13918800 z^4 + 88750830 z^2 - 293798925\right)\sin(5z) + 4 z\left(256 z^6 - 94080 z^4 + 9042600 z^2 + 336907305\right)\sin(7z) + 420 z\left(192 z^4 + 20072 z^2 + 162363\right)\sin(9z) + 2520 z\left(1260 z^2 - 11297\right)\sin(11z) + 258300 z \sin(13z)\Big) \sec^8(z)\, m^7 -$$
$$\frac{1}{173173081374720} \Big(-1196803584 z^8 - 11621272320 z^6 - 59265712800 z^4 + 51670006920 z^2 +$$

$$28\left(225\,685\,504\,z^6+1\,385\,289\,216\,z^4+3\,419\,854\,200\,z^2-5\,152\,437\,585\right)\sin(2\,z)\,z-$$
$$4\left(464\,069\,120\,z^6-3\,558\,439\,584\,z^4-18\,625\,005\,840\,z^2+39\,050\,472\,195\right)\sin(4\,z)\,z+$$
$$144\left(480\,320\,z^6-23\,703\,624\,z^4+113\,082\,060\,z^2-540\,952\,755\right)\sin(6\,z)\,z-$$
$$4\left(5888\,z^6-6\,093\,024\,z^4+375\,391\,800\,z^2+4\,243\,004\,955\right)\sin(8\,z)\,z+$$
$$32\left(32\,z^6+129\,948\,z^4-12\,802\,230\,z^2+2\,131\,605\right)\sin(10\,z)\,z+$$
$$756\left(864\,z^4-95\,200\,z^2+578\,675\right)\sin(12\,z)\,z+420\left(1000\,z^2-16\,371\right)\sin(14\,z)\,z+8820\sin(16\,z)\,z+$$
$$2\left(636\,233\,728\,z^8-1\,416\,993\,536\,z^6-30\,708\,972\,000\,z^4+53\,216\,291\,520\,z^2+17\,945\,797\,275\right)\cos(2\,z)-$$
$$4\left(42\,446\,336\,z^8-2\,036\,204\,800\,z^6-2\,051\,632\,800\,z^4-22\,241\,422\,350\,z^2+6\,187\,089\,195\right)\cos(4\,z)+$$
$$63\left(53\,248\,z^8-10\,174\,720\,z^6+163\,077\,600\,z^4+668\,248\,200\,z^2-553\,255\,345\right)\cos(6\,z)-$$
$$2\left(256\,z^8-760\,704\,z^6+12\,390\,000\,z^4-3\,620\,465\,100\,z^2+6\,758\,870\,265\right)\cos(8\,z)+$$
$$35\left(1792\,z^6+2\,028\,000\,z^4-25\,849\,368\,z^2-29\,453\,985\right)\cos(10\,z)+1260\left(8640\,z^4-195\,426\,z^2+251\,863\right)\cos(12\,z)+$$
$$1260\left(2300\,z^2-4459\right)\cos(14\,z)+16\,380\cos(16\,z)+37\,948\,733\,550\Big)\sec^9(z)\,m^8-\frac{1}{49\,873\,847\,435\,919\,360}$$
$$\Big(-63\left(5\,765\,895\,168\,z^8+64\,101\,931\,520\,z^6+358\,754\,198\,880\,z^4-461\,814\,328\,800\,z^2-210\,208\,477\,185\right)\cos(z)+$$
$$216\left(1\,654\,016\,128\,z^8+3\,795\,293\,152\,z^6-30\,335\,368\,280\,z^4+117\,788\,351\,835\,z^2+5\,787\,713\,820\right)\cos(3\,z)-$$
$$720\left(74\,894\,880\,z^8-1\,610\,434\,336\,z^6-3\,479\,640\,570\,z^4-22\,032\,817\,380\,z^2+9\,879\,190\,965\right)\cos(5\,z)+$$
$$63\left(17\,245\,696\,z^8-1\,590\,476\,032\,z^6+22\,333\,139\,520\,z^4+86\,714\,291\,880\,z^2-90\,423\,595\,035\right)\cos(7\,z)-$$
$$9\left(24\,064\,z^8+26\,967\,808\,z^6-5\,753\,905\,920\,z^4-53\,348\,533\,560\,z^2+184\,980\,199\,635\right)\cos(9\,z)-$$
$$36\left(128\,z^8+2\,892\,288\,z^6-583\,461\,480\,z^4+5\,821\,526\,340\,z^2+1\,616\,170\,815\right)\cos(11\,z)-$$
$$189\left(62\,208\,z^6-12\,659\,840\,z^4+187\,133\,400\,z^2-213\,575\,595\right)\cos(13\,z)-$$
$$945\left(20\,000\,z^4-800\,040\,z^2+997\,737\right)\cos(15\,z)-5670\left(196\,z^2-839\right)\cos(17\,z)-2835\cos(19\,z)+$$
$$8\,z\left(17\,769\,803\,264\,z^8+154\,337\,008\,896\,z^6+662\,044\,192\,992\,z^4+1\,413\,285\,787\,620\,z^2-2\,207\,834\,702\,955\right)\sin(z)-$$
$$4\,z\left(11\,114\,481\,664\,z^8-218\,002\,922\,496\,z^6-1\,813\,216\,327\,776\,z^4-4\,986\,817\,337\,880\,z^2+9\,150\,568\,206\,255\right)\sin(3\,z)+$$
$$20\,z\left(179\,849\,216\,z^8-17\,458\,283\,520\,z^6+75\,235\,589\,856\,z^4+516\,684\,231\,000\,z^2-1\,408\,419\,633\,915\right)\sin(5\,z)-$$
$$64\,z\left(629\,536\,z^8-211\,179\,024\,z^6+6\,915\,622\,266\,z^4-21\,606\,985\,575\,z^2+173\,396\,382\,075\right)\sin(7\,z)+$$
$$128\,z\left(16\,z^8-22\,536\,z^6+88\,216\,317\,z^4-3\,070\,844\,595\,z^2-14\,032\,657\,485\right)\sin(9\,z)+$$
$$36\,z\left(10\,240\,z^6+65\,446\,752\,z^4-2\,648\,877\,000\,z^2+3\,662\,620\,605\right)\sin(11\,z)+$$
$$11\,340\,z\left(23\,328\,z^4-1\,054\,072\,z^2+5\,123\,503\right)\sin(13\,z)+56\,700\,z\left(3400\,z^2-23\,959\right)\sin(15\,z)+4\,524\,660\,z\sin(17\,z)\Big)$$
$$\sec^{10}(z)\,m^9+\frac{1}{7\,979\,815\,589\,747\,097\,600}\Big(3\,127\,485\,374\,464\,z^{10}+43\,994\,001\,300\,480\,z^8+$$
$$330\,560\,432\,294\,400\,z^6+1\,585\,067\,158\,382\,400\,z^4-2\,197\,145\,541\,729\,600\,z^2-1080$$
$$\left(18\,545\,000\,960\,z^8+183\,880\,478\,592\,z^6+874\,548\,867\,248\,z^4+1\,915\,061\,306\,760\,z^2-3\,460\,899\,395\,415\right)\sin(2\,z)\,z+$$
$$120\left(69\,804\,123\,904\,z^8-410\,456\,968\,704\,z^6-5\,520\,309\,884\,208\,z^4-16\,449\,365\,164\,680\,z^2+36\,940\,206\,271\,815\right)$$
$$\sin(4\,z)\,z-160$$
$$\left(4\,557\,007\,168\,z^8-197\,971\,975\,584\,z^6+530\,033\,745\,528\,z^4+4\,517\,746\,166\,430\,z^2-16\,561\,877\,387\,265\right)\sin(6\,z)\,z+$$
$$40\left(206\,488\,576\,z^8-31\,450\,871\,808\,z^6+739\,283\,503\,392\,z^4-653\,416\,076\,880\,z^2+21\,042\,320\,833\,485\right)\sin(8\,z)\,z-$$
$$320\left(992\,z^8-17\,632\,080\,z^6+6\,037\,860\,528\,z^4-148\,398\,278\,175\,z^2-293\,396\,228\,055\right)\sin(10\,z)\,z+$$
$$20\left(512\,z^8+53\,001\,216\,z^6-18\,679\,375\,008\,z^4+500\,341\,983\,120\,z^2-880\,724\,436\,795\right)\sin(12\,z)\,z+$$
$$540\left(186\,624\,z^6-62\,952\,736\,z^4+1\,786\,255\,800\,z^2-7\,697\,400\,375\right)\sin(14\,z)\,z+$$
$$18\,900\left(20\,000\,z^4-1\,596\,816\,z^2+7\,072\,143\right)\sin(16\,z)\,z+18\,900\left(2744\,z^2-41\,535\right)\sin(18\,z)\,z+510\,300\sin(20\,z)\,z-$$
$$\Big(3\,714\,757\,763\,072\,z^{10}-725\,097\,369\,600\,z^8-264\,064\,801\,674\,240\,z^6-2\,031\,769\,019\,901\,600\,z^4+$$
$$4\,031\,463\,722\,718\,600\,z^2+969\,296\,803\,112\,925\Big)\cos(2\,z)+2\Big(365\,160\,251\,392\,z^{10}-18\,368\,995\,703\,040\,z^8-$$
$$80\,431\,555\,034\,880\,z^6+126\,842\,760\,399\,600\,z^4-1\,454\,744\,457\,558\,000\,z^2+200\,711\,093\,812\,425\Big)\cos(4\,z)-$$

$$\left(37\,339\,713\,536\,z^{10} - 6\,399\,354\,193\,920\,z^8 + 85\,954\,169\,468\,160\,z^6 + 302\,198\,583\,304\,800\,z^4 + 1\,412\,940\,934\,326\,600\,z^2 - 861\,620\,257\,661\,625\right)\cos(6 z) + 32\left(7\,556\,864\,z^{10} - 4\,110\,213\,600\,z^8 + 266\,226\,090\,480\,z^6 - 3\,641\,863\,424\,850\,z^4 - 10\,988\,271\,798\,525\,z^2 + 15\,273\,874\,335\,825\right)\cos(8 z) - \left(4096\,z^{10} - 32\,532\,480\,z^8 - 133\,951\,507\,200\,z^6 + 9\,643\,178\,109\,600\,z^4 - 4\,911\,089\,891\,400\,z^2 - 110\,501\,743\,056\,075\right)\cos(10 z) + 90\left(11\,520\,z^8 + 344\,065\,792\,z^6 - 27\,964\,693\,680\,z^4 + 238\,785\,870\,960\,z^2 - 6\,743\,301\,075\right)\cos(12 z) + 4725\left(622\,080\,z^6 - 47\,775\,520\,z^4 + 557\,948\,568\,z^2 - 597\,795\,471\right)\cos(14 z) + 4725\left(1\,120\,000\,z^4 - 18\,727\,968\,z^2 + 17\,561\,343\right)\cos(16 z) + 2\,778\,300\left(124\,z^2 - 221\right)\cos(18 z) + 963\,900\cos(20 z) - 889\,662\,247\,584\,075\Big)\sec^{11}(z)\,m^{10} + O\left(m^{11}\right)$$

09.28.06.0013.01

$$\operatorname{dc}(z\mid m)\propto\sec(z)\,(1+O(m))$$

Expansions at $m == 1$

09.28.06.0014.01

$$\operatorname{dc}(z\mid m)\propto 1-\frac{1}{2}\sinh^2(z)\,(m-1)+\frac{1}{32}\sinh(z)\,(-8\,z\cosh(z)+5\sinh(z)+\sinh(3 z))\,(m-1)^2+\ldots\;/;\;(m\to 1)$$

09.28.06.0015.01

$$\operatorname{dc}(z\mid m)\propto 1-\frac{1}{2}\sinh^2(z)\,(m-1)+\frac{1}{32}\sinh(z)\,(-8\,z\cosh(z)+5\sinh(z)+\sinh(3 z))\,(m-1)^2 + \frac{1}{1024}\left(-\left(32\,z^2+31\right)\cosh(2 z)-12\cosh(4 z)-\cosh(6 z)+72\,z\sinh(2 z)+16\,z\sinh(4 z)+44\right)(m-1)^3 + \frac{1}{49\,152}\Big(-256\sinh(2 z)\,z^3-2280\sinh(2 z)\,z-816\sinh(4 z)\,z-72\sinh(6 z)\,z + 12\left(112\,z^2+75\right)\cosh(2 z)+12\left(32\,z^2+37\right)\cosh(4 z)+60\cosh(6 z)+3\cosh(8 z)-1407\Big)(m-1)^4 + \frac{1}{786\,432}\Big(4864\sinh(2 z)\,z^3+2048\sinh(4 z)\,z^3-864\cosh(6 z)\,z^2+26\,400\sinh(2 z)\,z+12\,096\sinh(4 z)\,z + 1800\sinh(6 z)\,z+96\sinh(8 z)\,z-\left(512\,z^4+17\,664\,z^2+9765\right)\cosh(2 z) - 48\left(176\,z^2+119\right)\cosh(4 z)-984\cosh(6 z)-84\cosh(8 z)-3\cosh(10 z)+16\,548\Big)(m-1)^5 + \frac{1}{62\,914\,560}\Big(-4096\sinh(2 z)\,z^5-381\,440\sinh(2 z)\,z^3-276\,480\sinh(4 z)\,z^3-34\,560\sinh(6 z)\,z^3+129\,600\cosh(6 z)\,z^2 + 7680\cosh(8 z)\,z^2-1\,621\,320\sinh(2 z)\,z-873\,840\sinh(4 z)\,z-169\,920\sinh(6 z)\,z-15\,840\sinh(8 z)\,z - 600\sinh(10 z)\,z+60\left(1024\,z^4+19\,520\,z^2+9471\right)\cosh(2 z)+5\left(8192\,z^4+146\,304\,z^2+75\,933\right)\cosh(4 z) + 76\,320\cosh(6 z)+8580\cosh(8 z)+540\cosh(10 z)+15\cosh(12 z)-1\,033\,380\Big)(m-1)^6 + \frac{1}{3\,019\,898\,880}\Big(-\left(16\,384\,z^6+3\,356\,160\,z^4+47\,416\,320\,z^2+20\,807\,865\right)\cosh(2 z) - 3\Big(-118\,784\sinh(2 z)\,z^5-131\,072\sinh(4 z)\,z^5-5\,689\,600\sinh(2 z)\,z^3-5\,478\,400\sinh(4 z)\,z^3-1\,209\,600\sinh(6 z)\,z^3 - 81\,920\sinh(8 z)\,z^3+291\,840\cosh(8 z)\,z^2+12\,000\cosh(10 z)\,z^2-20\,742\,480\sinh(2 z)\,z - 12\,596\,880\sinh(4 z)\,z-2\,907\,720\sinh(6 z)\,z-362\,880\sinh(8 z)\,z-24\,600\sinh(10 z)\,z-720\sinh(12 z)\,z + 20\left(65\,536\,z^4+587\,328\,z^2+258\,687\right)\cosh(4 z)+90\left(2304\,z^4+30\,768\,z^2+12\,899\right)\cosh(6 z) + 155\,760\cosh(8 z)+13\,200\cosh(10 z)+660\cosh(12 z)+15\cosh(14 z)-13\,440\,240\Big)\Big)(m-1)^7 + \frac{1}{338\,228\,674\,560}\Big(-131\,072\sinh(2 z)\,z^7-51\,695\,616\sinh(2 z)\,z^5-101\,842\,944\sinh(4 z)\,z^5-20\,901\,888\sinh(6 z)\,z^5 + 174\,182\,400\cosh(6 z)\,z^4+13\,762\,560\cosh(8 z)\,z^4-1\,755\,425\,280\sinh(2 z)\,z^3-2\,034\,278\,400\sinh(4 z)\,z^3 - 606\,009\,600\sinh(6 z)\,z^3-73\,973\,760\sinh(8 z)\,z^3-3\,360\,000\sinh(10 z)\,z^3+1\,089\,184\,320\cosh(6 z)\,z^2 + 155\,635\,200\cosh(8 z)\,z^2+11\,592\,000\cosh(10 z)\,z^2+362\,880\cosh(12 z)\,z^2-5\,741\,358\,840\sinh(2 z)\,z - 3\,829\,104\,720\sinh(4 z)\,z-999\,144\,720\sinh(6 z)\,z-150\,645\,600\sinh(8 z)\,z-13\,910\,400\sinh(10 z)\,z -$$

$$740\,880\sinh(12\,z)\,z - 17\,640\sinh(14\,z)\,z + 28\left(139\,264\,z^6 + 13\,900\,800\,z^4 + 162\,684\,000\,z^2 + 65\,714\,805\right)\cosh(2\,z) + 28\left(262\,144\,z^6 + 21\,934\,080\,z^4 + 137\,187\,360\,z^2 + 53\,897\,355\right)\cosh(4\,z) + 368\,255\,160\cosh(6\,z) + 56\,071\,260\cosh(8\,z) + 5\,765\,760\cosh(10\,z) + 394\,380\cosh(12\,z) + 16\,380\cosh(14\,z) + 315\cosh(16\,z) - 3\,779\,643\,735\Big)(m-1)^8 +$$

$$\frac{1}{5\,411\,658\,792\,960}\Big(5\,111\,808\sinh(2\,z)\,z^7 + 16\,777\,216\sinh(4\,z)\,z^7 - 83\,607\,552\cosh(6\,z)\,z^6 + 941\,875\,200\sinh(2\,z)\,z^5 + 2\,526\,806\,016\sinh(4\,z)\,z^5 + 940\,584\,960\sinh(6\,z)\,z^5 + 88\,080\,384\sinh(8\,z)\,z^5 - 4\,589\,706\,240\cosh(6\,z)\,z^4 - 660\,602\,880\cosh(8\,z)\,z^4 - 33\,600\,000\cosh(10\,z)\,z^4 + 25\,705\,666\,560\sinh(2\,z)\,z^3 + 34\,074\,593\,280\sinh(4\,z)\,z^3 + 12\,337\,920\,000\sinh(6\,z)\,z^3 + 2\,062\,663\,680\sinh(8\,z)\,z^3 + 171\,360\,000\sinh(10\,z)\,z^3 + 5\,806\,080\sinh(12\,z)\,z^3 - 19\,101\,640\,320\cosh(6\,z)\,z^2 - 3\,329\,786\,880\cosh(8\,z)\,z^2 - 340\,956\,000\cosh(10\,z)\,z^2 - 19\,595\,520\cosh(12\,z)\,z^2 - 493\,920\cosh(14\,z)\,z^2 + 77\,414\,846\,040\sinh(2\,z)\,z + 55\,748\,468\,160\sinh(4\,z)\,z + 15\,973\,902\,000\sinh(6\,z)\,z + 2\,759\,299\,200\sinh(8\,z)\,z + 311\,938\,200\sinh(10\,z)\,z + 22\,921\,920\sinh(12\,z)\,z + 1\,005\,480\sinh(14\,z)\,z + 20\,160\sinh(16\,z)\,z - 2\left(65\,536\,z^8 + 45\,301\,760\,z^6 + 3\,093\,081\,600\,z^4 + 31\,716\,896\,400\,z^2 + 11\,928\,745\,395\right)\cosh(2\,z) - 336\left(917\,504\,z^6 + 35\,307\,520\,z^4 + 175\,588\,080\,z^2 + 63\,335\,115\right)\cosh(4\,z) - 5\,552\,602\,965\cosh(6\,z) - 931\,124\,880\cosh(8\,z) - 110\,065\,410\cosh(10\,z) - 9\,248\,400\cosh(12\,z) - 531\,720\cosh(14\,z) - 18\,900\cosh(16\,z) - 315\cosh(18\,z) + 51\,741\,682\,020\Big)(m-1)^9 +$$

$$\frac{1}{779\,278\,866\,186\,240}\Big(-1\,048\,576\sinh(2\,z)\,z^9 - 1\,184\,366\,592\sinh(2\,z)\,z^7 - 7\,096\,762\,368\sinh(4\,z)\,z^7 - 2\,579\,890\,176\sinh(6\,z)\,z^7 + 37\,623\,398\,400\cosh(6\,z)\,z^6 + 4\,227\,858\,432\cosh(8\,z)\,z^6 - 144\,973\,946\,880\sinh(2\,z)\,z^5 - 476\,600\,205\,312\sinh(4\,z)\,z^5 - 244\,175\,855\,616\sinh(6\,z)\,z^5 - 42\,014\,343\,168\sinh(8\,z)\,z^5 - 2\,419\,200\,000\sinh(10\,z)\,z^5 + 908\,605\,071\,360\cosh(6\,z)\,z^4 + 180\,592\,312\,320\cosh(8\,z)\,z^4 + 16\,934\,400\,000\cosh(10\,z)\,z^4 + 627\,056\,640\cosh(12\,z)\,z^4 - 3\,392\,002\,656\,000\sinh(2\,z)\,z^3 - 4\,988\,785\,213\,440\sinh(4\,z)\,z^3 - 2\,079\,581\,091\,840\sinh(6\,z)\,z^3 - 427\,513\,282\,560\sinh(8\,z)\,z^3 - 49\,200\,480\,000\sinh(10\,z)\,z^3 - 3\,083\,028\,480\sinh(12\,z)\,z^3 - 82\,978\,560\sinh(14\,z)\,z^3 + 2\,904\,470\,835\,840\cosh(6\,z)\,z^2 + 584\,559\,037\,440\cosh(8\,z)\,z^2 + 73\,823\,400\,000\cosh(10\,z)\,z^2 + 5\,894\,985\,600\cosh(12\,z)\,z^2 + 275\,607\,360\cosh(14\,z)\,z^2 + 5\,806\,080\cosh(16\,z)\,z^2 - 9\,564\,409\,335\,600\sinh(2\,z)\,z - 7\,348\,707\,374\,400\sinh(4\,z)\,z - 2\,268\,953\,217\,720\sinh(6\,z)\,z - 434\,954\,862\,720\sinh(8\,z)\,z - 56\,932\,923\,600\sinh(10\,z)\,z - 5\,174\,442\,000\sinh(12\,z)\,z - 316\,249\,920\sinh(14\,z)\,z - 11\,793\,600\sinh(16\,z)\,z - 204\,120\sinh(18\,z)\,z + 72\left(720\,896\,z^8 + 225\,075\,200\,z^6 + 12\,041\,433\,600\,z^4 + 111\,798\,981\,000\,z^2 + 39\,463\,527\,915\right)\cosh(2\,z) + 9\left(33\,554\,432\,z^8 + 8\,419\,016\,704\,z^6 + 212\,868\,956\,160\,z^4 + 900\,514\,460\,160\,z^2 + 303\,767\,329\,005\right)\cosh(4\,z) + 753\,874\,479\,540\cosh(6\,z) + 136\,530\,107\,280\cosh(8\,z) + 17\,958\,999\,240\cosh(10\,z) + 1\,752\,007\,320\cosh(12\,z) + 124\,921\,440\cosh(14\,z) + 6\,202\,980\cosh(16\,z) + 192\,780\cosh(18\,z) + 2835\cosh(20\,z) - 6\,485\,526\,884\,340\Big)(m-1)^{10} + O\left((m-1)^{11}\right)$$

09.28.06.0016.01

$$\operatorname{dc}(z \mid m) \propto 1 + O(m-1)$$

q-series

09.28.06.0002.01

$$\operatorname{dc}(z \mid m) = \frac{\pi}{2\,K(m)}\sec\left(\frac{\pi\,z}{2\,K(m)}\right) + \frac{2\,\pi}{K(m)}\sum_{k=0}^{\infty}\frac{(-1)^k\,q(m)^{2k+1}}{1-q(m)^{2k+1}}\cos\left(\frac{(2\,k+1)\,\pi\,z}{2\,K(m)}\right)$$

Other series representations

09.28.06.0003.01

$$\operatorname{dc}(z \mid m) = \frac{\pi}{2\,K(1-m)}\sum_{k=-\infty}^{\infty}(-1)^k\coth\left(\pi\,\frac{K(m)}{K(1-m)}\left(k+\frac{1}{2}+\frac{z}{2\,K(m)}\right)\right)$$

09.28.06.0004.01

$$\mathrm{dc}(z \mid m) \propto \frac{(-1)^{r-1}}{z - 2\,s\,i\,K(1-m) - (2\,r+1)\,K(m)} + O(1) \;/;\; (z \to (2\,s+1)\,i\,K(1-m) + (2\,r+1)\,K(m)) \bigwedge \{r, s\} \in \mathbb{Z}$$

Product representations

09.28.08.0001.01

$$\mathrm{dc}(z \mid m) = \frac{\sqrt[4]{m}}{2\sqrt[4]{q(m)}}\,\sec\!\left(\frac{\pi z}{2\,K(m)}\right)\prod_{k=1}^{\infty}\frac{1 + 2\,q(m)^{2k-1}\cos\!\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k-2}}{1 + 2\,q(m)^{2k}\cos\!\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k}}$$

Differential equations

Ordinary nonlinear differential equations

09.28.13.0001.01

$$w''(z) - w(z)\left(2\,w(z)^2 - m - 1\right) = 0 \;/;\; w(z) = \mathrm{dc}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.28.16.0001.01

$$\mathrm{dc}(i\,z \mid m) = \mathrm{dn}(z \mid 1-m)$$

09.28.16.0002.01

$$\mathrm{dc}(z \mid 1-m) = \mathrm{dn}(i\,z \mid m)$$

09.28.16.0003.01

$$\mathrm{dc}(i\,z \mid 1-m) = \mathrm{dn}(z \mid m)$$

09.28.16.0007.01

$$\mathrm{dc}(x + i\,y \mid m) = (\mathrm{dn}(x \mid m)\,\mathrm{cn}(y \mid 1-m)\,\mathrm{dn}(y \mid 1-m) - i\,m\,\mathrm{sn}(x \mid m)\,\mathrm{cn}(x \mid m)\,\mathrm{sn}(y \mid 1-m)) \,/$$
$$(\mathrm{cn}(x \mid m)\,\mathrm{cn}(y \mid 1-m) - i\,\mathrm{sn}(x \mid m)\,\mathrm{dn}(x \mid m)\,\mathrm{sn}(y \mid 1-m)\,\mathrm{dn}(y \mid 1-m)) \;/;\; \{x, y\} \in \mathbb{R}$$

09.28.16.0008.01

$$\mathrm{dc}\!\left(\sqrt{1-m}\,z \,\middle|\, \frac{m}{m-1}\right) = \mathrm{nc}(z \mid m)$$

09.28.16.0009.01

$$\mathrm{dc}\!\left(\sqrt{m}\,z \,\middle|\, \frac{1}{m}\right) = \mathrm{cd}(z \mid m)$$

09.28.16.0010.01

$$\mathrm{dc}\!\left(i\sqrt{m}\,z \,\middle|\, \frac{m-1}{m}\right) = \mathrm{cn}(z \mid m)$$

09.28.16.0011.01

$$\mathrm{dc}\!\left(i\sqrt{1-m}\,z \,\middle|\, \frac{1}{1-m}\right) = \mathrm{nd}(z \mid m)$$

Landen's transformation:

09.28.16.0012.01

$$\operatorname{dc}\left(\left(1+\sqrt{1-m}\right)z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) = \frac{1-\left(1-\sqrt{1-m}\right)\operatorname{sn}(z\mid m)^2}{1-\left(1+\sqrt{1-m}\right)\operatorname{sn}(z\mid m)^2}$$

Gauss' transformation:

09.28.16.0013.01

$$\operatorname{dc}\left(\left(1+\sqrt{m}\right)z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) = \frac{1-\sqrt{m}\,\operatorname{sn}(z\mid m)^2}{\operatorname{cn}(z\mid m)\operatorname{dn}(z\mid m)}$$

n th degree transformations:

09.28.16.0014.01

$$\operatorname{dc}\left(\frac{z}{M} \,\middle|\, l\right) = \operatorname{dc}(z\mid m)\prod_{r=1}^{\frac{n-1}{2}}\frac{1-m\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2\operatorname{sn}(z\mid m)^2}{1-\operatorname{ns}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2\operatorname{sn}(z\mid m)^2} \;/;$$

$$\frac{n+1}{2}\in\mathbb{Z}^+ \bigwedge l = m^n\prod_{r=1}^{\frac{n-1}{2}}\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n-1}{2}}\frac{\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2}$$

09.28.16.0015.01

$$\operatorname{cd}\left(\frac{z}{M}+\frac{K(m)}{nM} \,\middle|\, l\right) = -M\,\frac{\operatorname{cn}(z\mid m)}{\operatorname{dn}(z\mid m)\operatorname{sn}(z\mid m)}\prod_{r=1}^{\frac{n}{2}}\frac{1-m\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2\operatorname{sn}(z\mid m)^2}{1-\operatorname{ns}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2\operatorname{sn}(z\mid m)^2} \;/;$$

$$\frac{n}{2}\in\mathbb{Z}^+ \bigwedge l = m^n\prod_{r=1}^{\frac{n}{2}}\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n}{2}}\frac{\operatorname{sn}\left(\frac{(2r-1)K(m)}{n}\,\middle|\, m\right)^2}{\operatorname{sn}\left(\frac{2rK(m)}{n}\,\middle|\, m\right)^2}$$

Argument involving half-periods

09.28.16.0004.01

$$\operatorname{dc}(z+K(m)\mid m) = -\operatorname{ns}(z\mid m)$$

09.28.16.0020.01

$$\operatorname{dc}(z-K(m)\mid m) = \operatorname{ns}(z\mid m)$$

09.28.16.0021.01

$$\operatorname{dc}(z+3K(m)\mid m) = \operatorname{ns}(z\mid m)$$

09.28.16.0022.01

$$\operatorname{dc}(z+(2r+1)K(m)\mid m) = (-1)^{r-1}\operatorname{ns}(z\mid m) \;/;\; r\in\mathbb{Z}$$

09.28.16.0005.01

$$\operatorname{dc}(z+iK(1-m)\mid m) = \sqrt{m}\,\operatorname{cd}(z\mid m)$$

09.28.16.0023.01

$$\operatorname{dc}(z-iK(1-m)\mid m) = \sqrt{m}\,\operatorname{cd}(z\mid m)$$

09.28.16.0024.01

$$\mathrm{dc}(z + 3\,i\,K(1-m) \mid m) = \sqrt{m}\;\mathrm{cd}(z \mid m)\;/;\; s \in \mathbb{Z}$$

09.28.16.0025.01

$$\mathrm{dc}(z + (2\,s+1)\,i\,K(1-m) \mid m) = \sqrt{m}\;\mathrm{cd}(z \mid m)\;/;\; s \in \mathbb{Z}$$

09.28.16.0006.01

$$\mathrm{dc}(z + i\,K(1-m) + K(m) \mid m) == -\sqrt{m}\;\mathrm{sn}(z \mid m)$$

09.28.16.0026.01

$$\mathrm{dc}(z - i\,K(1-m) + K(m) \mid m) = -\sqrt{m}\;\mathrm{sn}(z \mid m)$$

09.28.16.0027.01

$$\mathrm{dc}(z + i\,K(1-m) - K(m) \mid m) = \sqrt{m}\;\mathrm{sn}(z \mid m)$$

09.28.16.0028.01

$$\mathrm{dc}(z - i\,K(1-m) - K(m) \mid m) = \sqrt{m}\;\mathrm{sn}(z \mid m)$$

09.28.16.0029.01

$$\mathrm{dc}(z + i\,K(1-m) + 3\,K(m) \mid m) = \sqrt{m}\;\mathrm{sn}(z \mid m)$$

09.28.16.0030.01

$$\mathrm{dc}(z + (2\,s+1)\,i\,K(1-m) + (4\,r+1)\,K(m) \mid m) = -\sqrt{m}\;\mathrm{sn}(z \mid m)\;/;\; \{r, s\} \in \mathbb{Z}$$

09.28.16.0031.01

$$\mathrm{dc}(z + (2\,s+1)\,i\,K(1-m) + (4\,r-1)\,K(m) \mid m) = \sqrt{m}\;\mathrm{sn}(z \mid m)\;/;\; \{r, s\} \in \mathbb{Z}$$

09.28.16.0032.01

$$\mathrm{dc}(z + (2\,s+1)\,i\,K(1-m) + (2\,r+1)\,K(m) \mid m) = (-1)^{r-1}\sqrt{m}\;\mathrm{sn}(z \mid m)\;/;\; \{r, s\} \in \mathbb{Z}$$

Argument involving inverse Jacobi functions

09.28.16.0033.01

$$\mathrm{dc}\left(\mathrm{cd}^{-1}(z \mid m) \,\middle|\, m\right) = \frac{1}{z}$$

09.28.16.0034.01

$$\mathrm{dc}\left(\mathrm{cn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m\,z^2 - m + 1}{z^2}$$

09.28.16.0035.01

$$\mathrm{dc}\left(\mathrm{cs}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2 - m + 1}{z^2}$$

09.28.16.0036.01

$$\mathrm{dc}\left(\mathrm{dn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m\,z^2}{z^2 + m - 1}$$

09.28.16.0037.01

$$\mathrm{dc}\left(\mathrm{ds}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2}{z^2 + m - 1}$$

09.28.16.0038.01

$$\mathrm{dc}\left(\mathrm{nc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = m - (m-1)\,z^2$$

09.28.16.0039.01

$$\mathrm{dc}\left(\mathrm{nd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m}{1-(1-m)\,z^2}$$

09.28.16.0040.01

$$\mathrm{dc}\left(\mathrm{ns}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2-m}{z^2-1}$$

09.28.16.0041.01

$$\mathrm{dc}\left(\mathrm{sc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = 1-(m-1)\,z^2$$

09.28.16.0042.01

$$\mathrm{dc}\left(\mathrm{sd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{1}{1-(1-m)\,z^2}$$

09.28.16.0043.01

$$\mathrm{dc}\left(\mathrm{sn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m\,z^2-1}{z^2-1}$$

Addition formulas

09.28.16.0016.01

$$\mathrm{dc}(u+v \mid m) = \frac{\mathrm{dn}(u \mid m)\,\mathrm{dn}(v \mid m) - m\,\mathrm{sn}(u \mid m)\,\mathrm{cn}(u \mid m)\,\mathrm{sn}(v \mid m)\,\mathrm{cn}(v \mid m)}{\mathrm{cn}(u \mid m)\,\mathrm{cn}(v \mid m) - \mathrm{sn}(u \mid m)\,\mathrm{dn}(u \mid m)\,\mathrm{sn}(v \mid m)\,\mathrm{dn}(v \mid m)}$$

09.28.16.0017.01

$$\mathrm{dc}(u+v \mid m)\,\mathrm{dc}(u-v \mid m) = \frac{\mathrm{dn}(v \mid m)^2 - m\,\mathrm{cn}(v \mid m)^2\,\mathrm{sn}(u \mid m)^2}{\mathrm{cn}(v \mid m)^2 - \mathrm{dn}(v \mid m)^2\,\mathrm{sn}(u \mid m)^2}$$

Half-angle formulas

09.28.16.0018.01

$$\mathrm{dc}\left(\frac{z}{2} \,\middle|\, m\right)^2 = \frac{1-m+\mathrm{dn}(z \mid m)+m\,\mathrm{cn}(z \mid m)}{\mathrm{cn}(z \mid m)+\mathrm{dn}(z \mid m)}$$

Multiple arguments

Double angle formulas

09.28.16.0019.01

$$\mathrm{dc}(2\,z \mid m) = \frac{\mathrm{dn}(z \mid m)^2 - m\,\mathrm{sn}(z \mid m)^2\,\mathrm{cn}(z \mid m)^2}{\mathrm{cn}(z \mid m)^2 - \mathrm{sn}(z \mid m)^2\,\mathrm{dn}(z \mid m)^2}$$

Identities

Functional identities

09.28.17.0001.01

$$w(z)^4 - 2\,m\,w(z)^2 + m + \left(w(z)^4 - 2\,w(z)^2 + m\right) w(2\,z) = 0 \;/;\; w(z) = \mathrm{dc}(z \mid m)$$

Complex characteristics

Real part

09.28.19.0001.01

$$\operatorname{Re}(\operatorname{dc}(x+i\,y\mid m)) = \left(\operatorname{cn}(x\mid m)\operatorname{dn}(x\mid m)\operatorname{dn}(y\mid 1-m)\operatorname{cn}(y\mid 1-m)^2 + m\operatorname{cn}(x\mid m)\operatorname{dn}(x\mid m)\operatorname{dn}(y\mid 1-m)\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right) / \left(\operatorname{cn}(x\mid m)^2\operatorname{cn}(y\mid 1-m)^2 + \operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right) /;\ \{x, y, m\} \in \mathbb{R}$$

Imaginary part

09.28.19.0002.01

$$\operatorname{Im}(\operatorname{dc}(x+i\,y\mid m)) = \frac{\operatorname{cn}(y\mid 1-m)\left(\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2 - m\operatorname{cn}(x\mid m)^2\right)\operatorname{sn}(x\mid m)\operatorname{sn}(y\mid 1-m)}{\operatorname{cn}(x\mid m)^2\operatorname{cn}(y\mid 1-m)^2 + \operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2} /;\ \{x, y, m\} \in \mathbb{R}$$

Absolute value

09.28.19.0003.01

$$|\operatorname{dc}(x+i\,y\mid m)| = \sqrt{\frac{\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2 + m^2\operatorname{cn}(x\mid m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2}{\operatorname{cn}(x\mid m)^2\operatorname{cn}(y\mid 1-m)^2 + \operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2}} /;\ \{x, y, m\} \in \mathbb{R}$$

Argument

09.28.19.0004.01

$$\arg(\operatorname{dc}(x+i\,y\mid m)) = \tan^{-1}\!\left(\operatorname{cn}(x\mid m)\operatorname{dn}(x\mid m)\operatorname{dn}(y\mid 1-m)\left(\operatorname{cn}(y\mid 1-m)^2 + m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right),\ \operatorname{cn}(y\mid 1-m)\left(\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2 - m\operatorname{cn}(x\mid m)^2\right)\operatorname{sn}(x\mid m)\operatorname{sn}(y\mid 1-m)\right) /;\ \{x, y, m\} \in \mathbb{R}$$

Conjugate value

09.28.19.0005.01

$$\overline{\operatorname{dc}(x+i\,y\mid m)} = \frac{\operatorname{cn}(y\mid 1-m)\operatorname{dn}(x\mid m)\operatorname{dn}(y\mid 1-m) + i\,m\operatorname{cn}(x\mid m)\operatorname{sn}(x\mid m)\operatorname{sn}(y\mid 1-m)}{\operatorname{cn}(x\mid m)\operatorname{cn}(y\mid 1-m) + i\operatorname{dn}(x\mid m)\operatorname{dn}(y\mid 1-m)\operatorname{sn}(x\mid m)\operatorname{sn}(y\mid 1-m)} /;\ \{x, y, m\} \in \mathbb{R}$$

Differentiation

Low-order differentiation

With respect to *z*

09.28.20.0001.01

$$\frac{\partial\operatorname{dc}(z\mid m)}{\partial z} = (1-m)\operatorname{nc}(z\mid m)\operatorname{sc}(z\mid m)$$

09.28.20.0002.01

$$\frac{\partial^2\operatorname{dc}(z\mid m)}{\partial z^2} = (1-m)\operatorname{dc}(z\mid m)\left(\operatorname{nc}(z\mid m)^2 + \operatorname{sc}(z\mid m)^2\right)$$

With respect to *m*

09.28.20.0003.01

$$\frac{\partial\operatorname{dc}(z\mid m)}{\partial m} = \frac{\operatorname{sc}(z\mid m)\operatorname{nc}(z\mid m)\left((1-m)\,z - E(\operatorname{am}(z\mid m)\mid m)\right)}{2\,m}$$

09.28.20.0004.01

$$\frac{\partial^2 \mathrm{dc}(z \mid m)}{\partial m^2} = -\frac{1}{4(m-1)m^2} \left(((m-1)z + E(\mathrm{am}(z \mid m) \mid m))\, \mathrm{dc}(z \mid m) \left(\mathrm{nc}(z \mid m)^2 + \mathrm{sc}(z \mid m)^2\right) ((m-1)z + E(\mathrm{am}(z \mid m) \mid m) - m\, \mathrm{cd}(z \mid m)\, \mathrm{sn}(z \mid m)) + \mathrm{nc}(z \mid m)\, \mathrm{sc}(z \mid m) \left((m-1)(2z - E(\mathrm{am}(z \mid m) \mid m) - F(\mathrm{am}(z \mid m) \mid m)) + ((m-1)z + E(\mathrm{am}(z \mid m) \mid m))\, \mathrm{dn}(z \mid m) \sqrt{1 - m\, \mathrm{sn}(z \mid m)^2} - m\, \mathrm{cn}(z \mid m)\, \mathrm{sn}(z \mid m) \sqrt{1 - m\, \mathrm{sn}(z \mid m)^2}\right)\right)$$

Symbolic differentiation

With respect to *z*

09.28.20.0007.01

$$\frac{\partial^n \mathrm{dc}(z \mid m)}{\partial z^n} = \mathrm{dc}(z \mid m)\, \delta_n + (1-m) \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{\partial^j \mathrm{sc}(z \mid m)}{\partial z^j} \frac{\partial^{-j+n-1} \mathrm{nc}(z \mid m)}{\partial z^{-j+n-1}} \;/;\; n \in \mathbb{N}$$

09.28.20.0005.01

$$\frac{\partial^n \mathrm{dc}(z \mid m)}{\partial z^n} = \frac{2^{1-n} \pi^{n+1}}{K(m)^{n+1}} \sum_{k=0}^{\infty} \frac{(-1)^k (2k+1)^n q(m)^{2k+1}}{1 - q(m)^{2k+1}} \cos\left(\frac{\pi n}{2} + \frac{(2k+1)\pi z}{2K(m)}\right) + \frac{\pi}{2 z^n K(m)} \sum_{k=0}^{\infty} \frac{(-1)^k E_{2k}}{(2k-n)!} \left(\frac{\pi z}{2K(m)}\right)^{2k} \;/;\; n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

09.28.20.0006.01

$$\frac{\partial^\alpha \mathrm{dc}(z \mid m)}{\partial z^\alpha} = \frac{\pi}{2K(m)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k} z^{2k-\alpha} K(m)^{-2k} E_{2k}}{\Gamma(2k-\alpha+1)} + \frac{2^{\alpha+1} \pi^{3/2} z^{-\alpha}}{K(m)} \sum_{k=0}^{\infty} \frac{(-1)^k q(m)^{2k+1}}{1 - q(m)^{2k+1}}\, {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1 - \frac{\alpha}{2}; -\frac{(2k+1)^2 \pi^2 z^2}{16 K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.28.21.0001.01

$$\int \mathrm{dc}(z \mid m)\, dz = \log(\mathrm{nc}(z \mid m) + \mathrm{sc}(z \mid m))$$

Representations through equivalent functions

With inverse function

09.28.27.0001.01

$$\mathrm{dc}\left(\mathrm{dc}^{-1}(z \mid m) \mid m\right) = z$$

With related functions

Involving am

09.28.27.0027.01

$$\operatorname{dc}(z \mid m)^2 = \frac{1 - m \sin^2(\operatorname{am}(z \mid m))}{\cos^2(\operatorname{am}(z \mid m))}$$

Involving one other Jacobi elliptic function

Involving cd

09.28.27.0002.01

$$\operatorname{dc}(z \mid m) == \frac{1}{\operatorname{cd}(z \mid m)}$$

Involving cn

09.28.27.0005.01

$$\operatorname{dc}(z \mid m)^2 == \frac{m \operatorname{cn}(z \mid m)^2 - m + 1}{\operatorname{cn}(z \mid m)^2}$$

Involving cs

09.28.27.0008.01

$$\operatorname{dc}(z \mid m)^2 == \frac{\operatorname{cs}(z \mid m)^2 - m + 1}{\operatorname{cs}(z \mid m)^2}$$

Involving dn

09.28.27.0010.01

$$\operatorname{dc}(z \mid m) == \operatorname{dn}(i\, z \mid 1 - m)$$

09.28.27.0011.01

$$\operatorname{dc}(z \mid m)^2 == \frac{m \operatorname{dn}(z \mid m)^2}{\operatorname{dn}(z \mid m)^2 + m - 1}$$

Involving ds

09.28.27.0013.01

$$\operatorname{dc}(z \mid m)^2 == \frac{\operatorname{ds}(z \mid m)^2}{\operatorname{ds}(z \mid m)^2 + m - 1}$$

Involving nc

09.28.27.0015.01

$$\operatorname{dc}(z \mid m)^2 == m - (m - 1) \operatorname{nc}(z \mid m)^2$$

Involving nd

09.28.27.0016.01

$$\mathrm{dc}(z \mid m) == \frac{1}{\mathrm{nd}(i\, z \mid 1-m)}$$

09.28.27.0017.01

$$\mathrm{dc}(z \mid m)^2 == \frac{m}{1-(1-m)\,\mathrm{nd}(z \mid m)^2}$$

Involving ns

09.28.27.0018.01

$$\mathrm{dc}(z \mid m)^2 == \frac{\mathrm{ns}(z \mid m)^2 - m}{\mathrm{ns}(z \mid m)^2 - 1}$$

Involving sc

09.28.27.0020.01

$$\mathrm{dc}(z \mid m)^2 == 1-(m-1)\,\mathrm{sc}(z \mid m)^2$$

Involving sd

09.28.27.0021.01

$$\mathrm{dc}(z \mid m)^2 == \frac{1}{1-(1-m)\,\mathrm{sd}(z \mid m)^2}$$

Involving sn

09.28.27.0022.01

$$\mathrm{dc}(z \mid m)^2 == \frac{m\,\mathrm{sn}(z \mid m)^2 - 1}{\mathrm{sn}(z \mid m)^2 - 1}$$

Involving two other Jacobi elliptic functions

Involving cn and dn

09.28.27.0003.01

$$\mathrm{dc}(z \mid m) == \frac{\mathrm{dn}(z \mid m)}{\mathrm{cn}(z \mid m)}$$

09.28.27.0028.01

$$\mathrm{dc}(z \mid m) = \frac{m\,\mathrm{cn}(z \mid m)\,\mathrm{dn}(z \mid m)}{\mathrm{dn}(z \mid m)^2 + m - 1}$$

Involving cn and nd

09.28.27.0004.01

$$\operatorname{dc}(z \mid m) = \frac{1}{\operatorname{nd}(z \mid m) \operatorname{cn}(z \mid m)}$$

Involving cd and nd

09.28.27.0029.01

$$\operatorname{dc}(z \mid m) = \frac{m \operatorname{cd}(z \mid m)}{m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1}$$

Involving cs and ds

09.28.27.0006.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{ds}(z \mid m)}{\operatorname{cs}(z \mid m)}$$

Involving cs and sd

09.28.27.0007.01

$$\operatorname{dc}(z \mid m) = \frac{1}{\operatorname{sd}(z \mid m) \operatorname{cs}(z \mid m)}$$

Involving dn and nc

09.28.27.0009.01

$$\operatorname{dc}(z \mid m) = \operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)$$

Involving ds and sc

09.28.27.0012.01

$$\operatorname{dc}(z \mid m) = \operatorname{ds}(z \mid m) \operatorname{sc}(z \mid m)$$

Involving nc and nd

09.28.27.0014.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{nc}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

Involving sc and sd

09.28.27.0019.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{sc}(z \mid m)}{\operatorname{sd}(z \mid m)}$$

Involving three other Jacobi elliptic functions

09.28.27.0030.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\operatorname{cs}(z \mid m)^2}$$

09.28.27.0031.01

$$\operatorname{dc}(z \mid m) = -\frac{(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1)\operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{cn}(z \mid m)^2}$$

09.28.27.0032.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1)\operatorname{ds}(z \mid m)^2}{\operatorname{cn}(z \mid m)^2}$$

09.28.27.0033.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)^2\operatorname{ds}(z \mid m)}{(\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m))(\operatorname{dn}(z \mid m) + \operatorname{ds}(z \mid m))}$$

09.28.27.0034.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m)^2\operatorname{ds}(z \mid m)^2}{(\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m))(\operatorname{dn}(z \mid m) + \operatorname{ds}(z \mid m))}$$

09.28.27.0035.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m)^2}{\operatorname{dn}(z \mid m)\left(\operatorname{ds}(z \mid m)^2 + m - 1\right)}$$

09.28.27.0036.01

$$\operatorname{dc}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{dn}(z \mid m)}{\operatorname{cs}(z \mid m)^2\operatorname{nc}(z \mid m)}$$

09.28.27.0037.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\left(\operatorname{ds}(z \mid m)^2 + m\right)}{\left(\operatorname{ds}(z \mid m)^2 + m - 1\right)\operatorname{nc}(z \mid m)}$$

09.28.27.0038.01

$$\operatorname{dc}(z \mid m) = \operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m)(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)$$

09.28.27.0039.01

$$\operatorname{dc}(z \mid m) = \operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)^2(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)$$

09.28.27.0040.01

$$\operatorname{dc}(z \mid m) = \frac{m\operatorname{cn}(z \mid m) - m\operatorname{nc}(z \mid m) + \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)}$$

09.28.27.0041.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m)^2\operatorname{nd}(z \mid m)^2}$$

09.28.27.0042.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m)^2\operatorname{nd}(z \mid m)}$$

09.28.27.0043.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{ds}(z \mid m)^2(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)\operatorname{nd}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.28.27.0044.01

$$\operatorname{dc}(z \mid m) = \frac{m \operatorname{cn}(z \mid m)}{\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.28.27.0045.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2}{(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) - 1)(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) + 1)}$$

09.28.27.0046.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m)}{(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) - 1)(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) + 1)}$$

09.28.27.0047.01

$$\operatorname{dc}(z \mid m) = m \operatorname{cd}(z \mid m) - m \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m) + \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)$$

09.28.27.0048.01

$$\operatorname{dc}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\operatorname{cs}(z \mid m) \operatorname{ns}(z \mid m)}$$

09.28.27.0049.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{ds}(z \mid m)(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1) \operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.28.27.0050.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2}{(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)}$$

09.28.27.0051.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ds}(z \mid m) \operatorname{ns}(z \mid m)}{(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)}$$

09.28.27.0052.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)^2}{(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)}$$

09.28.27.0053.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)^2}{\operatorname{nc}(z \mid m)(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)}$$

09.28.27.0054.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ns}(z \mid m)^2}{\operatorname{nd}(z \mid m)^2 (\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)}$$

09.28.27.0055.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m)^2}{\operatorname{nd}(z \mid m)(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)}$$

09.28.27.0056.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)}{(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1) \operatorname{sc}(z \mid m)}$$

09.28.27.0057.01

$$\operatorname{dc}(z \mid m) = \operatorname{cn}(z \mid m) \operatorname{dn}(z \mid m) \left(\operatorname{sc}(z \mid m)^2 + 1\right)$$

09.28.27.0058.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{nc}(z \mid m)}$$

09.28.27.0059.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{nd}(z \mid m)^2}$$

09.28.27.0060.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{nd}(z \mid m)}$$

09.28.27.0061.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{dn}(z \mid m)\left(\mathrm{sc}(z \mid m)^2 + 1\right)}{\mathrm{ns}(z \mid m)\,\mathrm{sc}(z \mid m)}$$

09.28.27.0062.01

$$\mathrm{dc}(z \mid m) = -\frac{\mathrm{cd}(z \mid m)\,(\mathrm{cn}(z \mid m) - 1)\,(\mathrm{cn}(z \mid m) + 1)}{\mathrm{cn}(z \mid m)^2\,\mathrm{sd}(z \mid m)^2}$$

09.28.27.0063.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cd}(z \mid m)\,(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)}{\mathrm{sd}(z \mid m)^2}$$

09.28.27.0064.01

$$\mathrm{dc}(z \mid m) = \frac{(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)\,\mathrm{nd}(z \mid m)}{\mathrm{nc}(z \mid m)\,\mathrm{sd}(z \mid m)^2}$$

09.28.27.0065.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cd}(z \mid m)}{(\mathrm{ns}(z \mid m) - 1)\,(\mathrm{ns}(z \mid m) + 1)\,\mathrm{sd}(z \mid m)^2}$$

09.28.27.0066.01

$$\mathrm{dc}(z \mid m) = -\frac{(\mathrm{cn}(z \mid m) - 1)\,(\mathrm{cn}(z \mid m) + 1)\,\mathrm{cs}(z \mid m)}{\mathrm{cn}(z \mid m)^2\,\mathrm{sd}(z \mid m)}$$

09.28.27.0067.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)}{\mathrm{sd}(z \mid m)}$$

09.28.27.0068.01

$$\mathrm{dc}(z \mid m) = \frac{(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)\,\mathrm{ns}(z \mid m)}{\mathrm{nc}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.28.27.0069.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,\mathrm{ns}(z \mid m)}{(\mathrm{ns}(z \mid m) - 1)\,(\mathrm{ns}(z \mid m) + 1)\,\mathrm{sd}(z \mid m)}$$

09.28.27.0070.01

$$\mathrm{dc}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{sd}(z \mid m)}{\mathrm{cs}(z \mid m)\,\mathrm{nd}(z \mid m)^2}$$

09.28.27.0071.01

$$\operatorname{dc}(z \mid m) = \frac{m \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m)}{(\operatorname{nd}(z \mid m) - 1)(\operatorname{nd}(z \mid m) + 1)}$$

09.28.27.0072.01

$$\operatorname{dc}(z \mid m) = \frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right) \operatorname{sd}(z \mid m)}{\operatorname{nd}(z \mid m)^2 \operatorname{sc}(z \mid m)}$$

09.28.27.0073.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)}{(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m))(\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}$$

09.28.27.0074.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{nd}(z \mid m)}{(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m))(\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}$$

09.28.27.0075.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{sd}(z \mid m)}{(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m))(\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}$$

09.28.27.0076.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) \operatorname{dn}(z \mid m)^2}{(\operatorname{dn}(z \mid m) \operatorname{sd}(z \mid m) - 1)(\operatorname{dn}(z \mid m) \operatorname{sd}(z \mid m) + 1)}$$

09.28.27.0077.01

$$\operatorname{dc}(z \mid m) = \operatorname{cd}(z \mid m) - m \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sd}(z \mid m)$$

09.28.27.0078.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{dn}(z \mid m)\left(m \operatorname{sd}(z \mid m)^2 - \operatorname{sd}(z \mid m)^2 + 1\right)}$$

09.28.27.0079.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ds}(z \mid m)}{\left(\operatorname{ds}(z \mid m)^2 + m - 1\right) \operatorname{sn}(z \mid m)}$$

09.28.27.0080.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{ds}(z \mid m) \operatorname{nc}(z \mid m)}{\left(\operatorname{ds}(z \mid m)^2 + m\right) \operatorname{sn}(z \mid m)}$$

09.28.27.0081.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{ds}(z \mid m)(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)}{\operatorname{nc}(z \mid m) \operatorname{sn}(z \mid m)}$$

09.28.27.0082.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{sd}(z \mid m)}{\left(m \operatorname{sd}(z \mid m)^2 - \operatorname{sd}(z \mid m)^2 + 1\right) \operatorname{sn}(z \mid m)}$$

09.28.27.0083.01

$$\operatorname{dc}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)}$$

09.28.27.0084.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{ds}(z \mid m)\left(\operatorname{ds}(z \mid m)^2 + m\right)\operatorname{sn}(z \mid m)}{\left(\operatorname{ds}(z \mid m)^2 + m - 1\right)\operatorname{nc}(z \mid m)}$$

09.28.27.0085.01

$$\operatorname{dc}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.28.27.0086.01

$$\operatorname{dc}(z \mid m) = \frac{m\operatorname{dn}(z \mid m)\operatorname{sn}(z \mid m)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)\operatorname{sc}(z \mid m)}$$

09.28.27.0087.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

09.28.27.0088.01

$$\operatorname{dc}(z \mid m) = \frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.28.27.0089.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)}{(\operatorname{sn}(z \mid m) - 1)(\operatorname{sn}(z \mid m) + 1)}$$

09.28.27.0090.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)}{\operatorname{nc}(z \mid m)(\operatorname{sn}(z \mid m) - 1)(\operatorname{sn}(z \mid m) + 1)}$$

09.28.27.0091.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)}{\operatorname{nd}(z \mid m)(\operatorname{sn}(z \mid m) - 1)(\operatorname{sn}(z \mid m) + 1)}$$

09.28.27.0092.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)\operatorname{sn}(z \mid m)}{(\operatorname{sn}(z \mid m) - 1)(\operatorname{sn}(z \mid m) + 1)}$$

09.28.27.0093.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{sd}(z \mid m)(\operatorname{sn}(z \mid m) - 1)(\operatorname{sn}(z \mid m) + 1)}$$

09.28.27.0094.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m)\operatorname{sn}(z \mid m)^2}{(\operatorname{sn}(z \mid m) - 1)(\operatorname{sn}(z \mid m) + 1)}$$

09.28.27.0095.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)^2\operatorname{sn}(z \mid m)^2}{(\operatorname{sn}(z \mid m) - 1)(\operatorname{sn}(z \mid m) + 1)}$$

09.28.27.0096.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{sn}(z \mid m)^2}{\operatorname{sd}(z \mid m)^2(\operatorname{sn}(z \mid m) - 1)(\operatorname{sn}(z \mid m) + 1)}$$

09.28.27.0097.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{sn}(z \mid m)^2}{\operatorname{sd}(z \mid m)(\operatorname{sn}(z \mid m) - 1)(\operatorname{sn}(z \mid m) + 1)}$$

Involving four other Jacobi elliptic functions

09.28.27.0098.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m)^2 (\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{cn}(z \mid m)}$$

09.28.27.0099.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{ds}(z \mid m)^2 (\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{dn}(z \mid m)}$$

09.28.27.0100.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{ds}(z \mid m)^2 (\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{dn}(z \mid m)}$$

09.28.27.0101.01

$$\operatorname{dc}(z \mid m) = -\operatorname{ds}(z \mid m)^2 (\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\operatorname{nd}(z \mid m)$$

09.28.27.0102.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{ds}(z \mid m)(\operatorname{cn}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m))}{\operatorname{cn}(z \mid m)}$$

09.28.27.0103.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m)}$$

09.28.27.0104.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m)^2}{\operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.28.27.0105.01

$$\operatorname{dc}(z \mid m) = -\operatorname{ds}(z \mid m)^2 (\operatorname{cd}(z \mid m) - \operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m))$$

09.28.27.0106.01

$$\operatorname{dc}(z \mid m) = \operatorname{ds}(z \mid m)(\operatorname{ds}(z \mid m)\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cs}(z \mid m))$$

09.28.27.0107.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{ds}(z \mid m)(\operatorname{cn}(z \mid m)\operatorname{cs}(z \mid m) - \operatorname{ns}(z \mid m))}{\operatorname{cn}(z \mid m)}$$

09.28.27.0108.01

$$\operatorname{dc}(z \mid m) = -\operatorname{ds}(z \mid m)(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\operatorname{ns}(z \mid m)$$

09.28.27.0109.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m)\operatorname{ns}(z \mid m)}$$

09.28.27.0110.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m)^2}{\operatorname{ds}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.28.27.0111.01

$$\operatorname{dc}(z \mid m) = -\operatorname{ds}(z \mid m)\,(\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m) - \operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m))$$

09.28.27.0112.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sc}(z \mid m)}$$

09.28.27.0113.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m)}$$

09.28.27.0114.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m)\operatorname{nc}(z \mid m)}$$

09.28.27.0115.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m)\operatorname{nd}(z \mid m)^2}$$

09.28.27.0116.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cs}(z \mid m)\operatorname{nd}(z \mid m)}$$

09.28.27.0117.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ns}(z \mid m)}$$

09.28.27.0118.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2 - \operatorname{sd}(z \mid m)}$$

09.28.27.0119.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sd}(z \mid m)}$$

09.28.27.0120.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\,(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{cn}(z \mid m)\operatorname{sd}(z \mid m)^2}$$

09.28.27.0121.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)^2}$$

09.28.27.0122.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)^2}$$

09.28.27.0123.01

$$\operatorname{dc}(z \mid m) = -\frac{(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))\operatorname{nd}(z \mid m)}{\operatorname{sd}(z \mid m)^2}$$

09.28.27.0124.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cd}(z \mid m) - \operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{sd}(z \mid m)^2}$$

09.28.27.0125.01

$$\mathrm{dc}(z \mid m) = -\frac{\mathrm{cn}(z \mid m)\,\mathrm{cs}(z \mid m) - \mathrm{ns}(z \mid m)}{\mathrm{cn}(z \mid m)\,\mathrm{sd}(z \mid m)}$$

09.28.27.0126.01

$$\mathrm{dc}(z \mid m) = -\frac{(\mathrm{cn}(z \mid m) - \mathrm{nc}(z \mid m))\,\mathrm{ns}(z \mid m)}{\mathrm{sd}(z \mid m)}$$

09.28.27.0127.01

$$\mathrm{dc}(z \mid m) = \frac{(\mathrm{cs}(z \mid m) + \mathrm{sc}(z \mid m))\,\mathrm{sd}(z \mid m)}{\mathrm{nd}(z \mid m)^2}$$

09.28.27.0128.01

$$\mathrm{dc}(z \mid m) = -\frac{m\,\mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m)\,\mathrm{sc}(z \mid m) - \mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m)\,\mathrm{sc}(z \mid m) - m\,\mathrm{sd}(z \mid m)}{\mathrm{sc}(z \mid m)}$$

09.28.27.0129.01

$$\mathrm{dc}(z \mid m) = -\frac{\mathrm{cn}(z \mid m)\,\mathrm{cs}(z \mid m)}{\mathrm{cn}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{cs}(z \mid m)\,\mathrm{nd}(z \mid m)}$$

09.28.27.0130.01

$$\mathrm{dc}(z \mid m) = -\frac{\mathrm{cs}(z \mid m)\,\mathrm{dn}(z \mid m)}{\mathrm{dn}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{ns}(z \mid m)}$$

09.28.27.0131.01

$$\mathrm{dc}(z \mid m) = -\frac{\mathrm{cn}(z \mid m)}{\mathrm{sd}(z \mid m)\,(\mathrm{dn}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{ns}(z \mid m))}$$

09.28.27.0132.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{nc}(z \mid m)\,\mathrm{ns}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{cd}(z \mid m)}{\mathrm{sd}(z \mid m)^2}$$

09.28.27.0133.01

$$\mathrm{dc}(z \mid m) = \mathrm{dn}(z \mid m)^2\,(\mathrm{cd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m))$$

09.28.27.0134.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{nd}(z \mid m)^2}$$

09.28.27.0135.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)\,\mathrm{nd}(z \mid m) + \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{nd}(z \mid m)^2}$$

09.28.27.0136.01

$$\mathrm{dc}(z \mid m) = -\frac{-\mathrm{cn}(z \mid m) + m\,\mathrm{dn}(z \mid m)\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{dn}(z \mid m)\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)}{\mathrm{dn}(z \mid m)}$$

09.28.27.0137.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cn}(z \mid m)}{\mathrm{nd}(z \mid m) - \mathrm{dn}(z \mid m)\,\mathrm{sd}(z \mid m)^2}$$

09.28.27.0138.01

$$\mathrm{dc}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{dn}(z \mid m)}{\mathrm{cs}(z \mid m)\,\mathrm{nc}(z \mid m) - \mathrm{sn}(z \mid m)}$$

09.28.27.0139.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.28.27.0140.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.28.27.0141.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)}{\operatorname{nd}(z \mid m)\left(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)\right)}$$

09.28.27.0142.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)\left(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)\right)}$$

09.28.27.0143.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{sd}(z \mid m)\left(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)\right)}$$

09.28.27.0144.01

$$\operatorname{dc}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{sd}(z \mid m)\operatorname{sn}(z \mid m)}$$

09.28.27.0145.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\left(\operatorname{ds}(z \mid m) + m\operatorname{sd}(z \mid m) - \operatorname{sd}(z \mid m)\right)\operatorname{sn}(z \mid m)}$$

09.28.27.0146.01

$$\operatorname{dc}(z \mid m) = \frac{m\operatorname{sn}(z \mid m)}{\left(\operatorname{dn}(z \mid m) + m\operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)\right)\operatorname{sc}(z \mid m)}$$

09.28.27.0147.01

$$\operatorname{dc}(z \mid m) = \operatorname{dn}(z \mid m)\left(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)\right)\operatorname{sn}(z \mid m)$$

09.28.27.0148.01

$$\operatorname{dc}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)\right)\operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

09.28.27.0149.01

$$\operatorname{dc}(z \mid m) = \frac{\left(\operatorname{ds}(z \mid m)^2 + m\right)\operatorname{sn}(z \mid m)}{\operatorname{nc}(z \mid m)\left(\operatorname{ds}(z \mid m) + m\operatorname{sd}(z \mid m) - \operatorname{sd}(z \mid m)\right)}$$

09.28.27.0150.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.28.27.0151.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.28.27.0152.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{sd}(z \mid m)\left(\operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)\right)}$$

09.28.27.0153.01

$$\mathrm{dc}(z\mid m)=\frac{\mathrm{dn}(z\mid m)\,(\mathrm{cd}(z\mid m)\,\mathrm{cs}(z\mid m)\,\mathrm{dn}(z\mid m)+\mathrm{sn}(z\mid m))}{\mathrm{cs}(z\mid m)}$$

09.28.27.0154.01

$$\mathrm{dc}(z\mid m)=-\frac{m\,\mathrm{nc}(z\mid m)\,\mathrm{sc}(z\mid m)-\mathrm{nc}(z\mid m)\,\mathrm{sc}(z\mid m)-m\,\mathrm{sn}(z\mid m)}{\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m)}$$

09.28.27.0155.01

$$\mathrm{dc}(z\mid m)=\frac{\mathrm{ds}(z\mid m)\,\mathrm{nc}(z\mid m)}{\mathrm{dn}(z\mid m)\,\mathrm{ds}(z\mid m)+m\,\mathrm{sn}(z\mid m)}$$

09.28.27.0156.01

$$\mathrm{dc}(z\mid m)=\frac{\mathrm{cn}(z\mid m)\,\mathrm{ds}(z\mid m)}{\mathrm{dn}(z\mid m)\,\mathrm{ds}(z\mid m)+m\,\mathrm{sn}(z\mid m)-\mathrm{sn}(z\mid m)}$$

09.28.27.0157.01

$$\mathrm{dc}(z\mid m)=\frac{\mathrm{nc}(z\mid m)\,\mathrm{sd}(z\mid m)-\mathrm{cd}(z\mid m)\,\mathrm{sn}(z\mid m)}{\mathrm{sd}(z\mid m)^2\,\mathrm{sn}(z\mid m)}$$

09.28.27.0158.01

$$\mathrm{dc}(z\mid m)=-\frac{\mathrm{cs}(z\mid m)\,\mathrm{dn}(z\mid m)^2}{\mathrm{dn}(z\mid m)\,\mathrm{sn}(z\mid m)-\mathrm{ds}(z\mid m)}$$

09.28.27.0159.01

$$\mathrm{dc}(z\mid m)=-\frac{\mathrm{ds}(z\mid m)\,(\mathrm{cd}(z\mid m)\,\mathrm{ds}(z\mid m)\,\mathrm{sn}(z\mid m)-\mathrm{nc}(z\mid m))}{\mathrm{sn}(z\mid m)}$$

09.28.27.0160.01

$$\mathrm{dc}(z\mid m)=\frac{\mathrm{cd}(z\mid m)\,\mathrm{cs}(z\mid m)+\mathrm{nd}(z\mid m)\,\mathrm{sn}(z\mid m)}{\mathrm{cs}(z\mid m)\,\mathrm{nd}(z\mid m)^2}$$

09.28.27.0161.01

$$\mathrm{dc}(z\mid m)=\frac{\mathrm{cn}(z\mid m)+\mathrm{sc}(z\mid m)\,\mathrm{sn}(z\mid m)}{\mathrm{nd}(z\mid m)}$$

09.28.27.0162.01

$$\mathrm{dc}(z\mid m)=\frac{\mathrm{cd}(z\mid m)\,(\mathrm{cn}(z\mid m)-m\,\mathrm{sc}(z\mid m)\,\mathrm{sn}(z\mid m)+\mathrm{sc}(z\mid m)\,\mathrm{sn}(z\mid m))}{\mathrm{cn}(z\mid m)}$$

09.28.27.0163.01

$$\mathrm{dc}(z\mid m)=\frac{\mathrm{cn}(z\mid m)-m\,\mathrm{sc}(z\mid m)\,\mathrm{sn}(z\mid m)+\mathrm{sc}(z\mid m)\,\mathrm{sn}(z\mid m)}{\mathrm{dn}(z\mid m)}$$

09.28.27.0164.01

$$\mathrm{dc}(z\mid m)=-\frac{\mathrm{sd}(z\mid m)\,(-\mathrm{cn}(z\mid m)+m\,\mathrm{sc}(z\mid m)\,\mathrm{sn}(z\mid m)-\mathrm{sc}(z\mid m)\,\mathrm{sn}(z\mid m))}{\mathrm{sn}(z\mid m)}$$

09.28.27.0165.01

$$\mathrm{dc}(z\mid m)=\frac{\mathrm{cn}(z\mid m)}{\mathrm{nd}(z\mid m)-\mathrm{sd}(z\mid m)\,\mathrm{sn}(z\mid m)}$$

09.28.27.0166.01

$$\mathrm{dc}(z\mid m)=-\frac{\mathrm{cs}(z\mid m)\,\mathrm{sn}(z\mid m)}{\mathrm{sd}(z\mid m)\,\mathrm{sn}(z\mid m)-\mathrm{nd}(z\mid m)}$$

09.28.27.0167.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cn}(z \mid m) - m\operatorname{sd}(z \mid m)\operatorname{sn}(z \mid m) + \operatorname{sd}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{cn}(z \mid m)}$$

Involving five other Jacobi elliptic functions

09.28.27.0168.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

09.28.27.0169.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{sn}(z \mid m)}$$

09.28.27.0170.01

$$\operatorname{dc}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{cs}(z \mid m)\operatorname{dn}(z \mid m) + \operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)\operatorname{nd}(z \mid m)}$$

Involving Weierstrass functions

09.28.27.0023.01

$$\operatorname{dc}(z \mid m) = \frac{\sigma_2\left(\frac{z}{\sqrt{e_1 - e_3}}; g_2, g_3\right)}{\sigma_1\left(\frac{z}{\sqrt{e_1 - e_3}}; g_2, g_3\right)} /;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

09.28.27.0024.01

$$\operatorname{dc}(z \mid m)^2 = \frac{\wp\left(\frac{z}{\sqrt{e_1 - e_3}}; g_2, g_3\right) - e_2}{\wp\left(\frac{z}{\sqrt{e_1 - e_3}}; g_2, g_3\right) - e_1} /;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

Involving theta functions

09.28.27.0025.01

$$\operatorname{dc}(z \mid m) = \sqrt[4]{m}\,\frac{\vartheta_3\left(\frac{\pi z}{2 K(m)}, q(m)\right)}{\vartheta_2\left(\frac{\pi z}{2 K(m)}, q(m)\right)}$$

09.28.27.0026.01

$$\operatorname{dc}(z \mid m) = \frac{\vartheta_d(z \mid m)}{\vartheta_c(z \mid m)}$$

Zeros

09.28.30.0001.01

$$\operatorname{dc}((2 r + 1) K(m) + (2 s + 1) i K(1 - m) \mid m) = 0 /; \{r, s\} \in \mathbb{Z}$$

History

– C. G. J. Jacobi (1827)
– N. H. Abel (1827)
– J. Glaisher (1882) introduced the notation dc

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