Im

Notations

Traditional name

Imaginary part

Traditional notation

$\mathrm{Im}(z)$

Mathematica StandardForm notation

`Im[z]`

Primary definition

Im (z) gives the imaginary part of the number z.

Specific values

Specialized values

12.04.03.0001.01

$\mathrm{Im}(x) = 0 \,/;\, x \in \mathbb{R}$

12.04.03.0002.01

$\mathrm{Im}(i\, x) = x \,/;\, x \in \mathbb{R}$

12.04.03.0003.01

$\mathrm{Im}(x + i\, y) = y \,/;\, x \in \mathbb{R} \bigwedge y \in \mathbb{R}$

Values at fixed points

12.04.03.0004.01

$\mathrm{Im}(0) = 0$

12.04.03.0005.01

$\mathrm{Im}(1) = 0$

12.04.03.0006.01

$\mathrm{Im}(-1) = 0$

12.04.03.0007.01

$\mathrm{Im}(i) = 1$

12.04.03.0008.01
$\operatorname{Im}(-i) = -1$

12.04.03.0020.01
$\operatorname{Im}(1 + i) = 1$

12.04.03.0021.01
$\operatorname{Im}(-1 + i) = 1$

12.04.03.0022.01
$\operatorname{Im}(-1 - i) = -1$

12.04.03.0023.01
$\operatorname{Im}(1 - i) = -1$

12.04.03.0024.01
$\operatorname{Im}\left(\sqrt{3} + i\right) = 1$

12.04.03.0025.01
$\operatorname{Im}\left(1 + i \sqrt{3}\right) = \sqrt{3}$

12.04.03.0026.01
$\operatorname{Im}\left(-1 + i \sqrt{3}\right) = \sqrt{3}$

12.04.03.0027.01
$\operatorname{Im}\left(-\sqrt{3} + i\right) = 1$

12.04.03.0028.01
$\operatorname{Im}\left(-\sqrt{3} - i\right) = -1$

12.04.03.0029.01
$\operatorname{Im}\left(-1 - i \sqrt{3}\right) = -\sqrt{3}$

12.04.03.0030.01
$\operatorname{Im}\left(1 - i \sqrt{3}\right) = \sqrt{3}$

12.04.03.0031.01
$\operatorname{Im}\left(\sqrt{3} - i\right) = -1$

12.04.03.0009.01
$\operatorname{Im}(2) = 0$

12.04.03.0010.01
$\operatorname{Im}(-2) = 0$

12.04.03.0011.01
$\operatorname{Im}(\pi) = 0$

12.04.03.0012.01
$\operatorname{Im}(3 i) = 3$

12.04.03.0013.01
$\operatorname{Im}(-2 i) = -2$

12.04.03.0014.01
$\operatorname{Im}(2 + i) = 1$

Values at infinities

12.04.03.0015.01

$\mathrm{Im}(\infty) == 0$

12.04.03.0016.01

$\mathrm{Im}(-\infty) == 0$

12.04.03.0017.01

$\mathrm{Im}(i\,\infty) == \infty$

12.04.03.0018.01

$\mathrm{Im}(-i\,\infty) == -\infty$

12.04.03.0019.01

$\mathrm{Im}(\tilde{\infty}) ==$ ¿

General characteristics

Domain and analyticity

$\mathrm{Im}(z)$ is a nonanalytical function; it is a real-analytic function of the variable z.

12.04.04.0001.01

$z \longrightarrow \mathrm{Im}(z) :: \mathbb{C} \longrightarrow \mathbb{R}$

Symmetries and periodicities

Parity

$\mathrm{Im}(z)$ is an odd function.

12.04.04.0002.01

$\mathrm{Im}(-z) == -\mathrm{Im}(z)$

Mirror symmetry

12.04.04.0003.01

$\mathrm{Im}(\bar{z}) == -\overline{\mathrm{Im}(z)}$

Periodicity

No periodicity

Homogeneity

12.04.04.0004.01

$\mathrm{Im}(a\,z) == a\,\mathrm{Im}(z)\;/;\; a \in \mathbb{R}$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

12.04.16.0001.01

$$\operatorname{Im}(-z) = -\operatorname{Im}(z)$$

12.04.16.0002.01

$$\operatorname{Im}(a\, z) = a \operatorname{Im}(z)\; /;\; a \in \mathbb{R}$$

12.04.16.0003.01

$$\operatorname{Im}(i\, x) = x\; /;\; x \in \mathbb{R}$$

12.04.16.0004.01

$$\operatorname{Im}(i\, z) = \operatorname{Re}(z)$$

12.04.16.0005.01

$$\operatorname{Im}(-i\, z) = -\operatorname{Re}(z)$$

12.04.16.0006.01

$$\operatorname{Im}\left(\frac{1}{z}\right) = -\frac{\operatorname{Im}(z)}{|z|^2}$$

Addition formulas

12.04.16.0007.01

$$\operatorname{Im}(x + i\, y) = y\; /;\; x \in \mathbb{R} \bigwedge y \in \mathbb{R}$$

12.04.16.0008.01

$$\operatorname{Im}\left(\sum_{k=1}^{n} z_k\right) = \sum_{k=1}^{n} \operatorname{Im}(z_k)$$

12.04.16.0009.01

$$\operatorname{Im}(z_1 + z_2) = \operatorname{Im}(z_1) + \operatorname{Im}(z_2)$$

Multiple arguments

12.04.16.0010.01

$$\operatorname{Im}(a\, z) = a \operatorname{Im}(z)\; /;\; a \in \mathbb{R}$$

12.04.16.0011.01

$$\operatorname{Im}(i\, x) = x\; /;\; x \in \mathbb{R}$$

12.04.16.0012.01

$$\operatorname{Im}(i\, z) = \operatorname{Re}(z)$$

12.04.16.0013.01

$$\operatorname{Im}(-i\, z) = -\operatorname{Re}(z)$$

12.04.16.0014.01

$$\operatorname{Im}(z_1\, z_2) = \operatorname{Im}(z_2) \operatorname{Re}(z_1) + \operatorname{Im}(z_1) \operatorname{Re}(z_2)$$

Ratio of arguments

12.04.16.0022.01

$$\operatorname{Im}\left(\frac{z_1}{z_2}\right) = \frac{\operatorname{Im}(z_1) \operatorname{Re}(z_2) - \operatorname{Re}(z_1) \operatorname{Im}(z_2)}{|z_2|^2}$$

Power of arguments

12.04.16.0015.01

$$\operatorname{Im}(x^a) = x^{\operatorname{Re}(a)} \sin(\operatorname{Im}(a) \log(x)) /; x \in \mathbb{R} \land x > 0$$

12.04.16.0016.01

$$\operatorname{Im}(z^a) = |z|^a \sin\left(a \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\right) /; a \in \mathbb{R}$$

12.04.16.0017.01

$$\operatorname{Im}(z^a) = |z|^a \sin(a \arg(z)) /; a \in \mathbb{R}$$

12.04.16.0018.01

$$\operatorname{Im}(z^n) = \sum_{j=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} (-1)^j \binom{n}{2j+1} \operatorname{Im}(z)^{2j+1} \operatorname{Re}(z)^{n-2j-1} /; n \in \mathbb{N}^+$$

12.04.16.0019.01

$$\operatorname{Im}(z^a) = |z|^{\operatorname{Re}(a)} e^{-\operatorname{Im}(a) \arg(z)} \sin(\operatorname{Im}(a) \log(|z|) + \arg(z) \operatorname{Re}(a))$$

12.04.16.0020.01

$$\operatorname{Im}(z^a) = \exp\left(-\operatorname{Im}(a) \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\right) |z|^{\operatorname{Re}(a)} \sin\left(\operatorname{Im}(a) \log(|z|) + \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)) \operatorname{Re}(a)\right)$$

Exponent of arguments

12.04.16.0023.01

$$\operatorname{Re}\left(e^{x+i y}\right) = e^x \cos(y)$$

12.04.16.0024.01

$$\operatorname{Im}(e^z) = e^{\operatorname{Re}(z)} \sin(\operatorname{Im}(z))$$

12.04.16.0025.01

$$\operatorname{Im}\left(e^{i z}\right) = e^{-\operatorname{Im}(z)} \sin(\operatorname{Re}(z))$$

Products, sums, and powers of the direct function

Sums of the direct function

12.04.16.0021.01

$$\operatorname{Im}(z_1) + \operatorname{Im}(z_2) = \operatorname{Im}(z_1 + z_2)$$

Complex characteristics

Real part

12.04.19.0001.01

$$\operatorname{Re}(\operatorname{Im}(x + i y)) = y$$

12.04.19.0002.01

$$\operatorname{Re}(\operatorname{Im}(z)) = \operatorname{Im}(z)$$

Imaginary part

12.04.19.0003.01

$$\operatorname{Im}(\operatorname{Im}(x + i y)) = 0$$

12.04.19.0004.01

$$\operatorname{Im}(\operatorname{Im}(z)) = 0$$

Absolute value

12.04.19.0005.01

$$|\mathrm{Im}(x+i\,y)| = \sqrt{y^2}$$

12.04.19.0009.01

$$|\mathrm{Im}(z)| = \sqrt{\mathrm{Im}(z)^2}$$

Argument

12.04.19.0006.01

$$\arg(\mathrm{Im}(x+i\,y)) = \tan^{-1}(y,\,0)$$

12.04.19.0010.01

$$\arg(\mathrm{Im}(x+i\,y)) = (1-\theta(y))\,\pi$$

12.04.19.0011.01

$$\arg(\mathrm{Im}(z)) = \tan^{-1}(\mathrm{Im}(z),\,0)$$

12.04.19.0012.01

$$\arg(\mathrm{Im}(z)) = (1-\theta(\mathrm{Im}(z)))\,\pi$$

Conjugate value

12.04.19.0007.01

$$\overline{\mathrm{Im}(x+i\,y)} = y$$

12.04.19.0008.01

$$\overline{\mathrm{Im}(z)} = \mathrm{Im}(z)$$

Signum value

12.04.19.0013.01

$$\mathrm{sgn}(\mathrm{Im}(x+i\,y)) = \mathrm{sgn}(y)$$

12.04.19.0014.01

$$\mathrm{sgn}(\mathrm{Im}(x+i\,y)) = \frac{y}{\sqrt{y^2}}$$

12.04.19.0015.01

$$\mathrm{sgn}(\mathrm{Im}(z)) = \frac{\mathrm{Im}(z)}{\sqrt{\mathrm{Im}(z)^2}}$$

Differentiation

Low-order differentiation

In a distributional sense for $x \in \mathbb{R}$.

12.04.20.0001.01

$$\frac{\partial\,\mathrm{Im}(x)}{\partial x} = 0$$

Representations through equivalent functions

With related functions

With Re

12.04.27.0002.01

$$\operatorname{Im}(z) = -\operatorname{Re}(i z)$$

12.04.27.0007.01

$$\operatorname{Im}(z) = i (\operatorname{Re}(z) - z)$$

12.04.27.0003.01

$$\operatorname{Im}(i z) = \operatorname{Re}(z)$$

With Abs

12.04.27.0008.01

$$\operatorname{Im}(z) = \frac{i \left(|z|^2 - z^2\right)}{2 z}$$

With Arg

12.04.27.0009.01

$$\operatorname{Im}(z) = \frac{1}{2} i e^{-2 i \arg(z)} \left(1 - e^{2 i \arg(z)}\right) z$$

12.04.27.0001.01

$$\operatorname{Im}(z) = |z| \sin(\arg(z))$$

With Conjugate

12.04.27.0004.01

$$\operatorname{Im}(z) = \frac{z - \bar{z}}{2 i}$$

12.04.27.0006.01

$$\operatorname{Im}(z) = i (\bar{z} - \operatorname{Re}(z))$$

With Sign

12.04.27.0010.01

$$\operatorname{Im}(z) = \frac{i z \left(1 - \operatorname{sgn}(z)^2\right)}{2 \operatorname{sgn}(z)^2}$$

12.04.27.0005.01

$$\operatorname{Im}(z) = \frac{z \sin(\arg(z))}{\operatorname{sgn}(z)} \;/;\; z \neq 0$$

Inequalities

12.04.29.0001.01

$$|\operatorname{Im}(z)| \leq |z|$$

Zeros

12.04.30.0001.01

$\mathrm{Im}(z) == 0 \;/;\; z \in \mathbb{R}$

History

The function Im is encountered often in mathematics and the natural sciences.

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