JacobiCD

Notations

Traditional name

Jacobi elliptic function cd

Traditional notation

$\operatorname{cd}(z \mid m)$

Mathematica StandardForm notation

`JacobiCD[z, m]`

Primary definition

09.25.02.0001.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{dn}(z \mid m)}$$

Specific values

Specialized values

For fixed *z*

Case $m = 0$

09.25.03.0001.01

$$\operatorname{cd}(z \mid 0) = \cos(z)$$

09.25.03.0002.01

$$\operatorname{cd}\left(z + \frac{\pi}{2} \,\middle|\, 0\right) = -\sin(z)$$

09.25.03.0025.01

$$\operatorname{cd}\left(z + \frac{\pi k}{2} \,\middle|\, 0\right) = \cos\left(z + \frac{\pi k}{2}\right) /;\ k \in \mathbb{Z}$$

Case $m = 1$

09.25.03.0003.01

$$\operatorname{cd}(z \mid 1) = 1$$

For fixed *m*

Values at quarter-period points in the fundamental period parallelogram

09.25.03.0004.01

$$\mathrm{cd}(0 \mid m) == 1$$

09.25.03.0005.01

$$\mathrm{cd}(K(m) \mid m) == 0$$

09.25.03.0006.01

$$\mathrm{cd}(2\,K(m) \mid m) == -1$$

09.25.03.0007.01

$$\mathrm{cd}(3\,K(m) \mid m) == 0$$

09.25.03.0008.01

$$\mathrm{cd}(4\,K(m) \mid m) == 1$$

09.25.03.0009.01

$$\mathrm{cd}(i\,K(1-m) \mid m) == \frac{1}{\sqrt{m}}$$

09.25.03.0010.01

$$\mathrm{cd}(2\,i\,K(1-m) \mid m) == 1$$

09.25.03.0011.01

$$\mathrm{cd}(3\,i\,K(1-m) \mid m) == \frac{1}{\sqrt{m}}$$

09.25.03.0012.01

$$\mathrm{cd}(4\,i\,K(1-m) \mid m) == 1$$

09.25.03.0013.01

$$\mathrm{cd}(K(m) + i\,K(1-m) \mid m) == \tilde{\infty}$$

09.25.03.0014.01

$$\mathrm{cd}(2\,K(m) + i\,K(1-m) \mid m) == -\frac{1}{\sqrt{m}}$$

09.25.03.0015.01

$$\mathrm{cd}(3\,K(m) + i\,K(1-m) \mid m) == \tilde{\infty}$$

09.25.03.0016.01

$$\mathrm{cd}(4\,K(m) + i\,K(1-m) \mid m) == \frac{1}{\sqrt{m}}$$

09.25.03.0017.01

$$\mathrm{cd}(K(m) + 2\,i\,K(1-m) \mid m) == 0$$

09.25.03.0018.01

$$\mathrm{cd}(2\,K(m) + 2\,i\,K(1-m) \mid m) == -1$$

09.25.03.0019.01

$$\mathrm{cd}(3\,K(m) + 2\,i\,K(1-m) \mid m) == 0$$

09.25.03.0020.01

$$\mathrm{cd}(4\,K(m) + 2\,i\,K(1-m) \mid m) == 1$$

09.25.03.0021.01

$$\mathrm{cd}((2r+1)K(m)+(2s+1)\,i\,K(1-m)\mid m) = \tilde{\infty} \;/;\; \{r,s\}\in\mathbb{Z}$$

Values at half-quarter-period points

09.25.03.0022.01

$$\mathrm{cd}\left(\frac{K(m)}{2}\,\middle|\, m\right) = \frac{1}{\sqrt{1+\sqrt{1-m}}}$$

09.25.03.0023.01

$$\mathrm{cd}\left(\frac{i\,K(1-m)}{2}\,\middle|\, m\right) = \frac{1}{\sqrt[4]{m}}$$

09.25.03.0024.01

$$\mathrm{cd}\left(\frac{K(m)}{2}+\frac{i\,K(1-m)}{2}\,\middle|\, m\right) = \frac{1-i}{\sqrt[4]{m}\left(\sqrt{1+\sqrt{1-m}}-i\sqrt{1-\sqrt{1-m}}\right)}$$

General characteristics

Domain and analyticity

$\mathrm{cd}(z\mid m)$ is a meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.25.04.0001.01

$$(z*m)\longrightarrow \mathrm{cd}(z\mid m) :: (\mathbb{C}\otimes\mathbb{C})\longrightarrow\mathbb{C}$$

Symmetries and periodicities

Parity

$\mathrm{cd}(z\mid m)$ is an even function with respect to z.

09.25.04.0002.01

$$\mathrm{cd}(-z\mid m) = \mathrm{cd}(z\mid m)$$

Mirror symmetry

09.25.04.0003.01

$$\mathrm{cd}(\bar{z}\mid \bar{m}) = \overline{\mathrm{cd}(z\mid m)}$$

Periodicity

$\mathrm{cd}(z\mid m)$ is a doubly periodic function with respect to z with periods $2\,i\,K(1-m)$ and $4\,K(m)$.

09.25.04.0004.01

$$\mathrm{cd}(z+2K(m)\mid m) = -\mathrm{cd}(z\mid m)$$

09.25.04.0005.01

$$\mathrm{cd}(z+4K(m)\mid m) = \mathrm{cd}(z\mid m)$$

09.25.04.0006.01

$$\mathrm{cd}(z+2\,i\,K(1-m)\mid m) = \mathrm{cd}(z\mid m)$$

09.25.04.0007.01

$\text{cd}(z + 2K(m) + 2iK(1-m) \mid m) == -\text{cd}(z \mid m)$

09.25.04.0008.01

$\text{cd}(z + 2rK(m) + 2siK(1-m) \mid m) == (-1)^r \text{cd}(z \mid m) /; \{r, s\} \in \mathbb{Z}$

Poles and essential singularities

With respect to *z*

For fixed m, the function $\text{cd}(z \mid m)$ has an infinite set of singular points:

a) $z = (2r+1)K(m) + (2s+1)iK(1-m)$, $\{r, s\} \in \mathbb{Z}$, are the simple poles with residues $\frac{(-1)^{r-1}}{\sqrt{m}}$;

b) $z == \tilde{\infty}$ is an essential singular point.

09.25.04.0009.01

$\mathcal{S}ing_z(\text{cd}(z \mid m)) == \{\{\{(2s+1)iK(1-m) + (2r+1)K(m), 1\} /; \{r, s\} \in \mathbb{Z}\}, \{\tilde{\infty}, \infty\}\}$

09.25.04.0010.01

$$\text{res}_z(\text{cd}(z \mid m))((2s+1)iK(1-m) + (2r+1)K(m)) == \frac{(-1)^{r-1}}{\sqrt{m}} /; \{r, s\} \in \mathbb{Z}$$

Branch points

With respect to *m*

For fixed z, the function $\text{cd}(z \mid m)$ is a meromorphic function in m that has no branch points.

09.25.04.0013.01

$\mathcal{BP}_m(\text{cd}(z \mid m)) == \{\}$

P. Walker

With respect to *z*

For fixed m, the function $\text{cd}(z \mid m)$ does not have branch points.

09.25.04.0011.01

$\mathcal{BP}_z(\text{cd}(z \mid m)) == \{\}$

Branch cuts

With respect to *m*

For fixed z, the function $\text{cd}(z \mid m)$ is a meromorphic function in m that has no branch cuts.

09.25.04.0014.01

$\mathcal{BC}_m(\text{cd}(z \mid m)) == \{\}$

P. Walker

With respect to *z*

For fixed m, the function $\text{cd}(z \mid m)$ does not have branch cuts.

09.25.04.0012.01

$\mathcal{BC}_z(\mathrm{cd}(z \mid m)) == \{\}$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

09.25.06.0005.01

$$\mathrm{cd}(z \mid m) \propto \mathrm{cd}(z_0 \mid m) + (m-1)\,\mathrm{nd}(z_0 \mid m)\,\mathrm{sd}(z_0 \mid m)\,(z-z_0) + \frac{1}{2}(m-1)\,\mathrm{cd}(z_0 \mid m)\left(\mathrm{nd}(z_0 \mid m)^2 + m\,\mathrm{sd}(z_0 \mid m)^2\right)(z-z_0)^2 + \frac{(m-1)\,\mathrm{sn}(z_0 \mid m)}{6\,\mathrm{dn}(z_0 \mid m)^4}\left(4\,m\,\mathrm{cn}(z_0 \mid m)^2 + (m-1)\left(m\,\mathrm{sn}(z_0 \mid m)^2+1\right)\right)(z-z_0)^3 + \frac{(m-1)\,\mathrm{cn}(z_0 \mid m)}{24\,\mathrm{dn}(z_0 \mid m)^5}\left(4\,m\left(m\,\mathrm{sn}(z_0 \mid m)^2+1\right)\mathrm{cn}(z_0 \mid m)^2 + (m-1)\left(m\left(m\,\mathrm{sn}(z_0 \mid m)^2+14\right)\mathrm{sn}(z_0 \mid m)^2+1\right)\right)(z-z_0)^4 + \frac{(m-1)\,\mathrm{sn}(z_0 \mid m)}{120\,\mathrm{dn}(z_0 \mid m)^6}\left(16\,m^2\,\mathrm{cn}(z_0 \mid m)^4 + 44\,(m-1)\,m\left(m\,\mathrm{sn}(z_0 \mid m)^2+1\right)\mathrm{cn}(z_0 \mid m)^2 + (m-1)^2\left(m\left(m\,\mathrm{sn}(z_0 \mid m)^2+14\right)\mathrm{sn}(z_0 \mid m)^2+1\right)\right)(z-z_0)^5 + \ldots \;/;\; (z \to z_0)$$

09.25.06.0006.01

$$\mathrm{cd}(z \mid m) \propto \mathrm{cd}(z_0 \mid m)\,(1 + O(z-z_0))$$

Expansions at $z == 0$

09.25.06.0007.01

$$\mathrm{cd}(z \mid m) \propto 1 + \frac{1}{2}(m-1)\,z^2 + \frac{1}{24}\left(1-6\,m+5\,m^2\right)z^4 + \ldots \;/;\; (z \to 0)$$

09.25.06.0001.02

$$\mathrm{cd}(z \mid m) \propto 1 + \frac{1}{2}(m-1)\,z^2 + \frac{1}{24}\left(1-6\,m+5\,m^2\right)z^4 + \frac{1}{720}\left(-1+47\,m-107\,m^2+61\,m^3\right)z^6 + \frac{\left(1-412\,m+2142\,m^2-3116\,m^3+1385\,m^4\right)z^8}{40\,320} + \frac{\left(-1+3693\,m-45\,530\,m^2+130\,250\,m^3-138\,933\,m^4+50\,521\,m^5\right)z^{10}}{3\,628\,800} + O\!\left(z^{12}\right)$$

09.25.06.0008.01

$$\mathrm{cd}(z \mid m) = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(j+1)\,(-1)^{k-j}\,\mathrm{cn}_{k-j}(m)}{(2\,k-2\,j)!}\sum_{r=0}^{j}\frac{(-1)^r}{r+1}\binom{j}{r}\,p_{r,j}\,z^{2\,k} \;/;\; p_{j,0}=1 \bigwedge p_{j,k} = \frac{1}{k}\sum_{i=1}^{k}\frac{(j\,i+i-k)\,(-1)^i\,\mathrm{dn}_i(m)\,p_{j,k-i}}{(2\,i)!} \bigwedge k \in \mathbb{N}^+ \bigwedge \mathrm{sn}_0(m)=1 \bigwedge \mathrm{sn}_n(m) = \sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2\,n}{2\,j}\mathrm{cn}_j(m)\,\mathrm{dn}_k(m)\,\delta_{j+k-n} \bigwedge \mathrm{cn}_0(m)=1 \bigwedge \mathrm{cn}_n(m) = \sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2\,n-1}{2\,j+1}\mathrm{sn}_j(m)\,\mathrm{dn}_k(m)\,\delta_{j+k-n+1} \bigwedge \mathrm{dn}_0(m)=1 \bigwedge \mathrm{dn}_n(m) = m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2\,n-1}{2\,j+1}\mathrm{sn}_j(m)\,\mathrm{cn}_k(m)\,\delta_{j+k-n+1}$$

09.25.06.0009.01

$$\operatorname{cd}(z \mid m) \propto 1 + O\left(z^2\right)$$

Expansions at $z = (2r+1)K(m) + (2s+1)\,i\,K(1-m)$

09.25.06.0010.01

$$\operatorname{cd}(z \mid m) \propto \frac{(-1)^{r-1}}{\sqrt{m}} \left(\frac{1}{z - z_0} + \frac{1}{6}(m+1)(z - z_0) + \frac{1}{360}\left(7m^2 - 22m + 7\right)(z - z_0)^3 + \ldots \right) /; (z \to z_0) \bigwedge z_0 = (2r+1)K(m) + (2s+1)\,i\,K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z}$$

09.25.06.0011.01

$$\operatorname{cd}(z \mid m) = \frac{(-1)^{r-1}}{\sqrt{m}} \sum_{k=0}^{\infty} (k+1) \sum_{r=0}^{k} \frac{(-1)^r}{r+1} \binom{k}{r} p_{r,k} (z - z_0)^{2k-1} /;$$

$$z_0 = (2r+1)K(m) + (2s+1)\,i\,K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z} \bigwedge p_{j,0} = 1 \bigwedge p_{j,k} = \frac{1}{k} \sum_{i=1}^{k} \frac{(j\,i + i - k)(-1)^i \operatorname{sn}_i(m)\, p_{j,k-i}}{(2i+1)!} \bigwedge$$

$$k \in \mathbb{N}^+ \bigwedge \operatorname{sn}_0(m) = 1 \bigwedge \operatorname{sn}_n(m) = \sum_{j=0}^{n} \sum_{k=0}^{n} \binom{2n}{2j} \operatorname{cn}_j(m) \operatorname{dn}_k(m)\, \delta_{j+k-n} \bigwedge \operatorname{cn}_0(m) = 1 \bigwedge$$

$$\operatorname{cn}_n(m) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \operatorname{sn}_j(m) \operatorname{dn}_k(m)\, \delta_{j+k-n+1} \bigwedge \operatorname{dn}_0(m) = 1 \bigwedge \operatorname{dn}_n(m) = m \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \operatorname{sn}_j(m) \operatorname{cn}_k(m)\, \delta_{j+k-n+1}$$

09.25.06.0012.01

$$\operatorname{cd}(z \mid m) \propto \frac{(-1)^{r-1}}{\sqrt{m}\,(z - z_0)} \left(1 + O\left((z - z_0)^2\right)\right) /; z_0 = (2r+1)K(m) + (2s+1)\,i\,K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z}$$

Expansions at generic point $m = m_0$

For the function itself

09.25.06.0013.01

$$\operatorname{cd}(z \mid m) \propto \operatorname{cd}(z \mid m_0) + \frac{\operatorname{nd}(z \mid m_0)\operatorname{sd}(z \mid m_0)}{2m_0}\left((m_0 - 1)z + E(\operatorname{am}(z \mid m_0) \mid m_0)\right)(m - m_0) +$$
$$\frac{1}{8m_0^2}\left(-2\left(E(\operatorname{am}(z \mid m_0) \mid m_0) + z(m_0 - 1)\right)\operatorname{sd}(z \mid m_0)\operatorname{nd}(z \mid m_0) + 2\operatorname{sd}(z \mid m_0)\left(z + \frac{E(\operatorname{am}(z \mid m_0) \mid m_0) - F(\operatorname{am}(z \mid m_0) \mid m_0)}{2m_0} + \right.\right.$$
$$\left.\frac{1}{2(m_0 - 1)m_0}\left(\left(E(\operatorname{am}(z \mid m_0) \mid m_0) + z(m_0 - 1)\right)\operatorname{dn}(z \mid m_0) - m_0\operatorname{cn}(z \mid m_0)\operatorname{sn}(z \mid m_0)\right)\sqrt{1 - m_0\operatorname{sn}(z \mid m_0)^2}\right)$$
$$m_0\operatorname{nd}(z \mid m_0) + \frac{1}{m_0 - 1}\left(E(\operatorname{am}(z \mid m_0) \mid m_0) + z(m_0 - 1)\right)\operatorname{cd}(z \mid m_0)\operatorname{nd}(z \mid m_0)^2$$
$$\left(E(\operatorname{am}(z \mid m_0) \mid m_0) - m_0\operatorname{dn}(z \mid m_0)\operatorname{sc}(z \mid m_0) + z(m_0 - 1)\right) + \frac{1}{m_0 - 1}m_0\left(E(\operatorname{am}(z \mid m_0) \mid m_0) + z(m_0 - 1)\right)$$
$$\left.\operatorname{cd}(z \mid m_0)\left(E(\operatorname{am}(z \mid m_0) \mid m_0) - \operatorname{dn}(z \mid m_0)\operatorname{sc}(z \mid m_0) + z(m_0 - 1)\right)\operatorname{sd}(z \mid m_0)^2\right)(m - m_0)^2 + \ldots /; (m \to m_0)$$

09.25.06.0014.01

$$\operatorname{cd}(z \mid m) \propto \operatorname{cd}(z \mid m_0)\left(1 + O(m - m_0)\right)$$

Expansions at $m == 0$

09.25.06.0015.01

$$\operatorname{cd}(z \mid m) \propto \cos(z) + \frac{1}{4}\sin(z)\,(z + \cos(z)\sin(z))\,m + \frac{1}{256}\left(\left(7 - 8z^2\right)\cos(z) - 8\cos(3z) + \cos(5z) + 24z\sin(z) - 12z\sin(3z)\right)m^2 + \ldots \;/;\; (m \to 0)$$

09.25.06.0016.01

$$\operatorname{cd}(z \mid m) \propto \cos(z) + \frac{1}{4}\sin(z)\,(z + \cos(z)\sin(z))\,m + \frac{1}{256}\left(\left(7 - 8z^2\right)\cos(z) - 8\cos(3z) + \cos(5z) + 24z\sin(z) - 12z\sin(3z)\right)m^2 - \frac{1}{12\,288}\left(3\left(88z^2 - 67\right)\cos(z) - 6\left(36z^2 - 41\right)\cos(3z) - 48\cos(5z) + 3\cos(7z) + 32z\left(z^2 - 21\right)\sin(z) + 468z\sin(3z) - 60z\sin(5z)\right)m^3 + \frac{1}{196\,608}\left(2\left(16z^4 - 1560z^2 + 1107\right)\cos(z) + 81\left(48z^2 - 35\right)\cos(3z) - 30\left(20z^2 - 23\right)\cos(5z) - 72\cos(7z) + 3\cos(9z) - 4z\left(128z^2 - 1845\right)\sin(z) + 72z\left(12z^2 - 83\right)\sin(3z) + 1260z\sin(5z) - 84z\sin(7z)\right)m^4 + \frac{1}{15\,728\,640}\left(120\left(28z^4 - 1628z^2 + 1101\right)\cos(z) - 45\left(288z^4 - 6600z^2 + 3887\right)\cos(3z) - 240\left(325z^2 - 204\right)\cos(5z) + 30\left(196z^2 - 221\right)\cos(7z) + 480\cos(9z) - 15\cos(11z) + 8z\left(16z^4 - 4580z^2 + 55\,245\right)\sin(z) + 1440z\left(69z^2 - 272\right)\sin(3z) - 200z\left(100z^2 - 531\right)\sin(5z) - 12\,180z\sin(7z) + 540z\sin(9z)\right)m^5 + \frac{1}{754\,974\,720}\left(-8\left(32z^6 - 21\,180z^4 + 955\,575z^2 - 622\,845\right)\cos(z) - 270\left(4032z^4 - 48\,564z^2 + 25\,081\right)\cos(3z) + 15\left(20\,000z^4 - 301\,800z^2 + 139\,557\right)\cos(5z) + 720\left(833z^2 - 465\right)\cos(7z) - 90\left(324z^2 - 359\right)\cos(9z) - 1800\cos(11z) + 45\cos(13z) + 12z\left(832z^4 - 129\,240z^2 + 1\,397\,055\right)\sin(z) - 648z\left(144z^4 - 8180z^2 + 24\,385\right)\sin(3z) - 300z\left(6200z^2 - 16\,761\right)\sin(5z) + 840z\left(196z^2 - 909\right)\sin(7z) + 59\,940z\sin(9z) - 1980z\sin(11z)\right)m^6 - \frac{1}{84\,557\,168\,640}\left(14\left(3968z^6 - 1\,331\,760z^4 + 51\,350\,220z^2 - 32\,480\,415\right)\cos(z) - 63\left(20\,736z^6 - 2\,453\,760z^4 + 21\,261\,240z^2 - 10\,022\,255\right)\cos(3z) - 315\left(240\,000z^4 - 1\,750\,000z^2 + 668\,241\right)\cos(5z) + 735\left(10\,976z^4 - 133\,560z^2 + 51\,573\right)\cos(7z) + 5040\left(1701z^2 - 874\right)\cos(9z) - 630\,(22z - 23)\,(22z + 23)\cos(11z) - 15\,120\cos(13z) + 315\cos(15z) + 4z\left(256z^6 - 337\,344z^4 + 38\,446\,800z^2 - 384\,987\,645\right)\sin(z) + 756z\left(28\,512z^4 - 801\,960z^2 + 2\,009\,245\right)\sin(3z) - 2100z\left(4000z^4 - 135\,400z^2 + 257\,937\right)\sin(5z) - 8820z\left(5096z^2 - 11\,175\right)\sin(7z) + 22\,680z\left(108z^2 - 461\right)\sin(9z) + 623\,700z\sin(11z) - 16\,380z\sin(13z)\right)m^7 + \frac{1}{1\,352\,914\,698\,240}\left(\left(512z^8 - 1\,211\,392z^6 + 285\,670\,560z^4 - 9\,875\,050\,920z^2 + 6\,089\,965\,245\right)\cos(z) + 63\left(787\,968z^6 - 44\,102\,880z^4 + 310\,448\,520z^2 - 136\,673\,455\right)\cos(3z) - 70\left(400\,000z^6 - 26\,280\,000z^4 + 130\,369\,500z^2 - 43\,550\,109\right)\cos(5z) - 5880\left(60\,368z^4 - 334\,950z^2 + 102\,195\right)\cos(7z) + 315\left(69\,984z^4 - 745\,848z^2 + 253\,307\right)\cos(9z) + 55\,440\left(275z^2 - 133\right)\cos(11z) - 630\left(676z^2 - 731\right)\cos(13z) - 17\,640\cos(15z) + 315\cos(17z) - 12z\left(3072z^6 - 1\,950\,368z^4 + 183\,014\,720z^2 - 1\,730\,146\,845\right)\sin(z) + 648z\left(3456z^6 - 758\,016z^4 + 14\,749\,280z^2 - 32\,740\,365\right)\sin(3z) + 21\,000z\left(16\,400z^4 - 257\,560z^2 + 392\,841\right)\sin(5z) - 588z\left(76\,832z^4 - 1\,977\,640z^2 + 2\,906\,355\right)\sin(7z) - 11\,340z\left(10\,152z^2 - 19\,451\right)\sin(9z) + 9240z\left(484z^2 - 1953\right)\sin(11z) + 868\,140z\sin(13z) - 18\,900z\sin(15z)\right)m^8 + \frac{1}{194\,819\,716\,546\,560}\left(18\left(10\,496z^8 - 11\,495\,680z^6 + 2\,165\,834\,160z^4 - 69\,005\,718\,180z^2 + 41\,633\,079\,075\right)\cos(z) - \right.$$

$$162\left(186\,624\,z^8-69\,745\,536\,z^6+2\,616\,757\,920\,z^4-15\,922\,895\,940\,z^2+6\,643\,081\,375\right)\cos(3\,z)-$$
$$630\left(18\,400\,000\,z^6-542\,880\,000\,z^4+2\,102\,352\,300\,z^2-635\,484\,717\right)\cos(5\,z)+$$
$$126\left(15\,059\,072\,z^6-719\,147\,520\,z^4+2\,605\,542\,660\,z^2-673\,078\,005\right)\cos(7\,z)+$$
$$45\,360\left(227\,448\,z^4-1\,059\,399\,z^2+274\,924\right)\cos(9\,z)-945\left(468\,512\,z^4-4\,562\,184\,z^2+1\,409\,247\right)\cos(11\,z)-$$
$$45\,360\left(4901\,z^2-2264\right)\cos(13\,z)+28\,350\left(180\,z^2-193\right)\cos(15\,z)+181\,440\cos(17\,z)-2835\cos(19\,z)+$$
$$8\,z\left(256\,z^8-1\,008\,000\,z^6+434\,436\,912\,z^4-35\,584\,337\,880\,z^2+321\,447\,804\,615\right)\sin(z)+$$
$$3888\,z\left(222\,912\,z^6-22\,302\,000\,z^4+342\,903\,750\,z^2-695\,415\,875\right)\sin(3\,z)-$$
$$900\,z\left(800\,000\,z^6-92\,400\,000\,z^4+959\,477\,400\,z^2-1\,251\,408\,501\right)\sin(5\,z)-$$
$$5292\,z\left(3\,764\,768\,z^4-42\,904\,400\,z^2+48\,678\,885\right)\sin(7\,z)+20\,412\,z\left(69\,984\,z^4-1\,517\,400\,z^2+1\,876\,715\right)\sin(9\,z)+$$
$$41\,580\,z\left(53\,240\,z^2-92\,781\right)\sin(11\,z)-98\,280\,z\left(676\,z^2-2619\right)\sin(13\,z)-$$
$$10\,376\,100\,z\sin(15\,z)+192\,780\,z\sin(17\,z)\Big)\,m^9+\frac{1}{15\,585\,577\,323\,724\,800}$$
$$\Big(-\left(4096\,z^{10}-25\,320\,960\,z^8+18\,304\,957\,440\,z^6-2\,944\,262\,714\,400\,z^4+88\,003\,507\,762\,800\,z^2-52\,079\,055\,504\,525\right)$$
$$\cos(z)-1215\left(5\,971\,968\,z^8-988\,533\,504\,z^6+28\,639\,981\,440\,z^4-156\,265\,593\,960\,z^2+62\,399\,823\,955\right)\cos(3\,z)+$$
$$225\left(40\,000\,000\,z^8-7\,554\,400\,000\,z^6+143\,925\,600\,000\,z^4-467\,583\,076\,800\,z^2+130\,698\,250\,767\right)\cos(5\,z)+$$
$$945\left(542\,126\,592\,z^6-11\,186\,739\,200\,z^4+30\,686\,626\,320\,z^2-6\,994\,157\,055\right)\cos(7\,z)-$$
$$17\,010\left(2\,519\,424\,z^6-98\,210\,880\,z^4+289\,458\,900\,z^2-61\,983\,655\right)\cos(9\,z)-$$
$$113\,400\left(1\,171\,280\,z^4-4\,826\,206\,z^2+1\,109\,661\right)\cos(11\,z)+4725\left(913\,952\,z^4-8\,335\,080\,z^2+2\,394\,237\right)\cos(13\,z)+$$
$$680\,400\left(2475\,z^2-1103\right)\cos(15\,z)-28\,350\left(1156\,z^2-1231\right)\cos(17\,z)-1\,020\,600\cos(19\,z)+14\,175\cos(21\,z)+$$
$$160\,z\left(2944\,z^8-5\,196\,384\,z^6+1\,745\,402\,148\,z^4-129\,145\,913\,190\,z^2+1\,124\,016\,300\,855\right)\sin(z)-$$
$$131\,220\,z\left(1536\,z^8-917\,760\,z^6+59\,976\,224\,z^4-780\,845\,800\,z^2+1\,475\,445\,055\right)\sin(3\,z)-$$
$$540\,000\,z\left(340\,000\,z^6-17\,170\,300\,z^4+136\,270\,295\,z^2-158\,031\,489\right)\sin(5\,z)+$$
$$17\,640\,z\left(2\,151\,296\,z^6-174\,254\,976\,z^4+1\,268\,918\,700\,z^2-1\,197\,550\,395\right)\sin(7\,z)+$$
$$918\,540\,z\left(443\,232\,z^4-4\,107\,600\,z^2+3\,812\,645\right)\sin(9\,z)-$$
$$41\,580\,z\left(468\,512\,z^4-9\,026\,600\,z^2+9\,864\,255\right)\sin(11\,z)-737\,100\,z\left(28\,392\,z^2-46\,115\right)\sin(13\,z)+$$
$$1\,701\,000\,z\left(300\,z^2-1127\right)\sin(15\,z)+66\,509\,100\,z\sin(17\,z)-1\,077\,300\,z\sin(19\,z)\Big)\,m^{10}+O\left(m^{11}\right)$$

09.25.06.0017.01

$$\operatorname{cd}(z\mid m)\propto\cos(z)\,(1+O(m))$$

Expansions at $m == 1$

09.25.06.0018.01

$$\operatorname{cd}(z\mid m)\propto 1+\frac{1}{2}\sinh^2(z)\,(m-1)+\frac{1}{32}\sinh(z)\,(8\,z\cosh(z)-11\sinh(z)+\sinh(3\,z))\,(m-1)^2+\ldots\;/;\;(m\to 1)$$

09.25.06.0019.01

$$\operatorname{cd}(z\mid m)\propto 1+\frac{1}{2}\sinh^2(z)\,(m-1)+\frac{1}{32}\sinh(z)\,(8\,z\cosh(z)-11\sinh(z)+\sinh(3\,z))\,(m-1)^2+$$
$$\frac{1}{1024}\left(\left(32\,z^2+159\right)\cosh(2\,z)-20\cosh(4\,z)+\cosh(6\,z)-136\,z\sinh(2\,z)+16\,z\sinh(4\,z)-140\right)(m-1)^3-$$
$$\frac{1}{49\,152}\Big(-12\left(176\,z^2+553\right)\cosh(2\,z)+12\left(32\,z^2+85\right)\cosh(4\,z)-84\cosh(6\,z)+$$
$$3\cosh(8\,z)+8\,z\left(32\,z^2+789\right)\sinh(2\,z)-1200\,z\sinh(4\,z)+72\,z\sinh(6\,z)+5697\Big)\,(m-1)^4+$$
$$\frac{1}{786\,432}\Big(\left(512\,z^4+37\,632\,z^2+95\,013\right)\cosh(2\,z)-720\left(16\,z^2+23\right)\cosh(4\,z)+24\left(36\,z^2+73\right)\cosh(6\,z)-108\cosh(8\,z)+$$

$$3\cosh(10z) - 96z\left(72z^2 + 1001\right)\sinh(2z) + 64z\left(32z^2 + 363\right)\sinh(4z) - 2376z\sinh(6z) + 96z\sinh(8z) - 80\,100\Big)$$

$$(m-1)^5 + \frac{1}{62\,914\,560}\Big(-20\left(4096z^4 + 156\,864z^2 + 346\,599\right)\cosh(2z) + 5\left(8192z^4 + 250\,752z^2 + 264\,093\right)\cosh(4z) -$$

$$480\left(342z^2 + 341\right)\cosh(6z) + 60\left(128z^2 + 223\right)\cosh(8z) - 660\cosh(10z) + 15\cosh(12z) +$$

$$8z\left(512z^4 + 87\,360z^2 + 913\,065\right)\sinh(2z) - 80z\left(4480z^2 + 25\,707\right)\sinh(4z) +$$

$$34\,560z\left(z^2 + 8\right)\sinh(6z) - 19\,680z\sinh(8z) + 600z\sinh(10z) + 5\,762\,460\Big)(m-1)^6 + \frac{1}{3\,019\,898\,880}$$

$$\Big(\left(16\,384z^6 + 5\,568\,000z^4 + 152\,570\,880z^2 + 307\,546\,425\right)\cosh(2z) - 60\left(81\,920z^4 + 1\,205\,568z^2 + 1\,044\,537\right)\cosh(4z) +$$

$$90\left(6912z^4 + 140\,688z^2 + 96\,553\right)\cosh(6z) - 720\left(1472z^2 + 1183\right)\cosh(8z) + 720\left(50z^2 + 79\right)\cosh(10z) -$$

$$2340\cosh(12z) + 45\cosh(14z) - 48z\left(9472z^4 + 794\,640z^2 + 6\,956\,025\right)\sinh(2z) +$$

$$48z\left(8192z^4 + 542\,080z^2 + 2\,182\,305\right)\sinh(4z) - 1080z\left(4128z^2 + 15\,533\right)\sinh(6z) +$$

$$1920z\left(128z^2 + 837\right)\sinh(8z) - 88\,200z\sinh(10z) + 2160z\sinh(12z) - 252\,766\,800\Big)(m-1)^7 +$$

$$\frac{1}{338\,228\,674\,560}\Big(-84\left(57\,344z^6 + 9\,164\,800z^4 + 203\,281\,440z^2 + 382\,829\,805\right)\cosh(2z) +$$

$$28\left(262\,144z^6 + 32\,747\,520z^4 + 326\,691\,360z^2 + 246\,944\,475\right)\cosh(4z) -$$

$$2520\left(82\,944z^4 + 766\,584z^2 + 414\,119\right)\cosh(6z) + 420\left(32\,768z^4 + 524\,160z^2 + 277\,311\right)\cosh(8z) -$$

$$100\,800\left(135z^2 + 94\right)\cosh(10z) + 1260\left(288z^2 + 425\right)\cosh(12z) - 18\,900\cosh(14z) + 315\cosh(16z) +$$

$$8z\left(16\,384z^6 + 9\,988\,608z^4 + 576\,959\,040z^2 + 4\,465\,358\,415\right)\sinh(2z) - 1680z\left(73\,728z^4 + 2\,254\,336z^2 + 7\,243\,269\right)$$

$$\sinh(4z) + 3024z\left(6912z^4 + 290\,640z^2 + 730\,025\right)\sinh(6z) - 10\,080z\left(8704z^2 + 25\,441\right)\sinh(8z) +$$

$$33\,600z\left(100z^2 + 573\right)\sinh(10z) - 861\,840z\sinh(12z) + 17\,640z\sinh(14z) + 26\,179\,325\,865\Big)(m-1)^8 +$$

$$\frac{1}{5\,411\,658\,792\,960}\Big(2\left(65\,536z^8 + 66\,404\,352z^6 + 7\,085\,084\,160z^4 + 135\,511\,382\,160z^2 + 242\,044\,097\,715\right)\cosh(2z) -$$

$$560\left(655\,360z^6 + 36\,642\,816z^4 + 284\,593\,104z^2 + 194\,260\,275\right)\cosh(4z) +$$

$$63\left(1\,327\,104z^6 + 101\,606\,400z^4 + 611\,222\,400z^2 + 278\,728\,555\right)\cosh(6z) -$$

$$1680\left(458\,752z^4 + 3\,196\,224z^2 + 1\,287\,903\right)\cosh(8z) + 210\left(160\,000z^4 + 2\,180\,400z^2 + 966\,489\right)\cosh(10z) -$$

$$15\,120\left(1488z^2 + 935\right)\cosh(12z) + 2520\left(196z^2 + 275\right)\cosh(14z) - 21\,420\cosh(16z) +$$

$$315\cosh(18z) - 8z\left(770\,048z^6 + 213\,126\,144z^4 + 9\,695\,105\,280z^2 + 68\,466\,371\,715\right)\sinh(2z) +$$

$$64z\left(262\,144z^6 + 56\,340\,480z^4 + 1\,135\,602\,720z^2 + 3\,112\,652\,025\right)\sinh(4z) -$$

$$3024z\left(366\,336z^4 + 6\,796\,800z^2 + 13\,139\,615\right)\sinh(6z) + 2688z\left(32\,768z^4 + 1\,048\,960z^2 + 1\,972\,635\right)\sinh(8z) -$$

$$21\,000z\left(9440z^2 + 23\,283\right)\sinh(10z) + 181\,440z\left(32z^2 + 167\right)\sinh(12z) -$$

$$1\,146\,600z\sinh(14z) + 20\,160z\sinh(16z) - 390\,888\,160\,740\Big)(m-1)^9 + \frac{1}{779\,278\,866\,186\,240}$$

$$\Big(-72\left(851\,968z^8 + 380\,821\,504z^6 + 31\,281\,384\,960z^4 + 535\,531\,809\,960z^2 + 916\,547\,119\,425\right)\cosh(2z) +$$

$$9\left(33\,554\,432z^8 + 11\,589\,910\,528z^6 + 417\,380\,597\,760z^4 + 2\,713\,786\,145\,280z^2 + 1\,710\,278\,838\,765\right)\cosh(4z) -$$

$$2268\left(19\,243\,008z^6 + 634\,798\,080z^4 + 2\,883\,765\,600z^2 + 1\,157\,463\,185\right)\cosh(6z) +$$

$$1008\left(4\,194\,304z^6 + 238\,141\,440z^4 + 1\,047\,009\,600z^2 + 347\,193\,495\right)\cosh(8z) -$$

$$7560\left(2\,560\,000z^4 + 14\,721\,000z^2 + 4\,843\,803\right)\cosh(10z) + 22\,680\left(27\,648z^4 + 335\,952z^2 + 130\,913\right)\cosh(12z) -$$

$$3\,175\,200\left(98z^2 + 57\right)\cosh(14z) + 11\,340\left(512z^2 + 691\right)\cosh(16z) - 215\,460\cosh(18z) + 2835\cosh(20z) +$$

$$16z\left(65\,536z^8 + 104\,103\,936z^6 + 18\,753\,928\,704z^4 + 720\,595\,355\,760z^2 + 4\,737\,962\,919\,375\right)\sinh(2z) -$$

$$2880z\left(2\,883\,584z^6 + 270\,219\,264z^4 + 4\,153\,099\,776z^2 + 10\,103\,019\,129\right)\sinh(4z) +$$

$$1944z\left(1\,327\,104z^6 + 169\,731\,072z^4 + 2\,006\,780\,160z^2 + 3\,218\,991\,965\right)\sinh(6z) -$$

$$24\,192z\left(1\,998\,848z^4 + 27\,271\,680z^2 + 38\,385\,765\right)\sinh(8z) + 75\,600z\left(32\,000z^4 + 852\,400z^2 + 1\,314\,741\right)\sinh(10z) -$$

$$136\,080z\left(25\,728z^2 + 56\,137\right)\sinh(12z) + 846\,720z\left(98z^2 + 477\right)\sinh(14z) -$$

$$13\,345\,120z\sinh(16z) + 204\,120z\sinh(18z) - 53\,907\,861\,017\,360\Big)(m-1)^{10} + O\left((m-1)^{11}\right)$$

09.25.06.0020.01

$$\operatorname{cd}(z \mid m) \propto 1 + O(m-1)$$

q-series

09.25.06.0002.01

$$\operatorname{cd}(z \mid m) = \frac{2\pi}{\sqrt{m}\, K(m)} \sum_{k=0}^{\infty} \frac{(-1)^k q(m)^{k+1/2}}{1 - q(m)^{2k+1}} \cos\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

Other series representations

09.25.06.0003.01

$$\operatorname{cd}(z \mid m) = \frac{\pi}{2\sqrt{m}\, K(1-m)} \sum_{k=-\infty}^{\infty} (-1)^k \tanh\left(\pi \frac{K(m)}{K(1-m)} \left(k + \frac{1}{2} + \frac{z}{2K(m)}\right)\right)$$

09.25.06.0004.01

$$\operatorname{cd}(z \mid m) \propto \frac{(-1)^{r-1}}{\sqrt{m}\,(z - i(2s+1)K(1-m) - (2r+1)K(m))} + O(1) \;/;\; (z \to (2s+1)\,i\,K(1-m) + (2r+1)K(m)) \land \{r, s\} \in \mathbb{Z}$$

Product representations

09.25.08.0001.01

$$\operatorname{cd}(z \mid m) = \frac{2\sqrt[4]{q(m)}}{\sqrt[4]{m}} \cos\left(\frac{\pi z}{2K(m)}\right) \prod_{k=1}^{\infty} \frac{1 + 2q(m)^{2k} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k}}{1 + 2q(m)^{2k-1} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k-2}}$$

Differential equations

Ordinary nonlinear differential equations

09.25.13.0001.01

$$w''(z) - w(z)\left(2m\,w(z)^2 - m - 1\right) = 0 \;/;\; w(z) = \operatorname{cd}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.25.16.0001.01

$$\operatorname{cd}(i z \mid m) = \operatorname{nd}(z \mid 1-m)$$

09.25.16.0002.01

$$\operatorname{cd}(z \mid 1-m) = \operatorname{nd}(i z \mid m)$$

09.25.16.0003.01

$$\operatorname{cd}(i z \mid 1-m) = \operatorname{nd}(z \mid m)$$

09.25.16.0007.01

$$\operatorname{cd}(x + i y \mid m) = \frac{\operatorname{cn}(x \mid m)\operatorname{cn}(y \mid 1-m) - i \operatorname{sn}(x \mid m)\operatorname{dn}(x \mid m)\operatorname{sn}(y \mid 1-m)\operatorname{dn}(y \mid 1-m)}{\operatorname{dn}(x \mid m)\operatorname{cn}(y \mid 1-m)\operatorname{dn}(y \mid 1-m) - i\, m \operatorname{sn}(x \mid m)\operatorname{cn}(x \mid m)\operatorname{sn}(y \mid 1-m)} \;/;\; \{x, y\} \in \mathbb{R}$$

09.25.16.0008.01

$$\mathrm{cd}\left(\sqrt{1-m}\, z \,\middle|\, \frac{m}{m-1}\right) = \mathrm{cn}(z \mid m)$$

09.25.16.0009.01

$$\mathrm{cd}\left(\sqrt{m}\, z \,\middle|\, \frac{1}{m}\right) = \mathrm{dc}(z \mid m)$$

09.25.16.0010.01

$$\mathrm{cd}\left(i \sqrt{m}\, z \,\middle|\, \frac{m-1}{m}\right) = \mathrm{nc}(z \mid m)$$

09.25.16.0011.01

$$\mathrm{cd}\left(i \sqrt{1-m}\, z \,\middle|\, \frac{1}{1-m}\right) = \mathrm{dn}(z \mid m)$$

Landen's transformation:

09.25.16.0012.01

$$\mathrm{cd}\left(\left(1+\sqrt{1-m}\right) z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) = \frac{1-\left(\sqrt{1-m}+1\right)\mathrm{sn}(z \mid m)^2}{1-\left(1-\sqrt{1-m}\right)\mathrm{sn}(z \mid m)^2}$$

Gauss' transformation:

09.25.16.0013.01

$$\mathrm{cd}\left(\left(1+\sqrt{m}\right) z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) = \frac{\mathrm{cn}(z \mid m)\, \mathrm{dn}(z \mid m)}{1-\sqrt{m}\, \mathrm{sn}(z \mid m)^2}$$

n th degree transformations:

09.25.16.0014.01

$$\mathrm{cd}\left(\frac{z}{M} \,\middle|\, l\right) = \mathrm{cd}(z \mid m) \prod_{r=1}^{\frac{n-1}{2}} \frac{1-\mathrm{ns}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^2 \mathrm{sn}(z \mid m)^2}{1-m\, \mathrm{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^2 \mathrm{sn}(z \mid m)^2} \;/;$$

$$\frac{n+1}{2} \in \mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n-1}{2}} \mathrm{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n-1}{2}} \frac{\mathrm{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^2}{\mathrm{sn}\left(\frac{2rK(m)}{n} \,\middle|\, m\right)^2}$$

09.25.16.0015.01

$$\mathrm{cd}\left(\frac{z}{M}+\frac{K(m)}{n M} \,\middle|\, l\right) = -\frac{1}{M} \frac{\mathrm{dn}(z \mid m)\, \mathrm{sn}(z \mid m)}{\mathrm{cn}(z \mid m)} \prod_{r=1}^{\frac{n}{2}} \frac{1-\mathrm{ns}\left(\frac{2rK(m)}{n} \,\middle|\, m\right)^2 \mathrm{sn}(z \mid m)^2}{1-m\, \mathrm{sn}\left(\frac{2rK(m)}{n} \,\middle|\, m\right)^2 \mathrm{sn}(z \mid m)^2} \;/;$$

$$\frac{n}{2} \in \mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n}{2}} \mathrm{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^8 \bigwedge M = \prod_{r=1}^{\frac{n}{2}} \frac{\mathrm{sn}\left(\frac{(2r-1)K(m)}{n} \,\middle|\, m\right)^2}{\mathrm{sn}\left(\frac{2rK(m)}{n} \,\middle|\, m\right)^2}$$

Argument involving half-periods

09.25.16.0004.01

$$\operatorname{cd}(z+K(m)\mid m) = -\operatorname{sn}(z\mid m)$$

09.25.16.0020.01

$$\operatorname{cd}(z-K(m)\mid m) = \operatorname{sn}(z\mid m)$$

09.25.16.0021.01

$$\operatorname{cd}(z+3\,K(m)\mid m) = \operatorname{sn}(z\mid m)$$

09.25.16.0022.01

$$\operatorname{cd}(z+(2\,r+1)\,K(m)\mid m) = (-1)^{r-1}\operatorname{sn}(z\mid m) /; r\in\mathbb{Z}$$

09.25.16.0005.01

$$\operatorname{cd}(z+i\,K(1-m)\mid m) = \frac{\operatorname{dc}(z\mid m)}{\sqrt{m}}$$

09.25.16.0023.01

$$\operatorname{cd}(z-i\,K(1-m)\mid m) = \frac{\operatorname{dc}(z\mid m)}{\sqrt{m}}$$

09.25.16.0024.01

$$\operatorname{cd}(z+3\,i\,K(1-m)\mid m) = \frac{1}{\sqrt{m}}\operatorname{dc}(z\mid m) /; s\in\mathbb{Z}$$

09.25.16.0025.01

$$\operatorname{cd}(z+(2\,s+1)\,i\,K(1-m)\mid m) = \frac{1}{\sqrt{m}}\operatorname{dc}(z\mid m) /; s\in\mathbb{Z}$$

09.25.16.0006.01

$$\operatorname{cd}(z+i\,K(1-m)+K(m)\mid m) = -\frac{\operatorname{ns}(z\mid m)}{\sqrt{m}}$$

09.25.16.0026.01

$$\operatorname{cd}(z-i\,K(1-m)+K(m)\mid m) = -\frac{\operatorname{ns}(z\mid m)}{\sqrt{m}}$$

09.25.16.0027.01

$$\operatorname{cd}(z+i\,K(1-m)-K(m)\mid m) = \frac{\operatorname{ns}(z\mid m)}{\sqrt{m}}$$

09.25.16.0028.01

$$\operatorname{cd}(z-i\,K(1-m)-K(m)\mid m) = \frac{\operatorname{ns}(z\mid m)}{\sqrt{m}}$$

09.25.16.0029.01

$$\operatorname{cd}(z+i\,K(1-m)+3\,K(m)\mid m) = \frac{\operatorname{ns}(z\mid m)}{\sqrt{m}}$$

09.25.16.0030.01

$$\operatorname{cd}(z+(2\,s+1)\,i\,K(1-m)+(4\,r+1)\,K(m)\mid m) = -\frac{\operatorname{ns}(z\mid m)}{\sqrt{m}} /; \{r,s\}\in\mathbb{Z}$$

09.25.16.0031.01

$$\operatorname{cd}(z+(2\,s+1)\,i\,K(1-m)+(4\,r-1)\,K(m)\mid m) = \frac{\operatorname{ns}(z\mid m)}{\sqrt{m}} /; \{r,s\}\in\mathbb{Z}$$

09.25.16.0032.01

$$\operatorname{cd}(z + (2s+1)\,i\,K(1-m) + (2r+1)\,K(m) \mid m) = \frac{(-1)^{r-1}\operatorname{ns}(z \mid m)}{\sqrt{m}} \;/;\; \{r, s\} \in \mathbb{Z}$$

Argument involving inverse Jacobi functions

09.25.16.0033.01

$$\operatorname{cd}\left(\operatorname{cn}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2}{m z^2 - m + 1}$$

09.25.16.0034.01

$$\operatorname{cd}\left(\operatorname{cs}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2}{z^2 - m + 1}$$

09.25.16.0035.01

$$\operatorname{cd}\left(\operatorname{dc}^{-1}(z \mid m) \mid m\right) = \frac{1}{z}$$

09.25.16.0036.01

$$\operatorname{cd}\left(\operatorname{dn}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2 + m - 1}{m z^2}$$

09.25.16.0037.01

$$\operatorname{cd}\left(\operatorname{ds}^{-1}(z \mid m) \mid m\right)^2 = 1 - \frac{1 - m}{z^2}$$

09.25.16.0038.01

$$\operatorname{cd}\left(\operatorname{nc}^{-1}(z \mid m) \mid m\right)^2 = \frac{1}{m - (m-1) z^2}$$

09.25.16.0039.01

$$\operatorname{cd}\left(\operatorname{nd}^{-1}(z \mid m) \mid m\right)^2 = \frac{1 - (1-m) z^2}{m}$$

09.25.16.0040.01

$$\operatorname{cd}\left(\operatorname{ns}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2 - 1}{z^2 - m}$$

09.25.16.0041.01

$$\operatorname{cd}\left(\operatorname{sc}^{-1}(z \mid m) \mid m\right)^2 = \frac{1}{1 - (m-1) z^2}$$

09.25.16.0042.01

$$\operatorname{cd}\left(\operatorname{sd}^{-1}(z \mid m) \mid m\right)^2 = 1 - (1-m) z^2$$

09.25.16.0043.01

$$\operatorname{cd}\left(\operatorname{sn}^{-1}(z \mid m) \mid m\right)^2 = \frac{z^2 - 1}{m z^2 - 1}$$

Addition formulas

09.25.16.0016.01

$$\operatorname{cd}(u + v \mid m) = \frac{\operatorname{cn}(u \mid m)\operatorname{cn}(v \mid m) - \operatorname{sn}(u \mid m)\operatorname{dn}(u \mid m)\operatorname{sn}(v \mid m)\operatorname{dn}(v \mid m)}{\operatorname{dn}(u \mid m)\operatorname{dn}(v \mid m) - m\operatorname{sn}(u \mid m)\operatorname{cn}(u \mid m)\operatorname{sn}(v \mid m)\operatorname{cn}(v \mid m)}$$

09.25.16.0017.01

$$\operatorname{cd}(u+v\mid m)\operatorname{cd}(u-v\mid m)=\frac{\operatorname{cn}(v\mid m)^2-\operatorname{dn}(v\mid m)^2\operatorname{sn}(u\mid m)^2}{\operatorname{dn}(v\mid m)^2-m\operatorname{cn}(v\mid m)^2\operatorname{sn}(u\mid m)^2}$$

Half-angle formulas

09.25.16.0018.01

$$\operatorname{cd}\left(\frac{z}{2}\,\middle|\,m\right)^2=\frac{\operatorname{cn}(z\mid m)+\operatorname{dn}(z\mid m)}{1-m+\operatorname{dn}(z\mid m)+m\operatorname{cn}(z\mid m)}$$

Multiple arguments

Double angle formulas

09.25.16.0019.01

$$\operatorname{cd}(2z\mid m)=\frac{\operatorname{cn}(z\mid m)^2-\operatorname{sn}(z\mid m)^2\operatorname{dn}(z\mid m)^2}{\operatorname{dn}(z\mid m)^2-m\operatorname{sn}(z\mid m)^2\operatorname{cn}(z\mid m)^2}$$

Identities

Functional identities

09.25.17.0001.01

$$m\,w(z)^4-2\,w(z)^2+\left(m\,w(z)^4-2\,m\,w(z)^2+1\right)w(2z)+1=0\;/;\;w(z)=\operatorname{cd}(z\mid m)$$

Complex characteristics

Real part

09.25.19.0001.01

$$\operatorname{Re}(\operatorname{cd}(x+i\,y\mid m))=\frac{\operatorname{cn}(x\mid m)\operatorname{dn}(x\mid m)\operatorname{dn}(y\mid 1-m)\left(\operatorname{cn}(y\mid 1-m)^2+m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)}{\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2+m^2\operatorname{cn}(x\mid m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2}\;/;\;\{x,y,m\}\in\mathbb{R}$$

Imaginary part

09.25.19.0002.01

$$\operatorname{Im}(\operatorname{cd}(x+i\,y\mid m))=\frac{\operatorname{cn}(y\mid 1-m)\left(m\operatorname{cn}(x\mid m)^2-\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2\right)\operatorname{sn}(x\mid m)\operatorname{sn}(y\mid 1-m)}{\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2+m^2\operatorname{cn}(x\mid m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2}\;/;\;\{x,y,m\}\in\mathbb{R}$$

Absolute value

09.25.19.0003.01

$$|\operatorname{cd}(x+i\,y\mid m)|=\sqrt{\frac{\operatorname{cn}(x\mid m)^2\operatorname{cn}(y\mid 1-m)^2+\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2}{\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{dn}(y\mid 1-m)^2+m^2\operatorname{cn}(x\mid m)^2\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2}}\;/;\;\{x,y,m\}\in\mathbb{R}$$

Argument

09.25.19.0004.01

$$\arg(\mathrm{cd}(x+i\,y\mid m)) = \tan^{-1}\Big(\mathrm{cn}(x\mid m)\,\mathrm{dn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)\left(\mathrm{cn}(y\mid 1-m)^2 + m\,\mathrm{sn}(x\mid m)^2\,\mathrm{sn}(y\mid 1-m)^2\right), \mathrm{cn}(y\mid 1-m)\,\mathrm{sn}(x\mid m)\,\mathrm{sn}(y\mid 1-m)\left(m\,\mathrm{cn}(x\mid m)^2 - \mathrm{dn}(x\mid m)^2\,\mathrm{dn}(y\mid 1-m)^2\right)\Big) /;\ \{x,y,m\}\in\mathbb{R}$$

Conjugate value

09.25.19.0005.01

$$\overline{\mathrm{cd}(x+i\,y\mid m)} = \frac{\mathrm{cn}(x\mid m)\,\mathrm{cn}(y\mid 1-m) + i\,\mathrm{dn}(x\mid m)\,\mathrm{dn}(y\mid 1-m)\,\mathrm{sn}(x\mid m)\,\mathrm{sn}(y\mid 1-m)}{\mathrm{cn}(y\mid 1-m)\,\mathrm{dn}(x\mid m)\,\mathrm{dn}(y\mid 1-m) + i\,m\,\mathrm{cn}(x\mid m)\,\mathrm{sn}(x\mid m)\,\mathrm{sn}(y\mid 1-m)} /;\ \{x,y,m\}\in\mathbb{R}$$

Differentiation

Low-order differentiation

With respect to *z*

09.25.20.0001.01

$$\frac{\partial\,\mathrm{cd}(z\mid m)}{\partial z} = (m-1)\,\mathrm{nd}(z\mid m)\,\mathrm{sd}(z\mid m)$$

09.25.20.0002.01

$$\frac{\partial^2\,\mathrm{cd}(z\mid m)}{\partial z^2} = (m-1)\,\mathrm{cd}(z\mid m)\left(\mathrm{nd}(z\mid m)^2 + m\,\mathrm{sd}(z\mid m)^2\right)$$

With respect to *m*

09.25.20.0003.01

$$\frac{\partial\,\mathrm{cd}(z\mid m)}{\partial m} = \frac{((m-1)\,z + E(\mathrm{am}(z\mid m)\mid m))\,\mathrm{nd}(z\mid m)\,\mathrm{sd}(z\mid m)}{2\,m}$$

09.25.20.0004.01

$$\frac{\partial^2\,\mathrm{cd}(z\mid m)}{\partial m^2} = \frac{1}{4\,m^2}\Bigg(-2\,((m-1)\,z + E(\mathrm{am}(z\mid m)\mid m))\,\mathrm{sd}(z\mid m)\,\mathrm{nd}(z\mid m) + 2\,m\,\mathrm{sd}(z\mid m)\Bigg(z + \frac{E(\mathrm{am}(z\mid m)\mid m) - F(\mathrm{am}(z\mid m)\mid m)}{2\,m} + \frac{1}{2\,(m-1)\,m}\left((((m-1)\,z + E(\mathrm{am}(z\mid m)\mid m))\,\mathrm{dn}(z\mid m) - m\,\mathrm{cn}(z\mid m)\,\mathrm{sn}(z\mid m))\sqrt{1-m\,\mathrm{sn}(z\mid m)^2}\right)\Bigg)\mathrm{nd}(z\mid m) + \frac{1}{m-1}\left(((m-1)\,z + E(\mathrm{am}(z\mid m)\mid m))\,\mathrm{cd}(z\mid m)\,\mathrm{nd}(z\mid m)^2\,((m-1)\,z + E(\mathrm{am}(z\mid m)\mid m) - m\,\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m))\right) + \frac{1}{m-1}\left(m\,((m-1)\,z + E(\mathrm{am}(z\mid m)\mid m))\,\mathrm{cd}(z\mid m)\,((m-1)\,z + E(\mathrm{am}(z\mid m)\mid m) - \mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m))\,\mathrm{sd}(z\mid m)^2\right)\Bigg)$$

Symbolic differentiation

With respect to *z*

09.25.20.0007.01

$$\frac{\partial^n\,\mathrm{cd}(z\mid m)}{\partial z^n} = \mathrm{cd}(z\mid m)\,\delta_n + (m-1)\sum_{j=0}^{n-1}\binom{n-1}{j}\frac{\partial^j\,\mathrm{sd}(z\mid m)}{\partial z^j}\,\frac{\partial^{-j+n-1}\,\mathrm{nd}(z\mid m)}{\partial z^{-j+n-1}} /;\ n\in\mathbb{N}$$

09.25.20.0005.02

$$\frac{\partial^n \operatorname{cd}(z \mid m)}{\partial z^n} == \frac{2^{1-n} \pi^{n+1}}{\sqrt{m}\; K(m)^{n+1}} \sum_{k=0}^{\infty} \frac{(-1)^k (2k+1)^n q(m)^{k+\frac{1}{2}}}{1-q(m)^{2k+1}} \cos\left(\frac{\pi n}{2} + \frac{(2k+1)\pi z}{2K(m)}\right) /; n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to *z*

09.25.20.0006.01

$$\frac{\partial^\alpha \operatorname{cd}(z \mid m)}{\partial z^\alpha} == \frac{2^{\alpha+1} \pi^{3/2} z^{-\alpha}}{\sqrt{m}\; K(m)} \sum_{k=0}^{\infty} \frac{(-1)^k q(m)^{k+\frac{1}{2}}}{1-q(m)^{2k+1}} \, {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{(2k+1)^2 \pi^2 z^2}{16 K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.25.21.0001.01

$$\int \operatorname{cd}(z \mid m) \, dz == \frac{\log\left(\operatorname{nd}(z \mid m) + \sqrt{m}\; \operatorname{sd}(z \mid m)\right)}{\sqrt{m}}$$

Representations through equivalent functions

With inverse function

09.25.27.0001.01

$$\operatorname{cd}\left(\operatorname{cd}^{-1}(z \mid m) \,\middle|\, m\right) == z$$

With related functions

Involving am

09.25.27.0026.01

$$\operatorname{cd}(z \mid m)^2 = \frac{\cos^2(\operatorname{am}(z \mid m))}{1 - m \sin^2(\operatorname{am}(z \mid m))}$$

Involving one other Jacobi elliptic function

Involving cn

09.25.27.0004.01

$$\operatorname{cd}(z \mid m)^2 == \frac{\operatorname{cn}(z \mid m)^2}{m \operatorname{cn}(z \mid m)^2 - m + 1}$$

Involving cs

09.25.27.0007.01

$$\operatorname{cd}(z \mid m)^2 == \frac{\operatorname{cs}(z \mid m)^2}{\operatorname{cs}(z \mid m)^2 - m + 1}$$

Involving dc

09.25.27.0008.01

$$\operatorname{cd}(z \mid m) == \frac{1}{\operatorname{dc}(z \mid m)}$$

Involving dn

09.25.27.0010.01

$$\operatorname{cd}(z \mid m)^2 == \frac{\operatorname{dn}(z \mid m)^2 + m - 1}{m \operatorname{dn}(z \mid m)^2}$$

Involving ds

09.25.27.0012.01

$$\operatorname{cd}(z \mid m)^2 == 1 - \frac{1 - m}{\operatorname{ds}(z \mid m)^2}$$

Involving nc

09.25.27.0014.01

$$\operatorname{cd}(z \mid m)^2 == \frac{1}{m - (m - 1) \operatorname{nc}(z \mid m)^2}$$

Involving nd

09.25.27.0015.01

$$\operatorname{cd}(z \mid m) == \operatorname{nd}(i z \mid 1 - m)$$

09.25.27.0016.01

$$\operatorname{cd}(z \mid m)^2 == \frac{1 - (1 - m) \operatorname{nd}(z \mid m)^2}{m}$$

Involving ns

09.25.27.0017.01

$$\operatorname{cd}(z \mid m)^2 == \frac{\operatorname{ns}(z \mid m)^2 - 1}{\operatorname{ns}(z \mid m)^2 - m}$$

Involving sc

09.25.27.0019.01

$$\operatorname{cd}(z \mid m)^2 == \frac{1}{1 - (m - 1) \operatorname{sc}(z \mid m)^2}$$

Involving sd

09.25.27.0020.01

$$\operatorname{cd}(z \mid m)^2 == 1 - (1 - m)\operatorname{sd}(z \mid m)^2$$

Involving sn

09.25.27.0021.01

$$\operatorname{cd}(z \mid m)^2 == \frac{\operatorname{sn}(z \mid m)^2 - 1}{m \operatorname{sn}(z \mid m)^2 - 1}$$

Involving two other Jacobi elliptic functions

Involving cn and dn

09.25.27.0002.01

$$\operatorname{cd}(z \mid m) == \frac{\operatorname{cn}(z \mid m)}{\operatorname{dn}(z \mid m)}$$

Involving cn and nd

09.25.27.0003.01

$$\operatorname{cd}(z \mid m) == \operatorname{cn}(z \mid m)\operatorname{nd}(z \mid m)$$

Involving cs and ds

09.25.27.0005.01

$$\operatorname{cd}(z \mid m) == \frac{\operatorname{cs}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

Involving cs and sd

09.25.27.0006.01

$$\operatorname{cd}(z \mid m) == \operatorname{cs}(z \mid m)\operatorname{sd}(z \mid m)$$

Involving dc and nd

09.25.27.0027.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\left(m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right)}{m}$$

Involving ds and sc

09.25.27.0011.01

$$\operatorname{cd}(z \mid m) == \frac{1}{\operatorname{sc}(z \mid m)\operatorname{ds}(z \mid m)}$$

Involving dn and nc

09.25.27.0028.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)}{m\operatorname{nc}(z \mid m)^2 - \operatorname{nc}(z \mid m)^2 - m}$$

Involving nc and dn

09.25.27.0009.01

$$\operatorname{cd}(z \mid m) = \frac{1}{\operatorname{nc}(z \mid m)\operatorname{dn}(z \mid m)}$$

Involving nc and nd

09.25.27.0013.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.25.27.0029.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\left(m\operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right)}{m\operatorname{nd}(z \mid m)}$$

Involving sc and sd

09.25.27.0018.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{sd}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

Involving three other Jacobi elliptic functions

Involving cn, dn and sn

09.25.27.0030.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)}{1 - m\operatorname{sn}(z \mid m)^2}$$

Involving cs, ds and ns

09.25.27.0031.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{ds}(z \mid m)}{\operatorname{ns}(z \mid m)^2 - m}$$

Involving other functions

09.25.27.0032.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)^2\operatorname{dc}(z \mid m)}{(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1)\operatorname{ds}(z \mid m)^2}$$

09.25.27.0033.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)(\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m))(\operatorname{dn}(z \mid m) + \operatorname{ds}(z \mid m))}{\operatorname{dn}(z \mid m)^2\operatorname{ds}(z \mid m)^2}$$

09.25.27.0034.01

$$\operatorname{cd}(z \mid m) = -\frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right) \operatorname{nc}(z \mid m)}$$

09.25.27.0035.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m)^2 \left(\operatorname{nc}(z \mid m) - 1\right) \left(\operatorname{nc}(z \mid m) + 1\right)}$$

09.25.27.0036.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)}{\operatorname{ds}(z \mid m)^2 \left(\operatorname{nc}(z \mid m) - 1\right) \left(\operatorname{nc}(z \mid m) + 1\right)}$$

09.25.27.0037.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right) \operatorname{nd}(z \mid m)}$$

09.25.27.0038.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)^2 \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)}{\operatorname{cs}(z \mid m)^2 + 1}$$

09.25.27.0039.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)^2 \operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m)^2}{\operatorname{cs}(z \mid m)^2 + 1}$$

09.25.27.0040.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m)^2}{\operatorname{cs}(z \mid m)^2 + 1}$$

09.25.27.0041.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)^2 \operatorname{nc}(z \mid m) \left(\operatorname{nd}(z \mid m) - 1\right) \left(\operatorname{nd}(z \mid m) + 1\right)}{m \operatorname{nd}(z \mid m)}$$

09.25.27.0042.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m) \left(\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)\right)}{m}$$

09.25.27.0043.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dc}(z \mid m) \left(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) - 1\right) \left(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) + 1\right)}{\operatorname{ds}(z \mid m)^2}$$

09.25.27.0044.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m) \left(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) - 1\right) \left(\operatorname{ds}(z \mid m) \operatorname{nd}(z \mid m) + 1\right)}{\operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m)}$$

09.25.27.0045.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dc}(z \mid m) + m \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m) - \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)}{m}$$

09.25.27.0046.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\left(\operatorname{cs}(z \mid m)^2 - m + 1\right) \operatorname{ns}(z \mid m)}$$

09.25.27.0047.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{ns}(z \mid m)}{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}$$

09.25.27.0048.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{cs}(z \mid m)^2 - m + 1}$$

09.25.27.0049.01

$$\operatorname{cd}(z \mid m) = \frac{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right) \operatorname{ns}(z \mid m)}{m \operatorname{cs}(z \mid m) \operatorname{dn}(z \mid m)}$$

09.25.27.0050.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{nd}(z \mid m) \operatorname{ns}(z \mid m)}{\operatorname{cs}(z \mid m)^2 + 1}$$

09.25.27.0051.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1) \operatorname{ns}(z \mid m)}{m \operatorname{nd}(z \mid m)}$$

09.25.27.0052.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dc}(z \mid m) (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}{\operatorname{ds}(z \mid m)^2}$$

09.25.27.0053.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1) (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}{m \operatorname{nd}(z \mid m)}$$

09.25.27.0054.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m) (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}{\operatorname{ns}(z \mid m)^2}$$

09.25.27.0055.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m)^2 (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}{\operatorname{ns}(z \mid m)^2}$$

09.25.27.0056.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m) (\operatorname{ns}(z \mid m) - 1) (\operatorname{ns}(z \mid m) + 1)}{\operatorname{ds}(z \mid m) \operatorname{ns}(z \mid m)}$$

09.25.27.0057.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)^2}{\operatorname{nc}(z \mid m) \left(\operatorname{ns}(z \mid m)^2 - m\right)}$$

09.25.27.0058.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ns}(z \mid m)^2}{\operatorname{nd}(z \mid m) \left(\operatorname{ns}(z \mid m)^2 - m\right)}$$

09.25.27.0059.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{ns}(z \mid m)}{\left(\operatorname{ns}(z \mid m)^2 - m\right) \operatorname{sc}(z \mid m)}$$

09.25.27.0060.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)^2 \operatorname{sc}(z \mid m)}{(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1)\operatorname{ds}(z \mid m)}$$

09.25.27.0061.01

$$\operatorname{cd}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m))(\operatorname{dn}(z \mid m) + \operatorname{ds}(z \mid m))\operatorname{sc}(z \mid m)}{\operatorname{dn}(z \mid m)^2 \operatorname{ds}(z \mid m)}$$

09.25.27.0062.01

$$\operatorname{cd}(z \mid m) = \frac{(\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) - 1)(\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m) + 1)\operatorname{sc}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

09.25.27.0063.01

$$\operatorname{cd}(z \mid m) = \frac{(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)\operatorname{sc}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

09.25.27.0064.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)\operatorname{sc}(z \mid m)}{\operatorname{ns}(z \mid m)}$$

09.25.27.0065.01

$$\operatorname{cd}(z \mid m) = \frac{(\operatorname{nd}(z \mid m) - 1)(\operatorname{nd}(z \mid m) + 1)(\operatorname{ns}(z \mid m) - 1)\operatorname{ns}(z \mid m)(\operatorname{ns}(z \mid m) + 1)\operatorname{sc}(z \mid m)}{m \operatorname{nd}(z \mid m)}$$

09.25.27.0066.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{sc}(z \mid m)^2 + 1}$$

09.25.27.0067.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m)^2}{\operatorname{sc}(z \mid m)^2 + 1}$$

09.25.27.0068.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2 \operatorname{sc}(z \mid m)}{\operatorname{sc}(z \mid m)^2 + 1}$$

09.25.27.0069.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{sc}(z \mid m)^2 + 1}$$

09.25.27.0070.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)}{m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1}$$

09.25.27.0071.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{nc}(z \mid m)\left(m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.25.27.0072.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{nd}(z \mid m)\left(m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.25.27.0073.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{ns}(z \mid m)\operatorname{sc}(z \mid m)\left(m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.25.27.0074.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ns}(z \mid m)}{\left(\operatorname{ns}(z \mid m)^2 - m\right)\operatorname{sd}(z \mid m)}$$

09.25.27.0075.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)\operatorname{sd}(z \mid m)}{\operatorname{ns}(z \mid m)}$$

09.25.27.0076.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)^2 \operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1)}$$

09.25.27.0077.01

$$\operatorname{cd}(z \mid m) = -\frac{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{(\operatorname{dn}(z \mid m) - 1)(\operatorname{dn}(z \mid m) + 1)}$$

09.25.27.0078.01

$$\operatorname{cd}(z \mid m) = \frac{\left(m \operatorname{nd}(z \mid m)^2 - \operatorname{nd}(z \mid m)^2 + 1\right)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{(\operatorname{nd}(z \mid m) - 1)(\operatorname{nd}(z \mid m) + 1)}$$

09.25.27.0079.01

$$\operatorname{cd}(z \mid m) = (\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)$$

09.25.27.0080.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)^2 \operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2}{(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1)}$$

09.25.27.0081.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2}{(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)}$$

09.25.27.0082.01

$$\operatorname{cd}(z \mid m) = \operatorname{dc}(z \mid m)(\operatorname{ns}(z \mid m) - 1)(\operatorname{ns}(z \mid m) + 1)\operatorname{sd}(z \mid m)^2$$

09.25.27.0083.01

$$\operatorname{cd}(z \mid m) = \operatorname{dc}(z \mid m)(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m))(\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))$$

09.25.27.0084.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m))(\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}{\operatorname{nd}(z \mid m)}$$

09.25.27.0085.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{sc}(z \mid m)(\operatorname{nd}(z \mid m) - \operatorname{sd}(z \mid m))(\operatorname{nd}(z \mid m) + \operatorname{sd}(z \mid m))}{\operatorname{sd}(z \mid m)}$$

09.25.27.0086.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)(\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m) - 1)(\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m) + 1)}{\operatorname{dn}(z \mid m)^2}$$

09.25.27.0087.01

$$\mathrm{cd}(z \mid m) = \mathrm{dc}(z \mid m) + m\,\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m) - \mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)$$

09.25.27.0088.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{nd}(z \mid m)}{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}$$

09.25.27.0089.01

$$\mathrm{cd}(z \mid m) = -\frac{\left(\mathrm{dn}(z \mid m)^2 + m - 1\right)\mathrm{sn}(z \mid m)}{\mathrm{cs}(z \mid m)\,(\mathrm{dn}(z \mid m) - 1)\,\mathrm{dn}(z \mid m)\,(\mathrm{dn}(z \mid m) + 1)}$$

09.25.27.0090.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{nc}(z \mid m)\,\mathrm{sn}(z \mid m)}{\mathrm{ds}(z \mid m)\,(\mathrm{nc}(z \mid m) - 1)\,(\mathrm{nc}(z \mid m) + 1)}$$

09.25.27.0091.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}{\left(\mathrm{cs}(z \mid m)^2 - m + 1\right)\mathrm{nd}(z \mid m)}$$

09.25.27.0092.01

$$\mathrm{cd}(z \mid m) = -\frac{\left(\mathrm{sc}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}{\mathrm{nd}(z \mid m)\,\mathrm{sc}(z \mid m)\left(m\,\mathrm{sc}(z \mid m)^2 - \mathrm{sc}(z \mid m)^2 - 1\right)}$$

09.25.27.0093.01

$$\mathrm{cd}(z \mid m) = -\frac{\mathrm{nc}(z \mid m)\,\mathrm{sn}(z \mid m)}{\left(m\,\mathrm{nc}(z \mid m)^2 - \mathrm{nc}(z \mid m)^2 - m\right)\mathrm{sd}(z \mid m)}$$

09.25.27.0094.01

$$\mathrm{cd}(z \mid m) = -\mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m)\,(\mathrm{sn}(z \mid m) - 1)\,(\mathrm{sn}(z \mid m) + 1)$$

09.25.27.0095.01

$$\mathrm{cd}(z \mid m) = -\frac{\mathrm{dc}(z \mid m)\,(\mathrm{sn}(z \mid m) - 1)\,(\mathrm{sn}(z \mid m) + 1)}{\mathrm{ds}(z \mid m)^2\,\mathrm{sn}(z \mid m)^2}$$

09.25.27.0096.01

$$\mathrm{cd}(z \mid m) = -\frac{\mathrm{sc}(z \mid m)\,(\mathrm{sn}(z \mid m) - 1)\,(\mathrm{sn}(z \mid m) + 1)}{\mathrm{ds}(z \mid m)\,\mathrm{sn}(z \mid m)^2}$$

09.25.27.0097.01

$$\mathrm{cd}(z \mid m) = -\frac{\mathrm{sc}(z \mid m)\,\mathrm{sd}(z \mid m)\,(\mathrm{sn}(z \mid m) - 1)\,(\mathrm{sn}(z \mid m) + 1)}{\mathrm{sn}(z \mid m)^2}$$

09.25.27.0098.01

$$\mathrm{cd}(z \mid m) = -\frac{\mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)^2\,(\mathrm{sn}(z \mid m) - 1)\,(\mathrm{sn}(z \mid m) + 1)}{\mathrm{sn}(z \mid m)^2}$$

09.25.27.0099.01

$$\mathrm{cd}(z \mid m) = -\frac{\mathrm{nd}(z \mid m)\,(\mathrm{sn}(z \mid m) - 1)\,(\mathrm{sn}(z \mid m) + 1)}{\mathrm{cs}(z \mid m)\,\mathrm{sn}(z \mid m)}$$

09.25.27.0100.01

$$\mathrm{cd}(z \mid m) = -\frac{\mathrm{cn}(z \mid m)}{\mathrm{nd}(z \mid m)\left(m\,\mathrm{sn}(z \mid m)^2 - 1\right)}$$

09.25.27.0101.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sn}(z \mid m)}{\operatorname{sd}(z \mid m)\left(m\operatorname{sn}(z \mid m)^2 - 1\right)}$$

Involving four other Jacobi elliptic functions

09.25.27.0102.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m)^2 (\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}$$

09.25.27.0103.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)\operatorname{dn}(z \mid m) - \operatorname{ds}(z \mid m)^2 \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{ds}(z \mid m)^2}$$

09.25.27.0104.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\left(\operatorname{ds}(z \mid m)^2 \operatorname{nd}(z \mid m) - \operatorname{dn}(z \mid m)\right)}{\operatorname{ds}(z \mid m)^2}$$

09.25.27.0105.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)^2 \operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m)^2}$$

09.25.27.0106.01

$$\operatorname{cd}(z \mid m) = \frac{(\operatorname{dn}(z \mid m) + m\operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{ns}(z \mid m)}{m\operatorname{cs}(z \mid m)}$$

09.25.27.0107.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)(\operatorname{ds}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{dn}(z \mid m))}{\operatorname{ds}(z \mid m)^2}$$

09.25.27.0108.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m)^2}$$

09.25.27.0109.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sc}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

09.25.27.0110.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)(\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sc}(z \mid m))}{\operatorname{ns}(z \mid m)}$$

09.25.27.0111.01

$$\operatorname{cd}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{ns}(z \mid m)}{m\operatorname{sc}(z \mid m)}$$

09.25.27.0112.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{ds}(z \mid m)(\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}$$

09.25.27.0113.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m)}{\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.25.27.0114.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m)^2}{\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.25.27.0115.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2}{\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.25.27.0116.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.25.27.0117.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - m\operatorname{sc}(z \mid m)}$$

09.25.27.0118.01

$$\operatorname{cd}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{dn}(z \mid m)}{\operatorname{ns}(z \mid m)(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.25.27.0119.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m)}$$

09.25.27.0120.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{dn}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{nc}(z \mid m)(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.25.27.0121.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{nd}(z \mid m)(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.25.27.0122.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{ds}(z \mid m)\operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)\operatorname{ds}(z \mid m)}$$

09.25.27.0123.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ns}(z \mid m)\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.25.27.0124.01

$$\operatorname{cd}(z \mid m) = \operatorname{sc}(z \mid m)\left(\operatorname{ds}(z \mid m)\operatorname{nd}(z \mid m)^2 - \operatorname{sd}(z \mid m)\right)$$

09.25.27.0125.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)(\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sd}(z \mid m))}{\operatorname{ns}(z \mid m)}$$

09.25.27.0126.01

$$\operatorname{cd}(z \mid m) = \operatorname{sc}(z \mid m)(\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sd}(z \mid m))$$

09.25.27.0127.01

$$\operatorname{cd}(z \mid m) = (\operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m) - \operatorname{sc}(z \mid m))\operatorname{sd}(z \mid m)$$

09.25.27.0128.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.25.27.0129.01

$$\operatorname{cd}(z \mid m) = -\frac{(\operatorname{dn}(z \mid m) + m\operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.25.27.0130.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)^2}{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.25.27.0131.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ns}(z \mid m)}{\operatorname{nd}(z \mid m)\operatorname{ns}(z \mid m) - m\operatorname{sd}(z \mid m)}$$

09.25.27.0132.01

$$\operatorname{cd}(z \mid m) = -\operatorname{sd}(z \mid m)(\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{nc}(z \mid m)\operatorname{ns}(z \mid m))$$

09.25.27.0133.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{sc}(z \mid m)\left(\operatorname{dn}(z \mid m)^2\operatorname{sd}(z \mid m) - \operatorname{ds}(z \mid m)\right)}{\operatorname{dn}(z \mid m)^2}$$

09.25.27.0134.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)(\operatorname{dn}(z \mid m) + m\operatorname{ns}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{ns}(z \mid m)\operatorname{sd}(z \mid m))}{m}$$

09.25.27.0135.01

$$\operatorname{cd}(z \mid m) = \operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)$$

09.25.27.0136.01

$$\operatorname{cd}(z \mid m) = \operatorname{dc}(z \mid m)\operatorname{nd}(z \mid m)^2 - \operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)$$

09.25.27.0137.01

$$\operatorname{cd}(z \mid m) = \operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m) + m\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m)$$

09.25.27.0138.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{nd}(z \mid m)}{\operatorname{cn}(z \mid m)}$$

09.25.27.0139.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{dn}(z \mid m)}$$

09.25.27.0140.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)^2\operatorname{sc}(z \mid m)\operatorname{sd}(z \mid m) - \operatorname{dc}(z \mid m)}{\operatorname{dn}(z \mid m)^2}$$

09.25.27.0141.01

$$\operatorname{cd}(z \mid m) = \frac{1}{\operatorname{nc}(z \mid m)}\left(\operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)^2 + m\operatorname{nc}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{nc}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sc}(z \mid m) + \operatorname{dn}(z \mid m)\right)$$

09.25.27.0142.01

$$\operatorname{cd}(z \mid m) = \frac{1}{\operatorname{nd}(z \mid m)}\left(\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)^2 + m\operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sc}(z \mid m) - \operatorname{nd}(z \mid m)\operatorname{sd}(z \mid m)\operatorname{sc}(z \mid m) + \operatorname{cn}(z \mid m)\right)$$

09.25.27.0143.01

$$\mathrm{cd}(z \mid m) = \mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{dc}(z \mid m)\,\mathrm{sd}(z \mid m)^2$$

09.25.27.0144.01

$$\mathrm{cd}(z \mid m) = -\frac{\mathrm{dc}(z \mid m)\,\mathrm{dn}(z \mid m)\,\mathrm{sd}(z \mid m)^2 - \mathrm{nc}(z \mid m)}{\mathrm{dn}(z \mid m)}$$

09.25.27.0145.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{ns}(z \mid m)\left(\mathrm{nc}(z \mid m)\,\mathrm{ns}(z \mid m)^2\,\mathrm{sd}(z \mid m)^2 - \mathrm{nc}(z \mid m)\,\mathrm{sd}(z \mid m)^2 - \mathrm{cn}(z \mid m)\right)}{m\,\mathrm{sd}(z \mid m)}$$

09.25.27.0146.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{cs}(z \mid m)\,\mathrm{nc}(z \mid m) - \mathrm{sn}(z \mid m)}{\mathrm{cs}(z \mid m)\,\mathrm{dn}(z \mid m)}$$

09.25.27.0147.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,(\mathrm{cs}(z \mid m)\,\mathrm{nc}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{cs}(z \mid m)}$$

09.25.27.0148.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,(\mathrm{cs}(z \mid m)\,\mathrm{dc}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{cs}(z \mid m)}$$

09.25.27.0149.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{nc}(z \mid m)\,(\mathrm{ds}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{ds}(z \mid m)}$$

09.25.27.0150.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,(\mathrm{ds}(z \mid m)\,\mathrm{nd}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{cs}(z \mid m)}$$

09.25.27.0151.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{ns}(z \mid m) - \mathrm{sn}(z \mid m)}{\mathrm{cs}(z \mid m)\,\mathrm{dn}(z \mid m)}$$

09.25.27.0152.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{nc}(z \mid m)\,(\mathrm{ns}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{ds}(z \mid m)}$$

09.25.27.0153.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{nd}(z \mid m)\,(\mathrm{ns}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{cs}(z \mid m)}$$

09.25.27.0154.01

$$\mathrm{cd}(z \mid m) = \frac{\mathrm{nc}(z \mid m)\,\mathrm{nd}(z \mid m)\,(\mathrm{ns}(z \mid m) - \mathrm{sn}(z \mid m))}{\mathrm{ns}(z \mid m)}$$

09.25.27.0155.01

$$\mathrm{cd}(z \mid m) = -\frac{(\mathrm{dn}(z \mid m) + m\,\mathrm{nd}(z \mid m) - \mathrm{nd}(z \mid m))\,\mathrm{sn}(z \mid m)}{\mathrm{cs}(z \mid m)\,(\mathrm{dn}(z \mid m) - 1)\,(\mathrm{dn}(z \mid m) + 1)}$$

09.25.27.0156.01

$$\mathrm{cd}(z \mid m) = \frac{\left(\mathrm{cs}(z \mid m)^2 + 1\right)\mathrm{sn}(z \mid m)}{\mathrm{nd}(z \mid m)\,(\mathrm{cs}(z \mid m) - m\,\mathrm{sc}(z \mid m) + \mathrm{sc}(z \mid m))}$$

09.25.27.0157.01

$$\operatorname{cd}(z \mid m) = -\frac{(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)) \operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m) \left(m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.25.27.0158.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m) (m \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) + \operatorname{sn}(z \mid m))}{m \operatorname{sd}(z \mid m)}$$

09.25.27.0159.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m) \left(\operatorname{sd}(z \mid m)^2 \operatorname{ns}(z \mid m)^3 - \operatorname{sd}(z \mid m)^2 \operatorname{ns}(z \mid m) - \operatorname{ns}(z \mid m) + \operatorname{sn}(z \mid m)\right)}{m \operatorname{sd}(z \mid m)}$$

09.25.27.0160.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{sd}(z \mid m) (\operatorname{ns}(z \mid m) - m \operatorname{sn}(z \mid m))}$$

09.25.27.0161.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{ds}(z \mid m) \operatorname{nc}(z \mid m) - \operatorname{dc}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{ds}(z \mid m)^2 \operatorname{sn}(z \mid m)}$$

09.25.27.0162.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m) \operatorname{dc}(z \mid m) - \operatorname{dn}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m) \operatorname{dn}(z \mid m)^2}$$

09.25.27.0163.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{dn}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{ds}(z \mid m)}{\operatorname{cs}(z \mid m) \operatorname{dn}(z \mid m)^2}$$

09.25.27.0164.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{cn}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m) \left(m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.25.27.0165.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{nc}(z \mid m)}{\operatorname{ds}(z \mid m) \operatorname{sn}(z \mid m)}$$

09.25.27.0166.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{sd}(z \mid m) (\operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{sn}(z \mid m)}$$

09.25.27.0167.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{dc}(z \mid m)}{\operatorname{cn}(z \mid m) - m \operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m)}$$

09.25.27.0168.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{sc}(z \mid m) (\operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m) - \operatorname{nd}(z \mid m))}{\operatorname{sn}(z \mid m)}$$

09.25.27.0169.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)}{\operatorname{nd}(z \mid m) - m \operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m)}$$

09.25.27.0170.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\,\operatorname{dc}(z \mid m) + m\,\operatorname{sd}(z \mid m)\,\operatorname{sn}(z \mid m) - \operatorname{sd}(z \mid m)\,\operatorname{sn}(z \mid m)}{\operatorname{cn}(z \mid m)}$$

09.25.27.0171.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{sd}(z \mid m)\,(\operatorname{dc}(z \mid m)\,\operatorname{sd}(z \mid m)\,\operatorname{sn}(z \mid m) - \operatorname{nc}(z \mid m))}{\operatorname{sn}(z \mid m)}$$

Involving five other Jacobi elliptic functions

09.25.27.0172.01

$$\operatorname{cd}(z \mid m) = -\frac{1}{m\,\operatorname{sd}(z \mid m)}\left(-\operatorname{nc}(z \mid m)\,\operatorname{ns}(z \mid m)\,\operatorname{nd}(z \mid m)^2 + \operatorname{nc}(z \mid m)\,\operatorname{sd}(z \mid m)\,\operatorname{nd}(z \mid m) + \operatorname{cn}(z \mid m)\,\operatorname{ns}(z \mid m)\right)$$

09.25.27.0173.01

$$\operatorname{cd}(z \mid m) = \frac{1}{\operatorname{nd}(z \mid m)}\,(\operatorname{cn}(z \mid m) + \operatorname{dn}(z \mid m)\,\operatorname{sc}(z \mid m)\,\operatorname{sd}(z \mid m) + m\,\operatorname{nd}(z \mid m)\,\operatorname{sc}(z \mid m)\,\operatorname{sd}(z \mid m) - \operatorname{nd}(z \mid m)\,\operatorname{sc}(z \mid m)\,\operatorname{sd}(z \mid m))$$

09.25.27.0174.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\,\operatorname{dc}(z \mid m)\,\operatorname{nd}(z \mid m) - \operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)\,\operatorname{dn}(z \mid m)}$$

09.25.27.0175.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{ds}(z \mid m)\,\operatorname{nd}(z \mid m) - \operatorname{sn}(z \mid m)}{\operatorname{cs}(z \mid m)\,\operatorname{dn}(z \mid m)}$$

09.25.27.0176.01

$$\operatorname{cd}(z \mid m) = -\frac{\operatorname{nc}(z \mid m)\left(-\operatorname{ns}(z \mid m)\,\operatorname{nd}(z \mid m)^2 + \operatorname{sd}(z \mid m)\,\operatorname{nd}(z \mid m) + \operatorname{ns}(z \mid m) - \operatorname{sn}(z \mid m)\right)}{m\,\operatorname{sd}(z \mid m)}$$

09.25.27.0177.01

$$\operatorname{cd}(z \mid m) = -\frac{-\operatorname{nc}(z \mid m)\,\operatorname{nd}(z \mid m)^2 + \operatorname{nc}(z \mid m)\,\operatorname{sd}(z \mid m)^2 + \operatorname{cn}(z \mid m)}{m\,\operatorname{sd}(z \mid m)\,\operatorname{sn}(z \mid m)}$$

09.25.27.0178.01

$$\operatorname{cd}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\,\operatorname{ns}(z \mid m)^2\,\operatorname{sd}(z \mid m)^2 - \operatorname{nc}(z \mid m)\,\operatorname{sd}(z \mid m)^2 - \operatorname{cn}(z \mid m)}{m\,\operatorname{sd}(z \mid m)\,\operatorname{sn}(z \mid m)}$$

09.25.27.0179.01

$$\operatorname{cd}(z \mid m) = -\operatorname{sn}(z \mid m)\,/\,(\operatorname{cs}(z \mid m)\,\operatorname{dn}(z \mid m) - \operatorname{cs}(z \mid m)\,\operatorname{nd}(z \mid m) + m\,\operatorname{nd}(z \mid m)\,\operatorname{sc}(z \mid m) - \operatorname{nd}(z \mid m)\,\operatorname{sc}(z \mid m))$$

09.25.27.0180.01

$$\operatorname{cd}(z \mid m) = \frac{1}{\operatorname{nd}(z \mid m)}\,(\operatorname{cn}(z \mid m) + m\,\operatorname{nd}(z \mid m)\,\operatorname{sc}(z \mid m)\,\operatorname{sd}(z \mid m) - \operatorname{nd}(z \mid m)\,\operatorname{sc}(z \mid m)\,\operatorname{sd}(z \mid m) + \operatorname{sc}(z \mid m)\,\operatorname{sn}(z \mid m))$$

Involving Weierstrass functions

09.25.27.0022.01

$$\operatorname{cd}(z \mid m) = \frac{\sigma_1\left(\frac{z}{\sqrt{e_1-e_3}};\, g_2,\, g_3\right)}{\sigma_2\left(\frac{z}{\sqrt{e_1-e_3}};\, g_2,\, g_3\right)} \;/;$$

$$\{\omega_1,\, \omega_2,\, \omega_3\} = \{\omega_1(g_2,\, g_3),\, -\omega_1(g_2,\, g_3) - \omega_3(g_2,\, g_3),\, \omega_3(g_2,\, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n;\, g_2,\, g_3) \bigwedge n \in \{1,\, 2,\, 3\}$$

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$$\operatorname{cd}(z \mid m)^2 = \frac{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_1}{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_2} \,/;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

Involving theta functions

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$$\operatorname{cd}(z \mid m) = \frac{1}{\sqrt[4]{m}} \frac{\vartheta_2\left(\frac{z\pi}{2K(m)}, q(m)\right)}{\vartheta_3\left(\frac{z\pi}{2K(m)}, q(m)\right)}$$

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$$\operatorname{cd}(z \mid m) = \frac{\vartheta_c(z \mid m)}{\vartheta_d(z \mid m)}$$

Zeros

09.25.30.0001.01

$$\operatorname{cd}((2r+1)K(m) + 2 s i K(1-m) \mid m) = 0 \,/; \{r, s\} \in \mathbb{Z}$$

History

– C. G. J. Jacobi (1827)
– N. H. Abel (1827)
– J. Glaisher (1882) introduced the notations cd, cs, dc, ds, nc, nd, ns, sc, and sd.

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