EllipticThetaPrime2

Notations

Traditional name

Derivative of the Jacobi theta function ϑ_2

Traditional notation

$\vartheta_2'(z, q)$

Mathematica StandardForm notation

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EllipticThetaPrime[2, z, q]
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Primary definition

09.06.02.0001.01

$$\vartheta_2'(z, q) == -2 \sqrt[4]{q} \sum_{k=0}^{\infty} q^{k(k+1)} (2k+1) \sin((2k+1) z) /; |q| < 1$$

Specific values

Specialized values

For fixed *z*

09.06.03.0001.01

$$\vartheta_2'(z, 0) == 0$$

For fixed *q*

09.06.03.0002.01

$$\vartheta_2'(0, q) == 0$$

09.06.03.0004.01

$$\vartheta_2'\left(-\frac{\pi}{2}, q\right) == 2\,\eta\left(-\frac{i \log(q)}{\pi}\right)^3$$

09.06.03.0005.01

$$\vartheta_2'\left(\frac{\pi}{2}, q\right) == -2\,\eta\left(-\frac{i \log(q)}{\pi}\right)^3$$

09.06.03.0003.01

$$\vartheta_2'(m \pi, q) == 0 /; m \in \mathbb{Z}$$

09.06.03.0006.01

$$\vartheta_2'\left(\pi m + \frac{\pi}{2}, q\right) == 2\,(-1)^{m-1}\,\eta\left(-\frac{i\log(q)}{\pi}\right)^3 \;/;\; m \in \mathbb{Z}$$

09.06.03.0007.01

$$\vartheta_2'(-i\log(q), q) == -\frac{2\,i}{q}\sqrt{\frac{2}{\pi}}\,\sqrt[4]{q^{-1}(q)}\,\sqrt{K\left(q^{-1}(q)\right)}$$

09.06.03.0008.01

$$\vartheta_2'\left(-\frac{1}{2}\,(i\log(q)), q\right) == -i\,\frac{1}{\sqrt[4]{q}}\,\vartheta_3(0, q)$$

09.06.03.0009.01

$$\vartheta_2'\left(\frac{1}{2}\,(\pi - i\log(q)), q\right) == -\frac{1}{\sqrt[4]{q}}\,\vartheta_4(0, q)$$

09.06.03.0010.01

$$\vartheta_2'(m\pi - n\,i\log(q), q) == (-1)^{m+1}\,2\,n\,i\,q^{-n^2}\sqrt{\frac{2}{\pi}}\,\sqrt[4]{q^{-1}(q)}\,\sqrt{K\left(q^{-1}(q)\right)} \;/;\; \{m, n\} \in \mathbb{Z}$$

General characteristics

Domain and analyticity

$\vartheta_2'(z, q)$ is an analytic function of z and q for $z, q \in \mathbb{C}$ and $|q| < 1$.

09.06.04.0001.01

$$(2 * z * q) \longrightarrow \vartheta_2'(z, q) :: (\{2\} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\vartheta_2'(z, q)$ is an odd function with respect to z.

09.06.04.0002.01

$$\vartheta_2'(-z, q) == -\vartheta_2'(z, q)$$

09.06.04.0003.01

$$\vartheta_2'(z, -q) == \exp\left(-\frac{i\pi}{4}\,\mathrm{sgn}(\mathrm{Im}(q))\right)\vartheta_2'(z, q)$$

Mirror symmetry

09.06.04.0004.01

$$\vartheta_2'(\bar{z}, \bar{q}) == \overline{\vartheta_2'(z, q)}$$

Periodicity

$\vartheta_2'(z, q)$, considered as a function of z, has a period of 2π.

09.06.04.0005.01

$$\vartheta_2'(z + \pi, q) == -\vartheta_2'(z, q)$$

09.06.04.0017.01

$$\vartheta_2'(z+2\pi, q) == \vartheta_2'(z, q)$$

09.06.04.0006.01

$$\vartheta_2'(z+m\pi, q) == (-1)^m \vartheta_2'(z, q) /; m \in \mathbb{Z}$$

Poles and essential singularities

With respect to *q*

The function $\vartheta_2'(z, q)$ does not have poles and essential singularities inside of the unit circle $|q| < 1$.

09.06.04.0007.01

$$\mathcal{S}ing_q(\vartheta_2'(z, q)) == \{\}$$

With respect to *z*

09.06.04.0008.01

$$\mathcal{S}ing_z(\vartheta_2'(z, q)) == \{\}$$

Branch points

With respect to *q*

For fixed z, the function $\vartheta_2'(z, q)$ has one branch point: $q = 0$. (The point $q == -1$ is the branch cut endpoint.)

09.06.04.0009.01

$$\mathcal{BP}_q(\vartheta_2'(z, q)) == \{0\}$$

09.06.04.0010.01

$$\mathcal{R}_q(\vartheta_2'(z, q), 0) == 4$$

With respect to *z*

For fixed q, the function $\vartheta_2'(z, q)$ does not have branch points.

09.06.04.0011.01

$$\mathcal{BP}_z(\vartheta_2'(z, q)) == \{\}$$

Branch cuts

With respect to *q*

For fixed z, the function $\vartheta_2'(z, q)$ is a single-valued function inside the unit circle of the complex q-plane, cut along the interval $(-1, 0)$, where it is continuous from above.

09.06.04.0012.01

$$\mathcal{BC}_q(\vartheta_2'(z, q)) == \{\{(-1, 0), -i\}\}$$

09.06.04.0013.01

$$\lim_{\epsilon \to +0} \vartheta_2'(z, q + i\epsilon) == \vartheta_2'(z, q) /; -1 < q < 0$$

09.06.04.0014.01

$$\lim_{\epsilon \to +0} \vartheta_2'(z, q - i\epsilon) == -i\, \vartheta_2'(z, q) /; -1 < q < 0$$

With respect to z

For fixed q, the function $\vartheta_2'(z, q)$ does not have branch cuts.

09.06.04.0015.01
$$\mathcal{BC}_z(\vartheta_2'(z, q)) = \{\}$$

Natural boundary of analyticity

The unit circle $|q| = 1$ is the natural boundary of the region of analyticity.

09.06.04.0016.01
$$\mathcal{AB}_z(\vartheta_2'(z, q)) = \left\{e^{i(-\pi,\pi)}\right\}$$

Branch cut endpoints

The function $\vartheta_2'(z, q)$ has one branch cut endpoint: $q = -1$.

Series representations

q-series

Expansions at generic point $q = q_0$

09.06.06.0005.01
$$\vartheta_2'(z, q) \propto \vartheta_2'(z, q_0) + \vartheta_2^{(1,1)}(z, q_0)(q - q_0) + \frac{\vartheta_2^{(1,2)}(z, q_0)}{2}(q - q_0)^2 + \frac{\vartheta_2^{(1,3)}(z, q_0)}{6}(q - q_0)^3 + O\left((q - q_0)^4\right)$$

09.06.06.0006.01
$$\vartheta_2'(z, q) = \sum_{k=0}^{\infty} \frac{\vartheta_2^{(1,k)}(z, q_0)}{k!}(q - q_0)^k$$

09.06.06.0007.01
$$\vartheta_2'(z, q) \propto \vartheta_2'(z, q_0)\,(1 + O(q - q_0))$$

Expansions on branch cuts

09.06.06.0008.01
$$\vartheta_2'(z, q) \propto e^{\frac{\pi i}{2}\left\lfloor \frac{\arg(q-x)}{2\pi} \right\rfloor}\left(\vartheta_2'(z, x) + \vartheta_2^{(1,1)}(z, x)(q - x) + \frac{\vartheta_2^{(1,2)}(z, x)}{2}(q - x)^2 + \frac{\vartheta_2^{(1,3)}(z, x)}{6}(q - x)^3 + O\left((q - x)^4\right)\right) /;$$
$$x \in \mathbb{R} \land -1 < x < 0$$

09.06.06.0009.01
$$\vartheta_2'(z, q) = e^{\frac{\pi i}{2}\left\lfloor \frac{\arg(q-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{\vartheta_2^{(1,k)}(z, x)}{k!}(q - x)^k /; x \in \mathbb{R} \land -1 < x < 0$$

09.06.06.0010.01
$$\vartheta_2'(z, q) \propto e^{\frac{\pi i}{2}\left\lfloor \frac{\arg(q-x)}{2\pi} \right\rfloor} \vartheta_2'(z, x)\,(1 + O(q - x)) /; x \in \mathbb{R} \land -1 < x < 0$$

Expansions at $q = 0$

09.06.06.0011.01

$$\vartheta_2'(z, q) \propto -2\sqrt[4]{q}\,\left(\sin(z) + 3\sin(3z)\,q^2 + 5\sin(5z)\,q^6 + 7\sin(7z)\,q^{12} + \ldots\right) /;\ (q \to 0)$$

09.06.06.0001.01

$$\vartheta_2'(z, q) = -2\sqrt[4]{q}\,\sum_{k=0}^{\infty} q^{k(k+1)}\,(2k+1)\sin((2k+1)z) /;\ |q| < 1$$

09.06.06.0002.01

$$\vartheta_2'(z, q) = i\sum_{k=-\infty}^{\infty} q^{\left(k+\frac{1}{2}\right)^2}\,(2k+1)\,e^{(2k+1)iz} /;\ |q| < 1$$

09.06.06.0012.01

$$\vartheta_2'(z, q) \propto -2\sqrt[4]{q}\,\left(\sin(z) + O\left(q^2\right)\right) /;\ (q \to 0)$$

Expansions at $q = 1$

09.06.06.0013.01

$$\vartheta_2'(z, q) \propto \frac{2\sqrt{\pi}\,i}{(q-1)^{3/2}}\,e^{-i\pi\left\lfloor -\frac{\arg(q-1)}{2\pi}\right\rfloor}\left(1 + \frac{3(q-1)}{4} - \frac{1}{32}(q-1)^2 + \frac{3}{128}(q-1)^3 + \ldots\right)e^{\frac{z^2}{\log(q)}} \left(z - 2\,e^{\frac{\pi^2}{\log(q)}}\left(z\cosh\left(\frac{2\pi z}{\log(q)}\right) + \pi\sinh\left(\frac{2\pi z}{\log(q)}\right)\right) + 2\,e^{\frac{4\pi^2}{\log(q)}}\left(z\cosh\left(\frac{4\pi z}{\log(q)}\right) + 2\pi\sinh\left(\frac{4\pi z}{\log(q)}\right)\right) + \ldots\right) /;\ (q \to 1) \land |q| < 1$$

09.06.06.0014.01

$$\vartheta_2'(z, q) = \frac{6\sqrt{\pi}\,i}{(q-1)^{3/2}}\,e^{-3i\pi\left\lfloor \frac{3}{4}-\frac{\arg(q-1)}{2\pi}\right\rfloor} \sum_{k=0}^{\infty}\binom{k+\frac{3}{2}}{k}\sum_{j=0}^{k}\frac{(-1)^j}{2j+3}\binom{k}{j}\,p_{j,k}\,(q-1)^k\,e^{\frac{z^2}{\log(q)}}\left(z + 2\sum_{m=1}^{\infty}(-1)^m\,e^{\frac{m^2\pi^2}{\log(q)}}\left(z\cosh\left(\frac{2m\pi z}{\log(q)}\right) + m\pi\sinh\left(\frac{2m\pi z}{\log(q)}\right)\right)\right) /;$$
$$(|q| < 1 \land |q-1| < 1) \bigwedge c_k = \frac{(-1)^{k-1}}{k+1} \bigwedge p_{j,0} = 1 \bigwedge p_{j,k} = -\frac{1}{k}\sum_{m=1}^{k}(j\,m - k + m)\,c_m\,p_{j,k-m} \bigwedge k \in \mathbb{N}^+$$

09.06.06.0015.01

$$\vartheta_2'(z, q) \propto \frac{2\sqrt{\pi}\,i}{(q-1)^{3/2}}\,e^{-i\pi\left\lfloor -\frac{\arg(q-1)}{2\pi}\right\rfloor}(1 + O(q-1))\,e^{\frac{z^2}{\log(q)}} \left(z - 2\,e^{\frac{\pi^2}{\log(q)}}\left(z\cosh\left(\frac{2\pi z}{\log(q)}\right) + \pi\sinh\left(\frac{2\pi z}{\log(q)}\right)\right) + O\left(e^{\frac{4\pi^2}{\log(q)}}\left(z\cosh\left(\frac{4\pi z}{\log(q)}\right) + 2\pi\sinh\left(\frac{4\pi z}{\log(q)}\right)\right)\right)\right) /;\ |q| < 1$$

Other q-series representations

09.06.06.0003.01

$$\frac{\vartheta_2'(z, q)}{\vartheta_2(z, q)} = -\tan(z) + 4\sum_{k=1}^{\infty}\frac{(-1)^k\,q^{2k}}{1-q^{2k}}\sin(2kz)$$

Other series representations

09.06.06.0016.01

$$\vartheta_2'(z, q) = -\frac{2(-1)^{3/4}\sqrt{\pi}}{(-i\log(q))^{3/2}}\,e^{\frac{z^2}{\log(q)}}\left(z + 2\sum_{k=1}^{\infty}(-1)^k\,e^{\frac{k^2\pi^2}{\log(q)}}\left(z\cosh\left(\frac{2k\pi z}{\log(q)}\right) + k\pi\sinh\left(\frac{2k\pi z}{\log(q)}\right)\right)\right)$$

09.06.06.0004.01

$$\vartheta_2'(z, q) = -\frac{2\,i^{3/2}}{\tau^{3/2}} \sum_{n=-\infty}^{\infty} (-1)^n \left(\frac{z}{\pi} + n\right) \exp\left(-\frac{\pi\,i}{\tau} \left(\frac{z}{\pi} + n\right)^2\right) /;\ q = e^{i\,\pi\,\tau}$$

Differential equations

Partial differential equations

The elliptic theta functions satisfy the one-dimensional heat equation:

09.06.13.0001.01

$$\frac{\partial \vartheta_2'(z, q)}{\partial \tau} = -\frac{\pi\,i}{4} \frac{\partial^2 \vartheta_2'(z, q)}{\partial z^2} /;\ q = e^{i\,\pi\,\tau}$$

09.06.13.0002.01

$$4\,q \frac{\partial \vartheta_2'(z, q)}{\partial q} + \frac{\partial^2 \vartheta_2'(z, q)}{\partial z^2} = 0$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.06.16.0002.01

$$\vartheta_2'(z, q) = -\frac{(-1)^{3/4}\, e^{\frac{z^2}{\log(q)}} \sqrt{\pi}}{(-i \log(q))^{3/2}} \left(2\,z\,\vartheta_4\left(\frac{i\,\pi\,z}{\log(q)}, e^{\frac{\pi^2}{\log(q)}}\right) + i\,\pi\,\vartheta_4'\left(\frac{i\,\pi\,z}{\log(q)}, e^{\frac{\pi^2}{\log(q)}}\right)\right)$$

09.06.16.0001.01

$$\vartheta_2'(z + \pi\,\tau, q) = \frac{e^{-2\,i\,z}}{q} (\vartheta_2'(z, q) - 2\,i\,\vartheta_2(z, q)) /;\ q = e^{i\,\pi\,\tau}$$

Differentiation

Low-order differentiation

With respect to z

09.06.20.0001.01

$$\frac{\partial \vartheta_2'(z, q)}{\partial z} = -2 \sqrt[4]{q} \sum_{k=0}^{\infty} q^{k\,(k+1)}\, (2\,k + 1)^2 \cos((2\,k + 1)\,z) /;\ |q| < 1$$

09.06.20.0002.01

$$\frac{\partial^2 \vartheta_2'(z, q)}{\partial z^2} = 2 \sqrt[4]{q} \sum_{k=0}^{\infty} q^{k\,(k+1)}\, (2\,k + 1)^3 \sin((2\,k + 1)\,z) /;\ |q| < 1$$

With respect to q

09.06.20.0003.01

$$\frac{\partial \vartheta_2'(z, q)}{\partial q} = \frac{\vartheta_2'(z, q)}{4\, q} - 2\, q^{-\frac{3}{4}} \sum_{k=1}^{\infty} q^{k(k+1)}\, k\, (k+1)\, (2\, k+1) \sin((2\, k+1)\, z) \,/;\, |q| < 1$$

09.06.20.0004.01

$$\frac{\partial^2 \vartheta_2'(z, q)}{\partial q^2} = -2\, q^{-\frac{7}{4}} \sum_{k=0}^{\infty} q^{k(k+1)}\, (2\, k+1) \left(k^2 + k - \frac{3}{4}\right) \left(k^2 + k + \frac{1}{4}\right) \sin((2\, k+1)\, z) \,/;\, |q| < 1$$

Symbolic differentiation

With respect to *z*

09.06.20.0005.01

$$\frac{\partial^n \vartheta_2'(z, q)}{\partial z^n} = -2 \sqrt[4]{q} \sum_{k=0}^{\infty} q^{k(k+1)}\, (2\, k+1)^{n+1} \sin\left(\frac{\pi\, n}{2} + (2\, k+1)\, z\right) /;\, |q| < 1 \bigwedge n \in \mathbb{N}^+$$

With respect to *q*

09.06.20.0006.01

$$\frac{\partial^n \vartheta_2(z, q)}{\partial q^n} = -2\, q^{\frac{1}{4}-n} \sum_{k=0}^{\infty} q^{k(k+1)}\, (2\, k+1) \left(k\, (k+1) - n + \frac{5}{4}\right)_n \sin((2\, k+1)\, z) \,/;\, |q| < 1 \bigwedge n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

09.06.20.0007.01

$$\frac{\partial^\alpha \vartheta_2'(z, q)}{\partial z^\alpha} = -2^\alpha\, a \sqrt{\pi}\, z^{1-\alpha} \sqrt[4]{q} \sum_{k=0}^{\infty} q^{k(k+1)}\, (2\, k+1)\, {}_1\tilde{F}_2\left(1;\, \frac{3-\alpha}{2},\, 1 - \frac{\alpha}{2};\, -\frac{1}{4}\, (2\, k+1)^2\, z^2\right) /;\, |q| < 1$$

With respect to *q*

09.06.20.0008.01

$$\frac{\partial^\alpha \vartheta_2'(z, q)}{\partial q^\alpha} = -2\, q^{\frac{1}{4}-\alpha} \sum_{k=0}^{\infty} \frac{q^{k(k+1)}\, \Gamma\left(k^2 + k + \frac{5}{4}\right) (2\, k+1) \sin((2\, k+1)\, z)}{\Gamma\left(k^2 + k - \alpha + \frac{5}{4}\right)} \,/;\, |q| < 1$$

Integration

Indefinite integration

Involving only one direct function

09.06.21.0001.01

$$\int \vartheta_2'(z, q)\, dz = \vartheta_2(z, q)$$

Involving only one direct function with respect to *q*

09.06.21.0002.01

$$\int \vartheta_2'(z,q)\,dq = -2\sum_{k=0}^{\infty}\frac{q^{(k+1)k+\frac{5}{4}}\,(2k+1)\sin((2k+1)z)}{(k+1)k+\frac{5}{4}} \;/;\; |q|<1$$

Representations through equivalent functions

With related functions

Involving theta functions

Involving $\vartheta_1'(z, q)$

09.06.27.0009.01

$$\vartheta_2'(z,q) = \frac{\vartheta_2(z,q)}{\vartheta_1(z,q)}\,\vartheta_1'(z,q) - \vartheta_2(0,q)^2\,\frac{\vartheta_3(z,q)\,\vartheta_4(z,q)}{\vartheta_1(z,q)}$$

09.06.27.0010.01

$$\vartheta_2'(z,q) = -\vartheta_1'\left(z-\frac{\pi}{2},\,q\right)$$

09.06.27.0004.01

$$\vartheta_2'(z,q) = \vartheta_1'\left(z+\frac{\pi}{2},\,q\right)$$

09.06.27.0011.01

$$\vartheta_2'(z,q) = (-1)^m\,\vartheta_1'\left(\frac{1}{2}\pi(2m+1)+z,\,q\right) \;/;\; m\in\mathbb{Z}$$

Involving $\vartheta_2(z, q)$

09.06.27.0001.02

$$\vartheta_2'\left(z,\,e^{i\pi\tau}\right) = e^{-i(2z-\pi\tau)}\,\vartheta_2'\left(z-\pi\tau,\,e^{i\pi\tau}\right) - 2\,e^{-i(2z-\pi\tau)}\,i\,\vartheta_2\left(z-\pi\tau,\,e^{i\pi\tau}\right) \;/;\; \mathrm{Im}(\tau)>0$$

09.06.27.0012.01

$$\vartheta_2'(z,q) = e^{2iz}\,q\,(2i\,\vartheta_2(z+\pi\tau,q)+\vartheta_2'(z+\pi\tau,q)) \;/;\; q = e^{i\pi\tau}$$

09.06.27.0013.01

$$\vartheta_2'(z,q) = e^{2inz}\,q^{n^2}\,(2in\,\vartheta_2(z+n\pi\tau,q)+\vartheta_2'(z+n\pi\tau,q)) \;/;\; n\in\mathbb{Z}\bigwedge q = e^{i\pi\tau}$$

09.06.27.0014.01

$$\vartheta_2'(z,q) = (-1)^m\,e^{2inz}\,q^{n^2}\,(2in\,\vartheta_2(z+m\pi+n\pi\tau,q)+\vartheta_2'(z+m\pi+n\pi\tau,q)) \;/;\; \{m,n\}\in\mathbb{Z}\bigwedge q = e^{i\pi\tau}$$

09.06.27.0015.01

$$\vartheta_2'(z,q) = e^{2iz}\,q\,(2i\,\vartheta_2(z-i\log(q),q)+\vartheta_2'(z-i\log(q),q))$$

09.06.27.0016.01

$$\vartheta_2'(z,q) = e^{-2iz}\,q\,(-2i\,\vartheta_2(z+i\log(q),q)+\vartheta_2'(z+i\log(q),q))$$

09.06.27.0017.01

$$\vartheta_2'(z,q) = e^{-2inz}\,q^{n^2}\,(-2in\,\vartheta_2(z+in\log(q),q)+\vartheta_2'(z+in\log(q),q)) \;/;\; n\in\mathbb{Z}$$

09.06.27.0018.01

$$\vartheta_2'(z,q) = (-1)^m\,e^{-2inz}\,q^{n^2}\,(-2in\,\vartheta_2(z+\pi m+in\log(q),q)+\vartheta_2'(z+\pi m+in\log(q),q)) \;/;\; \{m,n\}\in\mathbb{Z}$$

Involving $\vartheta_3(z, q)$

09.06.27.0019.01

$$\vartheta_2'(z, q) = e^{-i z} \sqrt[4]{q} \left(\vartheta_3'\left(z - \frac{\pi \tau}{2}, q\right) - i \vartheta_3\left(z - \frac{\pi \tau}{2}, q\right)\right) /; q = e^{i \pi \tau}$$

09.06.27.0020.01

$$\vartheta_2'(z, q) = e^{-i (2 m+1) z} q^{m^2+m+\frac{1}{4}} \left(\vartheta_3'\left(z - \frac{1}{2} (2 m+1) \pi \tau, q\right) - i (2 m+1) \vartheta_3\left(z - \frac{1}{2} (2 m+1) \pi \tau, q\right)\right) /; m \in \mathbb{Z} \bigwedge q = e^{i \pi \tau}$$

09.06.27.0021.01

$$\vartheta_2'(z, q) = e^{i z} \sqrt[4]{q} \left(i \vartheta_3\left(z - \frac{1}{2} i \log(q), q\right) + \vartheta_3'\left(z - \frac{1}{2} i \log(q), q\right)\right)$$

09.06.27.0022.01

$$\vartheta_2'(z, q) = e^{-i (2 m+1) z} q^{m^2+m+\frac{1}{4}} \left(\vartheta_3'\left(z + \frac{1}{2} i (2 m+1) \log(q), q\right) - i (2 m+1) \vartheta_3\left(z + \frac{1}{2} i (2 m+1) \log(q), q\right)\right) /; m \in \mathbb{Z}$$

Involving $\vartheta_4(z, q)$

09.06.27.0023.01

$$\vartheta_2'(z, q) = e^{i z} \sqrt[4]{q} \left(i \vartheta_4\left(z + \frac{1}{2} \pi (\tau+1), q\right) + \vartheta_4'\left(z + \frac{1}{2} \pi (\tau+1), q\right)\right) /; q = e^{i \pi \tau}$$

09.06.27.0024.01

$$\vartheta_2'(z, q) = e^{-i (2 m+1) z} q^{m^2+m+\frac{1}{4}} \left(\vartheta_4'\left(z - \frac{1}{2} (2 m+1) \pi (\tau-1), q\right) - i (2 m+1) \vartheta_4\left(z - \frac{1}{2} (2 m+1) \pi (\tau-1), q\right)\right) /; m \in \mathbb{Z} \bigwedge q = e^{i \pi \tau}$$

09.06.27.0025.01

$$\vartheta_2'(z, q) = e^{i z} \sqrt[4]{q} \left(i \vartheta_4\left(z + \frac{1}{2} (\pi - i \log(q)), q\right) + \vartheta_4'\left(z + \frac{1}{2} (\pi - i \log(q)), q\right)\right)$$

09.06.27.0026.01

$$\vartheta_2'(z, q) = e^{-i (2 m+1) z} q^{m^2+m+\frac{1}{4}} \left(\vartheta_4'\left(z + \frac{1}{2} (2 m+1) (i \log(q) + \pi), q\right) - i (2 m+1) \vartheta_4\left(z + \frac{1}{2} (2 m+1) (i \log(q) + \pi), q\right)\right) /; m \in \mathbb{Z}$$

Involving Weierstrass functions

09.06.27.0008.01

$$\frac{\vartheta_2'(z, q)}{\vartheta_2(z, q)} = \frac{2 \omega_1}{\pi} \zeta\left(\frac{2 \omega_1}{\pi} \left(z + \frac{\pi}{2}\right); g_2, g_3\right) - \frac{2 \omega_1 \eta_1}{\pi} - \frac{4 \omega_1 \eta_1 z}{\pi^2} /; \{\omega_1, \omega_3\} = \{\omega_1(g_2, g_3), \omega_{13}(g_2, g_3)\} \wedge \eta_1 = \zeta(\omega_1; g_2, g_3)$$

Zeros

09.06.30.0002.01

$$\vartheta_2'(z, 0) = 0$$

09.06.30.0003.01

$$\vartheta_2'(0, q) = 0 /; m \in \mathbb{Z}$$

09.06.30.0001.01

$$\vartheta_2'(m \pi, q) = 0 /; m \in \mathbb{Z}$$

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