Gamma2

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Notations

Traditional name

Incomplete gamma function

Traditional notation

$\Gamma(a, z)$

Mathematica StandardForm notation

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Gamma[a, z]
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Primary definition

06.06.02.0001.01

$$\Gamma(a, z) = \int_z^\infty t^{a-1} e^{-t} \, dt$$

Specific values

Specialized values

For fixed *a*

06.06.03.0001.01

$$\Gamma(a, 0) = \tilde{\infty} \;/;\; \operatorname{Re}(a) < 0$$

06.06.03.0002.01

$$\Gamma(a, 0) = \Gamma(a) \;/;\; \operatorname{Re}(a) > 0$$

06.06.03.0015.01

$$\Gamma(a, -1) = e \, \operatorname{Subfactorial}(a - 1)$$

For fixed *z*

06.06.03.0003.01

$$\Gamma(0, z) = -\operatorname{Ei}(-z) + \frac{1}{2}\left(\log(-z) - \log\left(-\frac{1}{z}\right)\right) - \log(z)$$

06.06.03.0004.01

$$\Gamma\left(\frac{1}{2}, z\right) = \sqrt{\pi} \, \operatorname{erfc}\left(\sqrt{z}\right)$$

06.06.03.0005.01

$$\Gamma\left(n+\frac{1}{2}, z\right) = \Gamma\left(n+\frac{1}{2}\right)\operatorname{erfc}\left(\sqrt{z}\right) + (-1)^{n-1} e^{-z} \sqrt{z} \sum_{k=0}^{n-1} \left(\frac{1}{2}-n\right)_{n-k-1} (-z)^k \;/;\; n \in \mathbb{N}$$

06.06.03.0006.01

$$\Gamma\left(-\frac{1}{2}, z\right) = \frac{2 e^{-z}}{\sqrt{z}} - 2\sqrt{\pi} \operatorname{erfc}\left(\sqrt{z}\right)$$

06.06.03.0007.01

$$\Gamma\left(\frac{1}{2}-n, z\right) = \frac{(-1)^n \sqrt{\pi}}{(1/2)_n} \operatorname{erfc}\left(\sqrt{z}\right) - z^{\frac{1}{2}-n} e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{(1/2-n)_{k+1}} \;/;\; n \in \mathbb{N}$$

06.06.03.0008.01

$$\Gamma(1, z) = e^{-z}$$

06.06.03.0009.01

$$\Gamma(n, z) = (n-1)!\, e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{k!} \;/;\; n \in \mathbb{N}^+$$

06.06.03.0010.01

$$\Gamma(-1, z) = \operatorname{Ei}(-z) + \frac{1}{2}\left(\log\left(-\frac{1}{z}\right) - \log(-z)\right) + \log(z) + \frac{e^{-z}}{z}$$

06.06.03.0011.01

$$\Gamma(-n, z) = \frac{(-1)^{n-1}}{n!}\left(\operatorname{Ei}(-z) - \frac{1}{2}\left(\log(-z) - \log\left(-\frac{1}{z}\right)\right) + \log(z)\right) - e^{-z} \sum_{k=1}^{n} \frac{z^{k-n-1}}{(-n)_k} \;/;\; n \in \mathbb{N}$$

06.06.03.0013.01

$$\Gamma(n, z) = \frac{(-1)^{n-1}}{(-n)!}\left(\operatorname{Ei}(-z) - \frac{1}{2}\left(\log(-z) - \log\left(-\frac{1}{z}\right)\right) + \log(z)\right) + e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{(n)_{k-n+1}} - e^{-z} \sum_{k=n}^{-1} \frac{z^k}{(n)_{k-n+1}} \;/;\; n \in \mathbb{Z}$$

06.06.03.0014.01

$$\Gamma\left(n+\frac{1}{2}, z\right) = \operatorname{erfc}\left(\sqrt{z}\right)\Gamma\left(n+\frac{1}{2}\right) + e^{-z} \sum_{k=0}^{n-1} \frac{z^{k+\frac{1}{2}}}{\left(n+\frac{1}{2}\right)_{k-n+1}} - e^{-z} \sum_{k=n}^{-1} \frac{z^{k+\frac{1}{2}}}{\left(n+\frac{1}{2}\right)_{k-n+1}} \;/;\; n \in \mathbb{Z}$$

Values at infinities

06.06.03.0012.01

$$\Gamma(a, \infty) = 0$$

General characteristics

Domain and analyticity

$\Gamma(a, z)$ is an analytical function of a and z which is defined in $\mathbb{C}^2$. For fixed z, it is an entire function of a.

06.06.04.0001.01

$$(a * z) \longrightarrow \Gamma(a, z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

06.06.04.0002.01

$\Gamma(\overline{a}, \overline{z}) = \overline{\Gamma(a, z)} /; z \notin (-\infty, 0)$

Periodicity

No periodicity

Poles and essential singularities

With respect to *z*

For fixed a, the function $\Gamma(a, z)$ has an essential singularity at $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is a branch point for generic a.

06.06.04.0003.01

$\mathcal{Sing}_z(\Gamma(a, z)) = \{\{\tilde{\infty}, \infty\}\}$

With respect to *a*

For fixed z, the function $\Gamma(a, z)$ has only one singular point at $a = \tilde{\infty}$. It is an essential singular point.

06.06.04.0004.01

$\mathcal{Sing}_a(\Gamma(a, z)) = \{\{\tilde{\infty}, \infty\}\}$

Branch points

With respect to *z*

For fixed a, not being positive integer, the function $\Gamma(a, z)$ has two branch points: $z = 0$, $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is an essential singularity.

06.06.04.0005.01

$\mathcal{BP}_z(\Gamma(a, z)) = \{0, \tilde{\infty}\} /; a \notin \mathbb{N}^+$

06.06.04.0006.01

$\mathcal{R}_z(\Gamma(a, z), 0) = \log /; a \notin \mathbb{Q}$

06.06.04.0007.01

$\mathcal{R}_z\left(\Gamma\left(\frac{p}{q}, z\right), 0\right) = q /; p \in \mathbb{Z} \wedge q - 1 \in \mathbb{N}^+ \wedge \gcd(p, q) = 1$

06.06.04.0008.01

$\mathcal{R}_z(\Gamma(a, z), \tilde{\infty}) = \log /; a \notin \mathbb{Q}$

06.06.04.0009.01

$\mathcal{R}_z\left(\Gamma\left(\frac{p}{q}, z\right), \tilde{\infty}\right) = q /; p \in \mathbb{Z} \wedge q - 1 \in \mathbb{N}^+ \wedge \gcd(p, q) = 1$

With respect to *a*

For fixed z, the function $\Gamma(a, z)$ does not have branch points.

06.06.04.0010.01

$\mathcal{BP}_a(\Gamma(a, z)) = \{\}$

Branch cuts

With respect to *z*

For fixed a, not being a positive integer, the function $\Gamma(a, z)$ has one infinitely long branch cut. It is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

06.06.04.0011.01

$$\mathcal{BC}_z(\Gamma(a, z)) == \{\{(-\infty, 0), -i\}\}$$

06.06.04.0012.01

$$\lim_{\epsilon\to+0} \Gamma(a, x + i\,\epsilon) == \Gamma(a, x) \;/;\; x < 0$$

06.06.04.0013.01

$$\lim_{\epsilon\to+0} \Gamma(a, x - i\,\epsilon) == \Gamma(a) - e^{-2\,i\,\pi\,a}\,(\Gamma(a) - \Gamma(a, x)) \;/;\; x < 0$$

With respect to *a*

For fixed z, the function $\Gamma(a, z)$ does not have branch cuts.

06.06.04.0014.01

$$\mathcal{BC}_a(\Gamma(a, z)) == \{\}$$

Series representations

Generalized power series

Expansions at generic point $a == a_0$

For the function itself

06.06.06.0017.01

$$\Gamma(a, z) \propto \Gamma(a_0, z) + \left(\Gamma(a_0, z)\log(z) + \Gamma(a_0)\,(\psi(a_0) - \log(z)) + \frac{z^{a_0}}{a_0^2}\,{}_2F_2(a_0, a_0; a_0+1, a_0+1; -z)\right)(a - a_0) - \frac{1}{2}\left(\Gamma(a_0)\left(\log^2(z) - \psi(a_0)^2 - \psi^{(1)}(a_0)\right) - \Gamma(a_0, z)\log^2(z) + \frac{2\,z^{a_0}}{a_0^3}\,({}_3F_3(a_0, a_0, a_0; a_0+1, a_0+1, a_0+1; -z) - a_0\log(z)\,{}_2F_2(a_0, a_0; a_0+1, a_0+1; -z))\right)(a - a_0)^2 + \ldots \;/;\; (a \to a_0)$$

06.06.06.0018.01

$$\Gamma(a, z) \propto \Gamma(a_0, z) + \left(\Gamma(a_0, z)\log(z) + \Gamma(a_0)\,(\psi(a_0) - \log(z)) + \frac{z^{a_0}}{a_0^2}\,{}_2F_2(a_0, a_0; a_0+1, a_0+1; -z)\right)(a - a_0) - \frac{1}{2}\left(\Gamma(a_0)\left(\log^2(z) - \psi(a_0)^2 - \psi^{(1)}(a_0)\right) - \Gamma(a_0, z)\log^2(z) + \frac{2\,z^{a_0}}{a_0^3}\,({}_3F_3(a_0, a_0, a_0; a_0+1, a_0+1, a_0+1; -z) - a_0\log(z)\,{}_2F_2(a_0, a_0; a_0+1, a_0+1; -z))\right)(a - a_0)^2 + O\left((a - a_0)^3\right)$$

06.06.06.0019.01

$$\Gamma(a,z) = \sum_{k=0}^{\infty}\left(\frac{\Gamma^{(k)}(a_0)}{k!} - z^{a_0}\sum_{j=0}^{k}\frac{(-1)^{k-j}\,\Gamma(a_0)^{k-j+1}\log^j(z)}{j!}\,{}_{k-j+1}\tilde{F}_{k-j+1}\left(c_1, c_2, \ldots, c_{k-j+1}; c_1+1, c_2+1, \ldots, c_{k-j+1}+1; -z\right)\right)(a-a_0)^k \,/;$$

$$c_1 = c_2 = \ldots = c_{k+1} = a_0 \bigwedge k \in \mathbb{N}$$

06.06.06.0020.01

$$\Gamma(a,z) \propto \Gamma(a_0, z)\,(1 + O(a - a_0))$$

Expansions at generic point $z = z_0$

For the function itself

06.06.06.0021.01

$$\Gamma(a,z) \propto -\frac{2\,i\,e^{-i a \pi}\,\pi}{\Gamma(1-a)}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor + \left(\frac{1}{z_0}\right)^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left(\Gamma(a,z_0) - e^{-z_0}\,z_0^{a-1}(z-z_0) + \frac{1}{2}\,e^{-z_0}\,(z_0-a+1)\,z_0^{a-2}(z-z_0)^2 + \ldots\right) /;\ (z \to z_0)$$

06.06.06.0022.01

$$\Gamma(a,z) \propto -\frac{2\,i\,e^{-i a \pi}\,\pi}{\Gamma(1-a)}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor + \left(\frac{1}{z_0}\right)^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left(\Gamma(a,z_0) - e^{-z_0}\,z_0^{a-1}(z-z_0) + \frac{1}{2}\,e^{-z_0}\,(z_0-a+1)\,z_0^{a-2}(z-z_0)^2 + O\left((z-z_0)^3\right)\right)$$

06.06.06.0023.01

$$\Gamma(a,z) = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(-1)^{k-j}\,(1-a)_j}{j!\,(k-j)!}\left((-1)^j\left(\frac{1}{z_0}\right)^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\Gamma(a-j, z_0) - \frac{2\,i\,\pi\,e^{-a\pi i}}{\Gamma(-a+j+1)}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor\right)(z-z_0)^k$$

06.06.06.0024.01

$$\Gamma(a,z) = \left(\frac{1}{z_0}\right)^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left(\Gamma(a,z_0) + e^{-z_0}\sum_{k=1}^{\infty}\sum_{j=0}^{k-1}\frac{(-1)^k\,(1-a)_j}{k\,j!\,(-j+k-1)!}\,z_0^{a-j-1}(z-z_0)^k\right) - \frac{2\,\pi\,i\,e^{-i a \pi}}{\Gamma(1-a)}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor$$

06.06.06.0025.01

$$\Gamma(a,z) \propto -\frac{2\,i\,e^{-i a \pi}\,\pi}{\Gamma(1-a)}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0)+\pi}{2\pi}\right\rfloor + \left(\frac{1}{z_0}\right)^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor} z_0^{a\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left(\Gamma(a,z_0) + O(z-z_0)\right)$$

Expansions on branch cuts

For the function itself

06.06.06.0026.01

$$\Gamma(a, z) \propto -\frac{2\pi i e^{-i a \pi}}{\Gamma(1-a)} \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + e^{2 i a \pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(\Gamma(a, x) - e^{-x} x^{a-1} (z-x) + \frac{1}{2} e^{-x} (x-a+1) x^{a-2} (z-x)^2 + \ldots\right) /; (z \to x) \land x \in \mathbb{R} \land x < 0$$

06.06.06.0027.01

$$\Gamma(a, z) \propto -\frac{2\pi i e^{-i a \pi}}{\Gamma(1-a)} \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + e^{2 i a \pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(\Gamma(a, x) - e^{-x} x^{a-1} (z-x) + \frac{1}{2} e^{-x} (x-a+1) x^{a-2} (z-x)^2 + O\left((z-x)^3\right)\right) /; x \in \mathbb{R} \land x < 0$$

06.06.06.0028.01

$$\Gamma(a, z) = e^{2\pi i a \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(\Gamma(a, x) + e^{-x} \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \frac{(-1)^k (1-a)_j}{k\, j!\, (-j+k-1)!} x^{a-j-1} (z-x)^k\right) - \frac{2 i e^{-i a \pi} \pi}{\Gamma(1-a)} \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor /; x \in \mathbb{R} \land x < 0$$

06.06.06.0029.01

$$\Gamma(a, z) \propto -\frac{2\pi i e^{-i a \pi}}{\Gamma(1-a)} \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + e^{2 i a \pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} (\Gamma(a, x) + O(z-x)) /; x \in \mathbb{R} \land x < 0$$

Expansions at $z = 0$

For the function itself

General case

06.06.06.0001.02

$$\Gamma(a, z) \propto \Gamma(a) - \frac{z^a}{a} \left(1 - \frac{a z}{a+1} + \frac{a z^2}{2(a+2)} - \ldots\right) /; (z \to 0)$$

06.06.06.0030.01

$$\Gamma(a, z) \propto \Gamma(a) - \frac{z^a}{a} \left(1 - \frac{a z}{a+1} + \frac{a z^2}{2(a+2)} - O\left(z^3\right)\right)$$

06.06.06.0002.01

$$\Gamma(a, z) = \Gamma(a) - z^a \sum_{k=0}^{\infty} \frac{(-z)^k}{(a+k)\, k!}$$

06.06.06.0003.01

$$\Gamma(a, z) = \Gamma(a) - \frac{z^a}{a} {}_1F_1(a; a+1; -z)$$

06.06.06.0004.02

$$\Gamma(a, z) \propto \Gamma(a) - \frac{z^a}{a} (1 + O(z))$$

06.06.06.0031.01

$$\Gamma(a, z) = F_\infty(z, a) /; \left(\left(F_n(z, a) = \Gamma(a) - z^a \sum_{k=0}^{n} \frac{(-z)^k}{(a+k)\, k!} = \Gamma(a, z) - \frac{(-1)^n z^{a+n+1}}{(a+n+1)(n+1)!} {}_2F_2(1, a+n+1; n+2, a+n+2; -z)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

06.06.06.0032.01

$$\Gamma(1, z) \propto 1 - z + O(z^3)$$

06.06.06.0033.01

$$\Gamma(2, z) \propto 1 - \frac{z^2}{2} + O(z^3)$$

06.06.06.0034.01

$$\Gamma(n, z) \propto (n-1)!\left(1 - \frac{z^n}{n!} + O(z^{n+1})\right) /; n \in \mathbb{N}^+$$

06.06.06.0005.01

$$\Gamma(n, z) = (n-1)!\, e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{k!} /; n \in \mathbb{N}^+$$

06.06.06.0006.02

$$\Gamma(n, z) \propto (n-1)!\,(1 + O(z^n)) /; n \in \mathbb{N}^+$$

06.06.06.0035.01

$$\Gamma(0, z) \propto -\log(z) - \gamma + z - \frac{z^2}{4} + \frac{z^3}{18} + O(z^4)$$

06.06.06.0036.01

$$\Gamma(-1, z) \propto \log(z) + \frac{1}{z} + \gamma - 1 - \frac{z}{2} + \frac{z^2}{12} - \frac{z^3}{72} + O(z^4)$$

06.06.06.0037.01

$$\Gamma(-2, z) \propto -\frac{\log(z)}{2} + \frac{1}{2z^2} - \frac{1}{z} - \gamma + \frac{3}{4} + \frac{z}{6} - \frac{z^2}{48} + \frac{z^3}{360} + O(z^4)$$

06.06.06.0007.01

$$\Gamma(-n, z) = \frac{(-1)^n}{n!}(\psi(n+1) - \log(z)) - z^{-n} \sum_{\substack{k=0 \\ k \neq n}}^{\infty} \frac{(-z)^k}{(k-n)\, k!} /; n \in \mathbb{N}$$

06.06.06.0008.01

$$\Gamma(-n, z) = \frac{(-1)^n}{n!}(\psi(n+1) - \log(z)) - z^{-n} \sum_{k=0}^{n-1} \frac{(-z)^k}{(k-n)\, k!} + \frac{(-1)^n z}{(n+1)!}\, {}_2F_2(1, 1; 2, n+2; -z) /; n \in \mathbb{N}$$

06.06.06.0038.01

$$\Gamma(-n, z) = \frac{(-1)^{n-1}}{n!}\left(\text{Ei}(-z) - \frac{1}{2}\left(\log(-z) - \log\left(-\frac{1}{z}\right)\right) + \log(z)\right) - z^{-n-1} e^{-z} \sum_{k=1}^{n} \frac{z^k}{(-n)_k} /; n \in \mathbb{N}$$

06.06.06.0009.02

$$\Gamma(0, z) \propto -\log(z) + O(1)$$

06.06.06.0010.02

$$\Gamma(-1, z) \propto \log(z) + \frac{1 + O(z)}{z}$$

06.06.06.0011.02

$$\Gamma(-2, z) \propto \frac{1 + O(z)}{2 z^2}$$

06.06.06.0012.01

$$\Gamma(-n, z) = \frac{(-1)^n}{n!} (\psi(n+1) - \log(z)) + \frac{z^{-n}}{n} (1 + O(z)) /; n - 1 \in \mathbb{N}^+$$

Asymptotic series expansions

Expansions at $a = \infty$

06.06.06.0039.01

$$\Gamma(a, z) \propto \sqrt{2\pi}\, a^{a-\frac{1}{2}} e^{-a} \left(1 + \frac{1}{12 a} + \frac{1}{288 a^2} + O\left(\frac{1}{a^3}\right)\right) - \frac{e^{-z} z^a}{a} \left(1 + \frac{z}{a} + \frac{(z-1) z}{a^2} + O\left(\frac{1}{a^3}\right)\right) /; (|a| \to \infty)$$

06.06.06.0040.01

$$\Gamma(a, z) \propto \Gamma(a) - \sum_{j=0}^{\infty} (-1)^j a^{-j-1} \sum_{k=1}^{\infty} \frac{k^j (-z)^k}{k!} z^a - \frac{z^a}{a} /; (|a| \to \infty)$$

06.06.06.0041.01

$$\Gamma(a, z) \propto \sqrt{2\pi}\, a^{a-\frac{1}{2}} e^{-a} \left(1 + O\left(\frac{1}{a}\right)\right) - \frac{e^{-z} z^a}{a} \left(1 + O\left(\frac{1}{a}\right)\right) /; (|a| \to \infty)$$

Expansions at $z = \infty$

06.06.06.0042.01

$$\Gamma(a, z) \propto e^{-z} z^{a-1} \left(1 - \frac{1-a}{z} + \frac{(2-a)(1-a)}{z^2} + O\left(\frac{1}{z^3}\right)\right) /; (|z| \to \infty)$$

06.06.06.0043.01

$$\Gamma(a, z) \propto e^{-z} z^{a-1} \left(\sum_{k=0}^{n} (-1)^k (1-a)_k z^{-k} + O\left(\frac{1}{z^{n+1}}\right)\right) /; (|z| \to \infty) \land n \in \mathbb{N}$$

06.06.06.0044.01

$$\Gamma(a, z) \propto e^{-z} z^{a-1} \sum_{k=0}^{\infty} (-1)^k (1-a)_k z^{-k} /; (|z| \to \infty)$$

06.06.06.0013.01

$$\Gamma(a, z) \propto e^{-z} z^{a-1} {}_2F_0\left(1, 1-a; ; -\frac{1}{z}\right) /; (|z| \to \infty)$$

06.06.06.0045.01

$$\Gamma(-n, z) = e^{-z} \left(\frac{(-1)^n}{n!\, z} {}_2F_0\left(1, 1; ; -\frac{1}{z}\right) - z^{-n-1} \sum_{k=1}^{n} \frac{z^k}{(-n)_k}\right) /; n \in \mathbb{N} \land (|z| \to \infty)$$

06.06.06.0014.01

$$\Gamma(a, z) \propto e^{-z} z^{a-1} \left(1 + O\left(\frac{1}{z}\right)\right) /; (|z| \to \infty)$$

Residue representations

06.06.06.0015.02

$$\Gamma(a, z) = \Gamma(a) + \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{z^{-s}}{s}\, \Gamma(a+s)\right)(-a-j)$$

06.06.06.0016.02

$$\Gamma(a, z) = \operatorname{res}_s\left(\Gamma(a+s)\, z^{-s}\, \frac{1}{s}\right)(0) + \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{z^{-s}}{s}\, \Gamma(a+s)\right)(-a-j)$$

Integral representations

On the real axis

Of the direct function

06.06.07.0001.01

$$\Gamma(a, z) = \int_z^{\infty} t^{a-1}\, e^{-t}\, dt$$

Contour integral representations

06.06.07.0002.01

$$\Gamma(a, z) = \Gamma(a) - \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma(s+a)\, \Gamma(-s)}{\Gamma(1-s)}\, z^{-s}\, ds$$

06.06.07.0003.01

$$\Gamma(a, z) = \Gamma(a) - \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\Gamma(s+a)\, \Gamma(-s)}{\Gamma(1-s)}\, z^{-s}\, ds \;/;\; -\operatorname{Re}(a) < \gamma < 1 \bigwedge |\arg(z)| < \frac{\pi}{2}$$

06.06.07.0004.01

$$\Gamma(a, z) = \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma(s+a)\, \Gamma(s)\, z^{-s}}{\Gamma(s+1)}\, ds$$

06.06.07.0005.01

$$\Gamma(a, z) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\Gamma(s+a)\, \Gamma(s)\, z^{-s}}{\Gamma(s+1)}\, ds \;/;\; \max(-\operatorname{Re}(a), 0) < \gamma \bigwedge |\arg(z)| < \frac{\pi}{2}$$

Continued fraction representations

06.06.10.0001.01

$$\Gamma(a, z) = \cfrac{z^a\, e^{-z}}{z + \cfrac{1-a}{1 + \cfrac{1}{z + \cfrac{2-a}{1 + \cfrac{2}{z + \cfrac{3-a}{1 + \cfrac{3}{z + \ldots}}}}}}} \;/;\; z \notin (-\infty, 0)$$

06.06.10.0002.01

$$\Gamma(a, z) = \frac{z^a e^{-z}}{z + \mathrm{K}_k\left(2^{\frac{1}{2}(-1-(-1)^k)} k^{\frac{1}{2}(1+(-1)^k)} \left(\frac{k+1}{2} - a\right)^{\frac{1}{2}(1-(-1)^k)}, z^{\frac{1}{2}((-1)^k+1)}\right)_1^\infty} \;/;\; z \notin (-\infty, 0)$$

06.06.10.0003.01

$$\Gamma(a, z) = \cfrac{z^a e^{-z}}{1 - a + z + \cfrac{a-1}{3 - a + z + \cfrac{2(a-2)}{5 - a + z + \cfrac{3(a-3)}{7 - a + z + \cfrac{4(a-4)}{9 - a + z + \cfrac{5(a-5)}{11 - a + z + \ldots}}}}}} \;/;\; z \notin (-\infty, 0)$$

06.06.10.0004.01

$$\Gamma(a, z) = \frac{z^a e^{-z}}{z - a + 1 + \mathrm{K}_k(-k(k-a), -a + 2k + z + 1)_1^\infty} \;/;\; z \notin (-\infty, 0)$$

06.06.10.0005.01

$$\Gamma(a, z) = \Gamma(a) - \cfrac{z^a e^{-z}}{a - \cfrac{a z}{a + 1 + \cfrac{z}{a + 2 - \cfrac{(a+1) z}{a + 3 + \cfrac{2 z}{a + 4 - \cfrac{(a+2) z}{a + 5 + \ldots}}}}}} \;/;\; z \notin (-\infty, 0)$$

06.06.10.0006.01

$$\Gamma(a, z) = \Gamma(a) - \frac{z^a e^{-z}}{a + \mathrm{K}_k\left((-1)^k \left(a^{\frac{1}{2}(1-(-1)^k)} + \left\lfloor \frac{k-1}{2} \right\rfloor\right) z, a + k\right)_1^\infty} \;/;\; z \notin (-\infty, 0)$$

06.06.10.0007.01

$$\Gamma(a, z) = \Gamma(a) - \cfrac{z^a e^{-z}}{a - \cfrac{a z}{a + z + 1 - \cfrac{(a+1) z}{a + z + 2 - \cfrac{(a+2) z}{a + z + 3 - \cfrac{(a+3) z}{a + z + 4 - \cfrac{(a+4) z}{a + z + 5 + \ldots}}}}}}$$

06.06.10.0008.01

$$\Gamma(a, z) = \Gamma(a) - \frac{z^a e^{-z}}{a + \mathrm{K}_k(-(a + k - 1) z, a + k + z)_1^\infty}$$

06.06.10.0009.01

$$\Gamma(a,z)=\Gamma(a)-\cfrac{z^a e^{-z}}{a-\cfrac{a z}{a+1+\cfrac{z}{a+2-\cfrac{(a+1) z}{a+3+\cfrac{2 z}{a+4-\cfrac{(a+2) z}{a+5+\ldots}}}}}}$$

06.06.10.0010.01

$$\Gamma(a,z)=\Gamma(a)-\frac{z^a e^{-z}}{a+\mathrm{K}_k\left((-1)^k \left(\frac{k}{2}\right)^{\frac{1}{2}(1+(-1)^k)} \left(\frac{k-1}{2}+a\right)^{\frac{1}{2}(1-(-1)^k)} z,\ a+k\right)_1^{\infty}}$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

06.06.13.0001.01

$$z\, w''(z)+(1-a+z)\, w'(z)=0 \;/;\; w(z)=c_1\, \Gamma(a,z)+c_2$$

06.06.13.0002.01

$$W_z(1,\Gamma(a,z))=-e^{-z} z^{a-1}$$

06.06.13.0003.01

$$w''(z)+\left(\frac{(g(z)-a+1)\, g'(z)}{g(z)}-\frac{g''(z)}{g'(z)}\right) w'(z)=0 \;/;\; w(z)=c_1\, \Gamma(a,g(z))+c_2$$

06.06.13.0004.01

$$W_z(\Gamma(a,g(z)),1)=e^{-g(z)}\, g(z)^{a-1}\, g'(z)$$

06.06.13.0005.01

$$w''(z)+\left(\frac{(g(z)-a+1)\, g'(z)}{g(z)}-\frac{2\, h'(z)}{h(z)}-\frac{g''(z)}{g'(z)}\right) w'(z)+\left(\frac{2\, h'(z)^2}{h(z)^2}+\frac{(a-1)\, g'(z)\, h'(z)}{g(z)\, h(z)}+\frac{g''(z)\, h'(z)}{h(z)\, g'(z)}-\frac{g'(z)\, h'(z)+h''(z)}{h(z)}\right) w(z)=0 \;/;\; w(z)=c_1\, h(z)\, \Gamma(a,g(z))+c_2\, h(z)$$

06.06.13.0006.01

$$W_z(h(z)\, \Gamma(a,g(z)),h(z))=e^{-g(z)}\, g(z)^{a-1}\, h(z)^2\, g'(z)$$

06.06.13.0007.01

$$z^2\, w''(z)+(d\, r\, z^r-a\, r-2\, s+1)\, z\, w'(z)+s\, (-d\, r\, z^r+a\, r+s)\, w(z)=0 \;/;\; w(z)=c_1\, z^s\, \Gamma(a,d\, z^r)+c_2\, z^s$$

06.06.13.0008.01

$$W_z(z^s\, \Gamma(a,d\, z^r),z^s)=e^{-d\, z^r}\, r\, z^{2 s-1}\, (d\, z^r)^a$$

06.06.13.0009.01

$$w''(z)-((a-d\, r^z) \log(r)+2 \log(s))\, w'(z)+\log(s)\, ((a-d\, r^z) \log(r)+\log(s))\, w(z)=0 \;/;\; w(z)=c_1\, s^z\, \Gamma(a,d\, r^z)+c_2\, s^z$$

06.06.13.0010.01

$$W_z(s^z\, \Gamma(a,d\, r^z),s^z)=e^{-d\, r^z}\, (d\, r^z)^a\, s^{2 z} \log(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.06.16.0001.01

$$\Gamma(a+1, z) = a\,\Gamma(a, z) + e^{-z} z^a$$

06.06.16.0002.01

$$\Gamma(a-1, z) = \frac{1}{a-1}\left(\Gamma(a, z) - e^{-z} z^{a-1}\right)$$

06.06.16.0003.01

$$\Gamma(a+n, z) = (a)_n\,\Gamma(a, z) + e^{-z} z^{a+n-1} \sum_{k=0}^{n-1} (1-a-n)_k\,(-z)^{-k} \;/;\; n \in \mathbb{N}$$

06.06.16.0004.01

$$\Gamma(a-n, z) = \frac{(-1)^n}{(1-a)_n}\,\Gamma(a, z) - e^{-z} z^{a-n} \sum_{k=0}^{n-1} \frac{z^k}{(a-n)_{k+1}} \;/;\; n \in \mathbb{N}$$

Identities

Recurrence identities

Consecutive neighbors

06.06.17.0001.01

$$\Gamma(a, z) = \frac{1}{a}\left(\Gamma(a+1, z) - e^{-z} z^a\right)$$

06.06.17.0002.01

$$\Gamma(a, z) = (a-1)\,\Gamma(a-1, z) + e^{-z} z^{a-1}$$

Distant neighbors

06.06.17.0003.01

$$\Gamma(a, z) = \frac{1}{(a)_n}\,\Gamma(a+n, z) - z^{a-1} e^{-z} \sum_{k=1}^{n} \frac{z^k}{(a)_k} \;/;\; n \in \mathbb{N}$$

06.06.17.0004.01

$$\Gamma(a, z) = (-1)^n (1-a)_n \left(\Gamma(a-n, z) + z^{a-n-1} e^{-z} \sum_{k=1}^{n} \frac{z^k}{(a-n)_k}\right) \;/;\; n \in \mathbb{N}$$

Functional identities

Relations of special kind

06.06.17.0005.01

$$\Gamma(-n, z) = \frac{(-1)^n}{n!}\,\Gamma(0, z) - e^{-z} \sum_{k=1}^{n} \frac{z^{k-n-1}}{(-n)_k} \;/;\; n \in \mathbb{N}$$

Differentiation

Low-order differentiation

With respect to *a*

06.06.20.0011.01

$$\frac{\partial \Gamma(a, z)}{\partial a} = \Gamma(a, z) \log(z) + G_{2,3}^{3,0}\left(z \,\middle|\, \begin{matrix} 1, 1 \\ 0, 0, a \end{matrix}\right)$$

06.06.20.0001.01

$$\frac{\partial \Gamma(a, z)}{\partial a} == \Gamma(a)^2 z^a \, {}_2\tilde{F}_2(a, a; a+1, a+1; -z) - \Gamma(a, 0, z) \log(z) + \Gamma(a)\, \psi(a)$$

06.06.20.0012.01

$$\frac{\partial^2 \Gamma(a, z)}{\partial a^2} = \Gamma(a, z) \log^2(z) + 2\, G_{2,3}^{3,0}\left(z \,\middle|\, \begin{matrix} 1, 1 \\ 0, 0, a \end{matrix}\right) \log(z) + 2\, G_{3,4}^{4,0}\left(z \,\middle|\, \begin{matrix} 1, 1, 1 \\ 0, 0, 0, a \end{matrix}\right)$$

06.06.20.0002.01

$$\frac{\partial^2 \Gamma(a, z)}{\partial a^2} == \Gamma(a, z) \log^2(z) + \Gamma(a) \left(\psi(a)^2 + \psi^{(1)}(a) - \log^2(z)\right) - \frac{2 z^a}{a^3} \left({}_3F_3(a, a, a; a+1, a+1, a+1; -z) - a \log(z)\, {}_2F_2(a, a; a+1, a+1; -z)\right)$$

With respect to *z*

06.06.20.0003.01

$$\frac{\partial \Gamma(a, z)}{\partial z} == -e^{-z} z^{a-1}$$

06.06.20.0004.01

$$\frac{\partial^2 \Gamma(a, z)}{\partial z^2} == e^{-z} z^{a-2} (1 - a + z)$$

Symbolic differentiation

With respect to *a*

06.06.20.0005.02

$$\frac{\partial^n \Gamma(a, z)}{\partial a^n} == \Gamma^{(n)}(a) - \sum_{k=0}^{\infty} \frac{(-1)^{n-k}\, \Gamma(n+1, -(a+k) \log(z))}{(a+k)^{n+1}\, k!} \;/;\; n \in \mathbb{N}$$

06.06.20.0013.01

$$\frac{\partial^n \Gamma(a, z)}{\partial a^n} = \Gamma(a, z) \log^n(z) + n! \sum_{k=1}^{n} \frac{\log^{n-k}(z)}{(n-k)!}\, G_{k+1,k+2}^{k+2,0}\left(z \,\middle|\, \begin{matrix} 1, 1, \ldots, 1 \\ 0, 0, \ldots, 0, a \end{matrix}\right) /;\; n \in \mathbb{N}$$

06.06.20.0006.02

$$\frac{\partial^n \Gamma(a,z)}{\partial a^n} = \Gamma^{(n)}(a) - z^a \sum_{j=0}^{n} (-1)^{n-j} \binom{n}{j} (n-j)!\, \Gamma(a)^{n-j+1} \log^j(z)\, {}_{n-j+1}\tilde{F}_{n-j+1}\big(a_1, a_2, \ldots, a_{n-j+1}; a_1+1, a_2+1, \ldots, a_{n-j+1}+1; -z\big) /; a_1 = a_2 = \ldots = a_{n+1} = a \bigwedge n \in \mathbb{N}$$

With respect to z

06.06.20.0014.01

$$\frac{\partial^n \Gamma(a,z)}{\partial z^n} = \Gamma(a,z)\, \delta_n + (-1)^n e^{-z} \sum_{k=0}^{n-1} \binom{n-1}{k} (1-a)_k\, z^{a-k-1} /; n \in \mathbb{N}$$

06.06.20.0007.02

$$\frac{\partial^n \Gamma(a,z)}{\partial z^n} = z^{-n} \sum_{k=0}^{n} (-1)^n \binom{n}{k} (-a)_k\, \Gamma(a-k+n, z) /; n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to a

06.06.20.0008.01

$$\frac{\partial^\alpha \Gamma(a,z)}{\partial a^\alpha} = a^{-\alpha} \int_z^\infty t^{a-1} (a \log(t))^\alpha\, e^{-t}\, Q(-\alpha, 0, a \log(t))\, dt$$

With respect to z

06.06.20.0009.01

$$\frac{\partial^\alpha \Gamma(a,z)}{\partial z^\alpha} = \frac{\Gamma(a)\, z^{-\alpha}}{\Gamma(1-\alpha)} - z^{a-\alpha}\, \Gamma(a)\, {}_1\tilde{F}_1(a; a-\alpha+1; -z) /; -a \notin \mathbb{N}^+$$

06.06.20.0010.01

$$\frac{\partial^\alpha \Gamma(a,z)}{\partial z^\alpha} = \frac{\Gamma(a)\, z^{-\alpha}}{\Gamma(1-\alpha)} - \sum_{k=0}^{\infty} \frac{(-1)^k\, \mathcal{FC}_{\exp}^{(\alpha)}(z, a+k)\, z^{a+k-\alpha}}{(a+k)\, k!}$$

Integration

Indefinite integration

Involving only one direct function

06.06.21.0001.01

$$\int \Gamma(a,z)\, dz = z\, \Gamma(a,z) - \Gamma(a+1, z)$$

Involving one direct function and elementary functions

Involving power function

06.06.21.0002.01

$$\int z^{\alpha-1}\,\Gamma(a,z)\,dz = \frac{z^{\alpha}\,\Gamma(a,z)-\Gamma(a+\alpha,z)}{\alpha}$$

Involving only one direct function with respect to *a*

06.06.21.0003.01

$$\int \Gamma(a,z)\,da = \int_z^{\infty}\frac{t^{a-1}\,e^{-t}}{\log(t)}\,dt$$

Integral transforms

Fourier cos transforms

06.06.22.0001.01

$$\mathcal{F}c_t[\Gamma(a,t)](x) = \Gamma(a)\sqrt{\frac{2}{\pi}}\,\frac{\left(x^2+1\right)^{-\frac{a}{2}}\sin\!\left(a\tan^{-1}(x)\right)}{x}+\sqrt{\frac{\pi}{2}}\,\Gamma(a)\,\delta(x)\,/;\,x\in\mathbb{R}\land\operatorname{Re}(a)>-1$$

Fourier sin transforms

06.06.22.0002.01

$$\mathcal{F}s_t[\Gamma(a,t)](x) = \Gamma(a)\sqrt{\frac{2}{\pi}}\,\frac{1+\left(1+x^2\right)^{-\frac{a}{2}}\cos\!\left(a\tan^{-1}(x)\right)}{x}\,/;\,x\in\mathbb{R}\land\operatorname{Re}(a)>-2$$

Laplace transforms

06.06.22.0003.01

$$\mathcal{L}_t[\Gamma(a,t)](z) = \Gamma(a)\,\frac{1-(1+z)^{-a}}{z}\,/;\,\operatorname{Re}(z)>0\land\operatorname{Re}(a)>-1$$

Representations through more general functions

Through hypergeometric functions

Involving $_1\tilde{F}_1$

06.06.26.0001.01

$$\Gamma(a,z) = \Gamma(a)\left(1-z^a\,{}_1\tilde{F}_1(a;a+1;-z)\right)/;\,-a\notin\mathbb{N}$$

Involving $_1F_1$

06.06.26.0002.01

$$\Gamma(a,z) = \Gamma(a)-\frac{z^a}{a}\,{}_1F_1(a;a+1;-z)\,/;\,-a\notin\mathbb{N}$$

Involving hypergeometric *U*

06.06.26.0003.01

$$\Gamma(a,z) = e^{-z}\,U(1-a,1-a,z)$$

Through Meijer *G*

Classical cases for the direct function itself

06.06.26.0004.01

$$\Gamma(a, z) = \Gamma(a) - G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1 \\ a, 0 \end{matrix}\right)$$

06.06.26.0005.01

$$\Gamma(a, z) = G_{1,2}^{2,0}\left(z \,\middle|\, \begin{matrix} 1 \\ 0, a \end{matrix}\right)$$

06.06.26.0006.01

$$\Gamma\left(a, \sqrt{z}\right) - \Gamma\left(a, -\sqrt{z}\right) = -2^{a-2} \sqrt{\pi} \sqrt{-z^2}\; G_{1,3}^{2,0}\left(-\frac{z}{4} \,\middle|\, \begin{matrix} 0 \\ \frac{a-1}{2}, \frac{a}{2} - 1, -1 \end{matrix}\right)$$

Classical cases involving exp

06.06.26.0007.01

$$e^z \Gamma(a, z) = \frac{1}{\Gamma(1-a)} G_{1,2}^{2,1}\left(z \,\middle|\, \begin{matrix} a \\ 0, a \end{matrix}\right)$$

06.06.26.0008.01

$$e^{-z} \Gamma(a, -z) + e^z \Gamma(a, z) = \frac{1}{\sqrt{\pi}\, \Gamma(1-a)} G_{2,4}^{3,2}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{a+1}{2}, \frac{a}{2} \\ \frac{a+1}{2}, \frac{a}{2}, 0, \frac{1}{2} \end{matrix}\right)$$

06.06.26.0009.01

$$e^{-z} \Gamma(a, -z) - e^z \Gamma(a, z) = -\frac{\sqrt{-z^2}}{\sqrt{\pi}\, \Gamma(1-a)\, z} G_{2,4}^{3,2}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{a+1}{2}, \frac{a}{2} \\ \frac{a+1}{2}, \frac{a}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

06.06.26.0010.01

$$e^{\frac{\pi i a}{2} - z} \Gamma(a, -z) + e^{-\frac{\pi i a}{2} + z} \Gamma(a, z) = \frac{1}{\sqrt{\pi}\, \Gamma(1-a)} G_{1,3}^{3,1}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{a}{2} \\ 0, \frac{1}{2}, \frac{a}{2} \end{matrix}\right) /; 0 < \arg(z) \leq \pi$$

06.06.26.0011.01

$$e^{\frac{\pi i a}{2} - z} \Gamma(a, -z) - e^{-\frac{\pi i a}{2} + z} \Gamma(a, z) = \frac{i}{\sqrt{\pi}\, \Gamma(1-a)} G_{1,3}^{3,1}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{a+1}{2} \\ 0, \frac{1}{2}, \frac{a+1}{2} \end{matrix}\right) /; 0 < \arg(z) \leq \pi$$

Classical cases for products of incomplete gamma functions ||| Classical cases for products of incomplete gamma functions

06.06.26.0012.01

$$\Gamma(a, -z)\, \Gamma(a, z) = \frac{2^{a-1}}{\sqrt{\pi}\, \Gamma(1-a)} G_{2,4}^{4,1}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} a, 1 \\ 0, \frac{a}{2}, \frac{a+1}{2}, a \end{matrix}\right)$$

Classical cases involving regularized gamma *Q*

06.06.26.0017.01

$$\Gamma(a, z)\, Q(a, -z) = \frac{2^{a-1} \sin(\pi a)}{\pi^{3/2}} G_{2,4}^{4,1}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} a, 1 \\ 0, \frac{a}{2}, \frac{a+1}{2}, a \end{matrix}\right)$$

Generalized cases for the direct function itself

06.06.26.0013.01

$$e^{\frac{\pi i a}{2}} \Gamma(a, -z) + e^{-\frac{\pi i z}{2}} \Gamma(a, z) = 2^a \sqrt{\pi}\, G_{1,3}^{2,0}\left(-\frac{i z}{2}, \frac{1}{2} \middle| \begin{matrix} 1 \\ 0, \frac{a}{2}, \frac{a+1}{2} \end{matrix}\right) /; 0 < \arg(z) \leq \pi$$

06.06.26.0014.01

$$e^{\frac{\pi i a}{2}} \Gamma(a, -z) - e^{-\frac{\pi i a}{2}} \Gamma(a, z) = i\, 2^a \sqrt{\pi}\, G_{1,3}^{2,0}\left(-\frac{i z}{2}, \frac{1}{2} \middle| \begin{matrix} 1 \\ 0, \frac{a+1}{2}, \frac{a}{2} \end{matrix}\right) /; 0 < \arg(z) \leq \pi$$

Generalized cases involving exp

06.06.26.0018.01

$$e^{-z} \Gamma(a, -z) - e^{z} \Gamma(a, z) = \frac{1}{\sqrt{\pi}\, \Gamma(1-a)} G_{2,4}^{3,2}\left(\frac{\sqrt{-z^2}}{2}, \frac{1}{2} \middle| \begin{matrix} \frac{a}{2}, \frac{a+1}{2} \\ \frac{a}{2}, \frac{a+1}{2}, 0, \frac{1}{2} \end{matrix}\right)$$

06.06.26.0019.01

$$e^{-z} \Gamma(a, -z) - e^{z} \Gamma(a, z) = -\frac{\sqrt{-z^2}}{\sqrt{\pi}\, \Gamma(1-a)\, z} G_{2,4}^{3,2}\left(\frac{\sqrt{-z^2}}{2}, \frac{1}{2} \middle| \begin{matrix} \frac{a}{2}, \frac{a+1}{2} \\ \frac{a}{2}, \frac{a+1}{2}, \frac{1}{2}, 0 \end{matrix}\right)$$

06.06.26.0020.01

$$e^{\frac{\pi i a}{2} - z} \Gamma(a, -z) + e^{-\frac{1}{2}(\pi i a) + z} \Gamma(a, z) = \frac{1}{\sqrt{\pi}\, \Gamma(1-a)} G_{1,3}^{3,1}\left(\frac{\sqrt{-z^2}}{2}, \frac{1}{2} \middle| \begin{matrix} \frac{a}{2} \\ 0, \frac{1}{2}, \frac{a}{2} \end{matrix}\right) /; 0 < \arg(z) \leq \pi$$

06.06.26.0021.01

$$e^{\frac{\pi i a}{2} - z} \Gamma(a, -z) - e^{-\frac{1}{2}(\pi i a) + z} \Gamma(a, z) = \frac{i}{\sqrt{\pi}\, \Gamma(1-a)} G_{1,3}^{3,1}\left(\frac{\sqrt{-z^2}}{2}, \frac{1}{2} \middle| \begin{matrix} \frac{a+1}{2} \\ 0, \frac{1}{2}, \frac{a+1}{2} \end{matrix}\right) /; 0 < \arg(z) \leq \pi$$

Through other functions

Involving some hypergeometric-type functions

06.06.26.0015.01

$$\Gamma(a, z) = \Gamma(a) + \Gamma(a, z, 0) /; \operatorname{Re}(a) > 0$$

06.06.26.0016.01

$$\Gamma(a, z) = \Gamma(a)\,(Q(a, z, 0) + 1) /; \operatorname{Re}(a) > 0$$

Representations through equivalent functions

With inverse function

06.06.27.0001.01

$$\Gamma\left(a, Q^{-1}(a, z)\right) = \Gamma(a)\, z$$

With related functions

06.06.27.0002.01

$$\Gamma(a, z) = \Gamma(a)\, Q(a, z)$$

06.06.27.0003.01

$$\Gamma(a, z) = z^a\, E_{1-a}(z)$$

History

– A. M. Legendre (1811)
– P. Schlömilch (1871) introduced the name "incomplete Gamma"
– J. Tannery (1882); F.E. Prym (1877)
– M. Lerch (1905) gave a series representation
– N. Nielsen (1906)

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