

# InverseErf

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## Notations

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### Traditional name

Inverse error function

### Traditional notation

$$\operatorname{erf}^{-1}(z)$$

### Mathematica StandardForm notation

InverseErf[z]

## Primary definition

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06.29.02.0001.01

$$z = \operatorname{erf}(w) \ ; \ w = \operatorname{erf}^{-1}(z)$$

## Specific values

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### Values at fixed points

06.29.03.0001.01

$$\operatorname{erf}^{-1}(0) = 0$$

06.29.03.0002.01

$$\operatorname{erf}^{-1}(1) = \infty$$

## General characteristics

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### Domain and analyticity

$\operatorname{erf}^{-1}(z)$  is an analytical function of  $z$  which is defined in the whole complex  $z$ -plane.

06.29.04.0001.01

$$z \rightarrow \operatorname{erf}^{-1}(z) :: \mathbb{C} \rightarrow \mathbb{C}$$

### Symmetries and periodicities

#### Symmetry

No symmetry

**Periodicity**

No periodicity

**Series representations**

**Generalized power series**

Expansions at generic point  $z = z_0$

**For the function itself**

06.29.06.0004.01

$$\begin{aligned} \operatorname{erf}^{-1}(z) \propto & \operatorname{erf}^{-1}(z_0) + \frac{\sqrt{\pi}}{2} e^{\operatorname{erf}^{-1}(z_0)^2} (z - z_0) + \frac{\pi}{4} e^{2 \operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) (z - z_0)^2 + \frac{\pi^{3/2}}{24} e^{3 \operatorname{erf}^{-1}(z_0)^2} \left(4 \operatorname{erf}^{-1}(z_0)^2 + 1\right) (z - z_0)^3 + \\ & \frac{\pi^2}{96} e^{4 \operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) \left(12 \operatorname{erf}^{-1}(z_0)^2 + 7\right) (z - z_0)^4 + \frac{\pi^{5/2}}{960} e^{5 \operatorname{erf}^{-1}(z_0)^2} \left(8 \operatorname{erf}^{-1}(z_0)^2 + 7\right) \left(12 \operatorname{erf}^{-1}(z_0)^2 + 1\right) (z - z_0)^5 + \\ & \frac{\pi^3}{5760} e^{6 \operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) \left(480 \operatorname{erf}^{-1}(z_0)^4 + 652 \operatorname{erf}^{-1}(z_0)^2 + 127\right) (z - z_0)^6 + \\ & \frac{\pi^{7/2}}{80640} e^{7 \operatorname{erf}^{-1}(z_0)^2} \left(5760 \operatorname{erf}^{-1}(z_0)^6 + 10224 \operatorname{erf}^{-1}(z_0)^4 + 3480 \operatorname{erf}^{-1}(z_0)^2 + 127\right) (z - z_0)^7 + \\ & \frac{\pi^4}{645120} e^{8 \operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) \left(40320 \operatorname{erf}^{-1}(z_0)^6 + 88848 \operatorname{erf}^{-1}(z_0)^4 + 44808 \operatorname{erf}^{-1}(z_0)^2 + 4369\right) (z - z_0)^8 + \\ & \frac{\pi^{9/2}}{11612160} e^{9 \operatorname{erf}^{-1}(z_0)^2} \left(645120 \operatorname{erf}^{-1}(z_0)^8 + 1703808 \operatorname{erf}^{-1}(z_0)^6 + 1161168 \operatorname{erf}^{-1}(z_0)^4 + 204328 \operatorname{erf}^{-1}(z_0)^2 + 4369\right) \\ & (z - z_0)^9 + \frac{\pi^5}{116121600} e^{10 \operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) \left(5806080 \operatorname{erf}^{-1}(z_0)^8 + 17914752 \operatorname{erf}^{-1}(z_0)^6 + \right. \\ & \left. 15561936 \operatorname{erf}^{-1}(z_0)^4 + 4161288 \operatorname{erf}^{-1}(z_0)^2 + 243649\right) (z - z_0)^{10} + \dots /; (z \rightarrow z_0) \end{aligned}$$

06.29.06.0005.01

$$\operatorname{erf}^{-1}(z) \propto \operatorname{erf}^{-1}(z_0) (1 + O(z - z_0))$$

**Expansions at  $z = 0$**

06.29.06.0001.02

$$\operatorname{erf}^{-1}(z) \propto \frac{\sqrt{\pi}}{2} \left( z + \frac{\pi z^3}{12} + \frac{7 \pi^2 z^5}{480} + \dots \right) /; (z \rightarrow 0)$$

06.29.06.0006.01

$$\operatorname{erf}^{-1}(z) \propto \frac{\sqrt{\pi}}{2} \left( z + \frac{\pi z^3}{12} + \frac{7 \pi^2 z^5}{480} \right) + O(z^7)$$

06.29.06.0007.01

$$\operatorname{erf}^{-1}(z) \propto \frac{\sqrt{\pi}}{2} z \left( 1 + \frac{\pi}{12} z^2 + \frac{7 \pi^2}{480} z^4 + \frac{127 \pi^3}{40320} z^6 + \frac{4369 \pi^4}{5806080} z^8 \right) + O(z^{11})$$

06.29.06.0003.01

$$\operatorname{erf}^{-1}(z) = \sum_{k=0}^{\infty} \frac{c_k}{2k+1} \left( \frac{\sqrt{\pi}}{2} z \right)^{2k+1} \quad ; c_0 = 1 \wedge c_k = \sum_{m=0}^{k-1} \frac{c_m c_{k-1-m}}{(m+1)(2m+1)}$$

Gyorgy Steinbrecher

06.29.06.0008.01

$$\operatorname{erf}^{-1}(z) = \frac{\sqrt{\pi} z}{2} + O(z^3)$$

## Asymptotic series expansions

06.29.06.0002.01

$$\operatorname{erf}^{-1}(z) \sim \frac{1}{\sqrt{2}} \sqrt{\log\left(\frac{2}{\pi(z-1)^2}\right) - \log\left(\log\left(\frac{2}{\pi(z-1)^2}\right)\right)} \quad ; (z \rightarrow 1)$$

## Differential equations

### Ordinary nonlinear differential equations

06.29.13.0001.01

$$w''(z) - 2w(z)w'(z)^2 = 0 \quad ; w(z) = \operatorname{erf}^{-1}(z)$$

## Differentiation

### Low-order differentiation

06.29.20.0001.01

$$\frac{\partial \operatorname{erf}^{-1}(z)}{\partial z} = \frac{\sqrt{\pi}}{2} e^{\operatorname{erf}^{-1}(z)^2}$$

06.29.20.0002.01

$$\frac{\partial^2 \operatorname{erf}^{-1}(z)}{\partial z^2} = \frac{1}{2} e^{2w^2} \pi w \quad ; w = \operatorname{erf}^{-1}(z)$$

06.29.20.0003.01

$$\frac{\partial^3 \operatorname{erf}^{-1}(z)}{\partial z^3} = \frac{1}{4} e^{3w^2} \pi^{3/2} (4w^2 + 1) \quad ; w = \operatorname{erf}^{-1}(z)$$

06.29.20.0004.01

$$\frac{\partial^4 \operatorname{erf}^{-1}(z)}{\partial z^4} = \frac{1}{4} e^{4w^2} \pi^2 w (12w^2 + 7) \quad ; w = \operatorname{erf}^{-1}(z)$$

### Symbolic differentiation

06.29.20.0005.01

$$\frac{\partial^n \operatorname{erf}^{-1}(z)}{\partial z^n} = \operatorname{erf}^{-1}(z) \delta_n + \frac{\pi^{n/2}}{2^n} e^{n \operatorname{erf}^{-1}(z)^2} \sum_{j_2=0}^n \dots \sum_{j_n=0}^n \delta_{\sum_{i=2}^n (i-1)j_i, n-1} (-1)^{\sum_{i=2}^n 2j_i} \left( n + \sum_{i=2}^n j_i - 1 \right)!$$

$$\prod_{i=2}^n \frac{1}{j_i!} \left( \frac{2^{i-1} e^{\operatorname{erf}^{-1}(z)^2} \sqrt{\pi} \operatorname{erf}^{-1}(z)^{1-i}}{i!} \right)^{j_i} {}_2\tilde{F}_2 \left( \frac{1}{2}, 1; 1 - \frac{i}{2}, \frac{3-i}{2}; -\operatorname{erf}^{-1}(z)^2 \right)^{j_i} /; n \in \mathbb{N}$$

## Integration

### Indefinite integration

Involving only one direct function

06.29.21.0001.01

$$\int \operatorname{erf}^{-1}(z) dz = -\frac{1}{\sqrt{\pi}} e^{-\operatorname{erf}^{-1}(z)^2}$$

## Representations through more general functions

### Through other functions

06.29.26.0001.01

$$\operatorname{erf}^{-1}(z) = \operatorname{erf}^{-1}(0, z)$$

## Representations through equivalent functions

### With inverse function

06.29.27.0001.01

$$\operatorname{erf}(\operatorname{erf}^{-1}(z)) = z$$

### With related functions

06.29.27.0002.01

$$\operatorname{erf}^{-1}(z) = \operatorname{erfc}^{-1}(1 - z)$$

## Zeros

06.29.30.0001.01

$$\operatorname{erf}^{-1}(z) = 0 /; z = 0$$

## History

- J. R. Philip (1960)
- A.J. Strecok (1968)

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