

InverseErf2

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Notations

Traditional name

Inverse of the generalized error function

Traditional notation

$$\operatorname{erf}^{-1}(z_1, z_2)$$

Mathematica StandardForm notation

`InverseErf[z1, z2]`

Primary definition

06.30.02.0001.01

$$z_2 = \operatorname{erf}(z_1, w) /; w = \operatorname{erf}^{-1}(z_1, z_2)$$

Specific values

Specialized values

06.30.03.0001.01

$$\operatorname{erf}^{-1}(0, z) = \operatorname{erf}^{-1}(z)$$

Values at fixed points

06.30.03.0002.01

$$\operatorname{erf}^{-1}(0, 0) = 0$$

06.30.03.0003.01

$$\operatorname{erf}^{-1}(0, 1) = \infty$$

06.30.03.0004.01

$$\operatorname{erf}^{-1}(1, 0) = 1$$

Values at infinities

06.30.03.0005.01

$$\operatorname{erf}^{-1}(\infty, z) = \operatorname{erfc}^{-1}(-z)$$

General characteristics

Domain and analyticity

$\operatorname{erf}^{-1}(z_1, z_2)$ is an analytical function of z_1 and z_2 which is defined in \mathbb{C}^2 .

06.30.04.0001.01

$$(z_1 * z_2) \rightarrow \operatorname{erf}^{-1}(z_1, z_2) :: (\mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at $z_1 = 0$

06.30.06.0001.01

$$\operatorname{erf}^{-1}(z_1, z_2) \propto \operatorname{erf}^{-1}(z_2) + e^{\operatorname{erf}^{-1}(z_2)^2} \left(z_1 + e^{\operatorname{erf}^{-1}(z_2)^2} \operatorname{erf}^{-1}(z_2) z_1^2 + \left(4 e^{2 \operatorname{erf}^{-1}(z_2)^2} \operatorname{erf}^{-1}(z_2)^2 + e^{2 \operatorname{erf}^{-1}(z_2)^2} - 1 \right) \frac{z_1^3}{3} + e^{\operatorname{erf}^{-1}(z_2)^2} \operatorname{erf}^{-1}(z_2) \left(12 e^{2 \operatorname{erf}^{-1}(z_2)^2} \operatorname{erf}^{-1}(z_2)^2 + 7 e^{2 \operatorname{erf}^{-1}(z_2)^2} - 4 \right) \frac{z_1^4}{6} + \left(96 e^{4 \operatorname{erf}^{-1}(z_2)^2} \operatorname{erf}^{-1}(z_2)^4 - 40 e^{2 \operatorname{erf}^{-1}(z_2)^2} \operatorname{erf}^{-1}(z_2)^2 + 92 e^{4 \operatorname{erf}^{-1}(z_2)^2} \operatorname{erf}^{-1}(z_2)^2 - 10 e^{2 \operatorname{erf}^{-1}(z_2)^2} + 7 e^{4 \operatorname{erf}^{-1}(z_2)^2} + 3 \right) \frac{z_1^5}{30} + \dots \right) /; (z_1 \rightarrow 0)$$

06.30.06.0002.01

$$\operatorname{erf}^{-1}(z_1, z_2) \propto \operatorname{erf}^{-1}(z_2) + e^{\operatorname{erf}^{-1}(z_2)^2} z_1 (1 + O(z_1)) /; (z_1 \rightarrow 0)$$

Expansions at $z_2 = 0$

06.30.06.0003.01

$$\operatorname{erf}^{-1}(z_1, z_2) \propto z_1 + \frac{1}{2} e^{z_1^2} \sqrt{\pi} z_2 + \frac{\pi z_1}{4} e^{2 z_1^2} z_2^2 + \frac{\pi^{3/2} (1 + 4 z_1^2)}{24} e^{3 z_1^2} z_2^3 + \frac{\pi^2 z_1 (7 + 12 z_1^2)}{96} e^{4 z_1^2} z_2^4 + \frac{\pi^{5/2} (7 + 8 z_1^2) (1 + 12 z_1^2)}{960} e^{5 z_1^2} z_2^5 + \frac{\pi^3 z_1 (127 + 652 z_1^2 + 480 z_1^4)}{5760} e^{6 z_1^2} z_2^6 + \frac{\pi^{7/2} (127 + 3480 z_1^2 + 10224 z_1^4 + 5760 z_1^6)}{80640} e^{7 z_1^2} z_2^7 + \frac{\pi^4 z_1 (4369 + 44808 z_1^2 + 88848 z_1^4 + 40320 z_1^6)}{645120} e^{8 z_1^2} z_2^8 + \dots /; (z_2 \rightarrow 0)$$

06.30.06.0004.01

$$\operatorname{erf}^{-1}(z_1, z_2) \propto z_1 (1 + O(z_2)) /; (z_2 \rightarrow 0)$$

Differential equations

Ordinary nonlinear differential equations

06.30.13.0001.01

$$w''(z_2) - 2 w(z_2) w'(z_2)^2 = 0 \ ; \ w(z_2) = \operatorname{erf}^{-1}(z_1, z_2)$$

Differentiation

Low-order differentiation

With respect to z_1

06.30.20.0001.01

$$\frac{\partial \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_1} = e^{\operatorname{erf}^{-1}(z_1, z_2)^2 - z_1^2}$$

06.30.20.0002.01

$$\frac{\partial^2 \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_1^2} = e^{\operatorname{erf}^{-1}(z_1, z_2)^2 - z_1^2} \left(2 e^{\operatorname{erf}^{-1}(z_1, z_2)^2 - z_1^2} \operatorname{erf}^{-1}(z_1, z_2) - 2 z_1 \right)$$

With respect to z_2

06.30.20.0003.01

$$\frac{\partial \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_2} = \frac{\sqrt{\pi}}{2} e^{\operatorname{erf}^{-1}(z_1, z_2)^2}$$

06.30.20.0004.01

$$\frac{\partial^2 \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_2^2} = \frac{\pi}{2} e^{2 \operatorname{erf}^{-1}(z_1, z_2)^2} \operatorname{erf}^{-1}(z_1, z_2)$$

Symbolic differentiation

With respect to z_2

06.30.20.0005.01

$$\frac{\partial^n \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_2^n} = \operatorname{erf}^{-1}(z_1, z_2) \delta_n + \frac{\pi^{n/2}}{2^n} e^{n \operatorname{erf}^{-1}(z_1, z_2)^2} \sum_{j_2=0}^n \dots \sum_{j_n=0}^n \delta_{\sum_{i=2}^n (i-1) j_i, n-1} (-1)^{\sum_{i=2}^n j_i} \left(n + \sum_{i=2}^n j_i - 1 \right)!$$

$$\prod_{i=2}^n \frac{1}{j_i!} \left(\frac{2^{i-1} e^{\operatorname{erf}^{-1}(z_1, z_2)^2} \sqrt{\pi} \operatorname{erf}^{-1}(z_1, z_2)^{1-i}}{i!} \right)^{j_i} {}_2\tilde{F}_2 \left(\frac{1}{2}, 1; 1 - \frac{i}{2}, \frac{3-i}{2}; -\operatorname{erf}^{-1}(z_1, z_2)^2 \right)^{j_i} \ ; \ n \in \mathbb{N}$$

Integration

Indefinite integration

Involving one direct function with respect to z_2

06.30.21.0001.01

$$\int \operatorname{erf}^{-1}(z_1, z_2) dz_2 = -\frac{1}{\sqrt{\pi}} e^{-\operatorname{erf}^{-1}(z_1, z_2)^2}$$

Representations through equivalent functions

With inverse function

06.30.27.0001.01

$$\operatorname{erf}(z_1, \operatorname{erf}^{-1}(z_1, z_2)) = z_2$$

Zeros

06.30.30.0001.01

$$\operatorname{erf}^{-1}(z_1, z_2) = 0 /; z_1 = 0 \wedge z_2 = 0$$

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