Erfc

Notations

Traditional name

Complementary error function

Traditional notation

$\text{erfc}(z)$

Mathematica StandardForm notation

`Erfc[z]`

Primary definition

06.27.02.0001.01

$$\text{erfc}(z) == 1 - \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k+1}}{k!\,(2k+1)}$$

Specific values

Values at fixed points

06.27.03.0001.01

$\text{erfc}(0) == 1$

Values at infinities

06.27.03.0002.01

$\text{erfc}(\infty) == 0$

06.27.03.0003.01

$\text{erfc}(-\infty) == 2$

06.27.03.0004.01

$\text{erfc}(i\,\infty) == -i\,\infty$

06.27.03.0005.01

$\text{erfc}(-i\,\infty) == i\,\infty$

06.27.03.0006.01

$\text{erfc}(\tilde{\infty}) ==$ ¿

General characteristics

Domain and analyticity

erfc(z) is an entire analytical function of z which is defined in the whole complex z-plane.

06.27.04.0001.01

$$z \longrightarrow \operatorname{erfc}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

06.27.04.0002.01

$$\operatorname{erfc}(\bar{z}) = \overline{\operatorname{erfc}(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function erfc(z) has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

06.27.04.0003.01

$$\mathcal{Sing}_z(\operatorname{erfc}(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function erfc(z) does not have branch points.

06.27.04.0004.01

$$\mathcal{BP}_z(\operatorname{erfc}(z)) = \{\}$$

Branch cuts

The function erfc(z) does not have branch cuts.

06.27.04.0005.01

$$\mathcal{BC}_z(\operatorname{erfc}(z)) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

06.27.06.0010.01

$$\operatorname{erfc}(z) \propto \operatorname{erfc}(z_0) - \frac{2\, e^{-z_0^2}}{\sqrt{\pi}} (z - z_0) + \frac{2\, e^{-z_0^2}\, z_0}{\sqrt{\pi}} (z - z_0)^2 + \ldots \;/;\; (z \to z_0)$$

06.27.06.0011.01

$$\operatorname{erfc}(z) \propto \operatorname{erfc}(z_0) - \frac{2\, e^{-z_0^2}}{\sqrt{\pi}} (z - z_0) + \frac{2\, e^{-z_0^2}\, z_0}{\sqrt{\pi}} (z - z_0)^2 + O\left((z - z_0)^3\right)$$

06.27.06.0012.01

$$\operatorname{erfc}(z) = \operatorname{erfc}(z_0) - \frac{2\, e^{-z_0^2}}{\sqrt{\pi}} \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \frac{(-1)^j\, (2\, j - k + 2)_{2\, (k-j-1)}}{k!\, (k - j - 1)!\, (2\, z_0)^{k-2\, j-1}} (z - z_0)^k$$

06.27.06.0013.01

$$\operatorname{erfc}(z) = -\sum_{k=0}^{\infty} \frac{2^k\, z_0^{1-k}}{k!}\, {}_2\tilde{F}_2\left(\frac{1}{2}, 1; 1 - \frac{k}{2}, \frac{3-k}{2}; -z_0^2\right) (z - z_0)^k$$

06.27.06.0014.01

$$\operatorname{erfc}(z) \propto \operatorname{erfc}(z_0)\, (1 + O(z - z_0))$$

Expansions at $z = 0$

For the function itself

06.27.06.0001.02

$$\operatorname{erfc}(z) \propto 1 - \frac{2}{\sqrt{\pi}} \left(z - \frac{z^3}{3} + \frac{z^5}{10} - \ldots\right) /;\; (z \to 0)$$

06.27.06.0015.01

$$\operatorname{erfc}(z) \propto 1 - \frac{2}{\sqrt{\pi}} \left(z - \frac{z^3}{3} + \frac{z^5}{10} - O\left(z^7\right)\right)$$

06.27.06.0002.01

$$\operatorname{erfc}(z) = 1 - \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k\, z^{2\, k+1}}{k!\, (2\, k + 1)}$$

06.27.06.0003.01

$$\operatorname{erfc}(z) = 1 - \frac{2\, z}{\sqrt{\pi}}\, {}_1F_1\left(\frac{1}{2}; \frac{3}{2}; -z^2\right)$$

06.27.06.0004.02

$$\operatorname{erfc}(z) \propto 1 + O(z)$$

06.27.06.0016.01

$$\operatorname{erfc}(z) = F_\infty(z) \;/;$$
$$\left(\left(F_n(z) = 1 - \frac{2\, z}{\sqrt{\pi}} \sum_{k=0}^{n} \frac{(-1)^k\, z^{2\, k}}{(2\, k + 1)\, k!} = \operatorname{erfc}(z) - \frac{(-1)^n\, 2\, z^{2\, n+3}}{\sqrt{\pi}\, (2\, n + 3)\, (n + 1)!}\, {}_2F_2\left(1, n + \frac{3}{2}; n + 2, n + \frac{5}{2}; -z^2\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

06.27.06.0005.01

$$\operatorname{erfc}(z) \propto 1 - \frac{z}{\sqrt{z^2}} + \frac{1}{\sqrt{\pi}\, z} e^{-z^2} {}_2F_0\left(1, \frac{1}{2};; -\frac{1}{z^2}\right) /; (|z| \to \infty)$$

06.27.06.0006.02

$$\operatorname{erfc}(z) \propto 1 - \frac{\sqrt{z^2}}{z} + \frac{1}{\sqrt{\pi}\, z} e^{-z^2} \left(1 + O\left(\frac{1}{z^2}\right)\right) /; (|z| \to \infty)$$

06.27.06.0017.01

$$\operatorname{erfc}(z) \propto \begin{cases} \frac{e^{-z^2}}{\sqrt{\pi}\, z} & -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2} \\ 2 + \frac{e^{-z^2}}{\sqrt{\pi}\, z} & \text{True} \end{cases} /; (|z| \to \infty)$$

Residue representations

06.27.06.0007.01

$$\operatorname{erfc}(z) = 1 - \frac{z}{\sqrt{\pi}} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\Gamma\left(\frac{1}{2} - s\right) \left(z^2\right)^{-s}}{\Gamma\left(\frac{3}{2} - s\right)} \Gamma(s) \right) (-j)$$

06.27.06.0008.01

$$\operatorname{erfc}(z) = 1 - \frac{1}{\sqrt{\pi}} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\Gamma(-s)\, z^{-2s}}{\Gamma(1-s)} \Gamma\left(s + \frac{1}{2}\right) \right) \left(-\frac{1}{2} - j\right)$$

Other series representations

06.27.06.0009.01

$$\operatorname{erfc}(z) = 1 - \frac{1}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k H_{2k+1}(z)}{2^{3k+\frac{1}{2}}\, k!\, (2k+1)}$$

Integral representations

On the real axis

Of the direct function

06.27.07.0001.01

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2}\, dt$$

06.27.07.0002.01

$$\operatorname{erfc}(x) = 1 - \frac{2}{\pi} \int_0^{\infty} \frac{e^{-t^2} \sin(2\, x\, t)}{t}\, dt /; x \in \mathbb{R}$$

Contour integral representations

06.27.07.0003.01

$$\operatorname{erfc}(z) = 1 - \frac{z}{\sqrt{\pi}\, 2\pi i} \int_{\mathcal{L}} \frac{\Gamma(s)\,\Gamma\left(\frac{1}{2}-s\right)}{\Gamma\left(\frac{3}{2}-s\right)} \left(z^2\right)^{-s} \, ds$$

06.27.07.0004.01

$$\operatorname{erfc}(z) = 1 - \frac{1}{\sqrt{\pi}\, 2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma\left(s+\frac{1}{2}\right)\Gamma(-s)}{\Gamma(1-s)} z^{-2s} \, ds \;/;\; -\frac{1}{2} < \gamma \bigwedge |\arg(z)| < \frac{\pi}{2}$$

06.27.07.0005.01

$$\operatorname{erfc}(z) = \frac{1}{\sqrt{\pi}\, 2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty}{}_{\mathcal{L}} \frac{\Gamma(s)\,\Gamma\left(s+\frac{1}{2}\right)}{\Gamma(s+1)} z^{-2s} \, ds \;/;\; 0 < \gamma \bigwedge |\arg(z)| < \frac{\pi}{2}$$

Continued fraction representations

Involving the function

06.27.10.0001.01

$$\operatorname{erfc}(z) = \frac{e^{-z^2}}{\sqrt{\pi}} \cfrac{1}{z + \cfrac{1/2}{z + \cfrac{1}{z + \cfrac{3/2}{z + \cfrac{2}{z + \cfrac{5/2}{z + \cfrac{3}{z + \ldots}}}}}}} \;/;\; \operatorname{Re}(z) > 0$$

06.27.10.0002.01

$$\operatorname{erfc}(z) = \frac{e^{-z^2}}{\sqrt{\pi}\left(z + K_k\left(\frac{k}{2}, z\right)_1^{\infty}\right)} \;/;\; \operatorname{Re}(z) > 0$$

06.27.10.0003.01

$$\operatorname{erfc}(z) = 1 - \frac{2z}{\sqrt{\pi}} e^{-z^2} \cfrac{1}{1 - \cfrac{2z^2}{3 + \cfrac{4z^2}{5 - \cfrac{6z^2}{7 + \cfrac{8z^2}{9 - \cfrac{10z^2}{11 + \cfrac{12z^2}{13 - \ldots}}}}}}}$$

06.27.10.0004.01

$$\operatorname{erfc}(z) = 1 - \frac{2z\, e^{-z^2}}{\sqrt{\pi}\left(1 + K_k\left((-1)^k\, 2k z^2, 2k+1\right)_1^{\infty}\right)}$$

06.27.10.0005.01

$$\operatorname{erfc}(z) = 1 - \frac{2z}{\sqrt{\pi}} e^{-z^2} 1 \Bigg/ \left(1 - 2z^2 + \cfrac{4z^2}{3 - 2z^2 + \cfrac{8z^2}{5 - 2z^2 - \cfrac{12z^2}{7 - 2z^2 + \cfrac{16z^2}{9 - 2z^2 - \cfrac{20z^2}{11 - 2z^2 + \cfrac{24z^2}{13 - 2z^2 + \ldots}}}}}}\right)$$

06.27.10.0006.01

$$\operatorname{erfc}(z) = 1 - \frac{2z\, e^{-z^2}}{\sqrt{\pi}\, \left(1 - 2z^2 + K_k\left(4kz^2, -2z^2 + 2k + 1\right)_1^\infty\right)}$$

06.27.10.0007.01

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} e^{-z^2} \cfrac{1}{2z + \cfrac{2}{2z + \cfrac{4}{2z + \cfrac{6}{2z + \cfrac{8}{2z + \cfrac{10}{2z + \cfrac{12}{2z + \ldots}}}}}}} \;/;\; \operatorname{Re}(z) > 0$$

06.27.10.0008.01

$$\operatorname{erfc}(z) = \frac{2\, e^{-z^2}}{\sqrt{\pi}\, \left(2z + K_k(2k, 2z)_1^\infty\right)} \;/;\; \operatorname{Re}(z) > 0$$

06.27.10.0009.01

$$\operatorname{erfc}(z) = \frac{2z}{\sqrt{\pi}} e^{-z^2} \cfrac{1}{1 + 2z^2 - \cfrac{2}{5 + 2z^2 - \cfrac{12}{9 + 2z^2 - \cfrac{30}{13 + 2z^2 - \cfrac{56}{17 + 2z^2 - \cfrac{90}{21 + 2z^2 - \cfrac{132}{2 \times 5 + 2z^2 - \ldots}}}}}}} \;/;\; \operatorname{Re}(z) > 0$$

06.27.10.0010.01

$$\operatorname{erfc}(z) = \frac{2 z e^{-z^2}}{\sqrt{\pi}\left(1 + 2 z^2 + \mathrm{K}_k\left(-2 k (2 k - 1), 2 z^2 + 4 k + 1\right)_1^\infty\right)} /; \operatorname{Re}(z) > 0$$

06.27.10.0011.01

$$\sqrt{\frac{e \pi}{2}} \operatorname{erfc}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{1 + \frac{1}{1 + \frac{2}{1 + \frac{3}{1 + \frac{4}{1 + \frac{5}{1 + \ldots}}}}}}$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

06.27.13.0001.01

$$w''(z) + 2 z w'(z) = 0 /; w(z) = \operatorname{erfc}(z) \bigwedge w(0) = 1 \bigwedge w'(0) = -\frac{2}{\sqrt{\pi}}$$

06.27.13.0002.01

$$w''(z) + 2 z w'(z) = 0 /; w(z) = c_1 \operatorname{erfc}(z) + c_2$$

06.27.13.0003.01

$$W_z(1, \operatorname{erfc}(z)) = -\frac{2 e^{-z^2}}{\sqrt{\pi}}$$

06.27.13.0004.01

$$w''(z) + \left(2 g(z) g'(z) - \frac{g''(z)}{g'(z)}\right) w'(z) = 0 /; w(z) = c_1 \operatorname{erfc}(g(z)) + c_2$$

06.27.13.0005.01

$$W_z(\operatorname{erfc}(g(z)), 1) = \frac{2 e^{-g(z)^2} g'(z)}{\sqrt{\pi}}$$

06.27.13.0006.01

$$w''(z) + \left(2 g(z) g'(z) - \frac{2 h'(z)}{h(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) + \left(\frac{2 h'(z)^2}{h(z)^2} + \frac{g''(z) h'(z)}{h(z) g'(z)} - \frac{2 g(z) g'(z) h'(z)}{h(z)} - \frac{h''(z)}{h(z)}\right) w(z) = 0 /;$$
$$w(z) = c_1 h(z) \operatorname{erfc}(g(z)) + c_2 h(z)$$

06.27.13.0007.01

$$W_z(h(z) \operatorname{erfc}(g(z)), h(z)) = \frac{2 e^{-g(z)^2} h(z)^2 g'(z)}{\sqrt{\pi}}$$

06.27.13.0008.01

$$z^2 w''(z) + \left(2 a^2 r z^{2 r} - r - 2 s + 1\right) z w'(z) + s \left(-2 a^2 r z^{2 r} + r + s\right) w(z) = 0 /; w(z) = c_1 z^s \operatorname{erfc}(a z^r) + c_2 z^s$$

06.27.13.0009.01

$$W_z(z^s \operatorname{erfc}(a z^r), z^s) = \frac{2 a e^{-a^2 z^{2 r}} r z^{r + 2 s - 1}}{\sqrt{\pi}}$$

06.27.13.0010.01

$$w''(z)+\left(\left(2a^2 r^{2z}-1\right)\log(r)-2\log(s)\right)w'(z)+\log(s)\left(-2a^2\log(r)\,r^{2z}+\log(r)+\log(s)\right)w(z)=0\;/;\;w(z)=c_1\,s^z\operatorname{erfc}(a\,r^z)+c_2\,s^z$$

06.27.13.0011.01

$$W_z(s^z\operatorname{erfc}(a\,r^z),\,s^z)=\frac{2\,a\,e^{-a^2 r^{2z}}\,r^z\,s^{2z}\log(r)}{\sqrt{\pi}}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.27.16.0001.01

$$\operatorname{erfc}(-z)=2-\operatorname{erfc}(z)$$

06.27.16.0002.01

$$\operatorname{erfc}(a\,(b\,z^c)^m)=1-\frac{(b\,z^c)^m}{b^m\,z^{m\,c}}\operatorname{erf}(a\,b^m\,z^{m\,c})\;/;\;2\,m\in\mathbb{Z}$$

06.27.16.0003.01

$$\operatorname{erfc}\left(\sqrt{z^2}\right)=1-\frac{\sqrt{z^2}}{z}\operatorname{erf}(z)$$

Complex characteristics

Real part

06.27.19.0001.01

$$\operatorname{Re}(\operatorname{erfc}(x+i\,y))=1-\frac{2\,x}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{y^{2k}}{k!}\,{}_1F_1\left(k+\frac{1}{2};\,\frac{3}{2};\,-x^2\right)$$

06.27.19.0002.01

$$\operatorname{Re}(\operatorname{erfc}(x+i\,y))=\operatorname{erfc}(x)-\frac{2}{\sqrt{\pi}}\,e^{-x^2}\sum_{k=0}^{\infty}\frac{(-1)^k\,y^{2k+2}}{(2\,k+2)!}\,H_{2k+1}(x)$$

06.27.19.0003.01

$$\operatorname{Re}(\operatorname{erfc}(x+i\,y))=\frac{1}{2}\left(\operatorname{erfc}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

06.27.19.0004.01

$$\operatorname{Im}(\operatorname{erfc}(x+i\,y))=-\frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{y^{2k+1}}{(2\,k+1)\,k!}\,{}_1F_1\left(k+\frac{1}{2};\,\frac{1}{2};\,-x^2\right)$$

06.27.19.0005.01

$$\operatorname{Im}(\operatorname{erfc}(x+i\,y))=-\frac{2}{\sqrt{\pi}}\,e^{-x^2}\sum_{k=0}^{\infty}\frac{(-1)^k\,y^{2k+1}}{(2\,k+1)!}\,H_{2k}(x)$$

06.27.19.0006.01

$$\operatorname{Im}(\operatorname{erfc}(x+i\,y)) = \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{erfc}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Absolute value

06.27.19.0007.01

$$|\operatorname{erfc}(x+i\,y)| = \sqrt{\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{erfc}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)}$$

Argument

06.27.19.0008.01

$$\arg(\operatorname{erfc}(x+i\,y)) = \tan^{-1}\left(\frac{1}{2}\left(\operatorname{erfc}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right), \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{erfc}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

Conjugate value

06.27.19.0009.01

$$\overline{\operatorname{erfc}(x+i\,y)} = \frac{1}{2}\left(\operatorname{erfc}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)-\frac{i\,x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{erfc}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Signum value

06.27.19.0010.01

$$\operatorname{sgn}(\operatorname{erfc}(x+i\,y)) = \left(\frac{i}{y}\,x\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{erfc}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)+\operatorname{erfc}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)\Big/ \left(2\sqrt{\operatorname{erfc}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{erfc}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)}\right)$$

Differentiation

Low-order differentiation

06.27.20.0001.01

$$\frac{\partial\operatorname{erfc}(z)}{\partial z} = -\frac{2\,e^{-z^2}}{\sqrt{\pi}}$$

06.27.20.0002.01

$$\frac{\partial^2 \operatorname{erfc}(z)}{\partial z^2} = \frac{4 e^{-z^2} z}{\sqrt{\pi}}$$

Symbolic differentiation

06.27.20.0006.01

$$\frac{\partial^n \operatorname{erfc}(z)}{\partial z^n} = \delta_n \operatorname{erfc}(z) - \frac{2}{\sqrt{\pi}} e^{-z^2} \sum_{k=0}^{n-1} \frac{(-1)^k (2k-n+2)_{2(n-k-1)}}{(-k+n-1)! \, (2z)^{n-2k-1}} \;/;\; n \in \mathbb{N}$$

06.27.20.0003.01

$$\frac{\partial^n \operatorname{erfc}(z)}{\partial z^n} = \operatorname{erfc}(z)\, \delta_n - \operatorname{Boole}\left(n \neq 0, \frac{2^{-n} (n-1)!}{\sqrt{\pi}} e^{-z^2} \sum_{k=1}^{n} \frac{(-1)^{k-1} 2^{2k} z^{2k-n-1}}{(2k-n-1)! \, (n-k)!}\right) /;\; n \in \mathbb{N}$$

06.27.20.0004.02

$$\frac{\partial^n \operatorname{erfc}(z)}{\partial z^n} = \delta_n - 2^n z^{1-n} \, {}_2\tilde{F}_2\left(\frac{1}{2}, 1; 1-\frac{n}{2}, \frac{3-n}{2}; -z^2\right) /;\; n \in \mathbb{N}$$

Fractional integro-differentiation

06.27.20.0005.01

$$\frac{\partial^\alpha \operatorname{erfc}(z)}{\partial z^\alpha} = \frac{z^{-\alpha}}{\Gamma(1-\alpha)} - 2^\alpha z^{1-\alpha} \, {}_2\tilde{F}_2\left(\frac{1}{2}, 1; 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -z^2\right)$$

Integration

Indefinite integration

Involving only one direct function

06.27.21.0001.01

$$\int \operatorname{erfc}(b + a z) \, dz = -\frac{b}{a} \operatorname{erf}(b + a z) + z \operatorname{erfc}(b + a z) - \frac{e^{-b^2 - 2 a z b - a^2 z^2}}{a \sqrt{\pi}}$$

06.27.21.0002.01

$$\int \operatorname{erfc}(a z) \, dz = z \operatorname{erfc}(a z) - \frac{e^{-a^2 z^2}}{a \sqrt{\pi}}$$

06.27.21.0003.01

$$\int \operatorname{erfc}(z) \, dz = z \operatorname{erfc}(z) - \frac{e^{-z^2}}{\sqrt{\pi}}$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear argument

06.27.21.0004.01

$$\int z^{\alpha-1}\operatorname{erfc}(a z)\,dz = \frac{z^{\alpha}\operatorname{erfc}(a z)}{\alpha} - \frac{a}{\sqrt{\pi}\,\alpha} z^{\alpha+1}\left(a^2 z^2\right)^{\frac{1}{2}(-\alpha-1)}\,\Gamma\!\left(\frac{\alpha+1}{2}, a^2 z^2\right)$$

06.27.21.0005.01

$$\int z^{\alpha-1}\operatorname{erfc}(z)\,dz = \frac{z^{\alpha}}{\alpha}\operatorname{erfc}(z) - \frac{z^{\alpha+1}}{\sqrt{\pi}\,\alpha}\left(z^2\right)^{-\frac{\alpha+1}{2}}\,\Gamma\!\left(\frac{\alpha+1}{2}, z^2\right)$$

06.27.21.0006.01

$$\int z\operatorname{erfc}(a z)\,dz = \frac{1}{4}\left(\frac{\operatorname{erf}(a z)}{a^2} + 2 z\left(z\operatorname{erfc}(a z) - \frac{e^{-a^2 z^2}}{a\sqrt{\pi}}\right)\right)$$

06.27.21.0007.01

$$\int \frac{\operatorname{erfc}(a z)}{z}\,dz = (\operatorname{erf}(a z) + \operatorname{erfc}(a z))\log(z) - \frac{2 a z}{\sqrt{\pi}}\,{}_2F_2\!\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -a^2 z^2\right)$$

06.27.21.0008.01

$$\int \frac{\operatorname{erfc}(a z)}{z^2}\,dz = -\frac{\operatorname{erfc}(a z)}{z} - \frac{a\operatorname{Ei}\left(-a^2 z^2\right)}{\sqrt{\pi}}$$

Power arguments

06.27.21.0009.01

$$\int z^{\alpha-1}\operatorname{erfc}(a z^r)\,dz = \frac{z^{\alpha}\operatorname{erfc}(a z^r)}{\alpha} - \frac{a}{\sqrt{\pi}\,\alpha} z^{r+\alpha}\left(a^2 z^{2r}\right)^{-\frac{r+\alpha}{2r}}\,\Gamma\!\left(\frac{r+\alpha}{2 r}, a^2 z^{2r}\right)$$

Involving rational functions

06.27.21.0010.01

$$\int \frac{\left(z^2-b\right)\operatorname{erfc}(a z)}{\left(z^2+b\right)^2}\,dz = -\frac{z\operatorname{erfc}(a z)}{z^2+b} - \frac{a}{\sqrt{\pi}} e^{a^2 b}\operatorname{Ei}\left(-z^2 a^2 - a^2 b\right)$$

Involving exponential function

Involving exp

06.27.21.0011.01

$$\int e^{b z}\operatorname{erfc}(a z)\,dz = \frac{1}{b}\left(e^{b z}\operatorname{erfc}(a z) - e^{\frac{b^2}{4 a^2}}\operatorname{erf}\!\left(\frac{b}{2 a} - a z\right)\right)$$

06.27.21.0012.01

$$\int e^{b z^2}\operatorname{erfc}(a z)\,dz = \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}\,z\right)}{2\sqrt{b}} - \frac{1}{\sqrt{\pi}\,b}\sum_{k=0}^{\infty}\frac{\left(b^{-k} a^{2k+1}\right)\Gamma\left(k+1, -b z^2\right)}{(2k+1)\,k!}$$

06.27.21.0013.01

$$\int e^{-a^2 z^2} \operatorname{erfc}(a z)\, dz == -\frac{\sqrt{\pi}\, \operatorname{erfc}(a z)^2}{4 a}$$

Involving exponential function and a power function

Involving exp and power

06.27.21.0014.01

$$\int z^{\alpha-1} e^{b z} \operatorname{erfc}(a z)\, dz == -z^{\alpha}\, \Gamma(\alpha, -b z)\, (-b z)^{-\alpha} - \frac{2 a z^{\alpha} (-b z)^{-\alpha}}{b \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} b^{-2k}\, \Gamma(2k+\alpha+1, -b z)}{(2k+1)\, k!}$$

06.27.21.0015.01

$$\int z^n e^{b z} \operatorname{erfc}(a z)\, dz == \frac{a\, n!\, (-b)^{-n-1}}{\sqrt{\pi}} \exp\left(\frac{b^2}{4 a^2}\right)$$

$$\sum_{m=0}^{n} \frac{(-b)^m \left(-a^2\right)^{\frac{1}{2}(-m-1)}}{m!} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b}{2\sqrt{-a^2}}\right)^{m-k} \left(\frac{b}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z\right)^{k+1} \left(-\left(\frac{b}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z\right)^2\right)^{\frac{1}{2}(-k-1)}$$

$$\Gamma\left(\frac{k+1}{2}, -\left(\frac{b}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z\right)^2\right) - (-b)^{-n-1} \operatorname{erfc}(a z)\, \Gamma(n+1, -b z) \;/;\; n \in \mathbb{N}$$

06.27.21.0016.01

$$\int z\, e^{b z} \operatorname{erfc}(a z)\, dz ==$$

$$\frac{1}{2 a^2 b^2 \sqrt{\pi}} \left(e^{-a^2 z^2} \left(\left(2 a^2 - b^2\right) \exp\left(\frac{b^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi}\, \operatorname{erf}\left(\frac{b}{2 a} - a z\right) - 2 a \left(b\, e^{b z} - a\, e^{z \left(z a^2 + b\right)} \sqrt{\pi}\, (b z - 1) \operatorname{erfc}(a z)\right)\right)\right)$$

06.27.21.0017.01

$$\int z^2 e^{b z} \operatorname{erfc}(a z)\, dz == \frac{1}{4 a^4 b^3 \sqrt{\pi}}\, e^{-a^2 z^2} \left(2 a\, e^{b z} \left(2 a^3 e^{a^2 z^2} \sqrt{\pi} \left(b^2 z^2 - 2 b z + 2\right) \operatorname{erfc}(a z) - b \left(2 (b z - 2) a^2 + b^2\right)\right) - \left(8 a^4 - 2 b^2 a^2 + b^4\right) e^{\frac{b^2}{4 a^2} + a^2 z^2} \sqrt{\pi}\, \operatorname{erf}\left(\frac{b}{2 a} - a z\right)\right)$$

06.27.21.0018.01

$$\int z^3 e^{b z} \operatorname{erfc}(a z)\, dz == \frac{1}{8 a^6 b^4 \sqrt{\pi}} \left(e^{-a^2 z^2} \left(\left(48 a^6 - 12 b^2 a^4 - b^6\right) e^{\frac{b^2}{4 a^2} + a^2 z^2} \sqrt{\pi}\, \operatorname{erf}\left(\frac{b}{2 a} - a z\right) - 2 a\, e^{b z} \left(b \left(4 \left(b^2 z^2 - 3 b z + 6\right) a^4 + 2 b^2 (b z - 1) a^2 + b^4\right) - 4 a^5 e^{a^2 z^2} \sqrt{\pi} \left(b^3 z^3 - 3 b^2 z^2 + 6 b z - 6\right) \operatorname{erfc}(a z)\right)\right)\right)$$

06.27.21.0019.01

$$\int z^{\alpha-1} e^{b z^2} \operatorname{erfc}(a z)\, dz == \frac{a z^{\alpha+1}}{\sqrt{\pi}\, \left(-b z^2\right)^{\frac{\alpha+1}{2}}} \sum_{k=0}^{\infty} \frac{b^{-k} a^{2k}}{(2k+1)\, k!}\, \Gamma\left(\frac{\alpha+1}{2} + k, -b z^2\right) - \frac{1}{2} z^{\alpha} \left(-b z^2\right)^{-\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, -b z^2\right)$$

06.27.21.0020.01

$$\int z^{\alpha-1} e^{a^2 z^2} \operatorname{erfc}(a z) \, dz = -\frac{1}{2} z^{\alpha} \left(E_{1-\frac{\alpha}{2}}(-a^2 z^2) + a z \, \Gamma\!\left(\frac{\alpha+1}{2}\right) {}_2\tilde{F}_2\!\left(1, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; a^2 z^2\right)\right)$$

06.27.21.0021.01

$$\int z \, e^{a^2 z^2} \operatorname{erfc}(a z) \, dz = \frac{1}{2 a^2} \left(\frac{2 a z}{\sqrt{\pi}} + e^{a^2 z^2} \operatorname{erfc}(a z)\right)$$

06.27.21.0022.01

$$\int z \, e^{b z^2} \operatorname{erfc}(c + a z) \, dz = \frac{1}{2 b} \left(e^{b z^2} \operatorname{erfc}(c + a z) + \frac{a}{\sqrt{b - a^2}} \exp\!\left(\frac{b c^2}{a^2 - b}\right) \operatorname{erfi}\!\left(\frac{-z a^2 - a c + b z}{\sqrt{b - a^2}}\right)\right)$$

06.27.21.0023.01

$$\int z \, e^{b z^2} \operatorname{erfc}(a z) \, dz = \frac{1}{2 b} \left(e^{b z^2} \operatorname{erfc}(a z) + \frac{a}{\sqrt{b - a^2}} \operatorname{erfi}\!\left(\sqrt{b - a^2} \, z\right)\right)$$

06.27.21.0024.01

$$\int z^3 e^{b z^2} \operatorname{erfc}(a z) \, dz = \frac{1}{2 b^2} \left(e^{b z^2} \left(b z^2 - 1\right) \operatorname{erfc}(a z) - \frac{a}{\sqrt{b - a^2}} \operatorname{erfi}\!\left(\sqrt{b - a^2} \, z\right) - \frac{a b z^3}{2 \sqrt{\pi} \left(\left(a^2 - b\right) z^2\right)^{3/2}} \left(-\sqrt{\pi} \operatorname{erf}\!\left(\sqrt{\left(a^2 - b\right) z^2}\right) + \sqrt{\pi} + 2 e^{\left(b - a^2\right) z^2} \sqrt{\left(a^2 - b\right) z^2}\right)\right)$$

06.27.21.0025.01

$$\int \frac{e^{b z^2} \operatorname{erfc}(a z)}{z} \, dz = \frac{1}{2} \operatorname{Ei}\!\left(b z^2\right) + \frac{a z}{\sqrt{-\pi b z^2}} \sum_{k=0}^{\infty} \frac{b^{-k} a^{2k} \, \Gamma\!\left(k + \frac{1}{2}, -b z^2\right)}{(2k + 1) \, k!}$$

Involving trigonometric functions

Involving sin

06.27.21.0026.01

$$\int \sin(b z) \operatorname{erfc}(a z) \, dz = -\frac{1}{2 b} \left(2 \cos(b z) \operatorname{erfc}(a z) + \exp\!\left(-\frac{b^2}{4 a^2}\right) \left(\operatorname{erf}\!\left(\frac{2 z a^2 + i b}{2 a}\right) - i \operatorname{erfi}\!\left(\frac{b}{2 a} + i a z\right)\right)\right)$$

06.27.21.0027.01

$$\int \sin\!\left(b z^2\right) \operatorname{erfc}(a z) \, dz = \frac{\sqrt{\pi}}{\sqrt{2 b}} S\!\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \, z\right) + \frac{1}{2 \sqrt{\pi} \, b} \left(\sum_{k=0}^{\infty} \frac{(i b)^{-k} a^{2k+1} \, \Gamma\!\left(k + 1, -i b z^2\right)}{(2k + 1) \, k!} + \sum_{k=0}^{\infty} \frac{(-i b)^{-k} a^{2k+1} \, \Gamma\!\left(k + 1, i b z^2\right)}{(2k + 1) \, k!}\right)$$

Involving cos

06.27.21.0028.01

$$\int \cos(b z) \operatorname{erfc}(a z) \, dz = \frac{1}{2 b} \left(i e^{-\frac{b^2}{4 a^2}} \left(\operatorname{erf}\!\left(\frac{2 z a^2 + i b}{2 a}\right) + i \operatorname{erfi}\!\left(\frac{b}{2 a} + i a z\right)\right) + 2 \operatorname{erfc}(a z) \sin(b z)\right)$$

06.27.21.0029.01

$$\int \cos(b z^2)\,\mathrm{erf}(a z)\,dz = \frac{\sqrt{\pi}}{\sqrt{2b}}\,C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,z\right) + \frac{i}{2\sqrt{\pi}\,b}\left(\sum_{k=0}^{\infty}\frac{(i b)^{-k} a^{2k+1}\,\Gamma(k+1, -i b z^2)}{(2k+1)\,k!} - \sum_{k=0}^{\infty}\frac{(-i b)^{-k} a^{2k+1}\,\Gamma(k+1, i b z^2)}{(2k+1)\,k!}\right)$$

Involving trigonometric functions and a power function

Involving sin and power

06.27.21.0030.01

$$\int z^{\alpha-1}\sin(b z)\,\mathrm{erfc}(a z)\,dz = -\frac{1}{2}\,i\,z^{\alpha}\left((i b z)^{-\alpha}\,\Gamma(\alpha, i b z) - (-i b z)^{-\alpha}\,\Gamma(\alpha, -i b z)\right) + \frac{a z^{\alpha}}{b\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{a^{2k} b^{-2k}}{(2k+1)\,k!}\left(\Gamma(2k+\alpha+1, -i b z)\,(-i b z)^{-\alpha} + (i b z)^{-\alpha}\,\Gamma(2k+\alpha+1, i b z)\right)$$

06.27.21.0031.01

$$\int z^{n}\sin(b z)\,\mathrm{erfc}(a z)\,dz = \frac{1}{2}\,b^{-n-1}\Bigg(-i^{n}\,\mathrm{erfc}(a z)\,(\Gamma(n+1, -i b z) + (-1)^{n}\,\Gamma(n+1, i b z)) +$$
$$\frac{a\,n!}{\sqrt{\pi}}\exp\left(-\frac{b^2}{4a^2}\right)\Bigg((-i)^{n}\sum_{m=0}^{n}\frac{1}{m!}\,(i b)^{m}\,(-a^2)^{\frac{1}{2}(-m-1)}\sum_{k=0}^{m}\binom{m}{k}\left(-\frac{-i b}{2\sqrt{-a^2}}\right)^{m-k}\left(\sqrt{-a^2}\,z - \frac{i b}{2\sqrt{-a^2}}\right)^{k+1}$$
$$\left(-\left(\sqrt{-a^2}\,z - \frac{i b}{2\sqrt{-a^2}}\right)^{2}\right)^{\frac{1}{2}(-k-1)}\Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\,z - \frac{i b}{2\sqrt{-a^2}}\right)^{2}\right) +$$
$$i^{n}\sum_{m=0}^{n}\frac{1}{m!}\,(-i b)^{m}\,(-a^2)^{\frac{1}{2}(-m-1)}\sum_{k=0}^{m}\binom{m}{k}\left(-\frac{i b}{2\sqrt{-a^2}}\right)^{m-k}\left(\sqrt{-a^2}\,z + \frac{i b}{2\sqrt{-a^2}}\right)^{k+1}$$
$$\left(-\left(\sqrt{-a^2}\,z + \frac{i b}{2\sqrt{-a^2}}\right)^{2}\right)^{\frac{1}{2}(-k-1)}\Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\,z + \frac{i b}{2\sqrt{-a^2}}\right)^{2}\right)\Bigg)\Bigg) \;/;\; n \in \mathbb{N}$$

06.27.21.0032.01

$$\int z\sin(b z)\,\mathrm{erfc}(a z)\,dz = \frac{1}{4a^2 b^2\sqrt{\pi}}\Bigg(e^{-\frac{b^2}{4a^2} - i z b - a^2 z^2}\Bigg(-2\,e^{\frac{b^2}{4a^2} + a^2 z^2}\sqrt{\pi}\left(-i + b z + e^{2 i b z}(i + b z)\right)\mathrm{erfc}(a z)\,a^2 - 2\,e^{z(z a^2 + b i)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2a} + a i z\right)a^2 +$$
$$2 b\,e^{\frac{b^2}{4a^2}}\,a + 2 b\,e^{\frac{1}{4}b\left(\frac{b}{a^2} + 8 i z\right)}\,a + (2a^2 + b^2)\,e^{z(z a^2 + b i)}\,i\sqrt{\pi}\,\mathrm{erf}\left(\frac{2 z a^2 + b i}{2a}\right) - b^2\,e^{z(z a^2 + b i)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2a} + a i z\right)\Bigg)\Bigg)$$

06.27.21.0033.01

$$\int z^2 \sin(b z) \operatorname{erfc}(a z) \, dz = \frac{1}{8 a^4 b^3 \sqrt{\pi}} \left(e^{-\frac{b^2}{4 a^2} - i z b - a^2 z^2} \left(-4 e^{\frac{b^2}{4 a^2} + a^2 z^2} \sqrt{\pi} \left(b^2 z^2 - 2 i b z + e^{2 i b z} \left(b^2 z^2 + 2 b i z - 2 \right) - 2 \right) \operatorname{erfc}(a z) a^4 - 8 i e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erfi}\left(\frac{b}{2 a} + a i z \right) a^4 - 8 b e^{\frac{b^2}{4 a^2}} i a^3 + 8 b e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} i a^3 + 4 b^2 e^{\frac{b^2}{4 a^2}} z a^3 + 4 b^2 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} z a^3 - 2 i b^2 e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erfi}\left(\frac{b}{2 a} + a i z \right) a^2 - 2 i b^3 e^{\frac{b^2}{4 a^2}} a + 2 b^3 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} i a + \left(8 a^4 + 2 b^2 a^2 + b^4 \right) e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erf}\left(\frac{2 z a^2 + b i}{2 a} \right) - i b^4 e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erfi}\left(\frac{b}{2 a} + a i z \right) \right) \right)$$

06.27.21.0034.01

$$\int z^3 \sin(b z) \operatorname{erfc}(a z) \, dz = \frac{1}{16 a^6 b^4 \sqrt{\pi}} \left(e^{-\frac{b^2}{4 a^2} - i z b - a^2 z^2} \left(48 e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erfi}\left(\frac{b}{2 a} + a i z \right) a^6 - 8 e^{\frac{b^2}{4 a^2} + a^2 z^2} \sqrt{\pi} \operatorname{erfc}(a z) \left(b z \left(b^2 z^2 - 3 i b z + e^{2 i b z} \left(b^2 z^2 + 3 b i z - 6 \right) - 6 \right) + 12 e^{i b z} \sin(b z) \right) a^6 - 48 b e^{\frac{b^2}{4 a^2}} a^5 - 48 b e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} a^5 + 8 b^3 e^{\frac{b^2}{4 a^2}} z^2 a^5 + 8 b^3 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} z^2 a^5 - 24 i b^2 e^{\frac{b^2}{4 a^2}} z a^5 + 24 b^2 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} i z a^5 + 12 b^2 e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erfi}\left(\frac{b}{2 a} + a i z \right) a^4 - 4 b^3 e^{\frac{b^2}{4 a^2}} a^3 - 4 b^3 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} a^3 - 4 i b^4 e^{\frac{b^2}{4 a^2}} z a^3 + 4 b^4 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} i z a^3 - 2 b^5 e^{\frac{b^2}{4 a^2}} a - 2 b^5 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} a - i \left(48 a^6 + 12 b^2 a^4 + b^6 \right) e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erf}\left(\frac{2 z a^2 + b i}{2 a} \right) + b^6 e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erfi}\left(\frac{b}{2 a} + a i z \right) \right) \right)$$

06.27.21.0035.01

$$\int z^{\alpha - 1} \sin\left(b z^2 \right) \operatorname{erfc}(a z) \, dz = \frac{1}{4} i z^\alpha \left(\left(-i b z^2 \right)^{-\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, -i b z^2 \right) - \left(i b z^2 \right)^{-\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, i b z^2 \right) \right) - \frac{i a z^{\alpha + 1}}{2 \sqrt{\pi}} \left(b^2 z^4 \right)^{\frac{1}{2} (-\alpha - 1)} \left(\left(i b z^2 \right)^{\frac{\alpha + 1}{2}} \sum_{k=0}^{\infty} \frac{(i b)^{-k} a^{2 k} \Gamma\left(\frac{\alpha + 1}{2} + k, -i b z^2 \right)}{(2 k + 1) k!} - \left(-i b z^2 \right)^{\frac{\alpha + 1}{2}} \sum_{k=0}^{\infty} \frac{(-i b)^{-k} a^{2 k} \Gamma\left(\frac{\alpha + 1}{2} + k, i b z^2 \right)}{(2 k + 1) k!} \right)$$

06.27.21.0036.01

$$\int z \sin\left(b z^2 \right) \operatorname{erfc}(c + a z) \, dz = \frac{1}{4 b \left(a^4 + b^2 \right)} e^{-c^2} \left(-a \sqrt{a^2 + b i} \left(a^2 - i b \right) e^{\frac{a^2 c^2}{a^2 + b i}} \operatorname{erf}\left(\frac{z a^2 + c a + b i z}{\sqrt{a^2 + b i}} \right) - \left(a^2 + b i \right) \left(2 \left(a^2 - i b \right) e^{c^2} \cos\left(b z^2 \right) \operatorname{erfc}(c + a z) - i a \sqrt{a^2 - i b} \, e^{\frac{a^2 c^2}{a^2 - i b}} \operatorname{erfi}\left(\frac{i z a^2 + c i a + b z}{\sqrt{a^2 - i b}} \right) \right) \right)$$

06.27.21.0037.01

$$\int z\sin(b z^2)\operatorname{erfc}(a z)\,dz = -\frac{1}{4 b (a^4+b^2)} \left(a\sqrt{a^2+b\,i}\,(a^2-i\,b)\operatorname{erf}\left(\sqrt{a^2+b\,i}\,z\right)+(a^2+b\,i)\left(2(a^2-i\,b)\cos(b z^2)\operatorname{erfc}(a z)-i\,a\sqrt{a^2-i\,b}\operatorname{erfi}\left(\frac{(i\,a^2+b)z}{\sqrt{a^2-i\,b}}\right)\right)\right)$$

06.27.21.0038.01

$$\int z^3\sin(b z^2)\operatorname{erfc}(a z)\,dz = \frac{1}{4 b^2}\left(-\frac{1}{2\sqrt{\pi}}\left(a b z^3\left(\left(\sqrt{\pi}\operatorname{erf}\left(\sqrt{(a^2+b\,i)z^2}\right)-\sqrt{\pi}-2e^{-(a^2+b\,i)z^2}\sqrt{(a^2+b\,i)z^2}\right)\Big/\left((a^2+b\,i)z^2\right)^{3/2}+\left(\sqrt{\pi}\operatorname{erf}\left(\sqrt{(a^2-i\,b)z^2}\right)-\sqrt{\pi}-2e^{-(a^2-i\,b)z^2}\sqrt{(a^2-i\,b)z^2}\right)\Big/\left((a^2-i\,b)z^2\right)^{3/2}\right)\right)+\frac{1}{a^4+b^2}\left(a\left(\sqrt{a^2+b\,i}\,(i\,a^2+b)\operatorname{erf}\left(\sqrt{a^2+b\,i}\,z\right)-\sqrt{a^2-i\,b}\,(a^2+b\,i)\operatorname{erfi}\left(\frac{(i\,a^2+b)z}{\sqrt{a^2-i\,b}}\right)\right)\right)-2\operatorname{erfc}(a z)\left(b z^2\cos(b z^2)-\sin(b z^2)\right)\right)$$

06.27.21.0039.01

$$\int\frac{\sin(b z^2)\operatorname{erfc}(a z)}{z}\,dz = \frac{1}{2}\operatorname{Si}(b z^2)-\frac{i\,a z}{2\sqrt{-\pi\,i\,b z^2}}\sum_{k=0}^{\infty}\frac{(i\,b)^{-k}a^{2k}\,\Gamma\left(k+\frac{1}{2},-i\,b z^2\right)}{(2k+1)\,k!}+\frac{i\,a z}{2\sqrt{\pi\,i\,b z^2}}\sum_{k=0}^{\infty}\frac{(-i\,b)^{-k}a^{2k}\,\Gamma\left(k+\frac{1}{2},i\,b z^2\right)}{(2k+1)\,k!}$$

Involving cos and power

06.27.21.0040.01

$$\int z^{\alpha-1}\cos(b z)\operatorname{erfc}(a z)\,dz = \frac{1}{2}z^{\alpha}\left(-(-i\,b z)^{-\alpha}\,\Gamma(\alpha,-i\,b z)-(i\,b z)^{-\alpha}\,\Gamma(\alpha,i\,b z)\right)-\frac{i\,a z^{\alpha}}{b\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{a^{2k}b^{-2k}}{(2k+1)\,k!}\left((i\,b z)^{-\alpha}\,\Gamma(2k+\alpha+1,i\,b z)-(-i\,b z)^{-\alpha}\,\Gamma(2k+\alpha+1,-i\,b z)\right)$$

06.27.21.0041.01

$$\int z^n \cos(b\,z)\,\mathrm{erfc}(a\,z)\,dz = \frac{i}{2} b^{-n-1} \Bigg(i^n \,\mathrm{erfc}(a\,z)\,(-\Gamma(n+1, -i\,b\,z) + (-1)^n\,\Gamma(n+1, i\,b\,z)) + \frac{a\,n!}{\sqrt{\pi}} \exp\left(-\frac{b^2}{4a^2}\right) \Bigg(-(-i)^n \sum_{m=0}^{n} \frac{1}{m!} (i\,b)^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{-i\,b}{2\sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2}\,z - \frac{i\,b}{2\sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2}\,z - \frac{i\,b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\,z - \frac{i\,b}{2\sqrt{-a^2}}\right)^2\right) + i^n \sum_{m=0}^{n} \frac{1}{m!} (-i\,b)^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{i\,b}{2\sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2}\,z + \frac{i\,b}{2\sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2}\,z + \frac{i\,b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\,z + \frac{i\,b}{2\sqrt{-a^2}}\right)^2\right)\Bigg)\Bigg) /; n \in \mathbb{N}$$

06.27.21.0042.01

$$\int z \cos(b\,z)\,\mathrm{erfc}(a\,z)\,dz = \frac{1}{4a^2 b^2 \sqrt{\pi}} \exp\left(-\frac{b^2}{4a^2} - i\,z\,b - a^2 z^2\right) \left(2 e^{\frac{b^2}{4a^2} + a^2 z^2} \sqrt{\pi}\,\mathrm{erfc}(a\,z)\left(2\,b\,e^{i\,b\,z} z \sin(b\,z) + e^{2\,i\,b\,z} + 1\right) a^2 + \left(2a^2 + b^2\right) e^{z\left(z\,a^2 + b\,i\right)} \sqrt{\pi}\,\mathrm{erf}\left(\frac{2\,z\,a^2 + b\,i}{2a}\right) - i\left(\left(2a^2 + b^2\right) e^{z\left(z\,a^2 + b\,i\right)} \sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2a} + a\,i\,z\right) - 2\,a\,b \exp\left(\frac{b^2}{4a^2}\right)\left(-1 + e^{2\,i\,b\,z}\right)\right)\right)$$

06.27.21.0043.01

$$\int z^2 \cos(b\,z)\,\mathrm{erfc}(a\,z)\,dz = \frac{1}{8a^4 b^3 \sqrt{\pi}} \exp\left(-\frac{b^2}{4a^2} - i\,z\,b - a^2 z^2\right) \Bigg(8 e^{z\left(z\,a^2 + b\,i\right)} \sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2a} + a\,i\,z\right) a^4 + 8 e^{\frac{b^2}{4a^2} + a^2 z^2} \sqrt{\pi}\,\mathrm{erfc}(a\,z)\left(b^2 e^{i\,b\,z} \sin(b\,z) z^2 + b\,z - i + e^{2\,i\,b\,z}(i + b\,z)\right) a^4 - 8\,b\,e^{\frac{b^2}{4a^2}} a^3 - 8\,b\,e^{\frac{1}{4} b\left(\frac{b}{a^2} + 8\,i\,z\right)} a^3 - 4\,i\,b^2 e^{\frac{b^2}{4a^2}} z\,a^3 + 4\,b^2 \exp\left(\frac{1}{4} b\left(\frac{b}{a^2} + 8\,i\,z\right)\right) i\,z\,a^3 + 2\,b^2 e^{z\left(z\,a^2 + b\,i\right)} \sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2a} + a\,i\,z\right) a^2 - 2\,b^3 e^{\frac{b^2}{4a^2}} a - 2\,b^3 \exp\left(\frac{1}{4} b\left(\frac{b}{a^2} + 8\,i\,z\right)\right) a - i\left(8a^4 + 2b^2 a^2 + b^4\right) e^{z\left(z\,a^2 + b\,i\right)} \sqrt{\pi}\,\mathrm{erf}\left(\frac{2\,z\,a^2 + b\,i}{2a}\right) + b^4 e^{z\left(z\,a^2 + b\,i\right)} \sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2a} + a\,i\,z\right)\Bigg)$$

06.27.21.0044.01

$$\int z^3 \cos(b z)\, \operatorname{erfc}(a z)\, dz = \frac{1}{16 a^6 b^4 \sqrt{\pi}} \exp\left(-\frac{b^2}{4 a^2} - i z b - a^2 z^2\right) \left(8 \exp\left(\frac{b^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi}\, \operatorname{erfc}(a z) \left(2 b^3 e^{i b z} \sin(b z) z^3 + 3 \left(b^2 z^2 - 2 i b z + e^{2 i b z} \left(b^2 z^2 + 2 b i z - 2\right) - 2\right)\right) a^6 - \left(48 a^6 + 12 b^2 a^4 + b^6\right) e^{z (z a^2 + b i)} \sqrt{\pi}\, \operatorname{erf}\left(\frac{2 z a^2 + b i}{2 a}\right) - i \left(-2 a b \exp\left(\frac{b^2}{4 a^2}\right) \left(4 \left(-b^2 z^2 + 3 b i z + e^{2 i b z} \left(b^2 z^2 + 3 b i z - 6\right) + 6\right) a^4 + 2 b^2 \left(b i z + e^{2 i b z} (i b z - 1) + 1\right) a^2 - b^4 \left(-1 + e^{2 i b z}\right)\right) - \left(48 a^6 + 12 b^2 a^4 + b^6\right) e^{z (z a^2 + b i)} \sqrt{\pi}\, \operatorname{erfi}\left(\frac{b}{2 a} + a i z\right)\right)\right)$$

06.27.21.0045.01

$$\int z^{\alpha-1} \cos\left(b z^2\right) \operatorname{erfc}(a z)\, dz = \frac{1}{4} z^\alpha \left(-\left(-i b z^2\right)^{-\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, -i b z^2\right) - \left(i b z^2\right)^{-\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, i b z^2\right)\right) - \frac{a z^{\alpha+1}}{2 \sqrt{\pi}} \left(b^2 z^4\right)^{\frac{1}{2}(-\alpha-1)} \left(-\left(i b z^2\right)^{\frac{\alpha+1}{2}} \sum_{k=0}^{\infty} \frac{(i b)^{-k} a^{2k}\, \Gamma\left(\frac{\alpha+1}{2} + k, -i b z^2\right)}{(2k+1)\, k!} - \left(-i b z^2\right)^{\frac{\alpha+1}{2}} \sum_{k=0}^{\infty} \frac{(-i b)^{-k} a^{2k}\, \Gamma\left(\frac{\alpha+1}{2} + k, i b z^2\right)}{(2k+1)\, k!}\right)$$

06.27.21.0046.01

$$\int z \cos\left(b z^2\right) \operatorname{erfc}(c + a z)\, dz = \frac{1}{b \left(a^4 + b^2\right)} \left(\left(\frac{1-i}{8}\right) \exp\left(-\frac{i b c^2}{a^2 + b i}\right) \left(\sqrt{2}\, a \left(b - i a^2\right) \sqrt{i a^2 + b}\, \exp\left(\frac{2 i a^2 b c^2}{a^4 + b^2}\right) \operatorname{erf}\left(\frac{(1+i) \left(z a^2 + c a - i b z\right)}{\sqrt{2} \sqrt{i a^2 + b}}\right) + \left(a^2 - i b\right) \left(\sqrt{2}\, a \sqrt{b - i a^2}\, i \operatorname{erfi}\left(\frac{(1+i)\left(z a^2 + c a + b i z\right)}{\sqrt{2} \sqrt{b - i a^2}}\right) + \left(a^2 + b i\right) \exp\left(\frac{i b c^2}{a^2 + b i}\right) (2 + 2 i) \operatorname{erfc}(c + a z) \sin\left(b z^2\right)\right)\right)\right)$$

06.27.21.0047.01

$$\int z \cos\left(b z^2\right) \operatorname{erfc}(a z)\, dz = \frac{1}{4 b \left(a^4 + b^2\right)} \left(\sqrt[4]{-1}\, a \sqrt{b - i a^2}\, \left(a^2 - i b\right) \operatorname{erfi}\left((-1)^{3/4} \sqrt{b - i a^2}\, z\right) + (-1)^{3/4} a \sqrt{i a^2 + b}\, \left(a^2 + b i\right) \operatorname{erfi}\left(\sqrt[4]{-1} \sqrt{i a^2 + b}\, z\right) + 2 \left(a^4 + b^2\right) \operatorname{erfc}(a z) \sin\left(b z^2\right)\right)$$

06.27.21.0048.01

$$\int z^3 \cos\left(b z^2\right) \operatorname{erfc}(a z)\, dz = \frac{1}{2 b^2} \left(-\frac{1}{4 \sqrt{\pi}} \left(i a b z^3 \left(\left(-\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{\left(a^2 + b i\right) z^2}\right) + \sqrt{\pi} + 2 e^{-\left(a^2 + b i\right) z^2} \sqrt{\left(a^2 + b i\right) z^2}\right) \Big/ \left(\left(a^2 + b i\right) z^2\right)^{3/2} + \left(\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{\left(a^2 - i b\right) z^2}\right) - \sqrt{\pi} - 2 e^{-\left(a^2 - i b\right) z^2} \sqrt{\left(a^2 - i b\right) z^2}\right) \Big/ \left(\left(a^2 - i b\right) z^2\right)^{3/2}\right)\right) - \frac{1}{2 \left(a^4 + b^2\right)} \left(\sqrt[4]{-1}\, a \left(\sqrt{b - i a^2}\, \left(i a^2 + b\right) \operatorname{erfi}\left((-1)^{3/4} \sqrt{b - i a^2}\, z\right) + \sqrt{i a^2 + b}\, \left(a^2 + b i\right) \operatorname{erfi}\left(\sqrt[4]{-1} \sqrt{i a^2 + b}\, z\right)\right)\right) + \operatorname{erfc}(a z) \left(b \sin\left(b z^2\right) z^2 + \cos\left(b z^2\right)\right)\right)$$

06.27.21.0049.01

$$\int \frac{\cos(b z^2)\,\text{erfc}(a z)}{z}\, dz = \frac{1}{2}\,\text{Ci}(b z^2) + \frac{a z}{2\sqrt{-\pi i b z^2}} \sum_{k=0}^{\infty} \frac{(i b)^{-k} a^{2k}\,\Gamma\left(k+\frac{1}{2}, -i b z^2\right)}{(2k+1)\,k!} + \frac{a z}{2\sqrt{\pi i b z^2}} \sum_{k=0}^{\infty} \frac{(-i b)^{-k} a^{2k}\,\Gamma\left(k+\frac{1}{2}, i b z^2\right)}{(2k+1)\,k!}$$

Involving exponential function and trigonometric functions

Involving exp and sin

06.27.21.0050.01

$$\int e^{b z} \sin(c z)\,\text{erfc}(a z)\, dz = \frac{1}{2(b^2+c^2)} \left((b + c i)\, e^{\frac{(b-i c)^2}{4 a^2}}\, i\,\text{erf}\left(\frac{2 z a^2 - b + c i}{2 a}\right) + (c + b i)\, e^{\frac{(b+c i)^2}{4 a^2}}\, \text{erf}\left(\frac{-2 z a^2 + b + c i}{2 a}\right) + 2 e^{b z}\,\text{erfc}(a z)\,(b \sin(c z) - c \cos(c z)) \right)$$

06.27.21.0051.01

$$\int e^{b z^2} \sin(c z^2)\,\text{erfc}(a z)\, dz = \frac{i\sqrt{\pi}}{4(b^2+c^2)} \left(\sqrt{b - i c}\,(b + c i)\,\text{erfi}\left(\sqrt{b - i c}\, z\right) - (b - i c)\sqrt{b + c i}\,\text{erfi}\left(\sqrt{b + c i}\, z\right) \right) - \frac{i}{2\sqrt{\pi}\,(b - i c)} \sum_{k=0}^{\infty} \frac{(b - i c)^{-k} a^{2k+1}\,\Gamma\left(k+1, -(b - i c) z^2\right)}{(2k+1)\,k!} + \frac{i}{2\sqrt{\pi}\,(b + c i)} \sum_{k=0}^{\infty} \frac{(b + c i)^{-k} a^{2k+1}\,\Gamma\left(k+1, -(b + c i) z^2\right)}{(2k+1)\,k!}$$

Involving exp and cos

06.27.21.0052.01

$$\int e^{b z} \cos(c z)\,\text{erfc}(a z)\, dz = \frac{1}{2(b^2+c^2)} \left((b + c i)\, e^{\frac{(b-i c)^2}{4 a^2}}\, \text{erf}\left(\frac{2 z a^2 - b + c i}{2 a}\right) - (b - i c)\, e^{\frac{(b+c i)^2}{4 a^2}}\, \text{erf}\left(\frac{-2 z a^2 + b + c i}{2 a}\right) + 2 e^{b z}\,\text{erfc}(a z)\,(b \cos(c z) + c \sin(c z)) \right)$$

06.27.21.0053.01

$$\int e^{b z^2} \cos(c z^2)\,\text{erfc}(a z)\, dz = \frac{\sqrt{\pi}}{4(b^2+c^2)} \left(\sqrt{b + c i}\,(b - i c)\,\text{erfi}\left(\sqrt{b + c i}\, z\right) + (b + c i)\sqrt{b - i c}\,\text{erfi}\left(\sqrt{b - i c}\, z\right) \right) - \frac{1}{2\sqrt{\pi}\,(b + c i)} \sum_{k=0}^{\infty} \frac{(b + c i)^{-k} a^{2k+1}\,\Gamma\left(k+1, -(b + c i) z^2\right)}{(2k+1)\,k!} - \frac{1}{2\sqrt{\pi}\,(b - i c)} \sum_{k=0}^{\infty} \frac{(b - i c)^{-k} a^{2k+1}\,\Gamma\left(k+1, -(b - i c) z^2\right)}{(2k+1)\,k!}$$

Involving power, exponential and trigonometric functions

Involving power, exp and sin

06.27.21.0054.01

$$\int z^{\alpha-1} e^{b z} \sin(c z) \operatorname{erfc}(a z) \, dz = -\frac{1}{2} i z^{\alpha} \left((-(b - i c) z)^{-\alpha} \Gamma(\alpha, -(b - i c) z) - (-(b + c i) z)^{-\alpha} \Gamma(\alpha, -(b + c i) z) \right) - \frac{i a z^{\alpha} (-(b - i c) z)^{-\alpha}}{(b - i c) \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b - i c)^{-2k}}{(2k+1) k!} \Gamma(2k + \alpha + 1, -(b - i c) z) + \frac{i a z^{\alpha} (-(b + c i) z)^{-\alpha}}{(b + c i) \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b + c i)^{-2k}}{(2k+1) k!} \Gamma(2k + \alpha + 1, -(b + c i) z)$$

06.27.21.0055.01

$$\int z^{n} e^{b z} \sin(c z) \operatorname{erfc}(a z) \, dz = \frac{1}{2} i \operatorname{erfc}(a z) \left((-b - i c)^{-n-1} \Gamma(n+1, -(b + c i) z) - (i c - b)^{-n-1} \Gamma(n+1, i c z - b z) \right) - \frac{1}{2\sqrt{\pi}} \left(i a (-b - i c)^{-n-1} e^{\frac{(b+c i)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b + c i))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b + c i}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^{k+1} \left(-\left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^2 \right) \right) \right) + \frac{1}{2\sqrt{\pi}} \left(i a (i c - b)^{-n-1} e^{\frac{(b-i c)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b - i c))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b - i c}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^{k+1} \left(-\left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^2 \right) \right) \right) /; n \in \mathbb{N}$$

06.27.21.0056.01

$$\int z e^{b z} \sin(c z) \operatorname{erfc}(a z) \, dz = \frac{1}{\left(b^2 + c^2\right)^2} \left(e^{b z} \left(\left(z b^3 - b^2 + c^2 z b + c^2 \right) \sin(c z) - c \left(z b^2 - 2 b + c^2 z \right) \cos(c z) \right) \right) - \frac{i}{4 a^2 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b - i c)^2} \left(2 e^{z \left(z a^2 + b - i c\right)} \sqrt{\pi} \, (b z - i c z - 1) \operatorname{erf}(a z) a^2 + 2 (b - i c) e^{(b - i c) z} a - \left(2 a^2 - (b - i c)^2 \right) \exp\left(\frac{(b - i c)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b - i c}{2 a} - a z \right) \right) - \frac{1}{(b + c i)^2} \left(2 e^{z \left(z a^2 + b + c i\right)} \sqrt{\pi} \, (b z + c i z - 1) \operatorname{erf}(a z) a^2 + 2 (b + c i) e^{(b + c i) z} a - \left(2 a^2 - (b + c i)^2 \right) \exp\left(\frac{(b + c i)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b + c i}{2 a} - a z \right) \right) \right)$$

06.27.21.0057.01

$$\int z^2 e^{b z} \sin(c z) \operatorname{erfc}(a z) \, dz = \frac{1}{\left(b^2+c^2\right)^3} \left(e^{b z} \left(\left(z^2 b^5 - 2 z b^4 + 2 \left(c^2 z^2+1\right) b^3 + c^2 \left(c^2 z^2-6\right) b + 2 c^4 z\right) \sin(c z) - c \left(z^2 b^4 - 4 z b^3 + 2 \left(c^2 z^2+3\right) b^2 - 4 c^2 z b + c^2 \left(c^2 z^2-2\right)\right) \cos(c z)\right)\right) - \frac{i}{8 a^4 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b - i c)^3} \left(4 e^{z \left(z a^2+b-i c\right)} \sqrt{\pi} \left((b-i c)^2 z^2 - 2 (b-i c) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b - i c) e^{(b-i c) z} \left(2 (b z - i c z - 2) a^2 + (b-i c)^2\right) a + \left(8 a^4 - 2 (b-i c)^2 a^2 + (b-i c)^4\right) \exp\left(\frac{(b-i c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b - i c}{2 a} - a z\right)\right) - \frac{1}{(b + c i)^3} \left(4 e^{z \left(z a^2+b+c i\right)} \sqrt{\pi} \left((b+c i)^2 z^2 - 2 (b+c i) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b + c i) e^{(b+c i) z} \left(2 (b z + c i z - 2) a^2 + (b+c i)^2\right) a + \left(8 a^4 - 2 (b+c i)^2 a^2 + (b+c i)^4\right) \exp\left(\frac{(b+c i)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b + c i}{2 a} - a z\right)\right)\right)$$

06.27.21.0058.01

$$\int z^3 e^{b z} \sin(c z) \operatorname{erfc}(a z) \, dz = \frac{1}{\left(b^2+c^2\right)^4} \left(e^{b z} \left(\left(z^3 b^7 - 3 z^2 b^6 + 3 z \left(c^2 z^2+2\right) b^5 - 3 \left(c^2 z^2+2\right) b^4 + 3 c^2 z \left(c^2 z^2-4\right) b^3 + 3 c^2 \left(c^2 z^2+12\right) b^2 + c^4 z \left(c^2 z^2-18\right) b + 3 c^4 \left(c^2 z^2-2\right)\right) \sin(c z) - c \left(z^3 b^6 - 6 z^2 b^5 + 3 z \left(c^2 z^2+6\right) b^4 - 12 \left(c^2 z^2+2\right) b^3 + 3 c^2 z \left(c^2 z^2+4\right) b^2 - 6 c^2 \left(c^2 z^2-4\right) b + c^4 z \left(c^2 z^2-6\right)\right) \cos(c z)\right)\right) - \frac{i}{16 a^6 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b - i c)^4} \left(8 e^{z \left(z a^2+b-i c\right)} \sqrt{\pi} \left((b-i c)^3 z^3 - 3 (b-i c)^2 z^2 + 6 (b-i c) z - 6\right) \operatorname{erf}(a z) a^6 + 2 (b - i c) e^{(b-i c) z} \left(4 \left((b-i c)^2 z^2 - 3 (b-i c) z + 6\right) a^4 + 2 (b-i c)^2 (b z - i c z - 1) a^2 + (b-i c)^4\right) a - \left(48 a^6 - 12 (b-i c)^2 a^4 - (b-i c)^6\right) \exp\left(\frac{(b-i c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b - i c}{2 a} - a z\right)\right) - \frac{1}{(b + c i)^4} \left(8 e^{z \left(z a^2+b+c i\right)} \sqrt{\pi} \left((b+c i)^3 z^3 - 3 (b+c i)^2 z^2 + 6 (b+c i) z - 6\right) \operatorname{erf}(a z) a^6 + 2 (b + c i) e^{(b+c i) z} \left(4 \left((b+c i)^2 z^2 - 3 (b+c i) z + 6\right) a^4 + 2 (b+c i)^2 (b z + c i z - 1) a^2 + (b+c i)^4\right) a - \left(48 a^6 - 12 (b+c i)^2 a^4 - (b+c i)^6\right) \exp\left(\frac{(b+c i)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b + c i}{2 a} - a z\right)\right)\right)$$

06.27.21.0059.01

$$\int z^{\alpha-1} e^{b z^2} \sin\left(c z^2\right) \operatorname{erfc}(a z) \, dz = -\frac{1}{4} i z^\alpha \left(\left(-(b - i c) z^2\right)^{-\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, -(b - i c) z^2\right) - \left(-(b + c i) z^2\right)^{-\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, -(b + c i) z^2\right)\right) + \frac{i}{2 \sqrt{\pi}} a z^{\alpha+1} \left(\left(-(b - i c) z^2\right)^{\frac{1}{2}(-\alpha-1)} \sum_{k=0}^{\infty} \frac{(b - i c)^{-k} a^{2 k} \Gamma\left(\frac{\alpha+1}{2} + k, -(b - i c) z^2\right)}{(2 k + 1) k!} - \left(-(b + c i) z^2\right)^{\frac{1}{2}(-\alpha-1)} \sum_{k=0}^{\infty} \frac{(b + c i)^{-k} a^{2 k} \Gamma\left(\frac{\alpha+1}{2} + k, -(b + c i) z^2\right)}{(2 k + 1) k!}\right)$$

06.27.21.0060.01

$$\int z\, e^{b z^2} \sin(c z^2)\, \mathrm{erfc}(a z)\, dz = -\frac{i}{4(b^2+c^2)} \left(\frac{a(b - i c)\, \mathrm{erf}\left(\sqrt{(a^2-b-i c) z^2}\right) z}{\sqrt{(a^2-b-i c) z^2}} + \frac{a b z}{\sqrt{(a^2-b+c i) z^2}} + \frac{a c i z}{\sqrt{(a^2-b+c i) z^2}} - \frac{a(b + c i)\, \mathrm{erf}\left(\sqrt{(a^2-b+c i) z^2}\right) z}{\sqrt{(a^2-b+c i) z^2}} - \frac{a b z}{\sqrt{(a^2-b-i c) z^2}} + \frac{a c i z}{\sqrt{(a^2-b-i c) z^2}} - 2 i c\, e^{b z^2} \cos(c z^2)\, \mathrm{erfc}(a z) + 2 b\, e^{b z^2}\, i\, \mathrm{erfc}(a z) \sin(c z^2) \right)$$

06.27.21.0061.01

$$\int \frac{e^{b z^2} \sin(c z^2)\, \mathrm{erfc}(a z)}{z}\, dz = \frac{1}{4} i \left(\mathrm{Ei}\left((b - i c) z^2\right) - \mathrm{Ei}\left((b + c i) z^2\right) \right) + \frac{i a z}{2\sqrt{\pi}} \left(\frac{1}{\sqrt{-(b-i c) z^2}} \sum_{k=0}^{\infty} \frac{(b - i c)^{-k} a^{2k}\, \Gamma\left(k+\frac{1}{2}, -(b-i c) z^2\right)}{(2k+1)\, k!} - \frac{1}{\sqrt{-(b+c i) z^2}} \sum_{k=0}^{\infty} \frac{(b + c i)^{-k} a^{2k}\, \Gamma\left(k+\frac{1}{2}, -(b+c i) z^2\right)}{(2k+1)\, k!} \right)$$

Involving power, exp and cos

06.27.21.0062.01

$$\int z^{\alpha-1} e^{b z} \cos(c z)\, \mathrm{erfc}(a z)\, dz = \frac{1}{2} z^{\alpha} \left(-\Gamma(\alpha, -(b + c i) z) (-(b + c i) z)^{-\alpha} - (-(b - i c) z)^{-\alpha}\, \Gamma(\alpha, -(b - i c) z) \right) - \frac{a z^{\alpha} (-(b + c i) z)^{-\alpha}}{(b + c i)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b + c i)^{-2k}}{(2k+1)\, k!} \Gamma(2k+\alpha+1, -(b + c i) z) - \frac{a z^{\alpha} (-(b - i c) z)^{-\alpha}}{(b - i c)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b - i c)^{-2k}}{(2k+1)\, k!} \Gamma(2k+\alpha+1, -(b - i c) z)$$

06.27.21.0063.01

$$\int z^n e^{b z} \cos(c z) \operatorname{erfc}(a z)\, dz = -\frac{1}{2} \operatorname{erfc}(a z) \left(\Gamma(n+1, i c z - b z) (i c - b)^{-n-1} + (-b - i c)^{-n-1} \Gamma(n+1, -(b + c i) z)\right) + \frac{1}{2\sqrt{\pi}} \left(a (-b - i c)^{-n-1} e^{\frac{(b+c i)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b + c i))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b + c i}{2\sqrt{-a^2}}\right)^{m-k} \left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z\right)^{k+1} \left(-\left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z\right)^2\right)\right)\right) + \frac{1}{2\sqrt{\pi}} \left(a (i c - b)^{-n-1} e^{\frac{(b-i c)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b - i c))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b - i c}{2\sqrt{-a^2}}\right)^{m-k} \left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z\right)^{k+1} \left(-\left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z\right)^2\right)\right)\right) /; n \in \mathbb{N}$$

06.27.21.0064.01

$$\int z\, e^{b z} \cos(c z) \operatorname{erfc}(a z)\, dz = \frac{1}{\left(b^2 + c^2\right)^2} \left(e^{b z} \left(\left(z b^3 - b^2 + c^2 z b + c^2\right) \cos(c z) + c \left(z b^2 - 2 b + c^2 z\right) \sin(c z)\right)\right) - \frac{1}{4 a^2 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b - i c)^2} \left(2 e^{z\left(z a^2 + b - i c\right)} \sqrt{\pi}\, (b z - i c z - 1) \operatorname{erf}(a z) a^2 + 2 (b - i c) e^{(b - i c) z} a - \left(2 a^2 - (b - i c)^2\right) \exp\left(\frac{(b - i c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b - i c}{2 a} - a z\right)\right) + \frac{1}{(b + c i)^2} \left(2 e^{z\left(z a^2 + b + c i\right)} \sqrt{\pi}\, (b z + c i z - 1) \operatorname{erf}(a z) a^2 + 2 (b + c i) e^{(b + c i) z} a - \left(2 a^2 - (b + c i)^2\right) \exp\left(\frac{(b + c i)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b + c i}{2 a} - a z\right)\right)\right)$$

06.27.21.0065.01

$$\int z^2 e^{bz}\cos(cz)\,\mathrm{erfc}(az)\,dz = \frac{1}{(b^2+c^2)^3}\left(e^{bz}\left(\left(z^2 b^5 - 2 z b^4 + 2\left(c^2 z^2+1\right) b^3 + c^2\left(c^2 z^2-6\right) b + 2 c^4 z\right)\cos(cz) + c\left(z^2 b^4 - 4 z b^3 + 2\left(c^2 z^2+3\right) b^2 - 4 c^2 z b + c^2\left(c^2 z^2-2\right)\right)\sin(cz)\right)\right) - \frac{1}{8 a^4\sqrt{\pi}} e^{-a^2 z^2}\left(\frac{1}{(b-ic)^3}\left(4 e^{z\left(z a^2+b-ic\right)}\sqrt{\pi}\left((b-ic)^2 z^2 - 2(b-ic)z+2\right)\mathrm{erf}(az)\,a^4 + 2(b-ic)\,e^{(b-ic)z}\left(2(bz-icz-2)a^2 + (b-ic)^2\right)a + \left(8a^4 - 2(b-ic)^2 a^2 + (b-ic)^4\right)\exp\left(\frac{(b-ic)^2}{4a^2}+a^2 z^2\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b-ic}{2a}-az\right)\right) + \frac{1}{(b+ci)^3}\left(4 e^{z\left(z a^2+b+ci\right)}\sqrt{\pi}\left((b+ci)^2 z^2 - 2(b+ci)z+2\right)\mathrm{erf}(az)\,a^4 + 2(b+ci)\,e^{(b+ci)z}\left(2(bz+ciz-2)a^2+(b+ci)^2\right)a + \left(8a^4 - 2(b+ci)^2 a^2 + (b+ci)^4\right)\exp\left(\frac{(b+ci)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b+ci}{2a}-az\right)\right)\right)$$

06.27.21.0066.01

$$\int z^3 e^{bz}\cos(cz)\,\mathrm{erfc}(az)\,dz = \frac{1}{(b^2+c^2)^4}\left(e^{bz}\left(\left(z^3 b^7 - 3z^2 b^6 + 3z\left(c^2z^2+2\right)b^5 - 3\left(c^2z^2+2\right)b^4 + 3c^2 z\left(c^2z^2-4\right)b^3 + 3c^2\left(c^2z^2+12\right)b^2 + c^4 z\left(c^2z^2-18\right)b + 3c^4\left(c^2z^2-2\right)\right)\cos(cz) + c\left(z^3 b^6 - 6z^2 b^5 + 3z\left(c^2z^2+6\right)b^4 - 12\left(c^2z^2+2\right)b^3 + 3c^2 z\left(c^2z^2+4\right)b^2 - 6c^2\left(c^2z^2-4\right)b + c^4 z\left(c^2z^2-6\right)\right)\sin(cz)\right)\right) - \frac{1}{16a^6\sqrt{\pi}} e^{-a^2z^2}\left(\frac{1}{(b-ic)^4}\left(8 e^{z\left(za^2+b-ic\right)}\sqrt{\pi}\left((b-ic)^3z^3 - 3(b-ic)^2z^2 + 6(b-ic)z - 6\right)\mathrm{erf}(az)\,a^6 + 2(b-ic)\,e^{(b-ic)z}\left(4\left((b-ic)^2z^2 - 3(b-ic)z+6\right)a^4 + 2(b-ic)^2(bz-icz-1)a^2 + (b-ic)^4\right)a - \left(48a^6 - 12(b-ic)^2a^4 - (b-ic)^6\right)\exp\left(\frac{(b-ic)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b-ic}{2a}-az\right)\right) + \frac{1}{(b+ci)^4}\left(8 e^{z\left(za^2+b+ci\right)}\sqrt{\pi}\left((b+ci)^3z^3 - 3(b+ci)^2z^2 + 6(b+ci)z - 6\right)\mathrm{erf}(az)\,a^6 + 2(b+ci)\,e^{(b+ci)z}\left(4\left((b+ci)^2z^2 - 3(b+ci)z+6\right)a^4 + 2(b+ci)^2(bz+ciz-1)a^2 + (b+ci)^4\right)a - \left(48a^6 - 12(b+ci)^2a^4 - (b+ci)^6\right)\exp\left(\frac{(b+ci)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b+ci}{2a}-az\right)\right)\right)$$

06.27.21.0067.01

$$\int z^{\alpha-1} e^{bz^2}\cos\left(cz^2\right)\mathrm{erfc}(az)\,dz = \frac{1}{4} z^\alpha\left(-\Gamma\left(\frac{\alpha}{2}, -(b+ci)z^2\right)\left(-(b+ci)z^2\right)^{-\frac{\alpha}{2}} - \left(-(b-ic)z^2\right)^{-\frac{\alpha}{2}}\Gamma\left(\frac{\alpha}{2}, -(b-ic)z^2\right)\right) + \frac{1}{2\sqrt{\pi}} a z^{\alpha+1}\left(\sum_{k=0}^{\infty}\frac{(b+ci)^{-k} a^{2k}\,\Gamma\left(\frac{\alpha+1}{2}+k, -(b+ci)z^2\right)}{(2k+1)\,k!}\left(-(b+ci)z^2\right)^{\frac{1}{2}(-\alpha-1)} + \left(-(b-ic)z^2\right)^{\frac{1}{2}(-\alpha-1)}\sum_{k=0}^{\infty}\frac{(b-ic)^{-k} a^{2k}\,\Gamma\left(\frac{\alpha+1}{2}+k, -(b-ic)z^2\right)}{(2k+1)\,k!}\right)$$

06.27.21.0068.01

$$\int z\, e^{b z^2} \cos(c z^2)\, \mathrm{erfc}(a z)\, dz = \frac{1}{4(b^2+c^2)} \left(\frac{a(b+c\,i)\, \mathrm{erf}\left(\sqrt{(a^2-b+c\,i)\, z^2}\right) z}{\sqrt{(a^2-b+c\,i)\, z^2}} + \frac{a(b-i\,c)\, \mathrm{erf}\left(\sqrt{(a^2-b-i\,c)\, z^2}\right) z}{\sqrt{(a^2-b-i\,c)\, z^2}} - \frac{a\,b\,z}{\sqrt{(a^2-b+c\,i)\, z^2}} - \frac{i\,a\,c\,z}{\sqrt{(a^2-b+c\,i)\, z^2}} - \frac{a\,b\,z}{\sqrt{(a^2-b-i\,c)\, z^2}} + \frac{a\,c\,i\,z}{\sqrt{(a^2-b-i\,c)\, z^2}} + 2\,b\, e^{b z^2} \cos(c z^2)\, \mathrm{erfc}(a z) + 2\,c\, e^{b z^2}\, \mathrm{erfc}(a z) \sin(c z^2) \right)$$

06.27.21.0069.01

$$\int \frac{e^{b z^2} \cos(c z^2)\, \mathrm{erfc}(a z)}{z}\, dz = \frac{1}{4}\left(\mathrm{Ei}\left((b+c\,i)\, z^2\right) + \mathrm{Ei}\left((b-i\,c)\, z^2\right)\right) + \frac{a\,z}{2\sqrt{\pi}} \left(\frac{1}{\sqrt{-(b-i\,c)\, z^2}} \sum_{k=0}^{\infty} \frac{(b-i\,c)^{-k} a^{2k}\, \Gamma\left(k+\frac{1}{2}, -(b-i\,c)\, z^2\right)}{(2k+1)\, k!} + \frac{1}{\sqrt{-(b+c\,i)\, z^2}} \sum_{k=0}^{\infty} \frac{(b+c\,i)^{-k} a^{2k}\, \Gamma\left(k+\frac{1}{2}, -(b+c\,i)\, z^2\right)}{(2k+1)\, k!} \right)$$

Involving hyperbolic functions

Involving sinh

06.27.21.0070.01

$$\int \sinh(b z)\, \mathrm{erfc}(a z)\, dz = \frac{1}{2b}\left(\exp\left(\frac{b^2}{4a^2}\right)\left(\mathrm{erf}\left(\frac{b}{2a} + a z\right) - \mathrm{erf}\left(\frac{b}{2a} - a z\right)\right) + 2\cosh(b z)\, \mathrm{erfc}(a z)\right)$$

06.27.21.0071.01

$$\int \sinh(b z^2)\, \mathrm{erfc}(a z)\, dz = \frac{\sqrt{\pi}\left(\mathrm{erfi}\left(\sqrt{b}\, z\right) - \mathrm{erf}\left(\sqrt{b}\, z\right)\right)}{4\sqrt{b}} - \frac{1}{2\sqrt{\pi}\, b}\left(\sum_{k=0}^{\infty} \frac{(-b)^{-k} a^{2k+1}\, \Gamma(k+1, b z^2)}{(2k+1)\, k!} + \sum_{k=0}^{\infty} \frac{b^{-k} a^{2k+1}\, \Gamma(k+1, -b z^2)}{(2k+1)\, k!}\right)$$

Involving cosh

06.27.21.0072.01

$$\int \cosh(b z)\, \mathrm{erfc}(a z)\, dz = \frac{1}{2b}\left(2\, \mathrm{erfc}(a z) \sinh(b z) - \exp\left(\frac{b^2}{4a^2}\right)\left(\mathrm{erf}\left(\frac{b}{2a} - a z\right) + \mathrm{erf}\left(\frac{b}{2a} + a z\right)\right)\right)$$

06.27.21.0073.01

$$\int \cosh(b z^2)\, \mathrm{erfc}(a z)\, dz = \frac{\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{b}\, z\right) + \mathrm{erfi}\left(\sqrt{b}\, z\right)\right)}{4\sqrt{b}} + \frac{1}{2\sqrt{\pi}\, b}\left(\sum_{k=0}^{\infty} \frac{(-b)^{-k} a^{2k+1}\, \Gamma(k+1, b z^2)}{(2k+1)\, k!} - \sum_{k=0}^{\infty} \frac{b^{-k} a^{2k+1}\, \Gamma(k+1, -b z^2)}{(2k+1)\, k!}\right)$$

Involving hyperbolic functions and a power function

Involving sinh and power

06.27.21.0074.01

$$\int z^{\alpha-1}\sinh(b\,z)\operatorname{erfc}(a\,z)\,dz = \frac{1}{2}z^{\alpha}\left((b\,z)^{-\alpha}\,\Gamma(\alpha,\,b\,z)-(-b\,z)^{-\alpha}\,\Gamma(\alpha,\,-b\,z)\right)- \frac{a\,z^{\alpha}}{b\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k\,a^{2k}\,b^{-2k}}{(2k+1)\,k!}\left((b\,z)^{-\alpha}\,\Gamma(2k+\alpha+1,\,b\,z)+(-b\,z)^{-\alpha}\,\Gamma(2k+\alpha+1,\,-b\,z)\right)$$

06.27.21.0075.01

$$\int z^{n}\sinh(b\,z)\operatorname{erfc}(a\,z)\,dz = \frac{1}{2}b^{-n-1}\operatorname{erfc}(a\,z)\left((-1)^n\,\Gamma(n+1,\,-b\,z)+\Gamma(n+1,\,b\,z)\right)+ \frac{(-b)^{-n-1}\,a\,n!}{2\sqrt{\pi}}\exp\left(\frac{b^2}{4a^2}\right)\left(\sum_{m=0}^{n}\frac{1}{m!}(-b)^m\left(-a^2\right)^{\frac{1}{2}(-m-1)}\sum_{k=0}^{m}\binom{m}{k}\left(-\frac{b}{2\sqrt{-a^2}}\right)^{m-k}\left(\sqrt{-a^2}\,z+\frac{b}{2\sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2}\,z+\frac{b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)}\Gamma\left(\frac{k+1}{2},\,-\left(\sqrt{-a^2}\,z+\frac{b}{2\sqrt{-a^2}}\right)^2\right)+ (-1)^n\sum_{m=0}^{n}\frac{1}{m!}b^m\left(-a^2\right)^{\frac{1}{2}(-m-1)}\sum_{k=0}^{m}\binom{m}{k}\left(\frac{b}{2\sqrt{-a^2}}\right)^{m-k}\left(\sqrt{-a^2}\,z-\frac{b}{2\sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2}\,z-\frac{b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)}\Gamma\left(\frac{k+1}{2},\,-\left(\sqrt{-a^2}\,z-\frac{b}{2\sqrt{-a^2}}\right)^2\right)\right)\;/;\;n\in\mathbb{N}$$

06.27.21.0076.01

$$\int z\sinh(b\,z)\operatorname{erfc}(a\,z)\,dz = \frac{1}{4a^2b^2\sqrt{\pi}}\left(e^{-z\left(z\,a^2+b\right)}\left(\left(2a^2-b^2\right)e^{\frac{\left(2za^2+b\right)^2}{4a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2a}+a\,z\right)+\left(2a^2-b^2\right)e^{\frac{\left(2za^2+b\right)^2}{4a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2a}-a\,z\right)+ 2a\left(a\,e^{a^2z^2}\sqrt{\pi}\left(b\,z+e^{2bz}(b\,z-1)+1\right)\operatorname{erfc}(a\,z)-b\left(1+e^{2bz}\right)\right)\right)\right)$$

06.27.21.0077.01

$$\int z^2 \sinh(b\,z)\,\operatorname{erfc}(a\,z)\,dz = \frac{1}{8\,a^4\,b^3\,\sqrt{\pi}}\Bigg(e^{-z\left(z\,a^2+b\right)}\Bigg(\left(8\,a^4-2\,b^2\,a^2+b^4\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}+a\,z\right)-\left(8\,a^4-2\,b^2\,a^2+b^4\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}-a\,z\right)+ 2\,a\left(2\,a^3\,e^{a^2\,z^2}\sqrt{\pi}\left(b^2\,z^2+2\,b\,z+e^{2\,b\,z}\left(b^2\,z^2-2\,b\,z+2\right)+2\right)\operatorname{erfc}(a\,z)- b\left(2\left(b\,z+e^{2\,b\,z}\,(b\,z-2)+2\right)a^2+b^2\left(-1+e^{2\,b\,z}\right)\right)\right)\Bigg)\Bigg)$$

06.27.21.0078.01

$$\int z^3 \sinh(b\,z)\,\operatorname{erfc}(a\,z)\,dz = \frac{1}{16\,a^6\,b^4\,\sqrt{\pi}}\Bigg(e^{-z\left(z\,a^2+b\right)}\Bigg(8\,b^3\,e^{a^2\,z^2}\sqrt{\pi}\,z^3\operatorname{erfc}(a\,z)\,a^6+8\,b^3\,e^{z\left(z\,a^2+2\,b\right)}\sqrt{\pi}\,z^3\operatorname{erfc}(a\,z)\,a^6+48\,b\,e^{a^2\,z^2}\sqrt{\pi}\,z\operatorname{erfc}(a\,z)\,a^6+ 48\,b\,e^{z\left(z\,a^2+2\,b\right)}\sqrt{\pi}\,z\operatorname{erfc}(a\,z)\,a^6-48\,b^2\,e^{z\left(z\,a^2+b\right)}\sqrt{\pi}\,z^2\operatorname{erfc}(a\,z)\sinh(b\,z)\,a^6- 96\,e^{z\left(z\,a^2+b\right)}\sqrt{\pi}\operatorname{erfc}(a\,z)\sinh(b\,z)\,a^6-48\,b\,e^{2\,b\,z}\,a^5-8\,b^3\,e^{2\,b\,z}\,z^2\,a^5-8\,b^3\,z^2\,a^5-48\,b\,a^5+ 24\,b^2\,e^{2\,b\,z}\,z\,a^5-24\,b^2\,z\,a^5+4\,b^3\,e^{2\,b\,z}\,a^3+4\,b^3\,a^3-4\,b^4\,e^{2\,b\,z}\,z\,a^3+4\,b^4\,z\,a^3-2\,b^5\,e^{2\,b\,z}\,a-2\,b^5\,a+ \left(48\,a^6-12\,b^2\,a^4-b^6\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}+a\,z\right)+\left(48\,a^6-12\,b^2\,a^4-b^6\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}-a\,z\right)\Bigg)\Bigg)$$

06.27.21.0079.01

$$\int z^{\alpha-1}\sinh\left(b\,z^2\right)\operatorname{erfc}(a\,z)\,dz = \frac{1}{4}\,z^\alpha\left(\left(b\,z^2\right)^{-\frac{\alpha}{2}}\,\Gamma\left(\frac{\alpha}{2},\,b\,z^2\right)-\left(-b\,z^2\right)^{-\frac{\alpha}{2}}\,\Gamma\left(\frac{\alpha}{2},\,-b\,z^2\right)\right)- \frac{a\,z^{\alpha+1}}{2\,\sqrt{\pi}}\left(-b^2\,z^4\right)^{\frac{1}{2}(-\alpha-1)}\left(\left(-b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{(-b)^{-k}\,a^{2\,k}\,\Gamma\left(\frac{\alpha+1}{2}+k,\,b\,z^2\right)}{(2\,k+1)\,k!}-\left(b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{b^{-k}\,a^{2\,k}\,\Gamma\left(\frac{\alpha+1}{2}+k,\,-b\,z^2\right)}{(2\,k+1)\,k!}\right)$$

06.27.21.0080.01

$$\int z\sinh\left(b\,z^2\right)\operatorname{erfc}(c+a\,z)\,dz = \frac{1}{4\,b\left(a^4-b^2\right)}\Bigg(e^{-c^2}\Bigg(a\,\sqrt{a^2-b}\,\left(a^2+b\right)e^{\frac{a^2\,c^2}{a^2-b}}\operatorname{erf}\left(\frac{z\,a^2+c\,a-b\,z}{\sqrt{a^2-b}}\right)+ \left(a^2-b\right)\left(a\,\sqrt{a^2+b}\,e^{\frac{a^2\,c^2}{a^2+b}}\operatorname{erf}\left(\frac{z\,a^2+c\,a+b\,z}{\sqrt{a^2+b}}\right)+2\left(a^2+b\right)e^{c^2}\cosh\left(b\,z^2\right)\operatorname{erfc}(c+a\,z)\right)\Bigg)\Bigg)$$

06.27.21.0081.01

$$\int z\sinh\left(b\,z^2\right)\operatorname{erfc}(a\,z)\,dz = \frac{1}{4\,b\left(a^4-b^2\right)}\left(a\,\sqrt{a^2-b}\,\left(a^2+b\right)\operatorname{erf}\left(\sqrt{a^2-b}\,z\right)+\left(a^2-b\right)\left(a\,\sqrt{a^2+b}\,\operatorname{erf}\left(\sqrt{a^2+b}\,z\right)+2\left(a^2+b\right)\cosh\left(b\,z^2\right)\operatorname{erfc}(a\,z)\right)\right)$$

06.27.21.0082.01

$$\int z^3 \sinh(b z^2)\operatorname{erfc}(a z)\,dz = \frac{1}{4b^2}\left(\frac{1}{2\sqrt{\pi}}\left(a b z^3\left(\left(\sqrt{\pi}\operatorname{erf}\left(\sqrt{(a^2-b)z^2}\right)-\sqrt{\pi}-2e^{(b-a^2)z^2}\sqrt{(a^2-b)z^2}\right)\Big/\left((a^2-b)z^2\right)^{3/2}+\left(\sqrt{\pi}\operatorname{erf}\left(\sqrt{(a^2+b)z^2}\right)-\sqrt{\pi}-2e^{-(a^2+b)z^2}\sqrt{(a^2+b)z^2}\right)\Big/\left((a^2+b)z^2\right)^{3/2}\right)\right)-\frac{1}{a^4-b^2}\left(a\left(\sqrt{a^2-b}\,(a^2+b)\operatorname{erf}\left(\sqrt{a^2-b}\,z\right)+(b-a^2)\sqrt{a^2+b}\operatorname{erf}\left(\sqrt{a^2+b}\,z\right)\right)\right)+2\operatorname{erfc}(a z)\left(b z^2\cosh(b z^2)-\sinh(b z^2)\right)\right)$$

06.27.21.0083.01

$$\int \frac{\sinh(b z^2)\operatorname{erfc}(a z)}{z}\,dz = \frac{1}{2}\operatorname{Shi}(b z^2)-\frac{a z}{2\sqrt{\pi b z^2}}\sum_{k=0}^{\infty}\frac{(-b)^{-k}a^{2k}\,\Gamma\left(k+\frac{1}{2},\,b z^2\right)}{(2k+1)\,k!}+\frac{a z}{2\sqrt{-\pi b z^2}}\sum_{k=0}^{\infty}\frac{b^{-k}a^{2k}\,\Gamma\left(k+\frac{1}{2},\,-b z^2\right)}{(2k+1)\,k!}$$

Involving cosh and power

06.27.21.0084.01

$$\int z^{\alpha-1}\cosh(b z)\operatorname{erfc}(a z)\,dz = \frac{1}{2}z^{\alpha}\left(-(-b z)^{-\alpha}\,\Gamma(\alpha,-b z)-(b z)^{-\alpha}\,\Gamma(\alpha,b z)\right)-\frac{a z^{\alpha}}{b\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k a^{2k}b^{-2k}}{(2k+1)\,k!}\left((-b z)^{-\alpha}\,\Gamma(2k+\alpha+1,-b z)-(b z)^{-\alpha}\,\Gamma(2k+\alpha+1,b z)\right)$$

06.27.21.0085.01

$$\int z^n\cosh(b z)\operatorname{erfc}(a z)\,dz = \frac{1}{2}(-1)^n b^{-n-1}\operatorname{erfc}(a z)\left(\Gamma(n+1,-b z)-(-1)^n\,\Gamma(n+1,b z)\right)-\frac{(-1)^n b^{-n-1}a\,n!}{2\sqrt{\pi}}\exp\left(\frac{b^2}{4a^2}\right)\left(\sum_{m=0}^{n}\frac{(-b)^m(-a^2)^{\frac{1}{2}(-m-1)}}{m!}\sum_{k=0}^{m}\binom{m}{k}\left(-\frac{b}{2\sqrt{-a^2}}\right)^{m-k}\left(\frac{b}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^{k+1}\left(-\left(\frac{b}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^2\right)^{\frac{1}{2}(-k-1)}\Gamma\left(\frac{k+1}{2},-\left(\frac{b}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^2\right)-(-1)^n\sum_{m=0}^{n}\frac{b^m(-a^2)^{\frac{1}{2}(-m-1)}}{m!}\sum_{k=0}^{m}\binom{m}{k}\left(\frac{b}{2\sqrt{-a^2}}\right)^{m-k}\left(\sqrt{-a^2}\,z-\frac{b}{2\sqrt{-a^2}}\right)^{k+1}\left(-\left(\sqrt{-a^2}\,z-\frac{b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)}\Gamma\left(\frac{k+1}{2},-\left(\sqrt{-a^2}\,z-\frac{b}{2\sqrt{-a^2}}\right)^2\right)\right)\,/;\,n\in\mathbb{N}$$

06.27.21.0086.01

$$\int z\cosh(b\,z)\operatorname{erfc}(a\,z)\,dz = \frac{1}{4\,a^2\,b^2\,\sqrt{\pi}}\left(e^{-z\left(z\,a^2+b\right)}\left(-\left(2\,a^2-b^2\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}+a\,z\right)+\left(2\,a^2-b^2\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}-a\,z\right)-2\,a\left(b\left(-1+e^{2\,b\,z}\right)+a\,e^{a^2\,z^2}\sqrt{\pi}\left(b\,z+e^{2\,b\,z}\left(1-b\,z\right)+1\right)\operatorname{erfc}(a\,z)\right)\right)\right)$$

06.27.21.0087.01

$$\int z^2\cosh(b\,z)\operatorname{erfc}(a\,z)\,dz = \frac{1}{8\,a^4\,b^3\,\sqrt{\pi}}\left(e^{-z\left(z\,a^2+b\right)}\left(-8\,b\,e^{a^2\,z^2}\sqrt{\pi}\,z\operatorname{erfc}(a\,z)\,a^4-8\,b\,e^{z\left(z\,a^2+2\,b\right)}\sqrt{\pi}\,z\operatorname{erfc}(a\,z)\,a^4-8\,e^{a^2\,z^2}\sqrt{\pi}\operatorname{erfc}(a\,z)\,a^4+8\,e^{z\left(z\,a^2+2\,b\right)}\sqrt{\pi}\operatorname{erfc}(a\,z)\,a^4+8\,b^2\,e^{z\left(z\,a^2+b\right)}\sqrt{\pi}\,z^2\operatorname{erfc}(a\,z)\sinh(b\,z)\,a^4+8\,b\,e^{2\,b\,z}\,a^3+8\,b\,a^3-4\,b^2\,e^{2\,b\,z}\,z\,a^3+4\,b^2\,z\,a^3-2\,b^3\,e^{2\,b\,z}\,a-2\,b^3\,a-\left(8\,a^4-2\,b^2\,a^2+b^4\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}+a\,z\right)-\left(8\,a^4-2\,b^2\,a^2+b^4\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}-a\,z\right)\right)\right)$$

06.27.21.0088.01

$$\int z^3\cosh(b\,z)\operatorname{erfc}(a\,z)\,dz = \frac{1}{16\,a^6\,b^4\,\sqrt{\pi}}\left(e^{-z\left(z\,a^2+b\right)}\left(-\left(48\,a^6-12\,b^2\,a^4-b^6\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}+a\,z\right)+\left(48\,a^6-12\,b^2\,a^4-b^6\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{b}{2\,a}-a\,z\right)-2\,a\left(4\,e^{a^2\,z^2}\sqrt{\pi}\left(b^3\,z^3+3\,b^2\,z^2+6\,b\,z+e^{2\,b\,z}\left(-b^3\,z^3+3\,b^2\,z^2-6\,b\,z+6\right)+6\right)\operatorname{erfc}(a\,z)\,a^5+b\left(4\left(-b^2\,z^2-3\,b\,z+e^{2\,b\,z}\left(b^2\,z^2-3\,b\,z+6\right)-6\right)a^4+2\,b^2\left(b\,z+e^{2\,b\,z}\left(b\,z-1\right)+1\right)a^2+b^4\left(-1+e^{2\,b\,z}\right)\right)\right)\right)\right)$$

06.27.21.0089.01

$$\int z^{\alpha-1}\cosh\left(b\,z^2\right)\operatorname{erfc}(a\,z)\,dz = \frac{1}{4}\,z^\alpha\left(-\left(-b\,z^2\right)^{-\frac{\alpha}{2}}\,\Gamma\left(\frac{\alpha}{2},-b\,z^2\right)-\left(b\,z^2\right)^{-\frac{\alpha}{2}}\,\Gamma\left(\frac{\alpha}{2},b\,z^2\right)\right)+\frac{a\,z^{\alpha+1}}{2\,\sqrt{\pi}}\left(-b^2\,z^4\right)^{\frac{1}{2}(-\alpha-1)}\left(\left(-b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{(-b)^{-k}\,a^{2\,k}\,\Gamma\left(\frac{\alpha+1}{2}+k,b\,z^2\right)}{(2\,k+1)\,k!}+\left(b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{b^{-k}\,a^{2\,k}\,\Gamma\left(\frac{\alpha+1}{2}+k,-b\,z^2\right)}{(2\,k+1)\,k!}\right)$$

06.27.21.0090.01

$$\int z\cosh\left(b\,z^2\right)\operatorname{erfc}(c+a\,z)\,dz = \frac{1}{4\left(b^3-a^4\,b\right)}\left(a\left(a^2-b\right)\sqrt{a^2+b}\;e^{-\frac{b\,c^2}{a^2+b}}\operatorname{erf}\left(\frac{z\,a^2+c\,a+b\,z}{\sqrt{a^2+b}}\right)+\left(a^2+b\right)\left(a\sqrt{b-a^2}\;e^{\frac{b\,c^2}{a^2-b}}\operatorname{erfi}\left(\frac{-z\,a^2-a\,c+b\,z}{\sqrt{b-a^2}}\right)+2\left(b-a^2\right)\operatorname{erfc}(c+a\,z)\sinh\left(b\,z^2\right)\right)\right)$$

06.27.21.0091.01

$$\int z \cosh(b z^2) \operatorname{erf}(a z) \, dz = \frac{1}{4(b^3 - a^4 b)} \left(a (a^2 - b) \sqrt{a^2 + b} \operatorname{erf}\left(\sqrt{a^2 + b} \, z\right) + (a^2 + b) \left(a \sqrt{b - a^2} \operatorname{erfi}\left(\sqrt{b - a^2} \, z\right) + 2 (b - a^2) \operatorname{erfc}(a z) \sinh(b z^2) \right) \right)$$

06.27.21.0092.01

$$\int z^3 \cosh(b z^2) \operatorname{erfc}(a z) \, dz = \frac{1}{4 b^2} \left(\frac{1}{2 \sqrt{\pi}} \left(a b z^3 \left(\left(\sqrt{\pi} \operatorname{erf}\left(\sqrt{(a^2 - b) z^2}\right) - \sqrt{\pi} - 2 e^{(b - a^2) z^2} \sqrt{(a^2 - b) z^2} \right) \Big/ \left((a^2 - b) z^2 \right)^{3/2} + \left(-\sqrt{\pi} \operatorname{erf}\left(\sqrt{(a^2 + b) z^2}\right) + \sqrt{\pi} + 2 e^{-(a^2 + b) z^2} \sqrt{(a^2 + b) z^2} \right) \Big/ \left((a^2 + b) z^2 \right)^{3/2} \right) \right) + \frac{1}{a^4 - b^2} \left(a (b - a^2) \sqrt{a^2 + b} \operatorname{erf}\left(\sqrt{a^2 + b} \, z\right) + a \sqrt{b - a^2} (a^2 + b) \operatorname{erfi}\left(\sqrt{b - a^2} \, z\right) \right) + 2 \operatorname{erfc}(a z) \left(b z^2 \sinh(b z^2) - \cosh(b z^2) \right) \right)$$

06.27.21.0093.01

$$\int \frac{\cosh(b z^2) \operatorname{erfc}(a z)}{z} \, dz = \frac{1}{2} \operatorname{Chi}(b z^2) + \frac{a z}{2 \sqrt{\pi b z^2}} \sum_{k=0}^{\infty} \frac{(-b)^{-k} a^{2k} \Gamma\left(k + \frac{1}{2}, b z^2\right)}{(2k + 1) k!} + \frac{a z}{2 \sqrt{-\pi b z^2}} \sum_{k=0}^{\infty} \frac{b^{-k} a^{2k} \Gamma\left(k + \frac{1}{2}, -b z^2\right)}{(2k + 1) k!}$$

Involving exponential function and hyperbolic functions

Involving exp and sinh

06.27.21.0094.01

$$\int e^{b z} \sinh(c z) \operatorname{erfc}(a z) \, dz = \frac{1}{2(c^2 - b^2)} \left((b + c) e^{\frac{(b - c)^2}{4 a^2}} \operatorname{erf}\left(\frac{2 z a^2 - b + c}{2 a}\right) + (b - c) e^{\frac{(b + c)^2}{4 a^2}} \operatorname{erf}\left(\frac{-2 z a^2 + b + c}{2 a}\right) + 2 e^{b z} \operatorname{erfc}(a z) (c \cosh(c z) - b \sinh(c z)) \right)$$

06.27.21.0095.01

$$\int e^{b z^2} \sinh(c z^2) \operatorname{erfc}(a z) \, dz = \frac{\sqrt{\pi}}{4(c^2 - b^2)} \left(\sqrt{b - c} \, (b + c) \operatorname{erfi}\left(\sqrt{b - c} \, z\right) + (c - b) \sqrt{b + c} \operatorname{erfi}\left(\sqrt{b + c} \, z\right) \right) - \frac{1}{2 \sqrt{\pi} \, (b + c)} \sum_{k=0}^{\infty} \frac{(b + c)^{-k} a^{2k+1} \Gamma\left(k + 1, -(b + c) z^2\right)}{(2k + 1) k!} + \frac{1}{2 \sqrt{\pi} \, (b - c)} \sum_{k=0}^{\infty} \frac{(b - c)^{-k} a^{2k+1} \Gamma\left(k + 1, -(b - c) z^2\right)}{(2k + 1) k!}$$

Involving exp and cosh

06.27.21.0096.01

$$\int e^{b z} \cosh(c z) \operatorname{erfc}(a z) \, dz = \frac{1}{2(b^2 - c^2)} \left((b + c) e^{\frac{(b - c)^2}{4 a^2}} \operatorname{erf}\left(\frac{2 z a^2 - b + c}{2 a}\right) - (b - c) e^{\frac{(b + c)^2}{4 a^2}} \operatorname{erf}\left(\frac{-2 z a^2 + b + c}{2 a}\right) + 2 e^{b z} \operatorname{erfc}(a z) (b \cosh(c z) - c \sinh(c z)) \right)$$

06.27.21.0097.01

$$\int e^{b z^2} \cosh(c z^2) \operatorname{erfc}(a z) \, dz = \frac{\sqrt{\pi}}{4 (b^2 - c^2)} \left(\sqrt{b-c} \, (b+c) \operatorname{erfi}\left(\sqrt{b-c} \, z\right) + (b-c) \sqrt{b+c} \, \operatorname{erfi}\left(\sqrt{b+c} \, z\right) \right) - \frac{1}{2 \sqrt{\pi} \, (b-c)} \sum_{k=0}^{\infty} \frac{(b-c)^{-k} a^{2k+1} \Gamma\left(k+1, -(b-c) z^2\right)}{(2k+1) k!} - \frac{1}{2 \sqrt{\pi} \, (b+c)} \sum_{k=0}^{\infty} \frac{(b+c)^{-k} a^{2k+1} \Gamma\left(k+1, -(b+c) z^2\right)}{(2k+1) k!}$$

Involving power, exponential and hyperbolic functions

Involving power, exp and sinh

06.27.21.0098.01

$$\int z^{\alpha-1} e^{b z} \sinh(c z) \operatorname{erfc}(a z) \, dz = \frac{1}{2} z^{\alpha} \left(((c-b) z)^{-\alpha} \Gamma(\alpha, (c-b) z) - (-(b+c) z)^{-\alpha} \Gamma(\alpha, -(b+c) z) \right) - \frac{a z^{\alpha} (-(b+c) z)^{-\alpha}}{(b+c) \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b+c)^{-2k}}{(2k+1) k!} \Gamma(2k+\alpha+1, -(b+c) z) + \frac{a z^{\alpha} (-(b-c) z)^{-\alpha}}{(b-c) \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b-c)^{-2k}}{(2k+1) k!} \Gamma(2k+\alpha+1, -(b-c) z)$$

06.27.21.0099.01

$$\int z^n e^{b z} \sinh(c z) \operatorname{erfc}(a z) \, dz = \frac{1}{2} \operatorname{erfc}(a z) \left((c-b)^{-n-1} \Gamma(n+1, (c-b) z) - (-b-c)^{-n-1} \Gamma(n+1, -(b+c) z) \right) - \frac{1}{2\sqrt{\pi}} \left(a (-b+c)^{-n-1} e^{\frac{(b-c)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b-c))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b-c}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b-c}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^{k+1} \left(-\left(\frac{b-c}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b-c}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^2 \right) \right) \right) + \frac{1}{2\sqrt{\pi}} \left(a (-c-b)^{-n-1} e^{\frac{(b+c)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b+c))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b+c}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b+c}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^{k+1} \left(-\left(\frac{b+c}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b+c}{2\sqrt{-a^2}} + \sqrt{-a^2} \, z \right)^2 \right) \right) \right) /; n \in \mathbb{N}$$

06.27.21.0100.01

$$\int z\, e^{b z} \sinh(c z) \operatorname{erfc}(a z)\, dz = \frac{1}{(b^2-c^2)^2} e^{b z} \left(\left(z c^3 + 2 b c - b^2 z c\right) \cosh(c z) + \left(z b^3 - b^2 - c^2 z b - c^2\right) \sinh(c z) \right) - \frac{1}{4 a^2 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b+c)^2} \left(2 e^{z (z a^2 + b + c)} \sqrt{\pi} (b z + c z - 1) \operatorname{erf}(a z) a^2 + 2 (b+c) e^{(b+c) z} a - \left(2 a^2 - (b+c)^2\right) \exp\left(\frac{(b+c)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b+c}{2 a} - a z \right) \right) - \frac{1}{(b-c)^2} \left(2 e^{z (z a^2 + b - c)} \sqrt{\pi} (b z - c z - 1) \operatorname{erf}(a z) a^2 + 2 (b-c) e^{(b-c) z} a - \left(2 a^2 - (b-c)^2\right) \exp\left(\frac{(b-c)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b-c}{2 a} - a z \right) \right) \right)$$

06.27.21.0101.01

$$\int z^2 e^{b z} \sinh(c z) \operatorname{erfc}(a z)\, dz = \frac{1}{(b^2-c^2)^3} e^{b z} \left(\left(z^2 b^5 - 2 z b^4 + \left(2 - 2 c^2 z^2\right) b^3 + c^2 \left(c^2 z^2 + 6\right) b + 2 c^4 z\right) \sinh(c z) - c \left(z^2 b^4 - 4 z b^3 + \left(6 - 2 c^2 z^2\right) b^2 + 4 c^2 z b + c^2 \left(c^2 z^2 + 2\right)\right) \cosh(c z) \right) - \frac{1}{8 a^4 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b+c)^3} \left(4 e^{z (z a^2 + b + c)} \sqrt{\pi} \left((b+c)^2 z^2 - 2 (b+c) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b+c) e^{(b+c) z} \left(2 (b z + c z - 2) a^2 + (b+c)^2\right) a + \left(8 a^4 - 2 (b+c)^2 a^2 + (b+c)^4\right) \exp\left(\frac{(b+c)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b+c}{2 a} - a z \right) \right) - \frac{1}{(b-c)^3} \left(4 e^{z (z a^2 + b - c)} \sqrt{\pi} \left((b-c)^2 z^2 - 2 (b-c) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b-c) e^{(b-c) z} \left(2 (b z - c z - 2) a^2 + (b-c)^2\right) a + \left(8 a^4 - 2 (b-c)^2 a^2 + (b-c)^4\right) \exp\left(\frac{(b-c)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b-c}{2 a} - a z \right) \right) \right)$$

06.27.21.0102.01

$$\int z^3 e^{bz}\sinh(cz)\operatorname{erfc}(az)\,dz = \frac{1}{(b^2-c^2)^4}\Big(e^{bz}\big(c\big(-z^3b^6+6z^2b^5+3z(c^2z^2-6)b^4-12(c^2z^2-2)b^3-3c^2z(c^2z^2-4)b^2+6c^2(c^2z^2+4)b+c^4z(c^2z^2+6)\big)\cosh(cz)+\big(z^3b^7-3z^2b^6+(6z-3c^2z^3)b^5+3(c^2z^2-2)b^4+3c^2z(c^2z^2+4)b^3+3c^2(c^2z^2-12)b^2-c^4z(c^2z^2+18)b-3c^4(c^2z^2+2)\big)\sinh(cz)\big)\Big) - \frac{1}{16a^6\sqrt{\pi}}e^{-a^2z^2}\Bigg(\frac{1}{(b+c)^4}\Big(8e^{z(za^2+b+c)}\sqrt{\pi}\left((b+c)^3z^3-3(b+c)^2z^2+6(b+c)z-6\right)\operatorname{erf}(az)a^6+2(b+c)e^{(b+c)z}\left(4\left((b+c)^2z^2-3(b+c)z+6\right)a^4+2(b+c)^2(bz+cz-1)a^2+(b+c)^4\right)a-\left(48a^6-12(b+c)^2a^4-(b+c)^6\right)\exp\left(\frac{(b+c)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\operatorname{erf}\left(\frac{b+c}{2a}-az\right)\Big)-\frac{1}{(b-c)^4}\Big(8e^{z(za^2+b-c)}\sqrt{\pi}\left((b-c)^3z^3-3(b-c)^2z^2+6(b-c)z-6\right)\operatorname{erf}(az)a^6+2(b-c)e^{(b-c)z}\left(4\left((b-c)^2z^2-3(b-c)z+6\right)a^4+2(b-c)^2(bz-cz-1)a^2+(b-c)^4\right)a-\left(48a^6-12(b-c)^2a^4-(b-c)^6\right)\exp\left(\frac{(b-c)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\operatorname{erf}\left(\frac{b-c}{2a}-az\right)\Big)\Bigg)$$

06.27.21.0103.01

$$\int z^{\alpha-1}e^{bz^2}\sinh(cz^2)\operatorname{erfc}(az)\,dz = \frac{1}{4}z^\alpha\left(\left(-(b-c)z^2\right)^{-\frac{\alpha}{2}}\Gamma\left(\frac{\alpha}{2},-(b-c)z^2\right)-\left(-(b+c)z^2\right)^{-\frac{\alpha}{2}}\Gamma\left(\frac{\alpha}{2},-(b+c)z^2\right)\right)+\frac{1}{2\sqrt{\pi}}az^{\alpha+1}\left(\left(-(b+c)z^2\right)^{\frac{1}{2}(-\alpha-1)}\sum_{k=0}^{\infty}\frac{(b+c)^{-k}a^{2k}\Gamma\left(\frac{\alpha+1}{2}+k,-(b+c)z^2\right)}{(2k+1)k!}-\left(-(b-c)z^2\right)^{\frac{1}{2}(-\alpha-1)}\sum_{k=0}^{\infty}\frac{(b-c)^{-k}a^{2k}\Gamma\left(\frac{\alpha+1}{2}+k,-(b-c)z^2\right)}{(2k+1)k!}\right)$$

06.27.21.0104.01

$$\int ze^{bz^2}\sinh(cz^2)\operatorname{erfc}(az)\,dz = \Big(-a(b+c)\sqrt{(a^2-b-c)z^2}\operatorname{erf}\left(\sqrt{(a^2-b+c)z^2}\right)z+a(b-c)\sqrt{(a^2-b+c)z^2}\operatorname{erf}\left(\sqrt{-(-a^2+b+c)z^2}\right)z+ab\sqrt{(a^2-b-c)z^2}\,z+ac\sqrt{(a^2-b-c)z^2}\,z-ab\sqrt{(a^2-b+c)z^2}\,z+ac\sqrt{(a^2-b+c)z^2}\,z-2ce^{bz^2}\sqrt{(a^2-b-c)z^2}\sqrt{(a^2-b+c)z^2}\cosh(cz^2)\operatorname{erfc}(az)+2be^{bz^2}\sqrt{(a^2-b-c)z^2}\sqrt{(a^2-b+c)z^2}\operatorname{erfc}(az)\sinh(cz^2)\Big)\Big/\Big(4(b^2-c^2)\sqrt{(a^2-b+c)z^2}\sqrt{-(-a^2+b+c)z^2}\Big)$$

06.27.21.0105.01

$$\int\frac{e^{bz^2}\sinh(cz^2)\operatorname{erfc}(az)}{z}\,dz = \frac{1}{4}\left(\operatorname{Ei}\left((b+c)z^2\right)-\operatorname{Ei}\left((b-c)z^2\right)\right)+\frac{az}{2\sqrt{\pi}}\left(\frac{1}{\sqrt{-(b+c)z^2}}\sum_{k=0}^{\infty}\frac{(b+c)^{-k}a^{2k}\Gamma\left(k+\frac{1}{2},-(b+c)z^2\right)}{(2k+1)k!}-\frac{1}{\sqrt{-(b-c)z^2}}\sum_{k=0}^{\infty}\frac{(b-c)^{-k}a^{2k}\Gamma\left(k+\frac{1}{2},-(b-c)z^2\right)}{(2k+1)k!}\right)$$

Involving power, exp and cosh

06.27.21.0106.01

$$\int z^{\alpha-1} e^{bz} \cosh(cz)\,\mathrm{erfc}(az)\,dz = \frac{1}{2} z^{\alpha} \left(-((c-b)z)^{-\alpha}\,\Gamma(\alpha, (c-b)z) - (-(b+c)z)^{-\alpha}\,\Gamma(\alpha, -(b+c)z)\right) - \frac{a z^{\alpha}(-(b-c)z)^{-\alpha}}{(b-c)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b-c)^{-2k}}{(2k+1)\,k!} \Gamma(2k+\alpha+1, -(b-c)z) - \frac{a z^{\alpha}(-(b+c)z)^{-\alpha}}{(b+c)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b+c)^{-2k}}{(2k+1)\,k!} \Gamma(2k+\alpha+1, -(b+c)z)$$

06.27.21.0107.01

$$\int z^{n} e^{bz} \cosh(cz)\,\mathrm{erfc}(az)\,dz = -\frac{1}{2}\mathrm{erfc}(az)\left(\Gamma(n+1, -(b+c)z)(-b-c)^{-n-1} + (c-b)^{-n-1}\,\Gamma(n+1, (c-b)z)\right) + \frac{1}{2\sqrt{\pi}}\left(a(-b+c)^{-n-1} e^{\frac{(b-c)^2}{4a^2}} n! \sum_{m=0}^{n} \frac{1}{m!}\left((-(b-c))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b-c}{2\sqrt{-a^2}}\right)^{m-k} \left(\frac{b-c}{2\sqrt{-a^2}} + \sqrt{-a^2}\,z\right)^{k+1} \left(-\left(\frac{b-c}{2\sqrt{-a^2}} + \sqrt{-a^2}\,z\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\!\left(\frac{k+1}{2}, -\left(\frac{b-c}{2\sqrt{-a^2}} + \sqrt{-a^2}\,z\right)^2\right)\right)\right) + \frac{1}{2\sqrt{\pi}}\left(a(-c-b)^{-n-1} e^{\frac{(b+c)^2}{4a^2}} n! \sum_{m=0}^{n} \frac{1}{m!}\left((-(b+c))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b+c}{2\sqrt{-a^2}}\right)^{m-k} \left(\frac{b+c}{2\sqrt{-a^2}} + \sqrt{-a^2}\,z\right)^{k+1} \left(-\left(\frac{b+c}{2\sqrt{-a^2}} + \sqrt{-a^2}\,z\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\!\left(\frac{k+1}{2}, -\left(\frac{b+c}{2\sqrt{-a^2}} + \sqrt{-a^2}\,z\right)^2\right)\right)\right) /;\ n \in \mathbb{N}$$

06.27.21.0108.01

$$\int z\, e^{bz} \cosh(cz)\,\mathrm{erfc}(az)\,dz = \frac{e^{bz}}{\left(b^2-c^2\right)^2}\left(\left(z b^3 - b^2 - c^2 z b - c^2\right)\cosh(cz) + c\left(-z b^2 + 2b + c^2 z\right)\sinh(cz)\right) - \frac{1}{4a^2\sqrt{\pi}} e^{-a^2 z^2}\left(\frac{1}{(b+c)^2}\left(2 e^{z(z a^2+b+c)}\sqrt{\pi}\,(bz+cz-1)\,\mathrm{erf}(az)\,a^2 + 2(b+c)e^{(b+c)z} a - \left(2a^2-(b+c)^2\right)\exp\!\left(\frac{(b+c)^2}{4a^2} + a^2 z^2\right)\sqrt{\pi}\,\mathrm{erf}\!\left(\frac{b+c}{2a} - az\right)\right) + \frac{1}{(b-c)^2}\left(2 e^{z(z a^2+b-c)}\sqrt{\pi}\,(bz-cz-1)\,\mathrm{erf}(az)\,a^2 + 2(b-c)e^{(b-c)z} a - \left(2a^2-(b-c)^2\right)\exp\!\left(\frac{(b-c)^2}{4a^2} + a^2 z^2\right)\sqrt{\pi}\,\mathrm{erf}\!\left(\frac{b-c}{2a} - az\right)\right)\right)$$

06.27.21.0109.01

$$\int z^2 e^{b z} \cosh(c z) \operatorname{erfc}(a z) \, dz = \frac{e^{b z}}{\left(b^2 - c^2\right)^3} \left(\left(z^2 b^5 - 2 z b^4 + \left(2 - 2 c^2 z^2\right) b^3 + c^2 \left(c^2 z^2 + 6\right) b + 2 c^4 z\right) \cosh(c z) - c \left(z^2 b^4 - 4 z b^3 + \left(6 - 2 c^2 z^2\right) b^2 + 4 c^2 z b + c^2 \left(c^2 z^2 + 2\right)\right) \sinh(c z)\right) - \frac{1}{8 a^4 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b+c)^3} \left(4 e^{z \left(z a^2 + b + c\right)} \sqrt{\pi} \left((b+c)^2 z^2 - 2 (b+c) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b+c) e^{(b+c) z} \left(2 (b z + c z - 2) a^2 + (b+c)^2\right) a + \left(8 a^4 - 2 (b+c)^2 a^2 + (b+c)^4\right) \exp\left(\frac{(b+c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b+c}{2 a} - a z\right)\right) + \frac{1}{(b-c)^3} \left(4 e^{z \left(z a^2 + b - c\right)} \sqrt{\pi} \left((b-c)^2 z^2 - 2 (b-c) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b-c) e^{(b-c) z} \left(2 (b z - c z - 2) a^2 + (b-c)^2\right) a + \left(8 a^4 - 2 (b-c)^2 a^2 + (b-c)^4\right) \exp\left(\frac{(b-c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b-c}{2 a} - a z\right)\right)\right)$$

06.27.21.0110.01

$$\int z^3 e^{b z} \cosh(c z) \operatorname{erfc}(a z) \, dz = \frac{e^{b z}}{\left(b^2 - c^2\right)^4} \left(\left(z^3 b^7 - 3 z^2 b^6 + \left(6 z - 3 c^2 z^3\right) b^5 + 3 \left(c^2 z^2 - 2\right) b^4 + 3 c^2 z \left(c^2 z^2 + 4\right) b^3 + 3 c^2 \left(c^2 z^2 - 12\right) b^2 - c^4 z \left(c^2 z^2 + 18\right) b - 3 c^4 \left(c^2 z^2 + 2\right)\right) \cosh(c z) + c \left(-z^3 b^6 + 6 z^2 b^5 + 3 z \left(c^2 z^2 - 6\right) b^4 - 12 \left(c^2 z^2 - 2\right) b^3 - 3 c^2 z \left(c^2 z^2 - 4\right) b^2 + 6 c^2 \left(c^2 z^2 + 4\right) b + c^4 z \left(c^2 z^2 + 6\right)\right) \sinh(c z)\right) - \frac{1}{16 a^6 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b+c)^4} \left(8 e^{z \left(z a^2 + b + c\right)} \sqrt{\pi} \left((b+c)^3 z^3 - 3 (b+c)^2 z^2 + 6 (b+c) z - 6\right) \operatorname{erf}(a z) a^6 + 2 (b+c) e^{(b+c) z} \left(4 \left((b+c)^2 z^2 - 3 (b+c) z + 6\right) a^4 + 2 (b+c)^2 (b z + c z - 1) a^2 + (b+c)^4\right) a - \left(48 a^6 - 12 (b+c)^2 a^4 - (b+c)^6\right) \exp\left(\frac{(b+c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b+c}{2 a} - a z\right)\right) + \frac{1}{(b-c)^4} \left(8 e^{z \left(z a^2 + b - c\right)} \sqrt{\pi} \left((b-c)^3 z^3 - 3 (b-c)^2 z^2 + 6 (b-c) z - 6\right) \operatorname{erf}(a z) a^6 + 2 (b-c) e^{(b-c) z} \left(4 \left((b-c)^2 z^2 - 3 (b-c) z + 6\right) a^4 + 2 (b-c)^2 (b z - c z - 1) a^2 + (b-c)^4\right) a - \left(48 a^6 - 12 (b-c)^2 a^4 - (b-c)^6\right) \exp\left(\frac{(b-c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b-c}{2 a} - a z\right)\right)\right)$$

06.27.21.0111.01

$$\int z^{\alpha-1} e^{b z^2} \cosh(c z^2)\, \operatorname{erfc}(a z)\, dz = \frac{1}{4} z^\alpha \left(-\left(-(b-c) z^2\right)^{-\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, -(b-c) z^2\right) - \left(-(b+c) z^2\right)^{-\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, -(b+c) z^2\right)\right) + \frac{1}{2\sqrt{\pi}} a z^{\alpha+1} \left(\left(-(b-c) z^2\right)^{\frac{1}{2}(-\alpha-1)} \sum_{k=0}^{\infty} \frac{(b-c)^{-k} a^{2k} \Gamma\left(\frac{\alpha+1}{2}+k, -(b-c) z^2\right)}{(2k+1)\, k!} + \left(-(b+c) z^2\right)^{\frac{1}{2}(-\alpha-1)} \sum_{k=0}^{\infty} \frac{(b+c)^{-k} a^{2k} \Gamma\left(\frac{\alpha+1}{2}+k, -(b+c) z^2\right)}{(2k+1)\, k!}\right)$$

06.27.21.0112.01

$$\int z e^{b z^2} \cosh(c z^2)\, \operatorname{erfc}(a z)\, dz = \left(a (b+c) \sqrt{(a^2-b-c) z^2}\, \operatorname{erf}\left(\sqrt{(a^2-b+c) z^2}\right) z + a (b-c) \sqrt{(a^2-b+c) z^2}\, \operatorname{erf}\left(\sqrt{-(-a^2+b+c) z^2}\right) z - a b \sqrt{(a^2-b-c) z^2}\, z - a c \sqrt{(a^2-b-c) z^2}\, z - a b \sqrt{(a^2-b+c) z^2}\, z + a c \sqrt{(a^2-b+c) z^2}\, z + 2 b e^{b z^2} \sqrt{(a^2-b-c) z^2} \sqrt{(a^2-b+c) z^2} \cosh(c z^2)\, \operatorname{erfc}(a z) - 2 c e^{b z^2} \sqrt{(a^2-b-c) z^2} \sqrt{(a^2-b+c) z^2}\, \operatorname{erfc}(a z) \sinh(c z^2)\right) \Big/ \left(4 (b^2-c^2) \sqrt{(a^2-b+c) z^2} \sqrt{-(-a^2+b+c) z^2}\right)$$

06.27.21.0113.01

$$\int \frac{e^{b z^2} \cosh(c z^2)\, \operatorname{erfc}(a z)}{z}\, dz = \frac{1}{4} \left(\operatorname{Ei}(b z^2 + c z^2) + \operatorname{Ei}(b z^2 - c z^2)\right) + \frac{a z}{2\sqrt{\pi}} \left(\frac{1}{\sqrt{-(b+c) z^2}} \sum_{k=0}^{\infty} \frac{(b+c)^{-k} a^{2k} \Gamma\left(k+\frac{1}{2}, -(b+c) z^2\right)}{(2k+1)\, k!} + \frac{1}{\sqrt{-(b-c) z^2}} \sum_{k=0}^{\infty} \frac{(b-c)^{-k} a^{2k} \Gamma\left(k+\frac{1}{2}, -(b-c) z^2\right)}{(2k+1)\, k!}\right)$$

Involving logarithm

Involving log

06.27.21.0114.01

$$\int \log(b z)\, \operatorname{erf}(a z)\, dz = \frac{1}{2 a \sqrt{\pi}} e^{-a^2 z^2} \left(e^{a^2 z^2} \operatorname{Ei}(-a^2 z^2) + 2 a e^{a^2 z^2} \sqrt{\pi}\, z\, \operatorname{erfc}(a z) (\log(b z) - 1) - 2 \log(b z) + 2\right)$$

Involving logarithm and a power function

Involving log and power

06.27.21.0115.01

$$\int z^{\alpha-1}\log(b\,z)\,\mathrm{erfc}(a\,z)\,dz = \frac{z^{\alpha}\left(a^2 z^2\right)^{\frac{1}{2}(-\alpha-1)}}{\sqrt{\pi}\;\alpha^2\,(\alpha+1)^2}$$
$$\left((\alpha+1)^2\left(\sqrt{\pi}\;\mathrm{erfc}(a\,z)\,(\alpha\log(b\,z)-1)\left(a^2 z^2\right)^{\frac{\alpha+1}{2}} + a\,z\left(\alpha\,\Gamma\!\left(\frac{\alpha+1}{2}\right)\log(z) + \Gamma\!\left(\frac{\alpha+1}{2},\,a^2 z^2\right)(1-\alpha\log(b\,z))\right)\right) - 2\,a\,z\left(a^2 z^2\right)^{\frac{\alpha+1}{2}}\alpha\;{}_2F_2\!\left(\frac{\alpha}{2}+\frac{1}{2},\,\frac{\alpha}{2}+\frac{1}{2};\,\frac{\alpha}{2}+\frac{3}{2},\,\frac{\alpha}{2}+\frac{3}{2};\,-a^2 z^2\right)\right)$$

06.27.21.0116.01

$$\int z\log(b\,z)\,\mathrm{erfc}(a\,z)\,dz = \frac{1}{4}z^2\,(2\log(b\,z)-1) - \frac{a\,z^3}{36\sqrt{\pi}\,\left(a^2 z^2\right)^{3/2}}\left(a\,z\sqrt{a^2 z^2}\left(4\,a\,z\;{}_2F_2\!\left(\frac{3}{2},\,\frac{3}{2};\,\frac{5}{2},\,\frac{5}{2};\,-a^2 z^2\right) + 9\sqrt{\pi}\;\mathrm{erf}(a\,z)\,(2\log(b\,z)-1)\right) - 9\left(\sqrt{\pi}\;\log(z) + \Gamma\!\left(\frac{3}{2},\,a^2 z^2\right)(1-2\log(b\,z))\right)\right)$$

06.27.21.0117.01

$$\int z^2\log(b\,z)\,\mathrm{erfc}(a\,z)\,dz = \frac{1}{9}z^3\,(3\log(b\,z)-1) - \frac{1}{18\,a^3\sqrt{\pi}}\,e^{-a^2 z^2}\left(2\,a^3\,e^{a^2 z^2}\sqrt{\pi}\;\mathrm{erf}(a\,z)\,(3\log(b\,z)-1)\,z^3 - 2\,a^2 z^2 + 6\,a^2\log(b\,z)\,z^2 - 3\,e^{a^2 z^2}\,\mathrm{Ei}\!\left(-a^2 z^2\right) + 6\log(b\,z) + 1\right)$$

06.27.21.0118.01

$$\int z^3\log(b\,z)\,\mathrm{erfc}(a\,z)\,dz = \frac{1}{16}z^4\,(4\log(b\,z)-1) - \frac{z}{400\,a^3\sqrt{\pi}\,\sqrt{a^2 z^2}}\left(a\,z\left(a^2 z^2\right)^{3/2}\left(8\,a\,z\;{}_2F_2\!\left(\frac{5}{2},\,\frac{5}{2};\,\frac{7}{2},\,\frac{7}{2};\,-a^2 z^2\right) + 25\sqrt{\pi}\;\mathrm{erf}(a\,z)\,(4\log(b\,z)-1)\right) - 25\left(3\sqrt{\pi}\;\log(z) + \Gamma\!\left(\frac{5}{2},\,a^2 z^2\right)(1-4\log(b\,z))\right)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

06.27.21.0119.01

$$\int \mathrm{erfc}(a\,z)^2\,dz = z\,\mathrm{erfc}(a\,z)^2 - \frac{2\,e^{-a^2 z^2}\,\mathrm{erfc}(a\,z)}{a\sqrt{\pi}} - \frac{\sqrt{\frac{2}{\pi}}\;\mathrm{erf}\!\left(\sqrt{2}\;a\,z\right)}{a}$$

Involving products of the direct function

06.27.21.0120.01

$$\int \operatorname{erfc}(a z) \operatorname{erfc}(b z) \, dz = -\frac{1}{a b} \left(\frac{\sqrt{a^2+b^2} \operatorname{erf}\left(\sqrt{a^2+b^2} \, z\right)}{\sqrt{\pi}} + \frac{b e^{-a^2 z^2} \operatorname{erfc}(b z)}{\sqrt{\pi}} + \operatorname{erfc}(a z) \left(\frac{a e^{-b^2 z^2}}{\sqrt{\pi}} - a b z \operatorname{erfc}(b z) \right) \right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

06.27.21.0121.01

$$\int z^{\alpha-1} \operatorname{erfc}(a z)^2 \, dz = \frac{4 z^\alpha \left(a^2 z^2\right)^{-\frac{\alpha}{2}}}{\pi \alpha} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma\left(\frac{\alpha+3}{2} + k, a^2 z^2\right)}{(2k+1) k!} + \frac{z^\alpha}{\alpha} \left(\operatorname{erfc}(a z)^2 - \frac{2 a z}{\sqrt{\pi}} \left(a^2 z^2\right)^{-\frac{\alpha+1}{2}} \Gamma\left(\frac{\alpha+1}{2}, a^2 z^2\right) \right)$$

06.27.21.0122.01

$$\int z \operatorname{erfc}(a z)^2 \, dz = \frac{1}{4 a^2 \pi} \left(\pi \left(2 a^2 z^2 - 1\right) \operatorname{erfc}(a z)^2 - 4 a e^{-a^2 z^2} \sqrt{\pi} \, z \operatorname{erfc}(a z) + 2 e^{-2 a^2 z^2} + \pi \right)$$

06.27.21.0123.01

$$\int z^2 \operatorname{erfc}(a z)^2 \, dz = \frac{1}{12 a^3 \pi} e^{-2 a^2 z^2} \left(4 a z - 8 e^{a^2 z^2} \sqrt{\pi} \left(a^2 z^2 + 1\right) \operatorname{erfc}(a z) + e^{2 a^2 z^2} \left(4 a^3 \pi z^3 \operatorname{erfc}(a z)^2 - 5 \sqrt{2 \pi} \operatorname{erf}\left(\sqrt{2} \, a z\right) \right) \right)$$

06.27.21.0124.01

$$\int z^3 \operatorname{erfc}(a z)^2 \, dz = \frac{1}{16 a^4 \pi} e^{-2 a^2 z^2} \left(4 a^2 z^2 - 4 a e^{a^2 z^2} \sqrt{\pi} \left(2 a^2 z^2 + 3\right) \operatorname{erfc}(a z) z + e^{2 a^2 z^2} \pi \left(\left(4 a^4 z^4 - 3\right) \operatorname{erfc}(a z)^2 + 3 \right) + 8 \right)$$

Involving products of the direct function and a power function

06.27.21.0125.01

$$\int z^{\alpha-1} \operatorname{erfc}(a z) \operatorname{erfc}(b z) \, dz = \frac{2 b z^\alpha \left(a^2 z^2\right)^{-\frac{\alpha}{2}}}{a \pi \alpha} \sum_{k=0}^{\infty} \frac{(-1)^k a^{-2k} b^{2k} \Gamma\left(k + \frac{\alpha}{2} + 1, a^2 z^2\right)}{(2k+1) k!} - \frac{a z^{\alpha+1} \left(a^2 z^2\right)^{-\frac{\alpha+1}{2}}}{\sqrt{\pi} \, \alpha} \Gamma\left(\frac{\alpha+1}{2}, a^2 z^2\right) +$$
$$\frac{z^\alpha \operatorname{erfc}(a z) \operatorname{erfc}(b z)}{\alpha} + \frac{2 a z^\alpha \left(b^2 z^2\right)^{-\frac{\alpha}{2}}}{b \pi \alpha} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} b^{-2k} \Gamma\left(k + \frac{\alpha}{2} + 1, b^2 z^2\right)}{(2k+1) k!} - \frac{b z^{\alpha+1} \left(b^2 z^2\right)^{-\frac{\alpha+1}{2}}}{\sqrt{\pi} \, \alpha} \Gamma\left(\frac{\alpha+1}{2}, b^2 z^2\right)$$

06.27.21.0126.01

$$\int z^2 \operatorname{erfc}(a z) \operatorname{erfc}(b z) \, dz =$$
$$\Big(e^{-\left(a^2+b^2\right) z^2} z^2 \Big(\left(a^2+b^2\right) \Big(2 e^{a^2 z^2} \sqrt{\pi} \sqrt{\left(a^2+b^2\right) z^2} \operatorname{erfc}(a z) \left(b^3 e^{b^2 z^2} \sqrt{\pi} \operatorname{erfc}(b z) z^3 - b^2 z^2 - 1 \right) a^3 -$$
$$a^2 b^2 e^{\left(a^2+b^2\right) z^2} \sqrt{\pi} \, z \operatorname{erf}\left(\sqrt{\left(a^2+b^2\right) z^2} \right) -$$
$$b^2 \left(2 b e^{b^2 z^2} \sqrt{\pi} \sqrt{\left(a^2+b^2\right) z^2} \left(a^2 z^2 + 1\right) \operatorname{erfc}(b z) - a^2 z \left(2 \sqrt{\left(a^2+b^2\right) z^2} + e^{\left(a^2+b^2\right) z^2} \sqrt{\pi} \right) \right) \Big) -$$
$$2 \sqrt{a^2+b^2} \left(a^4+b^4\right) e^{\left(a^2+b^2\right) z^2} \sqrt{\pi} \sqrt{\left(a^2+b^2\right) z^2} \operatorname{erf}\left(\sqrt{a^2+b^2} \, z \right) \Big) \Big) \Big/ \left(6 a^3 b^3 \pi \left(\left(a^2+b^2\right) z^2 \right)^{3/2} \right)$$

Involving power of the direct function and exponential function

06.27.21.0127.01

$$\int \frac{e^{-a^2 z^2}}{\operatorname{erfc}(a z)} \, dz = -\frac{\sqrt{\pi} \log(\operatorname{erfc}(a z))}{2 a}$$

06.27.21.0128.01

$$\int e^{-a^2 z^2} \operatorname{erfc}(a z)^r \, dz = -\frac{\sqrt{\pi} \operatorname{erfc}(a z)^{r+1}}{2 a (r+1)}$$

Involving direct function and Gamma-, Beta-, Erf-type functions

Involving erf-type functions

Involving erf

06.27.21.0129.01

$$\int \operatorname{erf}(b z) \operatorname{erfc}(a z) \, dz = \frac{1}{\sqrt{\pi} \sqrt{a^2+b^2}} \left(\frac{b}{a} \operatorname{erf}\left(z \sqrt{a^2+b^2}\right) + \frac{a}{b} \operatorname{erf}\left(z \sqrt{a^2+b^2}\right)\right) + \frac{\left(b \sqrt{\pi} z \operatorname{erf}(b z) + e^{-b^2 z^2}\right) \operatorname{erfc}(a z)}{b \sqrt{\pi}} - \frac{e^{-a^2 z^2} \operatorname{erf}(b z)}{a \sqrt{\pi}}$$

Involving erf-type functions and a power function

Involving erf and power

06.27.21.0130.01

$$\int z^{\alpha-1} \operatorname{erf}(b z) \operatorname{erfc}(a z) \, dz = -\frac{2 b z^{\alpha} \left(a^2 z^2\right)^{-\frac{\alpha}{2}}}{\pi \alpha a} \sum_{k=0}^{\infty} \frac{\left(-a^2\right)^{-k} b^{2k} \Gamma\left(k + \frac{\alpha}{2} + 1, a^2 z^2\right)}{(2k+1) k!} + \frac{z^{\alpha}}{\alpha} \left(\frac{b z \left(b^2 z^2\right)^{-\frac{\alpha+1}{2}}}{\sqrt{\pi}} \Gamma\left(\frac{\alpha+1}{2}, b^2 z^2\right) + \operatorname{erf}(b z) \operatorname{erfc}(a z)\right) - \frac{2 a z^{\alpha}}{\pi \alpha b} \left(b^2 z^2\right)^{-\frac{\alpha}{2}} \sum_{k=0}^{\infty} \frac{a^{2k} \left(-b^2\right)^{-k} \Gamma\left(k + \frac{\alpha}{2} + 1, b^2 z^2\right)}{(2k+1) k!}$$

06.27.21.0131.01

$$\int z^2 \operatorname{erf}(b z) \operatorname{erfc}(a z) \, dz = \Bigg(e^{-\left(a^2+b^2\right) z^2} z^2 \Bigg(2 \sqrt{a^2+b^2} \left(a^4+b^4\right) e^{\left(a^2+b^2\right) z^2} \sqrt{\pi} \operatorname{erf}\left(\sqrt{a^2+b^2} \, z\right) \sqrt{\left(a^2+b^2\right) z^2} + \frac{1}{z^2} \left(2 b^3 e^{b^2 z^2} \sqrt{\pi} \left(\left(a^2+b^2\right) z^2\right)^{3/2} \operatorname{erf}(b z) \left(a^3 e^{a^2 z^2} \sqrt{\pi} \operatorname{erfc}(a z) z^3 - a^2 z^2 - 1\right)\right) + a^2 \left(a^2+b^2\right) \left(e^{\left(a^2+b^2\right) z^2} \sqrt{\pi} z \operatorname{erf}\left(\sqrt{\left(a^2+b^2\right) z^2}\right) b^2 - b^2 z \left(2 \sqrt{\left(a^2+b^2\right) z^2} + e^{\left(a^2+b^2\right) z^2} \sqrt{\pi}\right) + 2 a e^{a^2 z^2} \sqrt{\pi} \sqrt{\left(a^2+b^2\right) z^2} \left(b^2 z^2 + 1\right) \operatorname{erfc}(a z)\right)\Bigg)\Bigg) \Big/ \left(6 a^3 b^3 \pi \left(\left(a^2+b^2\right) z^2\right)^{3/2}\right)$$

Definite integration

For the direct function itself

06.27.21.0132.01

$$\int_0^\infty t^{\alpha-1} \operatorname{erfc}(t) \, dt = \frac{1}{\sqrt{\pi}\ \alpha} \Gamma\left(\frac{\alpha+1}{2}\right) /; \operatorname{Re}(\alpha) > 0$$

Involving the direct function

06.27.21.0133.01

$$\int_0^\infty t^{\alpha-1} e^{-z t} \operatorname{erfc}(t) \, dt = \frac{1}{\sqrt{\pi}} \left(\frac{1}{\alpha} \Gamma\left(\frac{\alpha+1}{2}\right) {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}; \frac{1}{2}, \frac{\alpha}{2}+1; \frac{z^2}{4}\right) - \frac{z}{\alpha+1} \Gamma\left(\frac{\alpha}{2}+1\right) {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}; \frac{z^2}{4}\right)\right) /; \operatorname{Re}(z) > 0 \land \operatorname{Re}(\alpha) > 0$$

Integral transforms

Fourier exp transforms

06.27.22.0001.01

$$\mathcal{F}_t[\operatorname{erfc}(t)](x) = \sqrt{\frac{2}{\pi}} \frac{1}{x} e^{-\frac{x^2}{4}} \operatorname{erfi}\left(\frac{x}{2}\right) /; \operatorname{Im}(x) = 0$$

Fourier cos transforms

06.27.22.0002.01

$$\mathcal{F}c_t[\operatorname{erfc}(t)](x) = \sqrt{\frac{2}{\pi}} \frac{1}{x} e^{-\frac{x^2}{4}} \operatorname{erfi}\left(\frac{x}{2}\right) /; \operatorname{Im}(x) = 0$$

Fourier sin transforms

06.27.22.0003.01

$$\mathcal{F}s_t[\operatorname{erfc}(t)](x) = \sqrt{\frac{2}{\pi}} \frac{1}{x} \left(1 - e^{-\frac{x^2}{4}}\right) /; \operatorname{Im}(x) = 0$$

Laplace transforms

06.27.22.0004.01

$$\mathcal{L}_t[\operatorname{erfc}(t)](z) = \frac{1}{z} \left(1 - e^{\frac{z^2}{4}} \operatorname{erfc}\left(\frac{z}{2}\right)\right) /; \operatorname{Re}(z) > 0$$

Mellin transforms

06.27.22.0005.01

$$\mathcal{M}_t[\operatorname{erfc}(t)](z) = \frac{1}{\sqrt{\pi}\ z} \Gamma\left(\frac{z+1}{2}\right) /; \operatorname{Re}(z) > 0$$

Hankel transforms

06.27.22.0006.01

$$\mathcal{H}_{t,\nu}[\text{erfc}(t)](z) = \frac{2^{1-\nu} z^{\nu+\frac{1}{2}}}{\sqrt{\pi}\,(2\nu+3)\,\Gamma(\nu+1)}\,\Gamma\left(\frac{2\nu+5}{4}\right) {}_2F_2\left(\frac{2\nu+3}{4}, \frac{2\nu+5}{4}; \frac{2\nu+7}{4}, \nu+1; -\frac{z^2}{4}\right) /; \text{Re}(\nu) > -\frac{3}{2}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_1F_1$

06.27.26.0001.01

$$\text{erfc}(z) = 1 - \frac{2z}{\sqrt{\pi}}\,{}_1F_1\left(\frac{1}{2}; \frac{3}{2}; -z^2\right)$$

Involving hypergeometric U

06.27.26.0002.01

$$\text{erfc}(z) = \frac{z}{\sqrt{z^2}}\left(\frac{1}{\sqrt{\pi}}\,e^{-z^2}\,U\left(\frac{1}{2}, \frac{1}{2}, z^2\right) - 1\right) + 1$$

Through Meijer G

Classical cases for the direct function itself

06.27.26.0003.01

$$\text{erfc}(z) = 1 - \frac{z}{\sqrt{\pi}}\,G_{1,2}^{1,1}\left(z^2 \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, -\frac{1}{2} \end{matrix}\right)$$

06.27.26.0004.01

$$\text{erfc}(z) = 1 - \frac{\sqrt{z^2}}{\sqrt{\pi}\,z}\,G_{1,2}^{1,1}\left(z^2 \,\middle|\, \begin{matrix} 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

06.27.26.0005.01

$$\text{erfc}(z) = 1 - \frac{1}{\sqrt{\pi}}\,G_{1,2}^{1,1}\left(z^2 \,\middle|\, \begin{matrix} 1 \\ \frac{1}{2}, 0 \end{matrix}\right) /; \text{Re}(z) > 0$$

06.27.26.0006.01

$$\text{erfc}\left(\sqrt{z}\right) = \frac{1}{\sqrt{\pi}}\,G_{1,2}^{2,0}\left(z \,\middle|\, \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix}\right)$$

Classical cases involving exp

06.27.26.0007.01

$$e^{z^2}\,\text{erfc}(z) = \frac{1}{\pi}\,G_{1,2}^{2,1}\left(z^2 \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \frac{1}{2} \end{matrix}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

06.27.26.0008.01

$$e^{z}\,\text{erfc}\left(\sqrt{z}\right) = \frac{1}{\pi}\,G_{1,2}^{2,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \frac{1}{2} \end{matrix}\right)$$

Classical cases involving erfi

06.27.26.0009.01

$$\operatorname{erfi}\left(\sqrt[4]{z}\right)\operatorname{erfc}\left(\sqrt[4]{z}\right) = \frac{1}{\pi\sqrt{2}}\, G_{2,4}^{3,1}\left(\frac{z}{4} \,\middle|\, \begin{array}{c} \frac{1}{2}, 1 \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 0 \end{array}\right)$$

Generalized cases for the direct function itself

06.27.26.0010.01

$$\operatorname{erfc}(z) = \frac{1}{\sqrt{\pi}}\, G_{1,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 1 \\ 0, \frac{1}{2} \end{array}\right)$$

06.27.26.0011.01

$$\operatorname{erfc}(z) = 1 - \frac{1}{\sqrt{\pi}}\, G_{1,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 1 \\ \frac{1}{2}, 0 \end{array}\right)$$

Generalized cases involving exp

06.27.26.0012.01

$$e^{z^2}\operatorname{erfc}(z) = \frac{1}{\pi}\, G_{1,2}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{2} \\ 0, \frac{1}{2} \end{array}\right)$$

Generalized cases involving erfi

06.27.26.0013.01

$$\operatorname{erfi}(z)\operatorname{erfc}(z) = \frac{1}{\pi\sqrt{2}}\, G_{2,4}^{3,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1}{2}, 1 \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 0 \end{array}\right)$$

Through other functions

06.27.26.0014.01

$$\operatorname{erfc}(z) = \operatorname{erf}(z, \infty)$$

06.27.26.0015.01

$$\operatorname{erfc}(z) = 1 - \frac{\sqrt{z^2}}{z}\left(1 - \frac{1}{\sqrt{\pi}}\,\Gamma\left(\frac{1}{2}, z^2\right)\right)$$

06.27.26.0016.01

$$\operatorname{erfc}(z) = 1 - \frac{\sqrt{z^2}}{z}\left(1 - Q\left(\frac{1}{2}, z^2\right)\right)$$

06.27.26.0017.01

$$\operatorname{erfc}(z) = 1 - \frac{\sqrt{z^2}}{z} + \frac{z}{\sqrt{\pi}}\, E_{\frac{1}{2}}\left(z^2\right)$$

Representations through equivalent functions

With inverse function

06.27.27.0001.01

$$\operatorname{erfc}\left(\operatorname{erfc}^{-1}(z)\right) == z$$

With related functions

06.27.27.0002.01

$$\operatorname{erfc}(z) == 1 - \operatorname{erf}(z)$$

06.27.27.0003.01

$$\operatorname{erfc}(z) == 1 + i \operatorname{erfi}(i z)$$

06.27.27.0004.01

$$\operatorname{erfc}(z) == 1 - (1 + i)\left(C\left(\frac{(1 - i) z}{\sqrt{\pi}}\right) - i S\left(\frac{(1 - i) z}{\sqrt{\pi}}\right)\right)$$

Theorems

Solution of the one-dimensional heat equation

The function $w(x, t) == \operatorname{erfc}\left(\frac{x}{2\sqrt{t}}\right)$ fulfills the one-dimensional heat equation $\frac{\partial w(x,t)}{\partial t} == \frac{\partial^2 w(x,t)}{\partial x^2}$.

The iterated integral of the function erfc(z)

The iterated integral of the function erfc(z) can be expressed polynomially in erfc(z), exp(z), and z.

The solution to the initial value problem for the time-dependent Schrödinger equation

The functions $\psi_k(x, t) = e^{i (k x - k^2 t/2)} \operatorname{erfc}\left(e^{-\frac{i\pi}{4}} (x - k t) \big/ \sqrt{2 t}\right)$ are a solution to the initial value problem $\psi_k(x, 0) = \theta(-x)\, e^{i k x}$ for the time–dependent Schrödinger equation $i \frac{\partial \psi(x,t)}{\partial t} == -\frac{1}{2} \frac{\partial^2 \psi(x,t)}{\partial x^2}$.

History

– C. Kramp (1799)

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