

# Coth

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## Notations

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### Traditional name

Hyperbolic cotangent

### Traditional notation

$\operatorname{coth}(z)$

### Mathematica StandardForm notation

`Coth[z]`

## Primary definition

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01.22.02.0001.01

$$\operatorname{coth}(z) = \frac{\operatorname{cosh}(z)}{\operatorname{sinh}(z)} = \frac{e^z + e^{-z}}{e^z - e^{-z}}$$

## Specific values

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### Specialized values

01.22.03.0001.01

$$\operatorname{coth}(\pi i m) = \infty \text{ ; } m \in \mathbb{Z}$$

01.22.03.0002.01

$$\operatorname{coth}\left(\pi i \left(\frac{1}{2} + m\right)\right) = 0 \text{ ; } m \in \mathbb{Z}$$

### Values at fixed points

01.22.03.0003.01

$$\operatorname{coth}(0) = \infty$$

01.22.03.0004.01

$$\operatorname{coth}\left(\frac{\pi i}{12}\right) = -i (2 + \sqrt{3})$$

01.22.03.0005.01

$$\operatorname{coth}\left(\frac{\pi i}{12}\right) = (z; z^4 + 14z^2 + 1)_1^{-1}$$

01.22.03.0006.01

$$\operatorname{coth}\left(\frac{\pi i}{10}\right) = -i \sqrt{5 + 2\sqrt{5}}$$

01.22.03.0007.01

$$\operatorname{coth}\left(\frac{\pi i}{10}\right) = (z; z^4 + 10z^2 + 5)_1^{-1}$$

01.22.03.0008.01

$$\operatorname{coth}\left(\frac{\pi i}{9}\right) = -\frac{(-1 - i\sqrt{3})^{4/3} + (-1 + i\sqrt{3})^{4/3}}{-(-1 - i\sqrt{3})^{4/3} + (-1 + i\sqrt{3})^{4/3}}$$

01.22.03.0009.01

$$\operatorname{coth}\left(\frac{\pi i}{9}\right) = (z; 3z^6 + 27z^4 + 33z^2 + 1)_3^{-1}$$

01.22.03.0010.01

$$\operatorname{coth}\left(\frac{\pi i}{9}\right) = \frac{1 + (-1)^{2/9}}{-1 + (-1)^{2/9}}$$

01.22.03.0011.01

$$\operatorname{coth}\left(\frac{\pi i}{8}\right) = -i(1 + \sqrt{2})$$

01.22.03.0012.01

$$\operatorname{coth}\left(\frac{\pi i}{8}\right) = (z; z^4 + 6z^2 + 1)_3^{-1}$$

01.22.03.0013.01

$$\operatorname{coth}\left(\frac{\pi i}{8}\right) = \frac{1 + \sqrt[4]{-1}}{-1 + \sqrt[4]{-1}}$$

01.22.03.0014.01

$$\coth\left(\frac{\pi i}{7}\right) = \left( 2i 2^{2/3} \sqrt[3]{7-21i\sqrt{3}} + 2\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} (\sqrt{7}+i\sqrt{21}) - \right. \\ \left. 2\sqrt{7}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + 2i\sqrt{21}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + \right. \\ \left. i(14-i\sqrt{7}-3\sqrt{21})^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - \sqrt{3}(14-i\sqrt{7}-3\sqrt{21})^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + \right. \\ \left. i\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}(14+i\sqrt{7}+3\sqrt{21})^{2/3} + \sqrt{3}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}(14+i\sqrt{7}+3\sqrt{21})^{2/3} \right) / \\ \left( 2 \cdot 2^{2/3} \cdot 7^{5/6} \sqrt[3]{1-3i\sqrt{3}} - 2i\sqrt{7}(-i+\sqrt{3})\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} + i \left( 2\sqrt{7}(i+\sqrt{3})\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + \right. \right. \\ \left. \left. \sqrt[3]{14-42i\sqrt{3}} \left( (1+i\sqrt{3})\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}} + i(i+\sqrt{3})\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} \right) \right) \right)$$

01.22.03.0015.01

$$\coth\left(\frac{\pi i}{7}\right) = (z; 7z^6 + 35z^4 + 21z^2 + 1)_5^{-1}$$

01.22.03.0016.01

$$\coth\left(\frac{\pi i}{7}\right) = \frac{1 + (-1)^{2/7}}{-1 + (-1)^{2/7}}$$

01.22.03.0017.01

$$\coth\left(\frac{\pi i}{6}\right) = -\sqrt{3} i$$

01.22.03.0018.01

$$\coth\left(\frac{\pi i}{5}\right) = -i \sqrt{1 + \frac{2}{\sqrt{5}}}$$

01.22.03.0019.01

$$\coth\left(\frac{\pi i}{5}\right) = (z; 5z^4 + 10z^2 + 1)_1^{-1}$$

01.22.03.0020.01

$$\coth\left(\frac{2\pi i}{9}\right) = \frac{\sqrt[3]{-1-i\sqrt{3}} + \sqrt[3]{-1+i\sqrt{3}}}{-\sqrt[3]{-1-i\sqrt{3}} + \sqrt[3]{-1+i\sqrt{3}}}$$

01.22.03.0021.01

$$\coth\left(\frac{2\pi i}{9}\right) = (z; 3z^6 + 27z^4 + 33z^2 + 1)_1^{-1}$$

01.22.03.0022.01

$$\operatorname{coth}\left(\frac{2\pi i}{9}\right) = \frac{1 + (-1)^{4/9}}{-1 + (-1)^{4/9}}$$

01.22.03.0023.01

$$\operatorname{coth}\left(\frac{\pi i}{4}\right) = -i$$

01.22.03.0024.01

$$\operatorname{coth}\left(\frac{2\pi i}{7}\right) = -\left(2i\left(7 - 21i\sqrt{3} + 7 \cdot 2^{2/3} \sqrt[3]{7 - 21i\sqrt{3}} - \sqrt[3]{2} (7 - 21i\sqrt{3})^{2/3}\right)\right) /$$

$$\left(\sqrt[3]{\frac{7}{2}(1 - 3i\sqrt{3})} \left(2 \cdot 2^{2/3} \cdot 7^{5/6} \sqrt[3]{1 - 3i\sqrt{3}} + 4\sqrt{7} \sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}} - 2\sqrt{7} \sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} - \right.\right.$$

$$\left.\left.2i\sqrt{21} \sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} + i(14 - i\sqrt{7} - 3\sqrt{21})^{2/3} \sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}} + \right.\right.$$

$$\left.\left.\sqrt{3} (14 - i\sqrt{7} - 3\sqrt{21})^{2/3} \sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}} + 2i \sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}} (14 + i\sqrt{7} + 3\sqrt{21})^{2/3}\right)\right)$$

01.22.03.0025.01

$$\operatorname{coth}\left(\frac{2\pi i}{7}\right) = (z; 7z^6 + 35z^4 + 21z^2 + 1)_3^{-1}$$

01.22.03.0026.01

$$\operatorname{coth}\left(\frac{2\pi i}{7}\right) = \frac{1 + (-1)^{4/7}}{-1 + (-1)^{4/7}}$$

01.22.03.0027.01

$$\operatorname{coth}\left(\frac{3\pi i}{10}\right) = -i \sqrt{5 - 2\sqrt{5}}$$

01.22.03.0028.01

$$\operatorname{coth}\left(\frac{3\pi i}{10}\right) = (z; z^4 + 10z^2 + 5)_3^{-1}$$

01.22.03.0029.01

$$\operatorname{coth}\left(\frac{\pi i}{3}\right) = -\frac{i}{\sqrt{3}}$$

01.22.03.0030.01

$$\operatorname{coth}\left(\frac{3\pi i}{8}\right) = -i(\sqrt{2} - 1)$$

01.22.03.0031.01

$$\operatorname{coth}\left(\frac{3\pi i}{8}\right) = (z; z^4 + 6z^2 + 1)_1^{-1}$$

01.22.03.0032.01

$$\operatorname{coth}\left(\frac{3\pi i}{8}\right) = \frac{1 + (-1)^{3/4}}{-1 + (-1)^{3/4}}$$

01.22.03.0033.01

$$\operatorname{coth}\left(\frac{2\pi i}{5}\right) = -i \sqrt{1 - \frac{2}{\sqrt{5}}}$$

01.22.03.0034.01

$$\operatorname{coth}\left(\frac{2\pi i}{5}\right) = (z; 5z^4 + 10z^2 + 1)_3^{-1}$$

01.22.03.0035.01

$$\operatorname{coth}\left(\frac{5\pi i}{12}\right) = -i(2 - \sqrt{3})$$

01.22.03.0036.01

$$\operatorname{coth}\left(\frac{5\pi i}{12}\right) = (z; z^4 + 14z^2 + 1)_3^{-1}$$

01.22.03.0037.01

$$\begin{aligned} \operatorname{coth}\left(\frac{3\pi i}{7}\right) = & \left( -2^{2/3} \sqrt[3]{7 - 21i\sqrt{3}} + \sqrt{7}(i + \sqrt{3}) \sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}} + \right. \\ & 2i\sqrt{7} \sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} + (14 - i\sqrt{7} - 3\sqrt{21})^{2/3} \sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}} - \\ & \left. \frac{1}{2} \sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}} (14 + i\sqrt{7} + 3\sqrt{21})^{2/3} - \frac{1}{2} i\sqrt{3} \sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}} (14 + i\sqrt{7} + 3\sqrt{21})^{2/3} \right) / \\ & \left( -i2^{2/3} 7^{5/6} \sqrt[3]{1 - 3i\sqrt{3}} + \sqrt{7}(i + \sqrt{3}) \sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}} - \frac{1}{2} i \left( 4\sqrt{7} \sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} + \right. \right. \\ & \left. \left. \sqrt[3]{14 - 42i\sqrt{3}} \left( -2i \sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}} + (-i + \sqrt{3}) \sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}} \right) \right) \right) \end{aligned}$$

01.22.03.0038.01

$$\operatorname{coth}\left(\frac{3\pi i}{7}\right) = (z; 7z^6 + 35z^4 + 21z^2 + 1)_1^{-1}$$

01.22.03.0039.01

$$\operatorname{coth}\left(\frac{3\pi i}{7}\right) = \frac{1 + (-1)^{6/7}}{-1 + (-1)^{6/7}}$$

01.22.03.0040.01

$$\operatorname{coth}\left(\frac{4\pi i}{9}\right) = \frac{-\sqrt[3]{-1+i\sqrt{3}}(-i+\sqrt{3}) + \sqrt[3]{-1-i\sqrt{3}}(i+\sqrt{3})}{\sqrt[3]{-1+i\sqrt{3}}(-i+\sqrt{3}) + \sqrt[3]{-1-i\sqrt{3}}(i+\sqrt{3})}$$

01.22.03.0041.01

$$\operatorname{coth}\left(\frac{4\pi i}{9}\right) = (z; 3z^6 + 27z^4 + 33z^2 + 1)_5^{-1}$$

01.22.03.0042.01

$$\operatorname{coth}\left(\frac{4\pi i}{9}\right) = \frac{1 + (-1)^{8/9}}{-1 + (-1)^{8/9}}$$

01.22.03.0043.01

$$\operatorname{coth}\left(\frac{\pi i}{2}\right) = 0$$

01.22.03.0044.01

$$\operatorname{coth}\left(\frac{5\pi i}{9}\right) = \frac{\sqrt[3]{-1+i\sqrt{3}}(-i+\sqrt{3}) - \sqrt[3]{-1-i\sqrt{3}}(i+\sqrt{3})}{\sqrt[3]{-1+i\sqrt{3}}(-i+\sqrt{3}) + \sqrt[3]{-1-i\sqrt{3}}(i+\sqrt{3})}$$

01.22.03.0045.01

$$\operatorname{coth}\left(\frac{5\pi i}{9}\right) = (z; 3z^6 + 27z^4 + 33z^2 + 1)_6^{-1}$$

01.22.03.0046.01

$$\operatorname{coth}\left(\frac{5\pi i}{9}\right) = \frac{-1 + \sqrt[9]{-1}}{1 + \sqrt[9]{-1}}$$

01.22.03.0047.01

$$\operatorname{coth}\left(\frac{4\pi i}{7}\right) = \left( 7 + 7i\sqrt{3} + \left(\frac{7}{2}(1-3i\sqrt{3})\right)^{2/3} - i\sqrt{3} \left(\frac{7}{2} - \frac{21i\sqrt{3}}{2}\right)^{2/3} + 2^{2/3}\sqrt[3]{7-21i\sqrt{3}} \right) /$$

$$\left( -i2^{2/3}7^{5/6}\sqrt[3]{1-3i\sqrt{3}} + \sqrt{7}(i+\sqrt{3})\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} - \frac{1}{2}i \left( 4\sqrt{7}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + \right. \right.$$

$$\left. \left. \sqrt[3]{14-42i\sqrt{3}} \left( -2i\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}} + (-i+\sqrt{3})\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} \right) \right) \right)$$

01.22.03.0048.01

$$\operatorname{coth}\left(\frac{4\pi i}{7}\right) = (z; 7z^6 + 35z^4 + 21z^2 + 1)_2^{-1}$$

01.22.03.0049.01

$$\operatorname{coth}\left(\frac{4\pi i}{7}\right) = \frac{-1 + \sqrt[7]{-1}}{1 + \sqrt[7]{-1}}$$

01.22.03.0050.01

$$\operatorname{coth}\left(\frac{7\pi i}{12}\right) = i(2 - \sqrt{3})$$

01.22.03.0051.01

$$\operatorname{coth}\left(\frac{7\pi i}{12}\right) = (z; z^4 + 14z^2 + 1)_4^{-1}$$

01.22.03.0052.01

$$\operatorname{coth}\left(\frac{3\pi i}{5}\right) = i\sqrt{1 - \frac{2}{\sqrt{5}}}$$

01.22.03.0053.01

$$\operatorname{coth}\left(\frac{3\pi i}{5}\right) = (z; 5z^4 + 10z^2 + 1)_4^{-1}$$

01.22.03.0054.01

$$\operatorname{coth}\left(\frac{5\pi i}{8}\right) = i(\sqrt{2} - 1)$$

01.22.03.0055.01

$$\operatorname{coth}\left(\frac{5\pi i}{8}\right) = (z; z^4 + 6z^2 + 1)_2^{-1}$$

01.22.03.0056.01

$$\operatorname{coth}\left(\frac{5\pi i}{8}\right) = \frac{-1 + \sqrt[4]{-1}}{1 + \sqrt[4]{-1}}$$

01.22.03.0057.01

$$\operatorname{coth}\left(\frac{2\pi i}{3}\right) = \frac{i}{\sqrt{3}}$$

01.22.03.0058.01

$$\operatorname{coth}\left(\frac{7\pi i}{10}\right) = i\sqrt{5 - 2\sqrt{5}}$$

01.22.03.0059.01

$$\operatorname{coth}\left(\frac{7\pi i}{10}\right) = (z; z^4 + 10z^2 + 5)_4^{-1}$$

01.22.03.0060.01

$$\coth\left(\frac{5\pi i}{7}\right) = \left( -2i2^{2/3} \sqrt[3]{7-21i\sqrt{3}} + 4\sqrt{7} \sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} + 2\sqrt{7} \sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + \right. \\ \left. 2i\sqrt{21} \sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} - i(14-i\sqrt{7}-3\sqrt{21})^{2/3} \sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - \right. \\ \left. \sqrt{3}(14-i\sqrt{7}-3\sqrt{21})^{2/3} \sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + 2i\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}(14+i\sqrt{7}+3\sqrt{21})^{2/3} \right) / \\ \left( 22^{2/3}7^{5/6} \sqrt[3]{1-3i\sqrt{3}} + 4\sqrt{7} \sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} - 2\sqrt{7} \sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} - \right. \\ \left. 2i\sqrt{21} \sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + i(14-i\sqrt{7}-3\sqrt{21})^{2/3} \sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + \right. \\ \left. \sqrt{3}(14-i\sqrt{7}-3\sqrt{21})^{2/3} \sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + 2i\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}(14+i\sqrt{7}+3\sqrt{21})^{2/3} \right)$$

01.22.03.0061.01

$$\coth\left(\frac{5\pi i}{7}\right) = (z; 7z^6 + 35z^4 + 21z^2 + 1)_4^{-1}$$

01.22.03.0062.01

$$\coth\left(\frac{5\pi i}{7}\right) = \frac{-1 + (-1)^{3/7}}{1 + (-1)^{3/7}}$$

01.22.03.0063.01

$$\coth\left(\frac{3\pi i}{4}\right) = i$$

01.22.03.0064.01

$$\coth\left(\frac{7\pi i}{9}\right) = \frac{\sqrt[3]{-1-i\sqrt{3}} + \sqrt[3]{-1+i\sqrt{3}}}{\sqrt[3]{-1-i\sqrt{3}} - \sqrt[3]{-1+i\sqrt{3}}}$$

01.22.03.0065.01

$$\coth\left(\frac{7\pi i}{9}\right) = (z; 3z^6 + 27z^4 + 33z^2 + 1)_2^{-1}$$

01.22.03.0066.01

$$\coth\left(\frac{7\pi i}{9}\right) = \frac{-1 + (-1)^{5/9}}{1 + (-1)^{5/9}}$$



01.22.03.0067.01

$$\operatorname{coth}\left(\frac{4\pi i}{5}\right) = i \sqrt{1 + \frac{2}{\sqrt{5}}}$$

01.22.03.0068.01

$$\operatorname{coth}\left(\frac{4\pi i}{5}\right) = (z; 5z^4 + 10z^2 + 1)_2^{-1}$$

01.22.03.0069.01

$$\operatorname{coth}\left(\frac{5\pi i}{6}\right) = \sqrt{3} i$$

01.22.03.0070.01

$$\operatorname{coth}\left(\frac{6\pi i}{7}\right) = -\left(2\left(7 - 7i\sqrt{3} + \left(\frac{7}{2}(1 - 3i\sqrt{3})\right)^{2/3} + i\sqrt{3}\left(\frac{7}{2} - \frac{21i\sqrt{3}}{2}\right)^{2/3} + 2^{2/3}\sqrt[3]{7 - 21i\sqrt{3}}\right)\right) /$$

$$\left(\sqrt[3]{\frac{7}{2}(1 - 3i\sqrt{3})}\left(-4i\sqrt{7} + \frac{2\sqrt{7}(i + \sqrt{3})}{\sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}}} + (2 + 2i\sqrt{3})\sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}} - \frac{2\sqrt{7}(-i + \sqrt{3})}{\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}}} + 2i(i + \sqrt{3})\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}}\right)\right)$$

01.22.03.0071.01

$$\operatorname{coth}\left(\frac{6\pi i}{7}\right) = (z; 7z^6 + 35z^4 + 21z^2 + 1)_6^{-1}$$

01.22.03.0072.01

$$\operatorname{coth}\left(\frac{6\pi i}{7}\right) = \frac{-1 + (-1)^{5/7}}{1 + (-1)^{5/7}}$$

01.22.03.0073.01

$$\operatorname{coth}\left(\frac{7\pi i}{8}\right) = i(1 + \sqrt{2})$$

01.22.03.0074.01

$$\operatorname{coth}\left(\frac{7\pi i}{8}\right) = (z; z^4 + 6z^2 + 1)_4^{-1}$$

01.22.03.0075.01

$$\operatorname{coth}\left(\frac{7\pi i}{8}\right) = \frac{-1 + (-1)^{3/4}}{1 + (-1)^{3/4}}$$

01.22.03.0076.01

$$\operatorname{coth}\left(\frac{8\pi i}{9}\right) = \frac{(-1 - i\sqrt{3})^{4/3} + (-1 + i\sqrt{3})^{4/3}}{-(-1 - i\sqrt{3})^{4/3} + (-1 + i\sqrt{3})^{4/3}}$$

01.22.03.0077.01

$$\operatorname{coth}\left(\frac{8\pi i}{9}\right) = (z; 3z^6 + 27z^4 + 33z^2 + 1)_4^{-1}$$

01.22.03.0078.01

$$\operatorname{coth}\left(\frac{8\pi i}{9}\right) = \frac{-1 + (-1)^{7/9}}{1 + (-1)^{7/9}}$$

01.22.03.0079.01

$$\operatorname{coth}\left(\frac{9\pi i}{10}\right) = i\sqrt{5 + 2\sqrt{5}}$$

01.22.03.0080.01

$$\operatorname{coth}\left(\frac{9\pi i}{10}\right) = (z; z^4 + 10z^2 + 5)_2^{-1}$$

01.22.03.0081.01

$$\operatorname{coth}\left(\frac{11\pi i}{12}\right) = i(2 + \sqrt{3})$$

01.22.03.0082.01

$$\operatorname{coth}\left(\frac{11\pi i}{12}\right) = (z; z^4 + 14z^2 + 1)_2^{-1}$$

01.22.03.0083.01

$$\operatorname{coth}(\pi i) = \tilde{\infty}$$

01.22.03.0084.01

$$\operatorname{coth}\left(\frac{\pi i}{17}\right) = -i\sqrt{\left(\left(\sqrt{\left(2\left(\sqrt{34(17-\sqrt{17})} + 6\sqrt{17} - 8\sqrt{2(17+\sqrt{17})} - \sqrt{34-2\sqrt{17}} + 34\right)} + \sqrt{17} + \sqrt{34-2\sqrt{17}} + 15\right)\right) / \left(16 - 2\sqrt{\left(2\left(\sqrt{\left(2\left(-\sqrt{34(17-\sqrt{17})} + 6\sqrt{17} + 8\sqrt{2(17+\sqrt{17})} + \sqrt{34-2\sqrt{17}} + 34\right)} + \sqrt{17} - \sqrt{34-2\sqrt{17}} + 15\right)\right)}\right)}\right)}$$

01.22.03.0085.01

$$\operatorname{coth}\left(\frac{\pi i}{30}\right) = -i\sqrt{23 + 10\sqrt{5} + 2\sqrt{255 + 114\sqrt{5}}}$$

$\operatorname{coth}\left(\frac{n i \pi}{m}\right)$  can be expressed using only square roots if  $n \in \mathbb{Z}$  and  $m$  is a product of a power of 2 and distinct Fermat primes  $\{3, 5, 17, 257, \dots\}$ .

### Values at infinities

01.22.03.0086.01

$$\operatorname{coth}(\infty) = 1$$

01.22.03.0087.01

$$\operatorname{coth}(-\infty) = -1$$

01.22.03.0088.01

$$\coth(\tilde{\infty}) = i$$

## General characteristics

### Domain and analyticity

$\coth(z)$  is an analytical function of  $z$  which is defined over the whole complex  $z$ -plane..

01.22.04.0001.01

$$z \rightarrow \coth(z) :: \mathbb{C} \rightarrow \mathbb{C}$$

### Symmetries and periodicities

#### Parity

$\coth(z)$  is an odd function.

01.22.04.0002.01

$$\coth(-z) = -\coth(z)$$

#### Mirror symmetry

01.22.04.0003.01

$$\coth(\bar{z}) = \overline{\coth(z)}$$

#### Periodicity

$\coth(z)$  is a periodic function with period  $\pi i$ .

01.22.04.0009.01

$$\coth(z + \pi i) = \coth(z)$$

01.22.04.0004.01

$$\coth(z + \pi i m) = \coth(z) ; m \in \mathbb{Z}$$

### Poles and essential singularities

The function  $\coth(z)$  has an infinite set of singular points:

a)  $z = \pi i k ; k \in \mathbb{Z}$  are the simple poles with residues 1;

b)  $z = \tilde{\infty}$  is an essential singular point.

01.22.04.0005.01

$$Sing_z(\coth(z)) = \{\{\pi i k, 1\} ; k \in \mathbb{Z}\}, \{\tilde{\infty}, \infty\}$$

01.22.04.0006.01

$$res_z(\coth(z))(\pi i k) = 1 ; k \in \mathbb{Z}$$

### Branch points

The function  $\coth(z)$  does not have branch points.

01.22.04.0007.01

$$\mathcal{BP}_z(\coth(z)) = \{\}$$

## Branch cuts

The function  $\coth(z)$  does not have branch cuts.

01.22.04.0008.01

$$\mathcal{BC}_z(\coth(z)) = \{\}$$

## Series representations

### Generalized power series

Expansions at  $z = z_0$

#### For the function itself

01.22.06.0021.01

$$\coth(z) \propto \coth(z_0) - \operatorname{csch}^2(z_0)(z - z_0) + \frac{1}{2} \sinh(2z_0) \operatorname{csch}^4(z_0)(z - z_0)^2 + \dots /; (z \rightarrow z_0)$$

01.22.06.0022.01

$$\coth(z) \propto \coth(z_0) - \operatorname{csch}^2(z_0)(z - z_0) + \frac{1}{2} \sinh(2z_0) \operatorname{csch}^4(z_0)(z - z_0)^2 + O((z - z_0)^3)$$

01.22.06.0023.01

$$\coth(z) = \sum_{k=0}^{\infty} \frac{1}{k!} \left( \coth(z_0) \delta_k - \operatorname{csch}^2(z_0) \delta_{k-1} - \right. \\ \left. (-i)^k k \sum_{m=0}^{k-1} \sum_{j=0}^{m-1} \frac{(-1)^{j+m}}{m+1} \binom{k-1}{m} \sinh^{-2m-2}(z_0) 2^{k-2m} \binom{2m}{j} (m-j)^{k-1} \sinh\left(\frac{i\pi k}{2} + 2(m-j)z_0\right) \right) (z - z_0)^k$$

01.22.06.0024.01

$$\coth(z) \propto \coth(z_0) (1 + O(z - z_0))$$

Expansions at  $z = 0$

#### For the function itself

01.22.06.0001.02

$$\coth(z) \propto \frac{1}{z} + \frac{z}{3} - \frac{z^3}{45} + \frac{2z^5}{945} - \dots /; (z \rightarrow 0)$$

01.22.06.0025.01

$$\coth(z) \propto \frac{1}{z} + \frac{z}{3} - \frac{z^3}{45} + \frac{2z^5}{945} - O(z^7)$$

01.22.06.0002.01

$$\coth(z) = \frac{1}{z} + \sum_{k=1}^{\infty} \frac{2^{2k} B_{2k} z^{2k-1}}{(2k)!} /; |z| < \pi$$

01.22.06.0003.02

$$\coth(z) \propto \frac{1}{z} + \frac{z}{3} + O(z^3)$$

**Expansions at  $z = \frac{\pi i}{2}$**

### For the function itself

01.22.06.0004.02

$$\coth(z) \propto \left(z - \frac{\pi i}{2}\right) - \frac{1}{3} \left(z - \frac{\pi i}{2}\right)^3 + \frac{2}{15} \left(z - \frac{\pi i}{2}\right)^5 - \dots /; \left(z \rightarrow \frac{\pi i}{2}\right)$$

01.22.06.0026.01

$$\coth(z) \propto \left(z - \frac{\pi i}{2}\right) - \frac{1}{3} \left(z - \frac{\pi i}{2}\right)^3 + \frac{2}{15} \left(z - \frac{\pi i}{2}\right)^5 - O\left(\left(z - \frac{\pi i}{2}\right)^7\right)$$

01.22.06.0005.02

$$\coth(z) = \sum_{k=1}^{\infty} \frac{2^{2k} (2^{2k} - 1) B_{2k}}{(2k)!} \left(z - \frac{\pi i}{2}\right)^{2k-1} /; \left|z - \frac{\pi i}{2}\right| < \frac{\pi}{2}$$

01.22.06.0006.02

$$\coth(z) \propto \left(z - \frac{\pi i}{2}\right) + O\left(\left(z - \frac{\pi i}{2}\right)^3\right)$$

### q-series

01.22.06.0007.01

$$\coth(z) = -1 - 2 \sum_{k=1}^{\infty} q^{2k} /; q = e^z$$

### Dirichlet series

01.22.06.0008.01

$$\coth(z) = 2 \sum_{k=0}^{\infty} e^{-2z(k+1)} + 1 /; \operatorname{Re}(z) > 0$$

01.22.06.0009.01

$$\coth(z) = -2 \sum_{k=0}^{\infty} e^{2z(k+1)} - 1 /; \operatorname{Re}(z) < 0$$

### Asymptotic series expansions

01.22.06.0010.01

$$\coth(z) \propto 1 + 2 e^{-2z} {}_1F_0(1; ; e^{-2z}) /; \operatorname{Re}(z) > 0 \wedge (|z| \rightarrow \infty)$$

01.22.06.0011.01

$$\coth(z) \propto 1 + 2 e^{-2z} (1 + O(e^{-2z})) /; \operatorname{Re}(z) > 0 \wedge (|z| \rightarrow \infty)$$

01.22.06.0012.01

$$\coth(z) \propto -1 - 2 e^{2z} {}_1F_0(1; ; e^{2z}) /; \operatorname{Re}(z) < 0 \wedge (|z| \rightarrow \infty)$$

01.22.06.0013.01

$$\operatorname{coth}(z) \propto -1 - 2 e^{2z} (1 + O(e^{2z})) /; \operatorname{Re}(z) < 0 \wedge (|z| \rightarrow \infty)$$

01.22.06.0014.01

$$\operatorname{coth}(z) \propto \operatorname{coth}(z) /; \operatorname{Re}(z) = 0 \wedge (|z| \rightarrow \infty)$$

01.22.06.0015.01

$$\operatorname{coth}(z) \propto 1 /; (z \rightarrow e^{i\phi} \infty) \wedge -\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

01.22.06.0016.01

$$\operatorname{coth}(z) \propto -1 /; (z \rightarrow e^{i\phi} \infty) \wedge -\pi < \phi < -\frac{\pi}{2} \vee \frac{\pi}{2} < \phi \leq \pi$$

01.22.06.0027.01

$$\operatorname{coth}(z) \propto \begin{cases} -1 & -\pi < \arg(z) < -\frac{\pi}{2} \vee \frac{\pi}{2} < \arg(z) \leq \pi \\ 1 & -\frac{\pi}{2} < \arg(z) < \frac{\pi}{2} \\ \operatorname{coth}(z) & \text{True} \end{cases} /; (|z| \rightarrow \infty)$$

### Other series representations

01.22.06.0017.01

$$\operatorname{coth}(z) = 2z \sum_{k=1}^{\infty} \frac{1}{\pi^2 k^2 + z^2} + \frac{1}{z} /; \frac{iz}{\pi} \notin \mathbb{Z}$$

01.22.06.0018.01

$$\operatorname{coth}(z) = \frac{1}{z} - \frac{iz}{\pi} \left( \sum_{k=1}^{\infty} \frac{1}{k(z - i\pi k)} + \sum_{k=-\infty}^{-1} \frac{1}{k(z - i\pi k)} \right)$$

01.22.06.0019.01

$$\operatorname{coth}(z) = \sum_{k=-\infty}^{\infty} \frac{z}{\pi^2 k^2 + z^2} /; \frac{iz}{\pi} \notin \mathbb{Z}$$

01.22.06.0020.01

$$\operatorname{coth}(z) = \frac{1}{z} - \frac{iz}{\pi} \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \frac{1}{k(z - \pi i k)} /; \frac{iz}{\pi} \notin \mathbb{Z}$$

### Integral representations

#### On the real axis

##### Of the direct function

01.22.07.0001.01

$$\operatorname{coth}(z) = - \int_{\frac{\pi i}{2}}^z \operatorname{csch}^2(t) dt$$

01.22.07.0002.01

$$\operatorname{coth}(z) = - \frac{2i}{\pi} \int_0^{\infty} \frac{t^{\frac{2iz}{\pi} + 1} - 1}{t^2 - 1} dt /; 0 < \operatorname{Im}(z) < \frac{\pi}{2}$$

## Limit representations

01.22.09.0001.01

$$\operatorname{coth}(z) = \lim_{n \rightarrow \infty} \sum_{k=-n}^n \frac{1}{z + i \pi k} \quad /; \frac{i z}{\pi} \notin \mathbb{Z}$$

## Continued fraction representations

01.22.10.0003.01

$$\operatorname{coth}(z) = \frac{1}{z} + \frac{z}{3 + \frac{z^2}{5 + \frac{z^2}{7 + \frac{z^2}{9 + \frac{z^2}{11 + \dots}}}}}$$

Andreas Lauschke (2006)

01.22.10.0004.01

$$\operatorname{coth}(z) = \frac{1}{z} + \frac{1}{z} \operatorname{K}_k(z^2, 2k + 1)_1^\infty$$

Andreas Lauschke (2006)

01.22.10.0001.01

$$\operatorname{coth}(z) = \frac{1}{z} + \frac{4 \pi^{-2} z}{1 + \frac{1 + 4 \pi^{-2} z^2}{3 + \frac{4(4 + 4 \pi^{-2} z^2)}{5 + \frac{9(9 + 4 \pi^{-2} z^2)}{7 + \frac{16(16 + 4 \pi^{-2} z^2)}{9 + \frac{25(25 + 4 \pi^{-2} z^2)}{11 + \frac{36(36 + 4 \pi^{-2} z^2)}{13 + \dots}}}}}}$$

01.22.10.0002.01

$$\operatorname{coth}(z) = \frac{1}{z} + \frac{4 z}{\pi^2 \left( 1 + \operatorname{K}_k \left( k^2 \left( k^2 + \frac{4 z^2}{\pi^2} \right), 2k + 1 \right)_1^\infty \right)}$$

01.22.10.0005.01

$$\operatorname{coth}(z) = \frac{1}{z} - \frac{z/2}{-\frac{3}{2} + \frac{z^2/4}{-\frac{5}{2} + \frac{z^2/4}{-\frac{7}{2} + \frac{z^2/4}{-\frac{9}{2} + \frac{z^2/4}{-\frac{11}{2} + \frac{z^2/4}{-\frac{13}{2} + \frac{z^2/4}{-\frac{15}{2} + \frac{z^2/4}{-\frac{17}{2} + \dots}}}}}}}}$$

A.Lauschke (2006)

01.22.10.0006.01

$$\coth(z) = \frac{1}{z} - \frac{z}{-3 + 2 \operatorname{K}_k\left(\frac{z}{4}, -k - \frac{3}{2}\right)_1^\infty}$$

A.Lauschke (2006)

## Differential equations

### Ordinary nonlinear differential equations

01.22.13.0001.01

$$w'(z) + w(z)^2 - 1 = 0 \ ; \ w(z) = \coth(z) \wedge w\left(\frac{\pi i}{2}\right) = 0$$

01.22.13.0002.01

$$w'(z) - a w(z)^2 - b w(z) - c = 0 \ ; \ w(z) = -\frac{1}{2a} \left( b + \sqrt{b^2 - 4ac} \coth\left(\frac{a\sqrt{b^2 - 4ac} z + \sqrt{b^2 - 4ac} c_1}{2a}\right) \right)$$

## Transformations

### Transformations and argument simplifications

#### Argument involving basic arithmetic operations

01.22.16.0001.01

$$\coth(-z) = -\coth(z)$$

01.22.16.0002.01

$$\coth(a(bz^c)^m) = \frac{(bz^c)^m \coth(ab^m z^{mc})}{b^m z^{mc}} \ ; \ 2m \in \mathbb{Z}$$

01.22.16.0003.01

$$\coth\left(\sqrt{z^2}\right) = \frac{\sqrt{z^2} \coth(z)}{z}$$

#### Argument involving inverse trigonometric and hyperbolic functions

### Involving $\sin^{-1}$

01.22.16.0064.01

$$\coth(\sin^{-1}(z)) = \frac{1}{\frac{2}{(iz + \sqrt{1-z^2})^{2i} + 1} - 1}$$

01.22.16.0004.01

$$\coth(i \sin^{-1}(z)) = -\frac{i \sqrt{1-z^2}}{z}$$



01.22.16.0016.01

$$\operatorname{coth}\left(\frac{i}{2} \sin^{-1}(z)\right) = -i \frac{1 + \sqrt{1 - z^2}}{z}$$

01.22.16.0065.01

$$\operatorname{coth}(a \sin^{-1}(z)) = \frac{1}{\frac{2}{(iz + \sqrt{1 - z^2})^{2ia} + 1} - 1}$$

## Involving $\cos^{-1}$

01.22.16.0066.01

$$\operatorname{coth}(\cos^{-1}(z)) = 1 + \frac{2}{e^{\pi} (iz + \sqrt{1 - z^2})^{2i} - 1}$$

01.22.16.0005.01

$$\operatorname{coth}(i \cos^{-1}(z)) = -\frac{iz}{\sqrt{1 - z^2}}$$

01.22.16.0017.01

$$\operatorname{coth}\left(\frac{i}{2} \cos^{-1}(z)\right) = -\frac{i\sqrt{1+z}}{\sqrt{1-z}}$$

01.22.16.0067.01

$$\operatorname{coth}(a \cos^{-1}(z)) = 1 + \frac{2}{e^{a\pi} (iz + \sqrt{1 - z^2})^{2ai} - 1}$$

## Involving $\tan^{-1}$

01.22.16.0068.01

$$\operatorname{coth}(\tan^{-1}(z)) = \frac{1}{\frac{1}{2} - \frac{1}{2} (1 - iz)^{-i} (iz + 1)^i} - 1$$

01.22.16.0069.01

$$\operatorname{coth}(\tan^{-1}(x, y)) = \frac{1}{\frac{2}{\left(\frac{x+iy}{\sqrt{x^2+y^2}}\right)^{2i} + 1} - 1}$$

01.22.16.0006.01

$$\operatorname{coth}(i \tan^{-1}(z)) = -\frac{i}{z}$$

01.22.16.0070.01

$$\operatorname{coth}(i \tan^{-1}(x, y)) = -\frac{ix}{y}$$

01.22.16.0018.01

$$\operatorname{coth}\left(\frac{i}{2} \tan^{-1}(z)\right) = -\frac{i}{z} \left(1 + \sqrt{1 + z^2}\right)$$

01.22.16.0071.01

$$\operatorname{coth}\left(\frac{1}{2} i \tan^{-1}(x, y)\right) = -\frac{i \left(x + \sqrt{x^2 + y^2}\right)}{y}$$

01.22.16.0072.01

$$\operatorname{coth}(a \tan^{-1}(z)) = \frac{1}{\frac{1}{2} - \frac{1}{2} (1 - iz)^{-ia} (iz + 1)^{ia}} - 1$$

01.22.16.0073.01

$$\operatorname{coth}(a \tan^{-1}(x, y)) = \frac{1}{\frac{2}{\left(\frac{x+iy}{\sqrt{x^2+y^2}}\right)^{2ia} + 1} - 1}$$

### Involving $\cot^{-1}$

01.22.16.0074.01

$$\operatorname{coth}(\cot^{-1}(z)) = \frac{1}{\frac{1}{2} - \frac{1}{2} \left(\frac{-i+z}{z}\right)^{-i} \left(\frac{i+z}{z}\right)^i} - 1$$

01.22.16.0007.01

$$\operatorname{coth}(i \cot^{-1}(z)) = -iz$$

01.22.16.0019.01

$$\operatorname{coth}\left(\frac{i}{2} \cot^{-1}(z)\right) = -i \left(1 + \sqrt{1 + \frac{1}{z^2}}\right) z$$

01.22.16.0075.01

$$\operatorname{coth}(a \cot^{-1}(z)) = \frac{1}{\frac{1}{2} - \frac{1}{2} \left(\frac{-i+z}{z}\right)^{-ia} \left(\frac{i+z}{z}\right)^{ia}} - 1$$

### Involving $\csc^{-1}$

01.22.16.0076.01

$$\operatorname{coth}(\csc^{-1}(z)) = \frac{1}{\frac{2}{\left(\sqrt{1 - \frac{1}{z^2} + \frac{i}{z}}\right)^{2i} + 1} - 1}$$

01.22.16.0008.01

$$\operatorname{coth}(i \csc^{-1}(z)) = -i \sqrt{1 - \frac{1}{z^2}} z$$

01.22.16.0020.01

$$\operatorname{coth}\left(\frac{i}{2} \csc^{-1}(z)\right) = -\frac{i\sqrt{z^2}}{z} \left(\sqrt{z^2} + \sqrt{z^2 - 1}\right)$$

01.22.16.0077.01

$$\operatorname{coth}(a \csc^{-1}(z)) = \frac{1}{\frac{2}{\left(\sqrt{1 - \frac{1}{z^2} + \frac{i}{z}}\right)^{2ia} + 1} - 1}$$

### Involving $\sec^{-1}$

01.22.16.0078.01

$$\operatorname{coth}(\sec^{-1}(z)) = 1 + \frac{2}{e^{\pi \left(\sqrt{1 - \frac{1}{z^2} + \frac{i}{z}}\right)^{2i}} - 1}$$

01.22.16.0009.01

$$\operatorname{coth}(i \sec^{-1}(z)) = -\frac{i\sqrt{z^2}}{z\sqrt{z^2 - 1}}$$

01.22.16.0021.01

$$\operatorname{coth}\left(\frac{i}{2} \sec^{-1}(z)\right) = \frac{i\sqrt{-z-1}\sqrt{-z}}{\sqrt{z-1}\sqrt{z}}$$

01.22.16.0079.01

$$\operatorname{coth}(a \sec^{-1}(z)) = 1 + \frac{2}{e^{a\pi \left(\sqrt{1 - \frac{1}{z^2} + \frac{i}{z}}\right)^{2ai}} - 1}$$

### Involving $\sinh^{-1}$

01.22.16.0010.01

$$\operatorname{coth}(\sinh^{-1}(z)) = \frac{\sqrt{1+z^2}}{z}$$

01.22.16.0022.01

$$\operatorname{coth}\left(\frac{1}{2} \sinh^{-1}(z)\right) = \frac{\sqrt{z^2+1} + 1}{z}$$

01.22.16.0080.01

$$\operatorname{coth}(i \sinh^{-1}(z)) = \frac{1}{1 - \frac{2}{\left(z + \sqrt{z^2+1}\right)^{2i} + 1}}$$

01.22.16.0081.01

$$\operatorname{coth}(a \sinh^{-1}(z)) = \frac{1}{1 - \frac{2}{(z + \sqrt{z^2 + 1})^{2a} + 1}}$$

## Involving $\cosh^{-1}$

01.22.16.0011.01

$$\operatorname{coth}(\cosh^{-1}(z)) = \frac{z}{\sqrt{z-1} \sqrt{z+1}}$$

01.22.16.0023.01

$$\operatorname{coth}\left(\frac{1}{2} \cosh^{-1}(z)\right) = \frac{\sqrt{z+1}}{\sqrt{z-1}}$$

01.22.16.0082.01

$$\operatorname{coth}(i \cosh^{-1}(z)) = \frac{1}{1 - \frac{2}{(z + \sqrt{z-1} \sqrt{z+1})^{2i} + 1}}$$

01.22.16.0083.01

$$\operatorname{coth}(a \cosh^{-1}(z)) = \frac{1}{1 - \frac{2}{(z + \sqrt{z-1} \sqrt{z+1})^{2a} + 1}}$$

## Involving $\tanh^{-1}$

01.22.16.0012.01

$$\operatorname{coth}(\tanh^{-1}(z)) = \frac{1}{z}$$

01.22.16.0024.01

$$\operatorname{coth}\left(\frac{1}{2} \tanh^{-1}(z)\right) = \frac{\sqrt{1-z^2} + 1}{z}$$

01.22.16.0084.01

$$\operatorname{coth}(i \tanh^{-1}(z)) = 1 + \frac{2}{(1-z)^{-i} (z+1)^i - 1}$$

01.22.16.0085.01

$$\operatorname{coth}(a \tanh^{-1}(z)) = 1 + \frac{2}{(1-z)^{-a} (z+1)^a - 1}$$

## Involving $\operatorname{coth}^{-1}$

01.22.16.0013.01

$$\operatorname{coth}(\operatorname{coth}^{-1}(z)) = z$$

01.22.16.0025.01

$$\coth\left(\frac{1}{2} \coth^{-1}(z)\right) = z \sqrt{1 - \frac{1}{z^2} + 1}$$

01.22.16.0086.01

$$\coth(i \coth^{-1}(z)) = \frac{1}{\frac{1}{2} - \frac{1}{2} \left(1 + \frac{1}{z}\right)^{-i} \left(\frac{z-1}{z}\right)^i} - 1$$

01.22.16.0087.01

$$\coth(a \coth^{-1}(z)) = \frac{1}{\frac{1}{2} - \frac{1}{2} \left(1 + \frac{1}{z}\right)^{-a} \left(1 - \frac{1}{z}\right)^a} - 1$$

01.22.16.0028.01

$$\coth(n \coth^{-1}(z)) = \frac{(z-1)^n + (z+1)^n}{(z+1)^n - (z-1)^n} ; n \in \mathbb{N}^+$$

### Involving $\operatorname{csch}^{-1}$

01.22.16.0014.01

$$\coth(\operatorname{csch}^{-1}(z)) = z \sqrt{1 + \frac{1}{z^2}}$$

01.22.16.0026.01

$$\coth\left(\frac{1}{2} \operatorname{csch}^{-1}(z)\right) = z \sqrt{1 + \frac{1}{z^2} + 1}$$

01.22.16.0088.01

$$\coth(i \operatorname{csch}^{-1}(z)) = \frac{1}{1 - \frac{2}{\left(\sqrt{1 + \frac{1}{z^2} + \frac{1}{z}}\right)^{2i} + 1}}$$

01.22.16.0089.01

$$\coth(a \operatorname{csch}^{-1}(z)) = \frac{1}{1 - \frac{2}{\left(\sqrt{1 + \frac{1}{z^2} + \frac{1}{z}}\right)^{2a} + 1}}$$

### Involving $\operatorname{sech}^{-1}$

01.22.16.0015.01

$$\coth(\operatorname{sech}^{-1}(z)) = \frac{1}{1-z} \sqrt{\frac{1-z}{1+z}}$$

01.22.16.0027.01

$$\coth\left(\frac{1}{2} \operatorname{sech}^{-1}(z)\right) = \frac{z}{\sqrt{-z}} \sqrt{\frac{1}{z}} \sqrt{\frac{z+1}{z-1}}$$

01.22.16.0090.01

$$\coth(i \operatorname{sech}^{-1}(z)) = \frac{1}{1 - \frac{2}{\left(\sqrt{\frac{1}{z}-1} \sqrt{1+\frac{1}{z}+\frac{1}{z}}\right)^{2i} + 1}}$$

01.22.16.0091.01

$$\coth(a \operatorname{sech}^{-1}(z)) = \frac{1}{1 - \frac{2}{\left(\sqrt{\frac{1}{z}-1} \sqrt{1+\frac{1}{z}+\frac{1}{z}}\right)^{2a} + 1}}$$

## Addition formulas

01.22.16.0029.01

$$\coth(a+b) = \frac{\coth(a)\coth(b)+1}{\coth(a)+\coth(b)}$$

01.22.16.0030.01

$$\coth\left(a + \frac{i\pi}{4}\right) = \frac{(1-i\coth(a))i}{i\coth(a)+1}$$

01.22.16.0031.01

$$\coth(a-b) = \frac{1-\coth(a)\coth(b)}{\coth(a)-\coth(b)}$$

01.22.16.0032.01

$$\coth\left(a - \frac{i\pi}{4}\right) = \frac{(i\coth(a)+1)i}{i\coth(a)-1}$$

01.22.16.0033.01

$$\coth(a+bi) = \frac{\sinh(2a) - i\sin(2b)}{\cosh(2a) - \cos(2b)}$$

01.22.16.0034.01

$$\coth(a-ib) = \frac{i\sin(2b) + \sinh(2a)}{\cosh(2a) - \cos(2b)}$$

01.22.16.0035.01

$$\coth(z_1+z_2+z_3) = \frac{\coth(z_2)\coth(z_3)\coth(z_1) + \coth(z_1) + \coth(z_2) + \coth(z_3)}{\coth(z_1)\coth(z_2) + \coth(z_3)\coth(z_2) + \coth(z_1)\coth(z_3) + 1}$$

## Half-angle formulas

01.22.16.0036.01

$$\coth\left(\frac{z}{2}\right) = \coth(z) + \operatorname{csch}(z)$$

01.22.16.0037.01

$$\coth\left(\frac{z}{2}\right) = \frac{\sinh(z)}{\cosh(z)-1}$$

01.22.16.0038.01

$$\coth\left(\frac{z}{2}\right) = z \sqrt{\frac{1}{z^2}} \sqrt{\frac{\cosh(z) + 1}{\cosh(z) - 1}}$$

01.22.16.0039.01

$$\coth\left(\frac{z}{2}\right) = \frac{\sqrt{-z^2}}{z} \sqrt{\frac{1 + \cosh(z)}{1 - \cosh(z)}} \quad ; 0 < |\operatorname{Im}(z)| < \pi$$

## Multiple arguments

### Argument involving numeric multiples of variable

01.22.16.0040.01

$$\coth(2z) = \frac{1}{2} (\coth(z) + \tanh(z))$$

01.22.16.0041.01

$$\coth(3z) = \frac{\coth^3(z) + 3 \coth(z)}{3 \coth^2(z) + 1}$$

### Argument involving symbolic multiples of variable

01.22.16.0042.01

$$\coth(nz) = \frac{1}{n} \sum_{k=0}^{n-1} \coth\left(z + \frac{\pi i k}{n}\right) \quad ; n \in \mathbb{N}^+$$

01.22.16.0043.01

$$\coth(nz) = \frac{T_n(\cosh(z))}{\sinh(z) U_{n-1}(\cosh(z))}$$

01.22.16.0053.01

$$\coth(nz) = \frac{\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2k} \coth^{n-2k}(z)}{\sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n}{2k+1} \coth^{n-(2k+1)}(z)} \quad ; n \in \mathbb{N}^+$$

Roger Germundsson

## Products, sums, and powers of the direct function

### Products of the direct function

01.22.16.0044.01

$$\coth(a) \coth(b) = \frac{\cosh(a-b) + \cosh(a+b)}{\cosh(a+b) - \cosh(a-b)}$$

### Products involving the direct function

01.22.16.0045.01

$$\coth(a) \tanh(b) = \frac{\sinh(a+b) - \sinh(a-b)}{\sinh(a-b) + \sinh(a+b)}$$

**Sums of the direct function**

01.22.16.0046.01

$$\coth(a) + \coth(b) = \operatorname{csch}(a) \operatorname{csch}(b) \sinh(a + b)$$

01.22.16.0047.01

$$\coth(a) - \coth(b) = -\operatorname{csch}(a) \operatorname{csch}(b) \sinh(a - b)$$

**Sums involving the direct function****Involving other hyperbolic functions****Involving tanh**

01.22.16.0054.01

$$\coth(z) + \tanh(z) = \cosh(2z) \operatorname{csch}(z) \operatorname{sech}(z)$$

01.22.16.0055.01

$$\coth(z) - \tanh(z) = \operatorname{csch}(z) \operatorname{sech}(z)$$

01.22.16.0048.01

$$\coth(a) + \tanh(b) = \cosh(a + b) \operatorname{csch}(a) \operatorname{sech}(b)$$

01.22.16.0049.01

$$\coth(a) - \tanh(b) = \cosh(a - b) \operatorname{csch}(a) \operatorname{sech}(b)$$

**Involving trigonometric functions****Involving tan**

01.22.16.0056.01

$$\coth(z) + i \tan(z) = \cosh\left(\sqrt{2} e^{\frac{i\pi}{4}} z\right) \operatorname{csch}(z) \sec(z)$$

01.22.16.0057.01

$$\coth(z) - i \tan(z) = \cosh\left(\sqrt{2} e^{-\frac{1}{4}(i\pi)} z\right) \operatorname{csch}(z) \sec(z)$$

01.22.16.0058.01

$$\coth(a) + i \tan(b) = \cosh(a + b i) \operatorname{csch}(a) \sec(b)$$

01.22.16.0059.01

$$\coth(a) - i \tan(b) = \cosh(a - i b) \operatorname{csch}(a) \sec(b)$$

**Involving cot**

01.22.16.0060.01

$$\coth(z) + i \cot(z) = \csc(z) \operatorname{csch}(z) \sin\left(\sqrt{2} e^{\frac{i\pi}{4}} z\right)$$

01.22.16.0061.01

$$\coth(z) - i \cot(z) = \csc(z) \operatorname{csch}(z) \sin\left(\sqrt{2} e^{-\frac{i\pi}{4}} z\right)$$



01.22.16.0062.01

$$\coth(a) + i \cot(b) = \csc(b) \operatorname{csch}(a) \sin(b + a i)$$

01.22.16.0063.01

$$\coth(a) - i \cot(b) = \csc(b) \operatorname{csch}(a) \sin(b - a i)$$

### Powers of the direct function

01.22.16.0050.01

$$\coth^2(z) = \frac{\cosh(2z) + 1}{\cosh(2z) - 1}$$

01.22.16.0051.01

$$\coth^3(z) = \frac{3 \cosh(z) + \cosh(3z)}{\sinh(3z) - 3 \sinh(z)}$$

### Powers involving the direct function

01.22.16.0052.01

$$\coth^2(a) - \coth^2(b) = -\operatorname{csch}^2(a) \csc^2(b) \sinh(a - b) \sinh(a + b)$$

## Identities

### Functional identities

01.22.17.0001.01

$$\coth(z) \coth(2z) = \frac{1}{2} (\coth^2(z) + 1)$$

## Complex characteristics

### Real part

01.22.19.0001.01

$$\operatorname{Re}(\coth(x + i y)) = -\frac{\sinh(2x)}{\cos(2y) - \cosh(2x)}$$

01.22.19.0007.01

$$\operatorname{Re}(\coth(z)) = -\frac{\sinh(2 \operatorname{Re}(z))}{\cos(2 \operatorname{Im}(z)) - \cosh(2 \operatorname{Re}(z))}$$

### Imaginary part

01.22.19.0002.01

$$\operatorname{Im}(\coth(x + i y)) = \frac{\sin(2y)}{\cos(2y) - \cosh(2x)}$$

01.22.19.0008.01

$$\operatorname{Im}(\coth(z)) = \frac{\sin(2 \operatorname{Im}(z))}{\cos(2 \operatorname{Im}(z)) - \cosh(2 \operatorname{Re}(z))}$$

### Absolute value

01.22.19.0003.01

$$|\operatorname{coth}(x + i y)| = \sqrt{\frac{\cos(2 y) + \cosh(2 x)}{\cos(2 y) - \cosh(2 x)}}$$

01.22.19.0009.01

$$|\operatorname{coth}(z)| = \sqrt{\frac{\cos(2 \operatorname{Im}(z)) + \cosh(2 \operatorname{Re}(z))}{\cos(2 \operatorname{Im}(z)) - \cosh(2 \operatorname{Re}(z))}}$$

## Argument

01.22.19.0004.01

$$\arg(\operatorname{coth}(x + i y)) = \tan^{-1}\left(-\frac{\sinh(2 x)}{\cos(2 y) - \cosh(2 x)}, \frac{\sin(2 y)}{\cos(2 y) - \cosh(2 x)}\right)$$

01.22.19.0010.01

$$\arg(\operatorname{coth}(x + i y)) = \frac{1}{2} \left( \operatorname{sgn}\left(\frac{\operatorname{sgn}(\sin(2 y))}{\operatorname{sgn}(\cos(2 y) - \cosh(2 x))}\right) + \frac{1}{2} \right) \left( \pi - \frac{\pi \operatorname{sgn}(\sinh(2 x))}{\operatorname{sgn}(\cosh(2 x) - \cos(2 y))} \right) - 2 \tan^{-1}(\operatorname{csch}(2 x) \sin(2 y))$$

01.22.19.0011.01

$$\arg(\operatorname{coth}(z)) = \tan^{-1}\left(-\frac{\sinh(2 \operatorname{Re}(z))}{\cos(2 \operatorname{Im}(z)) - \cosh(2 \operatorname{Re}(z))}, \frac{\sin(2 \operatorname{Im}(z))}{\cos(2 \operatorname{Im}(z)) - \cosh(2 \operatorname{Re}(z))}\right)$$

01.22.19.0005.01

$$\arg(\operatorname{coth}(z)) = \frac{1}{2} \left( \operatorname{sgn}\left(\frac{\operatorname{sgn}(\sin(2 \operatorname{Im}(z)))}{\operatorname{sgn}(\cos(2 \operatorname{Im}(z)) - \cosh(2 \operatorname{Re}(z)))}\right) + \frac{1}{2} \right) \left( \pi - \frac{\pi \operatorname{sgn}(\sinh(2 \operatorname{Re}(z)))}{\operatorname{sgn}(\cosh(2 \operatorname{Re}(z)) - \cos(2 \operatorname{Im}(z)))} \right) - 2 \tan^{-1}(\operatorname{csch}(2 \operatorname{Re}(z)) \sin(2 \operatorname{Im}(z)))$$

## Conjugate value

01.22.19.0006.01

$$\overline{\operatorname{coth}(x + i y)} = \frac{\sinh(2 x) + i \sin(2 y)}{\cosh(2 x) - \cos(2 y)}$$

01.22.19.0012.01

$$\overline{\operatorname{coth}(z)} = -\frac{i \sin(2 \operatorname{Im}(z)) + \sinh(2 \operatorname{Re}(z))}{\cos(2 \operatorname{Im}(z)) - \cosh(2 \operatorname{Re}(z))}$$

## Signum value

01.22.19.0013.01

$$\operatorname{sgn}(\operatorname{coth}(x + i y)) = \frac{\sinh(2 x) - i \sin(2 y)}{\sqrt{\cosh^2(2 x) - \cos^2(2 y)}}$$

01.22.19.0014.01

$$\operatorname{sgn}(\operatorname{coth}(z)) = \frac{\sinh(2 \operatorname{Re}(z)) - i \sin(2 \operatorname{Im}(z))}{\sqrt{\cosh^2(2 \operatorname{Re}(z)) - \cos^2(2 \operatorname{Im}(z))}}$$

## Differentiation

## Low-order differentiation

01.22.20.0001.01

$$\frac{\partial \operatorname{coth}(z)}{\partial z} = -\operatorname{csch}^2(z)$$

01.22.20.0002.01

$$\frac{\partial^2 \operatorname{coth}(z)}{\partial z^2} = 2 \operatorname{coth}(z) \operatorname{csch}^2(z)$$

## Symbolic differentiation

01.22.20.0003.01

$$\frac{\partial^n \operatorname{coth}(z)}{\partial z^n} = (-1)^n n! z^{-n-1} + \sum_{k=1}^{\infty} \frac{2^{2k-1} B_{2k} z^{2k-n-1}}{k(2k-n-1)!} ; |z| < \pi \wedge n \in \mathbb{N}^+$$

01.22.20.0004.01

$$\begin{aligned} \frac{\partial^n \operatorname{coth}(z)}{\partial z^n} &= \operatorname{coth}(z) \delta_n + \operatorname{csch}^2(z) \delta_{n-1} - \\ &n (-i)^n \sum_{k=0}^{n-1} \sum_{j=0}^{k-1} \frac{(-1)^{j+k} (k-j)^{n-1} \sinh^{-2k-2}(z) 2^{n-2k}}{k+1} \binom{n-1}{k} \binom{2k}{j} \sinh\left(\frac{i\pi n}{2} + 2(k-j)z\right) ; n \in \mathbb{N} \end{aligned}$$

01.22.20.0006.01

$$\frac{\partial^n \operatorname{coth}(z)}{\partial z^n} = \delta_n + 2^n (\operatorname{coth}(z) - 1) \sum_{k=0}^n \frac{(-1)^k k! S_n^{(k)}}{2^k} (\operatorname{coth}(z) + 1)^k ; n \in \mathbb{N}$$

Victor Adamchik (2005)

## Fractional integro-differentiation

01.22.20.0005.01

$$\frac{\partial^\alpha \operatorname{coth}(z)}{\partial z^\alpha} = \mathcal{FC}_{\exp}^{(\alpha)}(z, -1) z^{-\alpha-1} + \sum_{k=1}^{\infty} \frac{2^{2k-1} B_{2k} z^{2k-\alpha-1}}{\Gamma(2k-\alpha)k} ; |z| < \pi$$

01.22.20.0007.01

$$\operatorname{coth}^{(\alpha)}(cz) = i^{\alpha+1} \pi^{-\alpha-1} \left( (-icz)^\alpha (icz)^{-\alpha} \psi^{(\alpha)}\left(-\frac{icz}{\pi}\right) - \psi^{(\alpha)}\left(\frac{icz}{\pi}\right) \right) + i^{\alpha+1} \lim_{\nu \rightarrow \alpha} \frac{(icz)^{-\nu-1}}{\Gamma(-\nu)} (2 \log(\pi) - \log(-icz) + \psi(-\nu) + \gamma)$$

## Integration

### Indefinite integration

#### Involving only one direct function

01.22.21.0013.01

$$\int \operatorname{coth}(b+az) dz = \frac{\log(\sinh(b+az))}{a}$$

$$\int \coth(az) dz = \frac{\log(\sinh(az))}{a}$$

$$\int \coth(z) dz = \log(\sinh(z))$$

**Involving one direct function and elementary functions**

### Involving power function

Involving power

### Involving $z^n$ and linear arguments

$$\int z \coth(b+az) dz = \frac{1}{2a^2} (a^2 z^2 + 2abz + 2a \log(1 - e^{-2(b+az)})z + \pi i az - i\pi \log(1 + e^{2az}) + 2b \log(1 - e^{-2(b+az)}) + i\pi \log(\cosh(az)) - 2b \log(i \sinh(b+az)) - \text{Li}_2(e^{-2(b+az)}))$$

$$\int z^n \coth(az) dz = -\frac{z^{n+1}}{n+1} - 2e^{2az} n! \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} a^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \dots, 1, 1; 2, \dots, 2; e^{2az}) /; n \in \mathbb{N}$$

$$\int z \coth(az) dz = \frac{az(a z + 2 \log(1 - e^{-2az})) - \text{Li}_2(e^{-2az})}{2a^2}$$

$$\int z^2 \coth(az) dz = \frac{-8a^3 z^3 + 24a^2 \log(1 - e^{2az}) z^2 + 24a \text{Li}_2(e^{2az}) z + i\pi^3 - 12 \text{Li}_3(e^{2az})}{24a^3}$$

$$\int z^3 \coth(az) dz = \frac{-16a^4 z^4 + 64a^3 \log(1 - e^{2az}) z^3 + 96a^2 \text{Li}_2(e^{2az}) z^2 - 96a \text{Li}_3(e^{2az}) z + \pi^4 + 48 \text{Li}_4(e^{2az})}{64a^4}$$

$$\int z^4 \coth(az) dz = -\frac{z^5}{5} + \frac{\log(1 - e^{2az}) z^4}{a} + \frac{2 \text{Li}_2(e^{2az}) z^3}{a^2} - \frac{3 \text{Li}_3(e^{2az}) z^2}{a^3} + \frac{3 \text{Li}_4(e^{2az}) z}{a^4} - \frac{3 \text{Li}_5(e^{2az})}{2a^5} - \frac{i\pi^5}{160a^5}$$

### Involving exponential function

Involving exp

### Involving $a^{bz}$

01.22.21.0022.01

$$\int a^{bz} \coth(cz) dz = -\frac{1}{b \log(a) (2c + b \log(a))} \left( a^{bz} \left( b e^{2cz} {}_2F_1 \left( \frac{b \log(a)}{2c} + 1, 1; \frac{b \log(a)}{2c} + 2; e^{2cz} \right) \log(a) + {}_2F_1 \left( \frac{b \log(a)}{2c}, 1; \frac{b \log(a)}{2c} + 1; e^{2cz} \right) (2c + b \log(a)) \right) \right)$$

01.22.21.0023.01

$$\int e^{bz} \coth(az) dz = -\frac{e^{bz}}{b(2a+b)} \left( (2a+b) {}_2F_1 \left( \frac{b}{2a}, 1; \frac{b}{2a} + 1; e^{2az} \right) + b e^{2az} {}_2F_1 \left( \frac{b}{2a} + 1, 1; \frac{b}{2a} + 2; e^{2az} \right) \right)$$

01.22.21.0024.01

$$\int e^{-az} \coth(az) dz = \frac{\log(-1 + e^{-az}) + e^{-az} - \log(1 + e^{-az})}{a}$$

01.22.21.0025.01

$$\int e^{az} \coth(az) dz = \frac{\log(-1 + e^{az}) + e^{az} - \log(1 + e^{az})}{a}$$

### Involving exponential function and a power function

Involving exp and power

### Involving $z^n e^{bz}$

01.22.21.0026.01

$$\int z^n e^{bz} \coth(cz) dz = -n! \left( e^{bz} \sum_{j=0}^n \frac{(-1)^j b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b}{2c}, \dots, \frac{b}{2c}, 1; \frac{b}{2c} + 1, \dots, \frac{b}{2c} + 1; e^{2cz} \right) + e^{(b+2c)z} \sum_{j=0}^n \frac{(-1)^j (b+2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b}{2c} + 1, \dots, \frac{b}{2c} + 1, 1; \frac{b}{2c} + 2, \dots, \frac{b}{2c} + 2; e^{2cz} \right) \right); n \in \mathbb{N}$$

### Arguments involving inverse trigonometric functions

Involving  $\sin^{-1}$

01.22.21.0027.01

$$\int \coth(\sin^{-1}(z)) dz = \frac{1}{10} e^{-i \sin^{-1}(z)} \left( 5 e^{2i \sin^{-1}(z)} i {}_2F_1 \left( \frac{i}{2}, 1; 1 + \frac{i}{2}; e^{2 \sin^{-1}(z)} \right) - 5 i {}_2F_1 \left( -\frac{i}{2}, 1; 1 - \frac{i}{2}; e^{2 \sin^{-1}(z)} \right) - (2-i) e^{(2+2i) \sin^{-1}(z)} {}_2F_1 \left( 1 + \frac{i}{2}, 1; 2 + \frac{i}{2}; e^{2 \sin^{-1}(z)} \right) - (2+i) e^{2 \sin^{-1}(z)} {}_2F_1 \left( 1 - \frac{i}{2}, 1; 2 - \frac{i}{2}; e^{2 \sin^{-1}(z)} \right) \right)$$

Involving  $\cos^{-1}$

01.22.21.0028.01

$$\int \coth(\cos^{-1}(z)) dz = \frac{1}{10} e^{-i \cos^{-1}(z)} \left( -5 e^{2i \cos^{-1}(z)} {}_2F_1\left(\frac{i}{2}, 1; 1 + \frac{i}{2}; e^{2 \cos^{-1}(z)}\right) - 5 {}_2F_1\left(-\frac{i}{2}, 1; 1 - \frac{i}{2}; e^{2 \cos^{-1}(z)}\right) - (1 + 2i) e^{(2+2i) \cos^{-1}(z)} {}_2F_1\left(1 + \frac{i}{2}, 1; 2 + \frac{i}{2}; e^{2 \cos^{-1}(z)}\right) - (1 - 2i) e^{2 \cos^{-1}(z)} {}_2F_1\left(1 - \frac{i}{2}, 1; 2 - \frac{i}{2}; e^{2 \cos^{-1}(z)}\right) \right)$$

### Arguments involving inverse hyperbolic functions

#### Involving $\sinh^{-1}$

01.22.21.0029.01

$$\int \coth(\sinh^{-1}(z)) dz = \log(z) - \log\left(\sqrt{z^2 + 1} + 1\right) + \sqrt{z^2 + 1}$$

01.22.21.0030.01

$$\int \coth(a \sinh^{-1}(z)) dz = -\frac{1}{8a^2 - 2} \left( e^{-2 \sinh^{-1}(z)} \left( (2a - 1) \left( (2a + 1) e^{\sinh^{-1}(z)} \left( e^{2 \sinh^{-1}(z)} {}_2F_1\left(\frac{1}{2a}, 1; 1 + \frac{1}{2a}; e^{2a \sinh^{-1}(z)}\right) - {}_2F_1\left(-\frac{1}{2a}, 1; 1 - \frac{1}{2a}; e^{2a \sinh^{-1}(z)}\right) \right) + e^{(2a+3) \sinh^{-1}(z)} {}_2F_1\left(1 + \frac{1}{2a}, 1; 2 + \frac{1}{2a}; e^{2a \sinh^{-1}(z)}\right) \right) + (2a + 1) e^{2a \sinh^{-1}(z) + \sinh^{-1}(z)} {}_2F_1\left(1 - \frac{1}{2a}, 1; 2 - \frac{1}{2a}; e^{2a \sinh^{-1}(z)}\right) \right)$$

#### Involving $\cosh^{-1}$

01.22.21.0031.01

$$\int \coth(\cosh^{-1}(z)) dz = \sqrt{z - 1} \sqrt{z + 1}$$

01.22.21.0032.01

$$\int \coth(a \cosh^{-1}(z)) dz = \frac{1}{8a^2 - 2} \left( e^{-2 \cosh^{-1}(z)} \left( (2a + 1) e^{2a \cosh^{-1}(z) + \cosh^{-1}(z)} {}_2F_1\left(1 - \frac{1}{2a}, 1; 2 - \frac{1}{2a}; e^{2a \cosh^{-1}(z)}\right) - (2a - 1) \left( (2a + 1) e^{\cosh^{-1}(z)} \left( e^{2 \cosh^{-1}(z)} {}_2F_1\left(\frac{1}{2a}, 1; 1 + \frac{1}{2a}; e^{2a \cosh^{-1}(z)}\right) + {}_2F_1\left(-\frac{1}{2a}, 1; 1 - \frac{1}{2a}; e^{2a \cosh^{-1}(z)}\right) \right) + e^{(2a+3) \cosh^{-1}(z)} {}_2F_1\left(1 + \frac{1}{2a}, 1; 2 + \frac{1}{2a}; e^{2a \cosh^{-1}(z)}\right) \right) \right)$$

#### Involving $\tanh^{-1}$

01.22.21.0033.01

$$\int \coth(\tanh^{-1}(z)) dz = \log(z)$$

#### Involving $\coth^{-1}$

01.22.21.0034.01

$$\int \coth(\coth^{-1}(z)) dz = \frac{z^2}{2}$$

Involving  $\operatorname{csch}^{-1}$

01.22.21.0035.01

$$\int \coth(\operatorname{csch}^{-1}(z)) dz = \frac{\sqrt{1 + \frac{1}{z^2}} z \left( \sqrt{z^2 + 1} z + \sinh^{-1}(z) \right)}{2 \sqrt{z^2 + 1}}$$

Involving  $\operatorname{sech}^{-1}$

01.22.21.0036.01

$$\int \coth(\operatorname{sech}^{-1}(z)) dz = \sqrt{\frac{1}{z+1}} \sqrt{z+1} \tan^{-1} \left( \frac{z}{\sqrt{1-z^2}} \right)$$

### Involving trigonometric functions

Involving  $\sin$

**Involving  $\sin(bz)$**

01.22.21.0037.01

$$\int \sin(bz) \coth(cz) dz = \frac{1}{2(b^3 + 4c^2b)} \left( e^{-ibz} \left( (4c^2 + b^2) e^{2ibz} {}_2F_1 \left( \frac{ib}{2c}, 1; 1 + \frac{ib}{2c}; e^{2cz} \right) + (4c^2 + b^2) {}_2F_1 \left( -\frac{ib}{2c}, 1; 1 - \frac{ib}{2c}; e^{2cz} \right) + b \left( (b + 2ic) e^{2(c+ib)z} {}_2F_1 \left( 1 + \frac{ib}{2c}, 1; 2 + \frac{ib}{2c}; e^{2cz} \right) + (b - 2ic) e^{2cz} {}_2F_1 \left( 1 - \frac{ib}{2c}, 1; 2 - \frac{ib}{2c}; e^{2cz} \right) \right) \right)$$

Involving power of  $\sin$

**Involving  $\sin^m(bz)$**

01.22.21.0038.01

$$\int \sin^m(bz) \coth(cz) dz = \frac{2^{-m} (1 - m \bmod 2) \log(\sinh(cz)) \left(\frac{m}{2}\right)}{c} +$$

$$2^{-m} i^{1-m} \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^s \binom{m}{s} \left( \left( e^{ib(m-2s)z} \left( (2c + bi(m-2s)) {}_2F_1\left(\frac{ib(m-2s)}{2c}, 1; \frac{bi(m-2s)}{2c} + 1; e^{2cz}\right) + \right. \right. \right.$$

$$\left. \left. \left. b e^{2cz} i(m-2s) {}_2F_1\left(\frac{bi(m-2s)}{2c} + 1, 1; \frac{bi(m-2s)}{2c} + 2; e^{2cz}\right) \right) \right) / (b(m-2s)(2c + bi(m-2s))) - \right.$$

$$\left. \left( (-1)^m e^{-ib(m-2s)z} \left( (2c - ib(m-2s)) {}_2F_1\left(-\frac{ib(m-2s)}{2c}, 1; 1 - \frac{ib(m-2s)}{2c}; e^{2cz}\right) - ib e^{2cz} (m-2s) \right. \right. \right.$$

$$\left. \left. \left. {}_2F_1\left(1 - \frac{ib(m-2s)}{2c}, 1; 2 - \frac{ib(m-2s)}{2c}; e^{2cz}\right) \right) \right) / (b(m-2s)(2c - ib(m-2s))) \right) /; m \in \mathbb{N}^+$$

Involving cos

Involving cos(bz)

01.22.21.0039.01

$$\int \cos(bz) \coth(cz) dz =$$

$$\frac{1}{2(b^3 + 4c^2b)} \left( i e^{-ibz} \left( (4c^2 + b^2) e^{2ibz} {}_2F_1\left(\frac{ib}{2c}, 1; 1 + \frac{ib}{2c}; e^{2cz}\right) - (4c^2 + b^2) {}_2F_1\left(-\frac{ib}{2c}, 1; 1 - \frac{ib}{2c}; e^{2cz}\right) + \right. \right.$$

$$\left. \left. b \left( (b + 2ic) e^{2(c+ib)z} {}_2F_1\left(1 + \frac{ib}{2c}, 1; 2 + \frac{ib}{2c}; e^{2cz}\right) + (2c + ib) e^{2cz} i {}_2F_1\left(1 - \frac{ib}{2c}, 1; 2 - \frac{ib}{2c}; e^{2cz}\right) \right) \right) \right)$$

Involving power of cos

Involving cos<sup>m</sup>(bz)

01.22.21.0040.01

$$\int \cos^m(bz) \coth(cz) dz = \frac{2^{-m} \log(\sinh(cz)) \left(\frac{m}{2}\right) (1 - m \bmod 2)}{c} +$$

$$2^{-m} i \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{s} \left( \left( e^{ib(m-2s)z} \left( (2c + bi(m-2s)) {}_2F_1\left(\frac{ib(m-2s)}{2c}, 1; \frac{bi(m-2s)}{2c} + 1; e^{2cz}\right) + \right. \right. \right.$$

$$\left. \left. \left. b e^{2cz} i(m-2s) {}_2F_1\left(\frac{bi(m-2s)}{2c} + 1, 1; \frac{bi(m-2s)}{2c} + 2; e^{2cz}\right) \right) \right) / (b(m-2s)(2c + bi(m-2s))) - \right.$$

$$\left. \left( e^{-ib(m-2s)z} \left( (2c - ib(m-2s)) {}_2F_1\left(-\frac{ib(m-2s)}{2c}, 1; 1 - \frac{ib(m-2s)}{2c}; e^{2cz}\right) - ib e^{2cz} (m-2s) \right. \right. \right.$$

$$\left. \left. \left. {}_2F_1\left(1 - \frac{ib(m-2s)}{2c}, 1; 2 - \frac{ib(m-2s)}{2c}; e^{2cz}\right) \right) \right) / (b(m-2s)(2c - ib(m-2s))) \right) /; m \in \mathbb{N}^+$$



## Involving trigonometric and a power functions

### Involving sin and power

#### Involving $z^n \sin(a + bz) \coth(cz)$

01.22.21.0041.01

$$\int z^n \sin(a + bz) \coth(cz) dz = \frac{1}{2} i n! \left( e^{-ia - ibz} \sum_{j=0}^n \frac{(ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{ib}{2c}, \dots, -\frac{ib}{2c}, 1; 1 - \frac{ib}{2c}, \dots, 1 - \frac{ib}{2c}; e^{2cz} \right) - \right. \\ e^{-ia + (2c - ib)z} \sum_{j=0}^n \frac{(-1)^j (2c - ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c - ib}{2c}, \dots, \frac{2c - ib}{2c}, 1; \frac{2c - ib}{2c} + 1, \dots, \frac{2c - ib}{2c} + 1; e^{2cz} \right) + \\ \left. e^{ia + ibz} \sum_{j=0}^n \frac{(-1)^j (ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib}{2c}, \dots, \frac{ib}{2c}, 1; \frac{ib}{2c} + 1, \dots, \frac{ib}{2c} + 1; e^{2cz} \right) + e^{ia + (2c + ib)z} \sum_{j=0}^n \frac{(-1)^j (2c + ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c + ib}{2c}, \dots, \frac{2c + ib}{2c}, 1; \frac{2c + ib}{2c} + 1, \dots, \frac{2c + ib}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

01.22.21.0042.01

$$\int z^n \sin(bz) \coth(cz) dz = \frac{1}{2} i n! \left( e^{-ibz} \sum_{j=0}^n \frac{(ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{ib}{2c}, \dots, -\frac{ib}{2c}, 1; 1 - \frac{ib}{2c}, \dots, 1 - \frac{ib}{2c}; e^{2cz} \right) - \right. \\ e^{(2c - ib)z} \sum_{j=0}^n \frac{(-1)^j (2c - ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c - ib}{2c}, \dots, \frac{2c - ib}{2c}, 1; \frac{2c - ib}{2c} + 1, \dots, \frac{2c - ib}{2c} + 1; e^{2cz} \right) + \\ \left. e^{ibz} \sum_{j=0}^n \frac{(-1)^j (ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib}{2c}, \dots, \frac{ib}{2c}, 1; \frac{ib}{2c} + 1, \dots, \frac{ib}{2c} + 1; e^{2cz} \right) + e^{(2c + ib)z} \sum_{j=0}^n \frac{(-1)^j (2c + ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c + ib}{2c}, \dots, \frac{2c + ib}{2c}, 1; \frac{2c + ib}{2c} + 1, \dots, \frac{2c + ib}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

### Involving powers of sin and power

#### Involving $z^n \operatorname{sih}^m(bz) \coth(cz)$

01.22.21.0043.01

$$\int z^n \sin^m(bz) \coth(cz) dz =$$

$$\begin{aligned}
 & -n! \left( \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{ib(m-2k)z} \sum_{j=0}^n \frac{(-1)^j (ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib(m-2k)}{2c}, \dots, \frac{ib(m-2k)}{2c}, 1; \right. \right. \right. \\
 & \left. \left. \left. \frac{ib(m-2k)}{2c} + 1, \dots, \frac{ib(m-2k)}{2c} + 1; e^{2cz} \right) + (-1)^m \left( e^{-ib(m-2k)z} \sum_{j=0}^n \frac{(-1)^j (-ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \right. \\
 & \left. \left. \left. {}_{j+2}F_{j+1} \left( -\frac{ib(m-2k)}{2c}, \dots, -\frac{ib(m-2k)}{2c}, 1; 1 - \frac{ib(m-2k)}{2c}, \dots, 1 - \frac{ib(m-2k)}{2c}; e^{2cz} \right) + \right. \right. \right. \\
 & \left. \left. \left. e^{(2c-ib(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c-ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-ib(m-2k)}{2c}, \dots, \right. \right. \right. \\
 & \left. \left. \left. \frac{2c-ib(m-2k)}{2c}, 1; \frac{2c-ib(m-2k)}{2c} + 1, \dots, \frac{2c-ib(m-2k)}{2c} + 1; e^{2cz} \right) + \right. \right. \right. \\
 & \left. \left. \left. e^{(2c+ib(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+ib(m-2k)}{2c}, \dots, \right. \right. \right. \\
 & \left. \left. \left. \frac{2c+ib(m-2k)}{2c}, 1; \frac{2c+ib(m-2k)}{2c} + 1, \dots, \frac{2c+ib(m-2k)}{2c} + 1; e^{2cz} \right) \right) \right) (2i)^{-m} - \\
 & 2^{-m} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + 2 e^{2cz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \dots, 1; 2, \dots, 2; e^{2cz}) \right); n \in
 \end{aligned}$$

$\mathbb{N} \wedge$   
 $m \in$   
 $\mathbb{N}^+$

Involving cos and power

Involving  $z^n \cos(a + bz) \coth(cz)$

01.22.21.0044.01

$$\int z^n \cos(a + b z) \coth(c z) dz =$$

$$-\frac{1}{2} n! \left( e^{-ia - ibz} \sum_{j=0}^n \frac{(-1)^j (-ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{ib}{2c}, \dots, -\frac{ib}{2c}, 1; 1 - \frac{ib}{2c}, \dots, 1 - \frac{ib}{2c}; e^{2cz} \right) + e^{-ia + (2c - ib)z} \right.$$

$$\sum_{j=0}^n \frac{(-1)^j (2c - ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ib + 2c}{2c}, \dots, \frac{-ib + 2c}{2c}, 1; \frac{-ib + 2c}{2c} + 1, \dots, \frac{-ib + 2c}{2c} + 1; e^{2cz} \right) +$$

$$e^{ia + ibz} \sum_{j=0}^n \frac{(-1)^j (ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib}{2c}, \dots, \frac{ib}{2c}, 1; \frac{ib}{2c} + 1, \dots, \frac{ib}{2c} + 1; e^{2cz} \right) + e^{ia + (2c + ib)z}$$

$$\left. \sum_{j=0}^n \frac{(-1)^j (2c + ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c + ib}{2c}, \dots, \frac{2c + ib}{2c}, 1; \frac{2c + ib}{2c} + 1, \dots, \frac{2c + ib}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

01.22.21.0045.01

$$\int z^n \cos(b z) \coth(c z) dz = -\frac{1}{2} n! \left( e^{-ibz} \sum_{j=0}^n \frac{(-1)^j (-ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{ib}{2c}, \dots, -\frac{ib}{2c}, 1; 1 - \frac{ib}{2c}, \dots, 1 - \frac{ib}{2c}; e^{2cz} \right) +$$

$$e^{(2c - ib)z} \sum_{j=0}^n \frac{(-1)^j (2c - ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ib + 2c}{2c}, \dots, \frac{-ib + 2c}{2c}, 1; \frac{-ib + 2c}{2c} + 1, \dots, \frac{-ib + 2c}{2c} + 1; e^{2cz} \right) +$$

$$e^{ibz} \sum_{j=0}^n \frac{(-1)^j (ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib}{2c}, \dots, \frac{ib}{2c}, 1; \frac{ib}{2c} + 1, \dots, \frac{ib}{2c} + 1; e^{2cz} \right) + e^{(2c + ib)z}$$

$$\left. \sum_{j=0}^n \frac{(-1)^j (2c + ib)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c + ib}{2c}, \dots, \frac{2c + ib}{2c}, 1; \frac{2c + ib}{2c} + 1, \dots, \frac{2c + ib}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

Involving powers of cos and power

Involving  $z^n \cos^m(bz) \coth(cz)$

01.22.21.0046.01

$$\int z^n \cos^m(bz) \coth(cz) dz =$$

$$-2^{-m} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + 2 e^{2cz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, 2; 2, \dots, 2; e^{2cz}) \right) -$$

$$2^{-m} n! \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{-ib(m-2k)z} \sum_{j=0}^n \frac{(-1)^j (-ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{ib(m-2k)}{2c}, \dots, -\frac{ib(m-2k)}{2c}, 1; 1 - \frac{ib(m-2k)}{2c}, \dots, 1 - \frac{ib(m-2k)}{2c}; e^{2cz} \right) + \right.$$

$$e^{ib(m-2k)z} \sum_{j=0}^n \frac{(-1)^j (ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib(m-2k)}{2c}, \dots, \frac{ib(m-2k)}{2c}, 1; \frac{ib(m-2k)}{2c} + 1, \dots, \frac{ib(m-2k)}{2c} + 1; e^{2cz} \right) +$$

$$e^{(2c-ib(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c-ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-ib(m-2k)}{2c}, \dots, \frac{2c-ib(m-2k)}{2c}, 1; \frac{2c-ib(m-2k)}{2c} + 1, \dots, \frac{2c-ib(m-2k)}{2c} + 1; e^{2cz} \right) +$$

$$e^{(2c+ib(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+ib(m-2k)}{2c}, \dots, \frac{2c+ib(m-2k)}{2c}, 1; \frac{2c+ib(m-2k)}{2c} + 1, \dots, \frac{2c+ib(m-2k)}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+$$

### Involving trigonometric and exponential functions

Involving sin and exp

### Involving $e^{pz} \sin(bz)$

01.22.21.0047.01

$$\int e^{pz} \sin(bz) \coth(cz) dz =$$

$$-\frac{1}{2} i \left( \frac{1}{(-ib+p)(2c-ib+p)} \left( e^{(-ib+p)z} \left( (2c-ib+p) {}_2F_1 \left( \frac{-ib+p}{2c}, 1; \frac{-ib+p}{2c} + 1; e^{2cz} \right) + e^{2cz} \right. \right. \right.$$

$$\left. \left. (-ib+p) {}_2F_1 \left( \frac{-ib+p}{2c} + 1, 1; \frac{-ib+p}{2c} + 2; e^{2cz} \right) \right) \right) - \frac{1}{(ib+p)(2c+ib+p)}$$

$$\left( e^{(ib+p)z} \left( (2c+ib+p) {}_2F_1 \left( \frac{ib+p}{2c}, 1; \frac{ib+p}{2c} + 1; e^{2cz} \right) + e^{2cz} (ib+p) {}_2F_1 \left( \frac{ib+p}{2c} + 1, 1; \frac{ib+p}{2c} + 2; e^{2cz} \right) \right) \right)$$

01.22.21.0048.01

$$\int e^{ibz} \sin(bz) \coth(cz) dz = \frac{1}{4bc(c+ib)} \left( bc e^{2(c+ib)z} {}_2F_1\left(1 + \frac{ib}{c}, 1; 2 + \frac{ib}{c}; e^{2cz}\right) + (c+ib) \left( c e^{2ibz} {}_2F_1\left(\frac{ib}{c}, 1; 1 + \frac{ib}{c}; e^{2cz}\right) + 2bi \log(\sinh(cz)) \right) \right)$$

01.22.21.0049.01

$$\int e^{-ibz} \sin(bz) \coth(cz) dz = -\frac{1}{4b(b+ic)c} \left( i e^{-2ibz} \left( bc e^{2cz} {}_2F_1\left(1 - \frac{ib}{c}, 1; 2 - \frac{ib}{c}; e^{2cz}\right) + (b+ic) \left( c i {}_2F_1\left(-\frac{ib}{c}, 1; 1 - \frac{ib}{c}; e^{2cz}\right) + 2b e^{2ibz} \log(\sinh(cz)) \right) \right) \right)$$

01.22.21.0050.01

$$\int e^{i(b-2c)z} \sin(az) \coth(cz) dz = \frac{1}{2} \left( \frac{e^{i(a+b)z} {}_2F_1\left(\frac{i(a+b)}{2c}, 1; 1 + \frac{i(a+b)}{2c}; e^{2cz}\right)}{a+b} + \frac{e^{i(a+b+2ic)z} {}_2F_1\left(\frac{i(a+b+2ic)}{2c}, 1; \frac{i(a+b)}{2c}; e^{2cz}\right)}{a+b+2ic} + \frac{e^{-i(a-b)z} {}_2F_1\left(-\frac{i(a-b)}{2c}, 1; 1 - \frac{i(a-b)}{2c}; e^{2cz}\right)}{a-b} + \frac{e^{-i(a-b-2ic)z} {}_2F_1\left(-\frac{i(a-b-2ic)}{2c}, 1; -\frac{i(a-b)}{2c}; e^{2cz}\right)}{a-b-2ic} \right)$$

01.22.21.0051.01

$$\int e^{-(2c+ib)z} \sin(az) \coth(cz) dz = \frac{1}{2} \left( \frac{e^{i(a-b)z} {}_2F_1\left(\frac{i(a-b)}{2c}, 1; 1 + \frac{i(a-b)}{2c}; e^{2cz}\right)}{a-b} + \frac{e^{i(a-b+2ic)z} {}_2F_1\left(\frac{i(a-b+2ic)}{2c}, 1; \frac{i(a-b)}{2c}; e^{2cz}\right)}{a-b+2ic} + \frac{e^{-i(a+b)z} {}_2F_1\left(-\frac{i(a+b)}{2c}, 1; 1 - \frac{i(a+b)}{2c}; e^{2cz}\right)}{a+b} + \frac{e^{-i(a+b-2ic)z} {}_2F_1\left(-\frac{i(a+b-2ic)}{2c}, 1; -\frac{i(a+b)}{2c}; e^{2cz}\right)}{a+b-2ic} \right)$$

Involving powers of sin and exp

Involving  $e^{pz} \sin^m(bz)$

01.22.21.0052.01

$$\int e^{pz} \sin^m(bz) \coth(cz) dz = \frac{1}{p(2c+p)} - \left(2^{-m} e^{pz} (-1)^{\left(\frac{m}{2}\right)} \left( (2c+p) {}_2F_1\left(\frac{p}{2c}, 1; \frac{p}{2c} + 1; e^{2cz}\right) + e^{2cz} p {}_2F_1\left(\frac{p}{2c} + 1, 1; \frac{p}{2c} + 2; e^{2cz}\right) \right) (1 - m \bmod 2) \right) - 2^{-m} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left( \left( e^{\frac{i\pi m}{2} + (p-ib(m-2k))z} \left( (2c-ib(m-2k)+p) {}_2F_1\left(\frac{p-ib(m-2k)}{2c}, 1; \frac{p-ib(m-2k)}{2c} + 1; e^{2cz}\right) + e^{2cz} (p-ib(m-2k)) {}_2F_1\left(\frac{p-ib(m-2k)}{2c} + 1, 1; \frac{p-ib(m-2k)}{2c} + 2; e^{2cz}\right) \right) \right) / \left( (p-ib(m-2k))(2c-ib(m-2k)+p) \right) + \left( e^{(bi(m-2k)+p)z - \frac{i\pi m}{2}} \left( (2c+bi(m-2k)+p) {}_2F_1\left(\frac{bi(m-2k)+p}{2c}, 1; \frac{bi(m-2k)+p}{2c} + 1; e^{2cz}\right) + e^{2cz} (bi(m-2k)+p) {}_2F_1\left(\frac{bi(m-2k)+p}{2c} + 1, 1; \frac{bi(m-2k)+p}{2c} + 2; e^{2cz}\right) \right) \right) / \left( (bi(m-2k)+p)(2c+bi(m-2k)+p) \right) \right) /; m \in \mathbb{N}^+$$

Involving cos and exp

Involving  $e^{pz} \cos(bz)$

01.22.21.0053.01

$$\int e^{pz} \cos(bz) \coth(cz) dz = \frac{1}{2} \left( \left( e^{(-ib+p)z} \left( (b+i(2c+p)) {}_2F_1\left(\frac{-ib+p}{2c}, 1; \frac{-ib+p}{2c} + 1; e^{2cz}\right) + e^{2cz} (b+ip) {}_2F_1\left(\frac{-ib+p}{2c} + 1, 1; \frac{-ib+p}{2c} + 2; e^{2cz}\right) \right) \right) / \left( (-2c+ib-p)(b+ip) \right) - \frac{1}{(ib+p)(2c+ib+p)} \right) + \left( e^{(ib+p)z} \left( (2c+ib+p) {}_2F_1\left(\frac{ib+p}{2c}, 1; \frac{ib+p}{2c} + 1; e^{2cz}\right) + e^{2cz} (ib+p) {}_2F_1\left(\frac{ib+p}{2c} + 1, 1; \frac{ib+p}{2c} + 2; e^{2cz}\right) \right) \right) / \left( (2c+ib+p)(2c+ib+p) \right) \right)$$

01.22.21.0054.01

$$\int e^{ibz} \cos(bz) \coth(cz) dz = \frac{1}{4bc(c+ib)} \left( (c+ib) \left( c e^{2ibz} {}_2F_1\left(\frac{ib}{c}, 1; 1 + \frac{ib}{c}; e^{2cz}\right) + 2b \log(\sinh(cz)) \right) - bc e^{2(c+ib)z} {}_2F_1\left(1 + \frac{ib}{c}, 1; 2 + \frac{ib}{c}; e^{2cz}\right) \right)$$

01.22.21.0055.01

$$\int e^{-ibz} \cos(bz) \coth(cz) dz = \frac{1}{4bc(c-ib)} \left( e^{-2ibz} \left( (c-ib) \left( 2b e^{2ibz} \log(\sinh(cz)) - ic {}_2F_1\left(-\frac{ib}{c}, 1; 1 - \frac{ib}{c}; e^{2cz}\right) \right) - bc e^{2cz} {}_2F_1\left(1 - \frac{ib}{c}, 1; 2 - \frac{ib}{c}; e^{2cz}\right) \right) \right)$$

01.22.21.0056.01

$$\int e^{i(b-2c)z} \cos(az) \coth(cz) dz = -\frac{1}{2} i \left( -\frac{e^{i(a+b)z} {}_2F_1\left(\frac{i(a+b)}{2c}, 1; 1 + \frac{i(a+b)}{2c}; e^{2cz}\right)}{a+b} + \frac{e^{-i(a-b)z} {}_2F_1\left(-\frac{i(a-b)}{2c}, 1; 1 - \frac{i(a-b)}{2c}; e^{2cz}\right)}{a-b} + \frac{e^{-i(a-b-2ic)z} {}_2F_1\left(-\frac{i(a-b-2ic)}{2c}, 1; -\frac{i(a-b)}{2c}; e^{2cz}\right)}{a-b-2ic} - \frac{e^{i(a+b+2ic)z} {}_2F_1\left(\frac{i(a+b+2ic)}{2c}, 1; \frac{i(a+b)}{2c}; e^{2cz}\right)}{a+b+2ic} \right)$$

01.22.21.0057.01

$$\int e^{-(2c+ib)z} \cos(az) \coth(cz) dz = \frac{1}{2} i \left( \frac{e^{i(a-b)z} {}_2F_1\left(\frac{i(a-b)}{2c}, 1; 1 + \frac{i(a-b)}{2c}; e^{2cz}\right)}{a-b} + \frac{e^{i(a-b+2ic)z} {}_2F_1\left(\frac{i(a-b+2ic)}{2c}, 1; \frac{i(a-b)}{2c}; e^{2cz}\right)}{a-b+2ic} - \frac{e^{-i(a+b-2ic)z} {}_2F_1\left(-\frac{i(a+b-2ic)}{2c}, 1; -\frac{i(a+b)}{2c}; e^{2cz}\right)}{a+b-2ic} - \frac{e^{-i(a+b)z} {}_2F_1\left(-\frac{i(a+b)}{2c}, 1; 1 - \frac{i(a+b)}{2c}; e^{2cz}\right)}{a+b} \right)$$

Involving powers of cos and exp

### Involving $e^{pz} \cos^m(bz)$

01.22.21.0058.01

$$\int e^{pz} \cos^m(bz) \coth(cz) dz = \frac{1}{p(2c+p)} \left( 2^{-m} e^{pz} (-1)^{\left\lfloor \frac{m}{2} \right\rfloor} \left( (2c+p) {}_2F_1\left(\frac{p}{2c}, 1; \frac{p}{2c} + 1; e^{2cz}\right) + e^{2cz} p {}_2F_1\left(\frac{p}{2c} + 1, 1; \frac{p}{2c} + 2; e^{2cz}\right) \right) (1 - m \bmod 2) \right) - 2^{-m} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left( \left( e^{(p-ib(m-2k))z} \left( (2c-ib(m-2k)+p) {}_2F_1\left(\frac{p-ib(m-2k)}{2c}, 1; \frac{p-ib(m-2k)}{2c} + 1; e^{2cz}\right) + e^{2cz} (p-ib(m-2k)) {}_2F_1\left(\frac{p-ib(m-2k)}{2c} + 1, 1; \frac{p-ib(m-2k)}{2c} + 2; e^{2cz}\right) \right) \right) / \left( (p-ib(m-2k))(2c-ib(m-2k)+p) \right) + \left( e^{(bi(m-2k)+p)z} \left( (2c+bi(m-2k)+p) {}_2F_1\left(\frac{bi(m-2k)+p}{2c}, 1; \frac{bi(m-2k)+p}{2c} + 1; e^{2cz}\right) + e^{2cz} (bi(m-2k)+p) {}_2F_1\left(\frac{bi(m-2k)+p}{2c}, 1, 1; \frac{bi(m-2k)+p}{2c} + 2; e^{2cz}\right) \right) \right) / \left( (bi(m-2k)+p)(2c+bi(m-2k)+p) \right) \right); m \in \mathbb{N}^+$$

Involving trigonometric, exponential and a power functions

Involving sin, exp and power

### Involving $z^n e^{pz} \sin(a+bz) \coth(cz)$

01.22.21.0059.01

$$\int z^n e^{p z} \sin(a + b z) \coth(c z) dz = -\frac{1}{2} i n!$$

$$\left( e^{-i a + (-i b + p) z} \sum_{j=0}^n \frac{(-1)^j (-i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + p}{2c}, \dots, \frac{-i b + p}{2c}, 1; \frac{-i b + p}{2c} + 1, \dots, \frac{-i b + p}{2c} + 1; e^{2c z} \right) + \right.$$

$$e^{-i a + (2c - i b + p) z} \sum_{j=0}^n \frac{(-1)^j (2c - i b + p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{2c - i b + p}{2c}, \dots, \frac{2c - i b + p}{2c}, 1; \frac{2c - i b + p}{2c} + 1, \dots, \frac{2c - i b + p}{2c} + 1; e^{2c z} \right) -$$

$$e^{i a + (i b + p) z} \sum_{j=0}^n \frac{(-1)^j (i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p}{2c}, \dots, \frac{i b + p}{2c}, 1; \frac{i b + p}{2c} + 1, \dots, \frac{i b + p}{2c} + 1; e^{2c z} \right) -$$

$$e^{i a + (2c + i b + p) z} \sum_{j=0}^n \frac{(-1)^j (2c + i b + p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{2c + i b + p}{2c}, \dots, \frac{2c + i b + p}{2c}, 1; \frac{2c + i b + p}{2c} + 1, \dots, \frac{2c + i b + p}{2c} + 1; e^{2c z} \right) \right) /; n \in \mathbb{N}$$

01.22.21.0060.01

$$\int z^n e^{p z} \sin(b z) \coth(c z) dz =$$

$$-\frac{1}{2} i n! \left( e^{(-i b + p) z} \sum_{j=0}^n \frac{(-1)^j (-i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p - i b}{2c}, \dots, \frac{p - i b}{2c}, 1; \frac{p - i b}{2c} + 1, \dots, \frac{p - i b}{2c} + 1; e^{2c z} \right) - \right.$$

$$e^{(i b + p) z} \sum_{j=0}^n \frac{(-1)^j (i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p}{2c}, \dots, \frac{i b + p}{2c}, 1; \frac{i b + p}{2c} + 1, \dots, \frac{i b + p}{2c} + 1; e^{2c z} \right) +$$

$$e^{(2c - i b + p) z} \sum_{j=0}^n \frac{(-1)^j (2c - i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + 2c + p}{2c}, \dots, \frac{-i b + 2c + p}{2c}, 1; \right.$$

$$\left. \frac{-i b + 2c + p}{2c} + 1, \dots, \frac{-i b + 2c + p}{2c} + 1; e^{2c z} \right) - e^{(2c + i b + p) z} \sum_{j=0}^n \frac{(-1)^j (2c + i b + p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{i b + 2c + p}{2c}, \dots, \frac{i b + 2c + p}{2c}, 1; \frac{i b + 2c + p}{2c} + 1, \dots, \frac{i b + 2c + p}{2c} + 1; e^{2c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of sin, exp and power

Involving  $z^n e^{p z} \sin^m(b z) \coth(c z)$



01.22.21.0061.01

$$\int z^n e^{p z} \sin^m(b z) \coth(c z) dz =$$

$$\begin{aligned}
 & -n! \left( \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( (-1)^m \left( e^{(p-i b(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (p-i b(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-i b(m-2k)}{2c}, \dots, \right. \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \frac{p-i b(m-2k)}{2c}, 1; \frac{p-i b(m-2k)}{2c} + 1, \dots, \frac{p-i b(m-2k)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \qquad \qquad \qquad \left. e^{(2c-i b(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (2c-i b(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-i b(m-2k)+p}{2c}, \dots, \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \frac{2c-i b(m-2k)+p}{2c}, 1; \frac{2c-i b(m-2k)+p}{2c} + 1, \dots, \frac{2c-i b(m-2k)+p}{2c} + 1; e^{2cz} \right) \right) \Bigg) (2i)^{-m} - \\
 & \qquad \qquad \qquad e^{(b i(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (b i(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b(m-2k)+p}{2c}, \dots, \frac{i b(m-2k)+p}{2c}, \right. \\
 & \qquad \qquad \qquad \left. 1; \frac{i b(m-2k)+p}{2c} + 1, \dots, \frac{i b(m-2k)+p}{2c} + 1; e^{2cz} \right) + \\
 & \qquad \qquad \qquad e^{(2c+b i(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+b i(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+i b(m-2k)+p}{2c}, \dots, \right. \\
 & \qquad \qquad \qquad \left. \frac{2c+i b(m-2k)+p}{2c}, 1; \frac{2c+i b(m-2k)+p}{2c} + 1, \dots, \frac{2c+i b(m-2k)+p}{2c} + 1; e^{2cz} \right) \Bigg) (2i)^{-m} - \\
 & 2^{-m} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \left( e^{p z} \sum_{j=0}^n \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p}{2c}, \dots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \dots, \frac{p}{2c} + 1; e^{2cz} \right) + \right. \\
 & \qquad \qquad \qquad \left. e^{(2c+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p}{2c}, \dots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c} + 1, \dots, \frac{2c+p}{2c} + 1; e^{2cz} \right) \right) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+
 \end{aligned}$$

Involving cos, exp and power

**Involving  $z^n e^{p z} \cos(a + b z) \coth(c z)$**

01.22.21.0062.01

$$\int z^n e^{p z} \cos(a + b z) \coth(c z) dz =$$

$$-\frac{1}{2} n! \left( e^{-i a + (-i b + p) z} \sum_{j=0}^n \frac{(-1)^j (-i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-i b}{2 c}, \dots, \frac{p-i b}{2 c}, 1; \frac{p-i b}{2 c} + 1, \dots, \frac{p-i b}{2 c} + 1; e^{2 c z} \right) + \right.$$

$$e^{i a + (i b + p) z} \sum_{j=0}^n \frac{(-1)^j (i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p}{2 c}, \dots, \frac{i b + p}{2 c}, 1; \frac{i b + p}{2 c} + 1, \dots, \frac{i b + p}{2 c} + 1; e^{2 c z} \right) +$$

$$e^{-i a + (2 c - i b + p) z} \sum_{j=0}^n \frac{(-1)^j (2 c - i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + 2 c + p}{2 c}, \dots, \frac{-i b + 2 c + p}{2 c}, 1; \right.$$

$$\left. \frac{-i b + 2 c + p}{2 c} + 1, \dots, \frac{-i b + 2 c + p}{2 c} + 1; e^{2 c z} \right) + e^{i a + (2 c + i b + p) z} \sum_{j=0}^n \frac{(-1)^j (2 c + i b + p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{i b + 2 c + p}{2 c}, \dots, \frac{i b + 2 c + p}{2 c}, 1; \frac{i b + 2 c + p}{2 c} + 1, \dots, \frac{i b + 2 c + p}{2 c} + 1; e^{2 c z} \right) \right) / ; n \in \mathbb{N}$$

01.22.21.0063.01

$$\int z^n e^{p z} \cos(b z) \coth(c z) dz =$$

$$-\frac{1}{2} n! \left( e^{(-i b + p) z} \sum_{j=0}^n \frac{(-1)^j (-i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-i b}{2 c}, \dots, \frac{p-i b}{2 c}, 1; \frac{p-i b}{2 c} + 1, \dots, \frac{p-i b}{2 c} + 1; e^{2 c z} \right) + \right.$$

$$e^{(i b + p) z} \sum_{j=0}^n \frac{(-1)^j (i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p}{2 c}, \dots, \frac{i b + p}{2 c}, 1; \frac{i b + p}{2 c} + 1, \dots, \frac{i b + p}{2 c} + 1; e^{2 c z} \right) +$$

$$e^{(2 c - i b + p) z} \sum_{j=0}^n \frac{(-1)^j (2 c - i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + 2 c + p}{2 c}, \dots, \frac{-i b + 2 c + p}{2 c}, 1; \right.$$

$$\left. \frac{-i b + 2 c + p}{2 c} + 1, \dots, \frac{-i b + 2 c + p}{2 c} + 1; e^{2 c z} \right) + e^{(2 c + i b + p) z} \sum_{j=0}^n \frac{(-1)^j (2 c + i b + p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{i b + 2 c + p}{2 c}, \dots, \frac{i b + 2 c + p}{2 c}, 1; \frac{i b + 2 c + p}{2 c} + 1, \dots, \frac{i b + 2 c + p}{2 c} + 1; e^{2 c z} \right) \right) / ; n \in \mathbb{N}$$

Involving powers of cos, exp and power

Involving  $z^n e^{p z} \cos^m(b z) \coth(c z)$

01.22.21.0064.01

$$\int z^n e^{p z} \cos^m(b z) \coth(c z) dz =$$

$$-2^{-m} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \left( e^{p z} \sum_{j=0}^n \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p}{2c}, \dots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \dots, \frac{p}{2c} + 1; e^{2cz} \right) + \right.$$

$$e^{(2c+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p}{2c} + 1, \dots, \frac{p}{2c} + 1, 1; \frac{p}{2c} + 2, \dots, \frac{p}{2c} + 2; e^{2cz} \right) \left. - \right.$$

$$2^{-m} n! \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{s} \left( e^{(p+2ibs-ibm)z} \sum_{j=0}^n \frac{(-1)^j (p+2ibs-ibm)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right.$$

$$\left( \frac{p+2ibs-ibm}{2c}, \dots, \frac{p+2ibs-ibm}{2c}, 1; \frac{p+2ibs-ibm}{2c} + 1, \dots, \frac{p+2ibs-ibm}{2c} + 1; e^{2cz} \right) +$$

$$e^{(p-2ibs+ibm)z} \sum_{j=0}^n \frac{(-1)^j (p-2ibs+ibm)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-2ibs+ibm}{2c}, \dots, \frac{p-2ibs+ibm}{2c}, \right.$$

$$1; \frac{p-2ibs+ibm}{2c} + 1, \dots, \frac{p-2ibs+ibm}{2c} + 1; e^{2cz} \left. \right) + e^{(2c+p+2ibs-ibm)z}$$

$$\sum_{j=0}^n \frac{(-1)^j (2c+p+2ibs-ibm)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p+2ibs-ibm}{2c}, \dots, \frac{2c+p+2ibs-ibm}{2c}, \right.$$

$$1; \frac{2c+p+2ibs-ibm}{2c} + 1, \dots, \frac{2c+p+2ibs-ibm}{2c} + 1; e^{2cz} \left. \right) + e^{(2c+p-2ibs+ibm)z}$$

$$\sum_{j=0}^n \frac{(-1)^j (2c+p-2ibs+ibm)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p-2ibs+ibm}{2c}, \dots, \frac{2c+p-2ibs+ibm}{2c}, \right.$$

$$1; \frac{2c+p-2ibs+ibm}{2c} + 1, \dots, \frac{2c+p-2ibs+ibm}{2c} + 1; e^{2cz} \left. \right) \Big/; n \in \mathbb{N} \wedge m \in \mathbb{N}^+$$

### Involving hyperbolic functions

#### Involving sinh

#### Involving sinh(b z)

01.22.21.0065.01

$$\int \sinh(b z) \coth(c z) dz =$$

$$\frac{1}{2(4c^2 b - b^3)} \left( e^{-(2c+b)z} \left( -(4c^2 - b^2) e^{2(c+b)z} {}_2F_1 \left( \frac{b}{2c}, 1; \frac{b}{2c} + 1; e^{2cz} \right) - (4c^2 - b^2) e^{2cz} {}_2F_1 \left( -\frac{b}{2c}, 1; 1 - \frac{b}{2c}; e^{2cz} \right) + \right.$$

$$b \left( (2c+b) e^{4cz} {}_2F_1 \left( 1 - \frac{b}{2c}, 1; 2 - \frac{b}{2c}; e^{2cz} \right) - (2c-b) e^{2(2c+b)z} {}_2F_1 \left( \frac{b}{2c} + 1, 1; \frac{b}{2c} + 2; e^{2cz} \right) \right) \left. \right)$$

01.22.21.0066.01

$$\int \sinh(c z) \coth(c z) dz = \frac{\sinh(c z)}{c}$$

01.22.21.0067.01

$$\int \sinh(c z) \coth(2 c z) dz = \frac{\sinh(c z) - \tan^{-1}\left(\tanh\left(\frac{c z}{2}\right)\right)}{c}$$

01.22.21.0068.01

$$\int \sinh(c z) \coth(3 c z) dz = \frac{3 \sinh(c z) - \sqrt{3} \tan^{-1}\left(\frac{2 \sinh(c z)}{\sqrt{3}}\right)}{3 c}$$

01.22.21.0069.01

$$\int \sinh(c z) \coth(4 c z) dz = \frac{-(1-i) \sqrt[4]{-1} \tan^{-1}\left(\frac{i+\tanh\left(\frac{c z}{2}\right)}{\sqrt{2}}\right) - 2 \tan^{-1}\left(\tanh\left(\frac{c z}{2}\right)\right) + \sqrt{2} i \tanh^{-1}\left(\frac{i \tanh\left(\frac{c z}{2}\right)+1}{\sqrt{2}}\right) + 4 \sinh(c z)}{4 c}$$

01.22.21.0070.01

$$\int \sinh(2 c z) \coth(c z) dz = z + \frac{\sinh(2 c z)}{2 c}$$

01.22.21.0071.01

$$\int \sinh(3 c z) \coth(c z) dz = \frac{6 \sinh(c z) + \sinh(3 c z)}{3 c}$$

01.22.21.0072.01

$$\int \sinh(4 c z) \coth(c z) dz = \frac{4 c z + 4 \sinh(2 c z) + \sinh(4 c z)}{4 c}$$

### Involving power of sinh

### Involving $\sinh^m(b z)$

01.22.21.0073.01

$$\int \sinh^m(b z) \coth(c z) dz = \frac{(1-m \bmod 2) \log(\sinh(c z))}{c} \binom{i}{2}^m \binom{m}{\frac{m}{2}} -$$

$$2^{-m} \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^s \binom{m}{s} \left( \left( e^{b(m-2s)z} \left( (2c+b(m-2s)) {}_2F_1\left(\frac{b(m-2s)}{2c}, 1; \frac{b(m-2s)}{2c} + 1; e^{2cz}\right) + \right. \right. \right.$$

$$\left. \left. \left. b e^{2cz} (m-2s) {}_2F_1\left(\frac{b(m-2s)}{2c} + 1, 1; \frac{b(m-2s)}{2c} + 2; e^{2cz}\right) \right) \right) / (b(m-2s)(2c+b(m-2s))) - \right.$$

$$\left. \left( (-1)^m e^{-b(m-2s)z} \left( (2c-b(m-2s)) {}_2F_1\left(-\frac{b(m-2s)}{2c}, 1; 1 - \frac{b(m-2s)}{2c}; e^{2cz}\right) - b e^{2cz} (m-2s) \right. \right. \right.$$

$$\left. \left. \left. {}_2F_1\left(1 - \frac{b(m-2s)}{2c}, 1; 2 - \frac{b(m-2s)}{2c}; e^{2cz}\right) \right) \right) / (b(m-2s)(2c-b(m-2s))) \right) /; m \in \mathbb{N}^+$$

01.22.21.0074.01

$$\int \sinh^\mu(c z) \coth(c z) dz = \frac{\sinh^\mu(c z)}{c \mu}$$

01.22.21.0075.01

$$\int \sinh^2(c z) \coth(c z) dz = \frac{\cosh^2(c z)}{2 c}$$

01.22.21.0076.01

$$\int \sinh^3(c z) \coth(c z) dz = \frac{\sinh^3(c z)}{3 c}$$

01.22.21.0077.01

$$\int \sinh^2(c z) \coth(2 c z) dz = \frac{\cosh(2 c z) - 2 \log(\cosh(c z))}{4 c}$$

01.22.21.0078.01

$$\int \sinh^3(c z) \coth(3 c z) dz = \frac{3 \sqrt{3} \tan^{-1}\left(\frac{2 \sinh(c z)}{\sqrt{3}}\right) - 9 \sinh(c z) + \sinh(3 c z)}{12 c}$$

01.22.21.0079.01

$$\int \sinh^3(c z) \coth(4 c z) dz = \frac{1}{24 c} \left( \sqrt[4]{-1} (3 - 3 i) \tan^{-1}\left(\frac{i + \tanh\left(\frac{c z}{2}\right)}{\sqrt{2}}\right) + \right. \\ \left. 12 \tan^{-1}\left(\tanh\left(\frac{c z}{2}\right)\right) + (-1)^{3/4} (3 - 3 i) \tanh^{-1}\left(\frac{-i \tanh\left(\frac{c z}{2}\right) - 1}{\sqrt{2}}\right) - 18 \sinh(c z) + 2 \sinh(3 c z) \right)$$

01.22.21.0080.01

$$\int \sinh^{\frac{1}{2}}(c z) \coth(c z) dz = \frac{2 \sinh^{\frac{1}{2}}(c z)}{c}$$

01.22.21.0081.01

$$\int \frac{\coth(c z)}{\sinh^{\frac{1}{2}}(c z)} dz = -\frac{2}{c \sinh^{\frac{1}{2}}(c z)}$$

01.22.21.0082.01

$$\int \frac{\coth(c z)}{\sqrt{\sinh^3(2 c z)}} dz = -\frac{\sinh(2 c z)}{3 c \sqrt{\sinh^3(2 c z)}} \left( \coth(c z) + i F\left(\frac{\pi}{4} - i c z \mid 2\right) \sqrt{i \sinh(2 c z)} \right)$$

Involving rational functions of sinh

**Involving  $(a + b \sinh(c z))^{-n}$**

01.22.21.0083.01

$$\int \frac{\coth(c z)}{a + b \sinh(c z)} dz = \frac{\log(\sinh(c z)) - \log(a + b \sinh(c z))}{a c}$$

01.22.21.0084.01

$$\int \frac{A + B \coth(c z)}{a + b \sinh(c z)} dz = \frac{1}{c} \left( \frac{2 A}{\sqrt{-a^2 - b^2}} \tan^{-1} \left( \frac{b - a \tanh\left(\frac{c z}{2}\right)}{\sqrt{-a^2 - b^2}} \right) + \frac{B (\log(\sinh(c z)) - \log(a + b \sinh(c z)))}{a} \right)$$

01.22.21.0085.01

$$\int \frac{\coth(c z)}{a + b \sinh(2 c z)} dz = \frac{1}{2 a c} \left( -\frac{2 b}{\sqrt{-a^2 - b^2}} \tan^{-1} \left( \frac{b - a \tanh(c z)}{\sqrt{-a^2 - b^2}} \right) + 2 \log(\sinh(c z)) - \log(a + b \sinh(2 c z)) \right)$$

01.22.21.0086.01

$$\int \frac{(A + B \sinh(c z)) \coth(c z)}{a + b \sinh(c z)} dz = \frac{A b \log(\sinh(c z)) + (A B - A b) \log(a + b \sinh(c z))}{a b c}$$

01.22.21.0087.01

$$\int \frac{\coth(c z)}{(a + b \sinh(c z))^2} dz = \frac{\frac{a}{a + b \sinh(c z)} + \log(\sinh(c z)) - \log(a + b \sinh(c z))}{a^2 c}$$

### Involving algebraic functions of sinh

#### Involving $(a + b \sinh(c z))^\beta$

01.22.21.0088.01

$$\int (a + b \sinh(c z))^\beta \coth(c z) dz = \frac{(a + b \sinh(c z))^\beta}{c \beta} \left( \frac{a \operatorname{csch}(c z)}{b} + 1 \right)^{-\beta} {}_2F_1 \left( -\beta, -\beta; 1 - \beta; -\frac{a \operatorname{csch}(c z)}{b} \right)$$

01.22.21.0089.01

$$\int \sqrt{a + b \sinh(c z)} \coth(c z) dz = \frac{2 \sqrt{a + b \sinh(c z)}}{c} \left( 1 - \frac{\sqrt{a} \operatorname{csch}^{\frac{1}{2}}(c z)}{\sqrt{b} \sqrt{\frac{a \operatorname{csch}(c z)}{b} + 1}} \sinh^{-1} \left( \frac{\sqrt{a} \operatorname{csch}^{\frac{1}{2}}(c z)}{\sqrt{b}} \right) \right)$$

01.22.21.0090.01

$$\int \sqrt{i \sinh(c z) a + a} \coth(c z) dz = \frac{1}{2 c (\cosh\left(\frac{c z}{2}\right) + i \sinh\left(\frac{c z}{2}\right))} \left( \left( 2 i \tan^{-1} \left( \coth\left(\frac{c z}{4}\right) \right) - 2 i \tan^{-1} \left( \tanh\left(\frac{c z}{4}\right) \right) + 4 \cosh\left(\frac{c z}{2}\right) - \log \left( \cosh^2\left(\frac{c z}{4}\right) \cosh\left(\frac{c z}{2}\right) \right) + \log \left( \cosh\left(\frac{c z}{2}\right) \sinh^2\left(\frac{c z}{4}\right) \right) + 4 i \sinh\left(\frac{c z}{2}\right) \right) \sqrt{i \sinh(c z) a + a} \right)$$

01.22.21.0091.01

$$\int \sqrt{a - i a \sinh(c z)} \coth(c z) dz = \frac{1}{2 c (\cosh\left(\frac{c z}{2}\right) - i \sinh\left(\frac{c z}{2}\right))} \left( \left( -2 i \tan^{-1} \left( \coth\left(\frac{c z}{4}\right) \right) + 2 i \tan^{-1} \left( \tanh\left(\frac{c z}{4}\right) \right) + 4 \cosh\left(\frac{c z}{2}\right) - \log \left( \cosh^2\left(\frac{c z}{4}\right) \cosh\left(\frac{c z}{2}\right) \right) + \log \left( \cosh\left(\frac{c z}{2}\right) \sinh^2\left(\frac{c z}{4}\right) \right) - 4 i \sinh\left(\frac{c z}{2}\right) \right) \sqrt{a - i a \sinh(c z)} \right)$$

01.22.21.0092.01

$$\int \frac{\coth(cz)}{\sqrt{a+b \sinh(cz)}} dz = -\frac{2\sqrt{b}}{\sqrt{a-c \operatorname{csch}^{\frac{1}{2}}(cz)} \sqrt{a+b \sinh(cz)}} \sinh^{-1}\left(\frac{\sqrt{a} \operatorname{csch}^{\frac{1}{2}}(cz)}{\sqrt{b}}\right) \sqrt{\frac{a \operatorname{csch}(cz)}{b} + 1}$$

01.22.21.0093.01

$$\int \frac{\coth(cz)}{\sqrt{i \sinh(cz) a + a}} dz = \frac{1}{2c \sqrt{i \sinh(cz) a + a}} \left( \left( 2i \tan^{-1}\left(\coth\left(\frac{cz}{4}\right)\right) - 2i \tan^{-1}\left(\tanh\left(\frac{cz}{4}\right)\right) - \log\left(\cosh^2\left(\frac{cz}{4}\right) \cosh\left(\frac{cz}{2}\right)\right) + \log\left(\cosh\left(\frac{cz}{2}\right) \sinh^2\left(\frac{cz}{4}\right)\right) \right) \left( \cosh\left(\frac{cz}{2}\right) + i \sinh\left(\frac{cz}{2}\right) \right) \right)$$

01.22.21.0094.01

$$\int \frac{\coth(cz)}{\sqrt{a - i a \sinh(cz)}} dz = \frac{1}{2c \sqrt{a - i a \sinh(cz)}} \left( \left( -2i \tan^{-1}\left(\coth\left(\frac{cz}{4}\right)\right) + 2i \tan^{-1}\left(\tanh\left(\frac{cz}{4}\right)\right) - \log\left(\cosh^2\left(\frac{cz}{4}\right) \cosh\left(\frac{cz}{2}\right)\right) + \log\left(\cosh\left(\frac{cz}{2}\right) \sinh^2\left(\frac{cz}{4}\right)\right) \right) \left( \cosh\left(\frac{cz}{2}\right) - i \sinh\left(\frac{cz}{2}\right) \right) \right)$$

### Involving cosh

#### Involving cosh(bz)

01.22.21.0095.01

$$\int \cosh(bz) \coth(cz) dz = \frac{1}{2(4c^2b - b^3)} \left( e^{-(2c+b)z} \left( -(4c^2 - b^2) e^{2(c+b)z} {}_2F_1\left(\frac{b}{2c}, 1; \frac{b}{2c} + 1; e^{2cz}\right) + (4c^2 - b^2) e^{2cz} {}_2F_1\left(-\frac{b}{2c}, 1; 1 - \frac{b}{2c}; e^{2cz}\right) - b \left( (2c - b) e^{2(2c+b)z} {}_2F_1\left(\frac{b}{2c} + 1, 1; \frac{b}{2c} + 2; e^{2cz}\right) + (2c + b) e^{4cz} {}_2F_1\left(1 - \frac{b}{2c}, 1; 2 - \frac{b}{2c}; e^{2cz}\right) \right) \right) \right)$$

01.22.21.0096.01

$$\int \cosh(cz) \coth(cz) dz = \frac{\cosh(cz) - \log\left(\cosh\left(\frac{cz}{2}\right)\right) + \log\left(\sinh\left(\frac{cz}{2}\right)\right)}{c}$$

01.22.21.0097.01

$$\int \cosh(cz) \coth(2cz) dz = \frac{2 \cosh(cz) - \log\left(\cosh\left(\frac{cz}{2}\right)\right) + \log\left(\sinh\left(\frac{cz}{2}\right)\right)}{2c}$$

01.22.21.0098.01

$$\int \cosh(cz) \coth(3cz) dz = \frac{6 \cosh(cz) - 2 \log\left(\cosh\left(\frac{cz}{2}\right)\right) + \log(2 \cosh(cz) - 1) - \log(2 \cosh(cz) + 1) + 2 \log\left(\sinh\left(\frac{cz}{2}\right)\right)}{6c}$$

01.22.21.0099.01

$$\int \cosh(c z) \coth(4 c z) d z = \frac{1}{4 c} \left( \sqrt[4]{-1} (1+i) \tan^{-1} \left( \frac{i + \tanh\left(\frac{c z}{2}\right)}{\sqrt{2}} \right) - \sqrt{2} \tanh^{-1} \left( \frac{i \tanh\left(\frac{c z}{2}\right) + 1}{\sqrt{2}} \right) + 4 \cosh(c z) - \log \left( \cosh\left(\frac{c z}{2}\right) \right) + \log \left( \sinh\left(\frac{c z}{2}\right) \right) \right)$$

01.22.21.0100.01

$$\int \cosh(2 c z) \coth(c z) d z = \frac{\cosh(2 c z) + 2 \log(\sinh(c z))}{2 c}$$

01.22.21.0101.01

$$\int \cosh(3 c z) \coth(c z) d z = \frac{6 \cosh(c z) + \cosh(3 c z) - 3 \log(\cosh\left(\frac{c z}{2}\right)) + 3 \log(\sinh\left(\frac{c z}{2}\right))}{3 c}$$

01.22.21.0102.01

$$\int \cosh(4 c z) \coth(c z) d z = \frac{4 \cosh(2 c z) + \cosh(4 c z) + 4 \log(\sinh(c z))}{4 c}$$

### Involving power of cosh

### Involving $\cosh^\mu(b z)$

01.22.21.0103.01

$$\int \cosh^m(b z) \coth(c z) d z = \frac{2^{-m} \log(\sinh(c z)) (1 - m \bmod 2)}{c} \binom{m}{\frac{m}{2}} - 2^{-m} \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{s} \left( \left( e^{b(m-2s)z} \left( (2c + b(m-2s)) {}_2F_1 \left( \frac{b(m-2s)}{2c}, 1; \frac{b(m-2s)}{2c} + 1; e^{2cz} \right) + b e^{2cz} (m-2s) {}_2F_1 \left( \frac{b(m-2s)}{2c} + 1, 1; \frac{b(m-2s)}{2c} + 2; e^{2cz} \right) \right) / (b(m-2s)(2c + b(m-2s))) - \left( e^{-b(m-2s)z} \left( (2c - b(m-2s)) {}_2F_1 \left( -\frac{b(m-2s)}{2c}, 1; 1 - \frac{b(m-2s)}{2c}; e^{2cz} \right) - b e^{2cz} (m-2s) {}_2F_1 \left( 1 - \frac{b(m-2s)}{2c}, 1; 2 - \frac{b(m-2s)}{2c}; e^{2cz} \right) \right) / (b(m-2s)(2c - b(m-2s))) \right) \right); m \in \mathbb{N}^+$$

01.22.21.0104.01

$$\int \cosh^\mu(c z) \coth(c z) d z = \frac{\cosh^{\mu+2}(c z) \coth^2(c z)^{\frac{1}{2}(-\mu-2)} \operatorname{csch}^2(c z)}{c \mu} {}_2F_1 \left( -\frac{\mu}{2}, -\frac{\mu}{2}; 1 - \frac{\mu}{2}; -\operatorname{csch}^2(c z) \right)$$

01.22.21.0105.01

$$\int \cosh^\mu(c z) \coth(c z) d z = \frac{((1 + e^{-2cz})^{-\mu} \cosh^\mu(c z))}{c} \left( \frac{e^{-2cz} F_1 \left( \frac{2-\mu}{2}; -\mu, 1; \frac{4-\mu}{2}; -e^{-2cz}, e^{-2cz} \right)}{\mu-2} + \frac{F_1 \left( -\frac{\mu}{2}; -\mu, 1; \frac{2-\mu}{2}; -e^{-2cz}, e^{-2cz} \right)}{\mu} \right)$$



01.22.21.0106.01

$$\int \cosh^2(c z) \coth(c z) dz = \frac{\cosh(2 c z) + 4 \log(\sinh(c z))}{4 c}$$

01.22.21.0107.01

$$\int \cosh^3(c z) \coth(c z) dz = \frac{15 \cosh(c z) + \cosh(3 c z) + 12 \left( \log\left(\sinh\left(\frac{c z}{2}\right)\right) - \log\left(\cosh\left(\frac{c z}{2}\right)\right) \right)}{12 c}$$

01.22.21.0108.01

$$\int \sqrt{\cosh^2(c z)} \coth(c z) dz = \frac{\sqrt{\cosh^2(c z)} \left( \cosh(c z) - \log\left(\cosh\left(\frac{c z}{2}\right)\right) + \log\left(\sinh\left(\frac{c z}{2}\right)\right) \right) \operatorname{sech}(c z)}{c}$$

01.22.21.0109.01

$$\int \cosh^2(c z) \coth(3 c z) dz = \frac{3 \cosh(2 c z) + \log(2 \cosh(2 c z) + 1) + 4 \log(\sinh(c z))}{12 c}$$

01.22.21.0110.01

$$\int \cosh^{\frac{1}{2}}(c z) \coth(c z) dz = \frac{2 \cosh^{\frac{5}{2}}(c z) \operatorname{csch}^2(c z)}{c \coth^2(c z)^{5/4}} {}_2F_1\left(-\frac{1}{4}, -\frac{1}{4}; \frac{3}{4}; -\operatorname{csch}^2(c z)\right)$$

01.22.21.0111.01

$$\int \frac{\coth(c z)}{\cosh^{\frac{1}{2}}(c z)} dz = -\frac{2 \cosh^{\frac{3}{2}}(c z) \operatorname{csch}^2(c z)}{c \coth^2(c z)^{3/4}} {}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}; -\operatorname{csch}^2(c z)\right)$$

01.22.21.0112.01

$$\int \cosh^{\frac{1}{2}}(2 c z) \coth(c z) dz = \frac{1}{c} \left( \cosh^{\frac{1}{2}}(2 c z) - \coth^{-1}\left(\cosh^{\frac{1}{2}}(2 c z)\right) \right)$$

01.22.21.0113.01

$$\int \frac{\coth(c z)}{\cosh^{\frac{1}{2}}(2 c z)} dz = -\frac{1}{c} \tanh^{-1}\left(\cosh^{\frac{1}{2}}(2 c z)\right)$$

### Involving rational functions of cosh

#### Involving $(a + b \cosh(c z))^{-n}$

01.22.21.0114.01

$$\int \frac{\coth(c z)}{a + b \cosh(c z)} dz = \frac{(a + b) \log\left(\cosh\left(\frac{c z}{2}\right)\right) - a \log(a + b \cosh(c z)) + (a - b) \log\left(\sinh\left(\frac{c z}{2}\right)\right)}{(a - b)(a + b) c}$$

01.22.21.0115.01

$$\int \frac{A + B \coth(c z)}{a + b \cosh(c z)} dz =$$

$$\left( (A + B \coth(c z)) \left( \frac{B \left( (a + b) \log\left(\cosh\left(\frac{c z}{2}\right)\right) - a \log(a + b \cosh(c z)) + (a - b) \log\left(\sinh\left(\frac{c z}{2}\right)\right)\right)}{(a - b)(a + b)} - \frac{2 A \tan^{-1}\left(\frac{(a - b) \tanh\left(\frac{c z}{2}\right)}{\sqrt{b^2 - a^2}}\right)}{\sqrt{b^2 - a^2}} \right) \right)$$

$$\sinh(c z) \Bigg/ (c (B \cosh(c z) + A \sinh(c z)))$$

01.22.21.0116.01

$$\int \frac{A + B \coth(z)}{1 - \cosh(z)} dz = \frac{1}{4} \left( 4 A \coth\left(\frac{z}{2}\right) + B \left( \operatorname{csch}^2\left(\frac{z}{2}\right) + 2 \log\left(\cosh\left(\frac{z}{2}\right)\right) - 2 \log\left(\sinh\left(\frac{z}{2}\right)\right) \right) \right)$$

01.22.21.0117.01

$$\int \frac{A + B \coth(z)}{\cosh(z) + 1} dz = \frac{-B \cosh(z) \left( \log\left(\cosh\left(\frac{z}{2}\right)\right) - \log\left(\sinh\left(\frac{z}{2}\right)\right) \right) + B \left( -\log\left(\cosh\left(\frac{z}{2}\right)\right) + \log\left(\sinh\left(\frac{z}{2}\right)\right) - 1 \right) + 2 A \sinh(z)}{2 (\cosh(z) + 1)}$$

01.22.21.0118.01

$$\int \frac{\coth(c z)}{a + b \cosh(2 c z)} dz = - \frac{\log(a + b \cosh(2 c z)) - 2 \log(\sinh(c z))}{2 (a + b) c}$$

01.22.21.0119.01

$$\int \frac{(A + B \cosh(c z)) \coth(c z)}{a + b \cosh(c z)} dz =$$

$$\frac{b (a + b) (A - B) \log\left(\cosh\left(\frac{c z}{2}\right)\right) + a (a B - A b) \log(a + b \cosh(c z)) + (a - b) b (A + B) \log\left(\sinh\left(\frac{c z}{2}\right)\right)}{(a - b) b (a + b) c}$$

01.22.21.0120.01

$$\int \frac{(A + B \cosh(z)) \coth(z)}{\cosh(z) - 1} dz = - \frac{2 \left( (A - B) \log\left(\cosh\left(\frac{z}{2}\right)\right) - (A + 3 B) \log\left(\sinh\left(\frac{z}{2}\right)\right) \right) \sinh^2\left(\frac{z}{2}\right) + A + B}{2 (\cosh(z) - 1)}$$

01.22.21.0121.01

$$\int \frac{(A + B \cosh(z)) \coth(z)}{\cosh(z) + 1} dz = \frac{\left( 2 (A + B) \log\left(\sinh\left(\frac{z}{2}\right)\right) - 2 (A - 3 B) \log\left(\cosh\left(\frac{z}{2}\right)\right) \right) \cosh^2\left(\frac{z}{2}\right) - A + B}{2 (\cosh(z) + 1)}$$

01.22.21.0122.01

$$\int \frac{\coth(c z)}{(a + b \cosh(c z))^2} dz = \frac{1}{c} \left( \frac{a}{(a^2 - b^2)(a + b \cosh(c z))} + \frac{\log\left(\cosh\left(\frac{c z}{2}\right)\right)}{(a - b)^2} + \frac{\log\left(\sinh\left(\frac{c z}{2}\right)\right)}{(a + b)^2} - \frac{(a^2 + b^2) \log(a + b \cosh(c z))}{(a^2 - b^2)^2} \right)$$

Involving algebraic functions of cosh

**Involving  $(a + b \cosh(c z))^{\beta}$**

01.22.21.0123.01

$$\int (a + b \cosh(c z))^{\beta} \coth(c z) dz = -\frac{(a + b \cosh(c z))^{\beta+1}}{2(a-b)(a+b)c(\beta+1)} \left( (a+b) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b \cosh(c z)}{a-b}\right) + (a-b) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b \cosh(c z)}{a+b}\right) \right)$$

01.22.21.0124.01

$$\int \sqrt{a + b \cosh(c z)} \coth(c z) dz = \frac{1}{\sqrt{-a-b} \sqrt{b-a} c} \left( \sqrt{b-a} (a+b) \tan^{-1}\left(\frac{\sqrt{a+b \cosh(c z)}}{\sqrt{-a-b}}\right) + \sqrt{-a-b} \left( (a-b) \tan^{-1}\left(\frac{\sqrt{a+b \cosh(c z)}}{\sqrt{b-a}}\right) + 2\sqrt{b-a} \sqrt{a+b \cosh(c z)} \right) \right)$$

01.22.21.0125.01

$$\int \sqrt{\cosh(c z) a + a} \coth(c z) dz = \frac{\sqrt{a(\cosh(c z) + 1)} \left( 2 \cosh\left(\frac{c z}{2}\right) - \log\left(\cosh\left(\frac{c z}{4}\right)\right) + \log\left(\sinh\left(\frac{c z}{4}\right)\right) \right) \operatorname{sech}\left(\frac{c z}{2}\right)}{c}$$

01.22.21.0126.01

$$\int \sqrt{a - a \cosh(c z)} \coth(c z) dz = -\frac{2\sqrt{a-a \cosh(c z)} \left( \tan^{-1}\left(\tanh\left(\frac{c z}{4}\right)\right) \operatorname{csch}\left(\frac{c z}{2}\right) - 1 \right)}{c}$$

01.22.21.0127.01

$$\int \frac{\coth(c z)}{\sqrt{a + b \cosh(c z)}} dz = -\frac{1}{c} \left( \frac{1}{\sqrt{a-b}} \tanh^{-1}\left(\frac{\sqrt{a+b \cosh(c z)}}{\sqrt{a-b}}\right) + \frac{1}{\sqrt{a+b}} \tanh^{-1}\left(\frac{\sqrt{a+b \cosh(c z)}}{\sqrt{a+b}}\right) \right)$$

01.22.21.0128.01

$$\int \frac{\coth(c z)}{\sqrt{\cosh(c z) a + a}} dz = \frac{\cosh\left(\frac{c z}{2}\right) \left( \log\left(\sinh\left(\frac{c z}{4}\right)\right) - \log\left(\cosh\left(\frac{c z}{4}\right)\right) \right) - 1}{c \sqrt{a(\cosh(c z) + 1)}}$$

01.22.21.0129.01

$$\int \frac{\coth(c z)}{\sqrt{a - a \cosh(c z)}} dz = \frac{2 \tan^{-1}\left(\tanh\left(\frac{c z}{4}\right)\right) \sinh\left(\frac{c z}{2}\right) - 1}{c \sqrt{a - a \cosh(c z)}}$$

### Involving $(a + b \cosh(2 c z))^{\beta}$

01.22.21.0130.01

$$\int (a + b \cosh(2 c z))^{\beta} \coth(c z) dz = -\frac{(a + b(2 \cosh^2(c z) - 1))^{\beta+1}}{2(a+b)c(\beta+1)} {}_2F_1\left(\beta+1, 1; \beta+2; \frac{2 b \cosh^2(c z) + a - b}{a + b}\right)$$

01.22.21.0131.01

$$\int \sqrt{a + b \cosh(2 c z)} \coth(c z) dz = \frac{1}{c} \left( \sqrt{a + b \cosh(2 c z)} - \sqrt{a + b} \tanh^{-1}\left(\frac{\sqrt{a + b \cosh(2 c z)}}{\sqrt{a + b}}\right) \right)$$

01.22.21.0132.01

$$\int \sqrt{\cosh(2 c z) a + a} \coth(c z) dz = \frac{\sqrt{\cosh(2 c z) a + a} \left( \cosh(c z) - \log\left(\cosh\left(\frac{c z}{2}\right)\right) + \log\left(\sinh\left(\frac{c z}{2}\right)\right) \right) \operatorname{sech}(c z)}{c}$$

01.22.21.0133.01

$$\int \sqrt{a - a \cosh(2 c z)} \coth(c z) dz = \frac{\sqrt{a - a \cosh(2 c z)}}{c}$$

01.22.21.0134.01

$$\int \frac{\coth(c z)}{\sqrt{a + b \cosh(2 c z)}} dz = -\frac{1}{\sqrt{a + b} c} \tanh^{-1} \left( \frac{\sqrt{a + b \cosh(2 c z)}}{\sqrt{a + b}} \right)$$

01.22.21.0135.01

$$\int \frac{\coth(c z)}{\sqrt{\cosh(2 c z) a + a}} dz = \frac{\cosh(c z) \left( \log \left( \sinh \left( \frac{c z}{2} \right) \right) - \log \left( \cosh \left( \frac{c z}{2} \right) \right) \right)}{c \sqrt{\cosh(2 c z) a + a}}$$

01.22.21.0136.01

$$\int \frac{\coth(c z)}{\sqrt{a - a \cosh(2 c z)}} dz = -\frac{1}{c \sqrt{a - a \cosh(2 c z)}}$$

### Involving $\cosh(c z) (a + b \cosh(2 c z))^\beta$

01.22.21.0137.01

$$\int \cosh(c z) (a + b \cosh(2 c z))^\beta \coth(c z) dz = \frac{1}{(2 \beta c + c) \sqrt{\coth^2(c z)}} \cosh(c z) (a + b \cosh(2 c z))^\beta$$

$$\left( \frac{(a + b) \operatorname{csch}^2(c z)}{2 b} + 1 \right)^{-\beta} F_1 \left( -\beta - \frac{1}{2}; -\frac{1}{2}, -\beta; \frac{1}{2} - \beta; -\operatorname{csch}^2(c z), -\frac{(a + b) \operatorname{csch}^2(c z)}{2 b} \right)$$

01.22.21.0138.01

$$\int \cosh(c z) \sqrt{a + b \cosh(2 c z)} \coth(c z) dz = \frac{1}{4 \sqrt{b} \sqrt{a + b} c} \left( \sqrt{a + b} \left( 2 \sqrt{b} \sqrt{a + b \cosh(2 c z)} \cosh(c z) + \sqrt{2} (a + 3 b) \log \left( \sqrt{2} \sqrt{b} \cosh(c z) + \sqrt{a + b \cosh(2 c z)} \right) \right) - 4 \sqrt{b} (a + b) \tanh^{-1} \left( \frac{\sqrt{a + b} \cosh(c z)}{\sqrt{a + b \cosh(2 c z)}} \right) \right)$$

01.22.21.0139.01

$$\int \frac{\cosh(c z) \coth(c z)}{\sqrt{a + b \cosh(2 c z)}} dz = \frac{1}{2 c} \left( \frac{\sqrt{2} \log \left( \sqrt{2} \sqrt{b} \cosh(c z) + \sqrt{a + b \cosh(2 c z)} \right)}{\sqrt{b}} - \frac{2 \tanh^{-1} \left( \frac{\sqrt{a + b} \cosh(c z)}{\sqrt{a + b \cosh(2 c z)}} \right)}{\sqrt{a + b}} \right)$$

01.22.21.0140.01

$$\int \frac{\cosh^2(c z) \coth(c z)}{\sqrt{a + b \cosh(2 c z)}} dz = \frac{\frac{\sqrt{a + b \cosh(2 c z)}}{b} - \frac{2 \tanh^{-1} \left( \frac{\sqrt{a + b \cosh(2 c z)}}{\sqrt{a + b}} \right)}{\sqrt{a + b}}}{2 c}$$

### Involving $\cosh(2 c z) (a + b \cosh(2 c z))^\beta$

01.22.21.0141.01

$$\int \cosh(2cz) (a + b \cosh(2cz))^\beta \coth(cz) dz = \frac{(a + b \cosh(2cz))^{\beta+1}}{2b(a+b)c(\beta+1)} \left( a + b - b {}_2F_1 \left( \beta + 1, 1; \beta + 2; \frac{a + b \cosh(2cz)}{a + b} \right) \right)$$

01.22.21.0142.01

$$\int \cosh(2cz) \sqrt{a + b \cosh(2cz)} \coth(cz) dz = \frac{\sqrt{a+b} \sqrt{a + b \cosh(2cz)} (a + 3b + b \cosh(2cz)) - 3b(a+b) \tanh^{-1} \left( \frac{\sqrt{a+b \cosh(2cz)}}{\sqrt{a+b}} \right)}{3b \sqrt{a+b} c}$$

01.22.21.0143.01

$$\int \frac{\cosh(2cz) \coth(cz)}{\sqrt{a + b \cosh(2cz)}} dz = \frac{1}{c} \left( \frac{\sqrt{a + b \cosh(2cz)}}{b} - \frac{1}{\sqrt{a+b}} \tanh^{-1} \left( \frac{\sqrt{a + b \cosh(2cz)}}{\sqrt{a+b}} \right) \right)$$

Involving tanh

Involving tanh(cz)

01.22.21.0144.01

$$\int \tanh(cz) \coth(cz) dz = z$$

01.22.21.0145.01

$$\int \tanh(cz) \coth(2cz) dz = z - \frac{\tanh(cz)}{2c}$$

Involving power of tanh

Involving tanh<sup>μ</sup>(cz)

01.22.21.0146.01

$$\int \tanh^\mu(cz) \coth(cz) dz = \frac{\tanh^\mu(cz)}{c\mu} {}_2F_1 \left( \frac{\mu}{2}, 1; \frac{\mu}{2} + 1; \tanh^2(cz) \right)$$

01.22.21.0147.01

$$\int \tanh^2(cz) \coth(cz) dz = \frac{\log(\cosh(cz))}{c}$$

01.22.21.0148.01

$$\int \tanh^3(cz) \coth(cz) dz = z - \frac{\tanh(cz)}{c}$$

Involving sinh and cosh

Involving sinh(cz) (a + b cosh(2cz))<sup>β</sup>

01.22.21.0149.01

$$\int \sinh(c z) (a + b \cosh(2 c z))^{\beta} \coth(c z) dz = \frac{(a + b \cosh(2 c z))^{\beta+1} \operatorname{csch}(c z)}{2 \sqrt{2} b c (\beta + 1)} \sqrt{-\frac{b \sinh^2(c z)}{a + b}} {}_2F_1\left(\beta + 1, \frac{1}{2}; \beta + 2; \frac{a + b \cosh(2 c z)}{a + b}\right)$$

01.22.21.0150.01

$$\int \sinh(c z) \sqrt{a + b \cosh(2 c z)} \coth(c z) dz = \frac{\sqrt{2} (a + b) \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{b} \sinh(c z)}{\sqrt{a + b \cosh(2 c z)}}\right) + 2 \sqrt{b} \sqrt{a + b \cosh(2 c z)} \sinh(c z)}{4 \sqrt{b} c}$$

01.22.21.0151.01

$$\int \frac{\sinh(c z) \coth(c z)}{\sqrt{a + b \cosh(2 c z)}} dz = \frac{\tanh^{-1}\left(\frac{\sqrt{2} \sqrt{b} \sinh(c z)}{\sqrt{a + b \cosh(2 c z)}}\right)}{\sqrt{2} \sqrt{b} c}$$

### Other integrals

01.22.21.0152.01

$$\int \frac{(A + B \cosh(c z)) \coth(c z)}{a + b \sinh(c z)} dz = \frac{1}{a b \sqrt{-a^2 - b^2} c} \left( \sqrt{-a^2 - b^2} \left( A B c z + b (A - B) \log\left(\cosh\left(\frac{c z}{2}\right)\right) + b (A + B) \log\left(\sinh\left(\frac{c z}{2}\right)\right) - A b \log(a + b \sinh(c z)) \right) - 2 (a^2 + b^2) B \tan^{-1}\left(\frac{b - a \tanh\left(\frac{c z}{2}\right)}{\sqrt{-a^2 - b^2}}\right) \right)$$

01.22.21.0153.01

$$\int \frac{(A + B \sinh(c z)) \coth(c z)}{a + b \cosh(c z)} dz = \frac{B z}{b} + \frac{2 a B \tan^{-1}\left(\frac{(a-b) \tanh\left(\frac{c z}{2}\right)}{\sqrt{b^2 - a^2}}\right)}{b \sqrt{b^2 - a^2} c} + \frac{A \log\left(\cosh\left(\frac{c z}{2}\right)\right)}{a c - b c} + \frac{a A \log(a + b \cosh(c z))}{b^2 c - a^2 c} + \frac{A \log\left(\sinh\left(\frac{c z}{2}\right)\right)}{(a + b) c}$$

01.22.21.0154.01

$$\int \sqrt{\cosh(c z) \sinh(c z)} \coth(c z) dz = -\frac{2 \coth(c z) (\cosh(c z) \sinh(c z))^{3/2}}{5 c \sqrt[4]{-\sinh^2(c z)}} {}_2F_1\left(\frac{5}{4}, \frac{3}{4}; \frac{9}{4}; \cosh^2(c z)\right)$$

01.22.21.0155.01

$$\int \frac{\coth(ez)}{\sqrt{a \cosh^2(ez) + b \sinh(ez) \cosh(ez) + c \sinh^2(ez)}} dz =$$

$$\left( (b - 2c - \sqrt{b^2 - 4ac}) \left( F \left[ \sin^{-1} \left( \frac{\sqrt{(-b - 2c + \sqrt{b^2 - 4ac})(\tanh(ez) + 1)}}{\sqrt{(-b + 2c + \sqrt{b^2 - 4ac})(\tanh(ez) - 1)}} \right) \right] - \frac{-a + c + \sqrt{b^2 - 4ac}}{a - c + \sqrt{b^2 - 4ac}} \right) - \right.$$

$$\left. 2\Pi \left( \frac{-b + 2c + \sqrt{b^2 - 4ac}}{b + 2c - \sqrt{b^2 - 4ac}}; \sin^{-1} \left( \frac{\sqrt{(-b - 2c + \sqrt{b^2 - 4ac})(\tanh(ez) + 1)}}{\sqrt{(-b + 2c + \sqrt{b^2 - 4ac})(\tanh(ez) - 1)}} \right) \right] - \frac{-a + c + \sqrt{b^2 - 4ac}}{a - c + \sqrt{b^2 - 4ac}} \right) \right)$$

$$(\tanh(ez) - 1) \sqrt{\frac{(-b - 2c + \sqrt{b^2 - 4ac})(\tanh(ez) + 1)}{(-b + 2c + \sqrt{b^2 - 4ac})(\tanh(ez) - 1)}}$$

$$\sqrt{-\frac{2a - b + (b - 2c + \sqrt{b^2 - 4ac}) \tanh(ez) + \sqrt{b^2 - 4ac}}{(a - b + c)(\tanh(ez) - 1)}}$$

$$\sqrt{\frac{-2a + b + (-b + 2c + \sqrt{b^2 - 4ac}) \tanh(ez) + \sqrt{b^2 - 4ac}}{(a - b + c)(\tanh(ez) - 1)}} \Bigg/$$

$$\left( (b + 2c - \sqrt{b^2 - 4ac}) e (\tanh(ez) + 1) \sqrt{a \cosh^2(ez) + b \sinh(ez) \cosh(ez) + c \sinh^2(ez)} \right)$$

**Involving sinh and tanh**

01.22.21.0156.01

$$\int \frac{A + C \coth(z) + B \tanh(z)}{a + b \sinh(z)} dz =$$

$$\left( 2 \cosh(z) \left( 2aA(a^2 + b^2) \tan^{-1} \left( \frac{b - a \tanh(\frac{z}{2})}{\sqrt{-a^2 - b^2}} \right) + 2ab \sqrt{-a^2 - b^2} B \tan^{-1} \left( \tanh \left( \frac{z}{2} \right) \right) + \sqrt{-a^2 - b^2} \right. \right.$$

$$\left. (B \log(\cosh(z)) a^2 + (a^2 + b^2) C \log(\sinh(z)) - ((B + C) a^2 + b^2 C) \log(a + b \sinh(z))) \right) \sinh(z)$$

$$(A + C \coth(z) + B \tanh(z)) \Bigg/ \left( (a - ib)(a + ib) \sqrt{-a^2 - b^2} (-B + C + (B + C) \cosh(2z) + A \sinh(2z)) \right)$$

**Involving cosh and tanh**

01.22.21.0157.01

$$\int \frac{A + B \tanh(z) + C \coth(z)}{a + b \cosh(z)} dz =$$

$$\left( 2 \cosh(z) \left( -\frac{2A}{\sqrt{b^2 - a^2}} \tan^{-1} \left( \frac{(a-b) \tanh\left(\frac{z}{2}\right)}{\sqrt{b^2 - a^2}} \right) + \frac{C \log(\cosh\left(\frac{z}{2}\right))}{a-b} + \frac{B \log(\cosh(z))}{a} + \frac{C \log(\sinh\left(\frac{z}{2}\right))}{a+b} - \frac{(a^2(B+C) - b^2 B) \log(a + b \cosh(z))}{a^3 - a b^2} \right) \sinh(z) (A + C \coth(z) + B \tanh(z)) \right) / (-B + C + (B + C) \cosh(2z) + A \sinh(2z))$$

### Involving hyperbolic and a power functions

#### Involving sinh and power

#### Involving $z^n \sinh(a + bz) \coth(cz)$

01.22.21.0158.01

$$\int z^n \sinh(a + bz) \coth(cz) dz = -\frac{1}{2} n! \left( e^{-a-bz} \sum_{j=0}^n \frac{b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{b}{2c}, \dots, -\frac{b}{2c}, 1; 1 - \frac{b}{2c}, \dots, 1 - \frac{b}{2c}; e^{2cz} \right) + e^{a+bz} \sum_{j=0}^n \frac{(-1)^j b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b}{2c}, \dots, \frac{b}{2c}, 1; \frac{b}{2c} + 1, \dots, \frac{b}{2c} + 1; e^{2cz} \right) - e^{(2c-b)z-a} \sum_{j=0}^n \frac{(-1)^j (2c-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-b}{2c}, \dots, \frac{2c-b}{2c}, 1; \frac{2c-b}{2c} + 1, \dots, \frac{2c-b}{2c} + 1; e^{2cz} \right) + e^{a+(b+2c)z} \sum_{j=0}^n \frac{(-1)^j (b+2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c}{2c}, \dots, \frac{b+2c}{2c}, 1; \frac{b+2c}{2c} + 1, \dots, \frac{b+2c}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

01.22.21.0159.01

$$\int z^n \sinh(bz) \coth(cz) dz = -\frac{1}{2} n! \left( e^{-bz} \sum_{j=0}^n \frac{b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{b}{2c}, \dots, -\frac{b}{2c}, 1; 1 - \frac{b}{2c}, \dots, 1 - \frac{b}{2c}; e^{2cz} \right) + e^{bz} \sum_{j=0}^n \frac{(-1)^j b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b}{2c}, \dots, \frac{b}{2c}, 1; \frac{b}{2c} + 1, \dots, \frac{b}{2c} + 1; e^{2cz} \right) - e^{(2c-b)z} \sum_{j=0}^n \frac{(-1)^j (2c-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-b}{2c}, \dots, \frac{2c-b}{2c}, 1; \frac{2c-b}{2c} + 1, \dots, \frac{2c-b}{2c} + 1; e^{2cz} \right) + e^{(b+2c)z} \sum_{j=0}^n \frac{(-1)^j (b+2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c}{2c}, \dots, \frac{b+2c}{2c}, 1; \frac{b+2c}{2c} + 1, \dots, \frac{b+2c}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

#### Involving powers of sinh and power

#### Involving $z^n \sinh^u(bz) \coth(cz)$



01.22.21.0160.01

$$\int z^n \sinh^u(bz) \coth(cz) dz =$$

$$2^{-u} n! \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^{k+1} \binom{u}{k} \left( e^{b(u-2k)z} \sum_{j=0}^n \frac{(-1)^j (b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2k)}{2c}, \dots, \frac{b(u-2k)}{2c}, 1; \frac{b(u-2k)}{2c} + 1, \dots, \frac{b(u-2k)}{2c} + 1; e^{2cz} \right) + e^{(2c+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(-2k+u)+2c}{2c}, \dots, \frac{b(-2k+u)+2c}{2c}, 1; \frac{b(-2k+u)+2c}{2c} + 1, \dots, \frac{b(-2k+u)+2c}{2c} + 1; e^{2cz} \right) + (-1)^u \left( e^{-b(u-2k)z} \sum_{j=0}^n \frac{(-1)^j (-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b(-2k+u)}{2c}, \dots, \frac{-b(-2k+u)}{2c}, 1; \frac{-b(-2k+u)}{2c} + 1, \dots, \frac{-b(-2k+u)}{2c} + 1; e^{2cz} \right) + e^{(2c-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b(-2k+u)+2c}{2c}, \dots, \frac{-b(-2k+u)+2c}{2c}, 1; \frac{-b(-2k+u)+2c}{2c} + 1, \dots, \frac{-b(-2k+u)+2c}{2c} + 1; e^{2cz} \right) \right) \Bigg) - \left( \frac{i}{2} \right)^u \binom{u}{\frac{u}{2}} n! (1-u \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + 2 e^{2cz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \dots, 1, 1; 2, \dots, 2; e^{2cz}) \right) /; n \in \mathbb{N} \wedge u \in \mathbb{N}^+$$

Involving cosh and power

Involving  $z^n \cosh(a + bz) \coth(cz)$

01.22.21.0161.01

$$\int z^n \cosh(a + bz) \coth(cz) dz = -\frac{1}{2} n! \left( -e^{-a-bz} \sum_{j=0}^n \frac{b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{b}{2c}, \dots, -\frac{b}{2c}, 1; 1 - \frac{b}{2c}, \dots, 1 - \frac{b}{2c}; e^{2cz} \right) + e^{a+bz} \sum_{j=0}^n \frac{(-1)^j b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b}{2c}, \dots, \frac{b}{2c}, 1; \frac{b}{2c} + 1, \dots, \frac{b}{2c} + 1; e^{2cz} \right) + e^{(2c-b)z-a} \sum_{j=0}^n \frac{(-1)^j (2c-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-b}{2c}, \dots, \frac{2c-b}{2c}, 1; \frac{2c-b}{2c} + 1, \dots, \frac{2c-b}{2c} + 1; e^{2cz} \right) + e^{a+(b+2c)z} \sum_{j=0}^n \frac{(-1)^j (b+2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c}{2c}, \dots, \frac{b+2c}{2c}, 1; \frac{b+2c}{2c} + 1, \dots, \frac{b+2c}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N}$$

01.22.21.0162.01

$$\int z^n \cosh(bz) \coth(cz) dz = -\frac{1}{2} n! \left( -e^{-bz} \sum_{j=0}^n \frac{b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{b}{2c}, \dots, -\frac{b}{2c}, 1; 1 - \frac{b}{2c}, \dots, 1 - \frac{b}{2c}; e^{2cz} \right) + \right. \\ \left. e^{bz} \sum_{j=0}^n \frac{(-1)^j b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b}{2c}, \dots, \frac{b}{2c}, 1; \frac{b}{2c} + 1, \dots, \frac{b}{2c} + 1; e^{2cz} \right) + \right. \\ \left. e^{(2c-b)z} \sum_{j=0}^n \frac{(-1)^j (2c-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-b}{2c}, \dots, \frac{2c-b}{2c}, 1; \frac{2c-b}{2c} + 1, \dots, \frac{2c-b}{2c} + 1; e^{2cz} \right) + \right. \\ \left. e^{(b+2c)z} \sum_{j=0}^n \frac{(-1)^j (b+2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c}{2c}, \dots, \frac{b+2c}{2c}, 1; \frac{b+2c}{2c} + 1, \dots, \frac{b+2c}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

Involving powers of cosh and power

### Involving $z^n \cosh^u(bz) \coth(cz)$

01.22.21.0163.01

$$\int z^n \cosh^u(bz) \coth(cz) dz = -2^{-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + 2 e^{2cz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} (1, \dots, 1; 2, \dots, 2; e^{2cz}) \right) - \\ 2^{-u} n! \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{k} \left( e^{-b(u-2k)z} \sum_{j=0}^n \frac{(-1)^j (-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{b(u-2k)}{2c}, \dots, -\frac{b(u-2k)}{2c}, 1; 1 - \frac{b(u-2k)}{2c}, \dots, \right. \right. \\ \left. \left. 1 - \frac{b(u-2k)}{2c}; e^{2cz} \right) + e^{b(u-2k)z} \sum_{j=0}^n \frac{(-1)^j (b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2k)}{2c}, \dots, \frac{b(u-2k)}{2c}, \right. \right. \\ \left. \left. 1; \frac{b(u-2k)}{2c} + 1, \dots, \frac{b(u-2k)}{2c} + 1; e^{2cz} \right) + e^{(2c-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} \right. \\ \left. {}_{j+2}F_{j+1} \left( \frac{2c-b(u-2k)}{2c}, \dots, \frac{2c-b(u-2k)}{2c}, 1; \frac{2c-b(u-2k)}{2c} + 1, \dots, \frac{2c-b(u-2k)}{2c} + 1; e^{2cz} \right) + \right. \\ \left. e^{(2c+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+b(u-2k)}{2c}, \dots, \frac{2c+b(u-2k)}{2c}, \right. \right. \\ \left. \left. 1; \frac{2c+b(u-2k)}{2c} + 1, \dots, \frac{2c+b(u-2k)}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N} \wedge u \in \mathbb{N}^+$$

Involving hyperbolic and exponential functions

Involving sinh and exp

### Involving $e^{Pz} \sinh(bz) \coth(cz)$

01.22.21.0164.01

$$\int e^{Pz} \sinh(bz) \coth(cz) dz = \frac{1}{4} \left( \frac{2 e^{(p-b)z} {}_2F_1\left(\frac{p-b}{2c}, 1; \frac{-b+2c+p}{2c}; e^{2cz}\right)}{p-b} + \frac{2 e^{(-b+2c+p)z} {}_2F_1\left(\frac{-b+2c+p}{2c}, 1; \frac{-b+4c+p}{2c}; e^{2cz}\right)}{-b+2c+p} - \frac{2 e^{(b+p)z} {}_2F_1\left(\frac{b+p}{2c}, 1; \frac{b+2c+p}{2c}; e^{2cz}\right)}{b+p} - \frac{2 e^{(b+2c+p)z} {}_2F_1\left(\frac{b+2c+p}{2c}, 1; \frac{b+4c+p}{2c}; e^{2cz}\right)}{b+2c+p} \right)$$

### Involving powers of sinh and exp

### Involving $e^{Pz} \sinh^u(bz) \coth(cz)$

01.22.21.0165.01

$$\int e^{Pz} \sinh^u(bz) \coth(cz) dz = -\frac{1}{p(2c+p)} \left(\frac{i}{2}\right)^u e^{Pz} \left(\frac{u}{2}\right) \left( (2c+p) {}_2F_1\left(\frac{p}{2c}, 1; \frac{p}{2c} + 1; e^{2cz}\right) + e^{2cz} p {}_2F_1\left(\frac{p}{2c} + 1, 1; \frac{p}{2c} + 2; e^{2cz}\right) \right) (1 - u \bmod 2) - 2^{-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^s \binom{u}{s} \left( \left( e^{(p+b(u-2s))z} \left( (2c+p+b(u-2s)) {}_2F_1\left(\frac{p+b(u-2s)}{2c}, 1; \frac{p+b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (p+b(u-2s)) {}_2F_1\left(\frac{p+b(u-2s)}{2c} + 1, 1; \frac{p+b(u-2s)}{2c} + 2; e^{2cz}\right) \right) \right) / \left( (p+b(u-2s))(2c+p+b(u-2s)) \right) + \left( (-1)^u e^{(p-b(u-2s))z} \left( (2c+p-b(u-2s)) {}_2F_1\left(\frac{p-b(u-2s)}{2c}, 1; \frac{p-b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (p-b(u-2s)) {}_2F_1\left(\frac{p-b(u-2s)}{2c} + 1, 1; \frac{p-b(u-2s)}{2c} + 2; e^{2cz}\right) \right) \right) / \left( (p-b(u-2s))(2c+p-b(u-2s)) \right) \right) / (u \in \mathbb{N}^+)$$

01.22.21.0166.01

$$\int e^{Pz} \sinh^u(cz) \coth(cz) dz = -2^{-u} (-e^{-cz} + e^{cz})^\mu (1 - e^{2cz})^{-\mu} \left( \frac{e^{Pz} {}_2F_1\left(\frac{p-c\mu}{2c}, 1-\mu; \frac{-\mu c+2c+p}{2c}; e^{2cz}\right)}{p-c\mu} + \frac{e^{(2c+p)z} {}_2F_1\left(\frac{-\mu c+2c+p}{2c}, 1-\mu; \frac{-\mu c+4c+p}{2c}; e^{2cz}\right)}{-\mu c+2c+p} \right)$$

### Involving cosh and exp

### Involving $e^{Pz} \cosh(bz) \coth(cz)$

01.22.21.0167.01

$$\int e^{pz} \cosh(bz) \coth(cz) dz = -\frac{1}{4} \left( \frac{2 e^{(p-b)z} {}_2F_1\left(\frac{p-b}{2c}, 1; \frac{-b+2c+p}{2c}; e^{2cz}\right)}{p-b} + \frac{2 e^{(b+p)z} {}_2F_1\left(\frac{b+p}{2c}, 1; \frac{b+2c+p}{2c}; e^{2cz}\right)}{b+p} + \frac{2 e^{(-b+2c+p)z} {}_2F_1\left(\frac{-b+2c+p}{2c}, 1; \frac{-b+4c+p}{2c}; e^{2cz}\right)}{-b+2c+p} + \frac{2 e^{(b+2c+p)z} {}_2F_1\left(\frac{b+2c+p}{2c}, 1; \frac{b+4c+p}{2c}; e^{2cz}\right)}{b+2c+p} \right)$$

01.22.21.0168.01

$$\int e^{bz} \coth(cz) \cosh(bz) dz = \frac{1}{4bc(b+c)} \left( b \left( 2(b+c) \log(\sinh(cz)) - c e^{2(b+c)z} {}_2F_1\left(\frac{b+c}{c}, 1; \frac{b}{c} + 2; e^{2cz}\right) \right) - c(b+c) e^{2bz} {}_2F_1\left(\frac{b}{c}, 1; \frac{b+c}{c}; e^{2cz}\right) \right)$$

01.22.21.0169.01

$$\int e^{-bz} \coth(cz) \cosh(bz) dz = \frac{e^{-2bz}}{4b(b-c)c} \left( bc e^{2cz} {}_2F_1\left(1 - \frac{b}{c}, 1; 2 - \frac{b}{c}; e^{2cz}\right) + (b-c) \left( c {}_2F_1\left(-\frac{b}{c}, 1; 1 - \frac{b}{c}; e^{2cz}\right) + 2b e^{2bz} \log(\sinh(cz)) \right) \right)$$

01.22.21.0170.01

$$\int e^{cz} \coth(cz) \cosh(cz) dz = \frac{-2cz + e^{2cz} + 4 \log(-1 + e^{2cz})}{4c}$$

Involving powers of cosh and exp

**Involving  $e^{pz} \cosh^u(bz) \coth(cz)$**

01.22.21.0171.01

$$\int e^{pz} \cosh^u(bz) \coth(cz) dz = -\frac{1}{p(2c+p)} \left( 2^{-u} e^{pz} \left( \frac{u}{2} \right) \left( (2c+p) {}_2F_1\left(\frac{p}{2c}, 1; \frac{p}{2c} + 1; e^{2cz}\right) + e^{2cz} p {}_2F_1\left(\frac{p}{2c} + 1, 1; \frac{p}{2c} + 2; e^{2cz}\right) \right) (1 - u \bmod 2) \right) - 2^{-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{s} \left( \left( e^{(p+b(u-2s))z} \left( (2c+p+b(u-2s)) {}_2F_1\left(\frac{p+b(u-2s)}{2c}, 1; \frac{p+b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (p+b(u-2s)) {}_2F_1\left(\frac{p+b(u-2s)}{2c} + 1, 1; \frac{p+b(u-2s)}{2c} + 2; e^{2cz}\right) \right) \right) / ((p+b(u-2s))(2c+p+b(u-2s))) + \left( e^{(p-b(u-2s))z} \left( (2c+p-b(u-2s)) {}_2F_1\left(\frac{p-b(u-2s)}{2c}, 1; \frac{p-b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (p-b(u-2s)) {}_2F_1\left(\frac{p-b(u-2s)}{2c} + 1, 1; \frac{p-b(u-2s)}{2c} + 2; e^{2cz}\right) \right) \right) / ((p-b(u-2s))(2c+p-b(u-2s))) \right) /; u \in \mathbb{N}^+$$

01.22.21.0172.01

$$\int e^{pz} \cosh^\mu(cz) \coth(cz) dz = (1 + e^{-2cz})^{-\mu} \cosh^\mu(cz) \left( \frac{e^{pz} F_1\left(-\frac{p+c\mu}{2c}; -\mu, 1; \frac{(3-\mu)c-c-p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{(\mu-1)c+c+p} + \frac{e^{(p-2c)z} F_1\left(-\frac{\mu c-2c+p}{2c}; -\mu, 1; \frac{(3-\mu)c+c-p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{(\mu-1)c-c+p} \right)$$

Involving powers of tanh and exp

**Involving  $e^{pz} \tanh^\mu(cz) \coth(cz)$**

01.22.21.0173.01

$$\int e^{pz} \tanh^\mu(cz) \coth(cz) dz = \frac{e^{pz} (1 - e^{-2cz})^{-\mu} (1 + e^{-2cz})^\mu \tanh^\mu(cz)}{p} F_1\left(-\frac{p}{2c}; \mu-1, 1-\mu; 1-\frac{p}{2c}; -e^{-2cz}, e^{-2cz}\right)$$

**Involving hyperbolic and trigonometric functions**

Involving sin and sinh

**Involving  $\sin(az) \sinh(bz) \coth(cz)$**

01.22.21.0174.01

$$\int \sin(az) \sinh(bz) \coth(cz) dz = \frac{1}{4} i \left( \frac{e^{(-b-ia)z} {}_2F_1\left(\frac{-b-ia}{2c}, 1; \frac{-b+2c-ia}{2c}; e^{2cz}\right)}{-b-ia} + \frac{e^{(b+ia)z} {}_2F_1\left(\frac{b+ia}{2c}, 1; \frac{b+2c+ia}{2c}; e^{2cz}\right)}{b+ia} + \frac{e^{(-b+2c-ia)z} {}_2F_1\left(\frac{-b+2c-ia}{2c}, 1; \frac{-b+4c-ia}{2c}; e^{2cz}\right)}{-b+2c-ia} + \frac{e^{(b+2c+ia)z} {}_2F_1\left(\frac{b+2c+ia}{2c}, 1; \frac{b+4c+ia}{2c}; e^{2cz}\right)}{b+2c+ia} - \frac{e^{(i-a-b)z} {}_2F_1\left(\frac{ia-b}{2c}, 1; \frac{-b+2c+ia}{2c}; e^{2cz}\right)}{ia-b} - \frac{e^{(b-ia)z} {}_2F_1\left(\frac{b-ia}{2c}, 1; \frac{b+2c-ia}{2c}; e^{2cz}\right)}{b-ia} - \frac{e^{(-b+2c+ia)z} {}_2F_1\left(\frac{-b+2c+ia}{2c}, 1; \frac{-b+4c+ia}{2c}; e^{2cz}\right)}{-b+2c+ia} - \frac{e^{(b+2c-ia)z} {}_2F_1\left(\frac{b+2c-ia}{2c}, 1; \frac{b+4c-ia}{2c}; e^{2cz}\right)}{b+2c-ia} \right)$$

Involving powers of sin and powers of sinh

**Involving  $\sin^m(az) \sinh^u(bz) \coth(cz)$**

01.22.21.0175.01

$$\int \sin^m(az) \sinh^u(bz) \coth(cz) dz = \frac{i^u 2^{-m-u} \log(\sinh(cz)) (1 - m \bmod 2) (1 - u \bmod 2)}{c} \left( \frac{m}{\frac{1}{2}} \right) \left( \frac{u}{\frac{1}{2}} \right) - i^{u+1} 2^{-m-u} \left( \frac{u}{\frac{1}{2}} \right) (1 - u \bmod 2)$$

$$\begin{aligned}
 & \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \left( e^{\frac{i\pi m}{2} - ia(m-2k)z} \left( (2c - ia(m-2k)) {}_2F_1 \left( -\frac{ia(m-2k)}{2c}, 1; 1 - \frac{ia(m-2k)}{2c}; e^{2cz} \right) - ia e^{2cz} (m-2k) \right. \right. \right. \\
 & \quad \left. \left. {}_2F_1 \left( 1 - \frac{ia(m-2k)}{2c}, 1; 2 - \frac{ia(m-2k)}{2c}; e^{2cz} \right) \right) \right) / (a(m-2k)(2c - ia(m-2k))) - \\
 & \quad \left( e^{ia(m-2k)z - \frac{i\pi m}{2}} \left( (2c + ai(m-2k)) {}_2F_1 \left( \frac{ia(m-2k)}{2c}, 1; \frac{ai(m-2k)}{2c} + 1; e^{2cz} \right) + a e^{2cz} i(m-2k) \right. \right. \\
 & \quad \left. \left. {}_2F_1 \left( \frac{ai(m-2k)}{2c} + 1, 1; \frac{ai(m-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / (a(m-2k)(2c + ai(m-2k))) \Big) - \\
 & 2^{-m-u} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( \left( e^{b(u-2k)z} \left( (2c + b(u-2k)) {}_2F_1 \left( \frac{b(u-2k)}{2c}, 1; \frac{b(u-2k)}{2c} + 1; e^{2cz} \right) + \right. \right. \right. \\
 & \quad \left. \left. b e^{2cz} (u-2k) {}_2F_1 \left( \frac{b(u-2k)}{2c} + 1, 1; \frac{b(u-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / (b(u-2k)(2c + b(u-2k))) - \\
 & \quad \left( (-1)^u e^{-b(u-2k)z} \left( (2c - b(u-2k)) {}_2F_1 \left( -\frac{b(u-2k)}{2c}, 1; 1 - \frac{b(u-2k)}{2c}; e^{2cz} \right) - \right. \right. \\
 & \quad \left. \left. b e^{2cz} (u-2k) {}_2F_1 \left( 1 - \frac{b(u-2k)}{2c}, 1; 2 - \frac{b(u-2k)}{2c}; e^{2cz} \right) \right) \right) / (b(u-2k)(2c - b(u-2k))) \Big) - \\
 & 2^{-m-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{k+s} \binom{m}{k} \binom{u}{s} \left( \left( e^{\frac{i\pi m}{2} + (b(u-2s) - ia(m-2k))z} \left( (2c - ia(m-2k) + b(u-2s)) {}_2F_1 \right. \right. \right. \\
 & \quad \left( \frac{b(u-2s) - ia(m-2k)}{2c}, 1; \frac{b(u-2s) - ia(m-2k)}{2c} + 1; e^{2cz} \right) + e^{2cz} (b(u-2s) - ia(m-2k)) {}_2F_1 \left( \frac{b(u-2s) - ia(m-2k)}{2c} + 1, 1; \frac{b(u-2s) - ia(m-2k)}{2c} + 2; e^{2cz} \right) \right) \Big) / \\
 & \quad ((b(u-2s) - ia(m-2k))(2c - ia(m-2k) + b(u-2s))) + \left( e^{(ai(m-2k) + b(u-2s))z - \frac{i\pi m}{2}} \right. \\
 & \quad \left( (2c + ai(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{ai(m-2k) + b(u-2s)}{2c}, 1; \frac{ai(m-2k) + b(u-2s)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{2cz} (ai(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{ai(m-2k) + b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k) + b(u-2s)}{2c} + 2; \right. \right. \\
 & \quad \left. \left. e^{2cz} \right) \right) \Big) / ((ai(m-2k) + b(u-2s))(2c + ai(m-2k) + b(u-2s))) + \\
 & \quad \left( (-1)^u e^{\frac{i\pi m}{2} + (-ia(m-2k) - b(u-2s))z} \left( (2c - ia(m-2k) - b(u-2s)) {}_2F_1 \left( \frac{-ia(m-2k) - b(u-2s)}{2c}, \right. \right. \right. \\
 & \quad \left. \left. 1; \frac{-ia(m-2k) - b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (-ia(m-2k) - b(u-2s)) {}_2F_1 \left( \frac{-ia(m-2k) - b(u-2s)}{2c} + 1, 1; \frac{-ia(m-2k) - b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \Big) / \\
 & \quad ((-ia(m-2k) - b(u-2s))(2c - ia(m-2k) - b(u-2s))) + \left( (-1)^u e^{(ia(m-2k) - b(u-2s))z - \frac{i\pi m}{2}} \right.
 \end{aligned}$$

$$\left( (2c + a i(m-2k) - b(u-2s)) {}_2F_1\left(\frac{ia(m-2k) - b(u-2s)}{2c}, 1; \frac{ia(m-2k) - b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (ia(m-2k) - b(u-2s)) {}_2F_1\left(\frac{ia(m-2k) - b(u-2s)}{2c} + 1, 1; \frac{ia(m-2k) - b(u-2s)}{2c} + 2; e^{2cz}\right) \right) / ((ia(m-2k) - b(u-2s))(2c + a i(m-2k) - b(u-2s))) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.22.21.0176.01

$$\int \sin^m(az) \sinh^\mu(cz) \coth(cz) dz = -2^{-m-\mu} (-e^{-cz} + e^{cz})^\mu (1 - e^{2cz})^{-\mu}$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \frac{e^{\frac{i\pi m}{2} + (2c - ia(m-2k))z} {}_2F_1\left(\frac{-(\mu-2)c - ia(m-2k)}{2c}, 1 - \mu; \frac{-(\mu-4)c - ia(m-2k)}{2c}; e^{2cz}\right)}{ia(2k - m) - c(\mu - 2)} + \frac{e^{(2c + ia(m-2k))z - \frac{i\pi m}{2}} {}_2F_1\left(\frac{ia(m-2k) - c(\mu-2)}{2c}, 1 - \mu; \frac{ia(m-2k) - c(\mu-4)}{2c}; e^{2cz}\right)}{ia(m-2k) - c(\mu - 2)} + \frac{e^{\frac{i\pi m}{2} - ia(m-2k)z} {}_2F_1\left(\frac{-ia(m-2k) - c\mu}{2c}, 1 - \mu; \frac{-(\mu-2)c - ia(m-2k)}{2c}; e^{2cz}\right)}{-ia(m-2k) - c\mu} + \frac{e^{ia(m-2k)z - \frac{i\pi m}{2}} {}_2F_1\left(\frac{ia(m-2k) - c\mu}{2c}, 1 - \mu; \frac{ia(m-2k) - c(\mu-2)}{2c}; e^{2cz}\right)}{ia(m-2k) - c\mu} \right) - \frac{1}{c(\mu - 2)\mu} 2^{-m-\mu} (-e^{-cz} + e^{cz})^\mu (1 - e^{2cz})^{-\mu} \left( \frac{m}{2} \right) \left( (\mu - 2) {}_2F_1\left(-\frac{\mu}{2}, 1 - \mu; 1 - \frac{\mu}{2}; e^{2cz}\right) + e^{2cz} \mu {}_2F_1\left(1 - \frac{\mu}{2}, 1 - \mu; 2 - \frac{\mu}{2}; e^{2cz}\right) \right) /; m \in \mathbb{N}^+$$

Involving cos and sinh

Involving cos(az) sinh(bz) coth(cz)

01.22.21.0177.01

$$\int \cos(az) \sinh(bz) \coth(cz) dz = \frac{1}{4} \left( \frac{e^{(-b-ia)z} {}_2F_1\left(\frac{-b-ia}{2c}, 1; \frac{-b+2c-ia}{2c}; e^{2cz}\right)}{-b-ia} + \frac{e^{(ia-b)z} {}_2F_1\left(\frac{ia-b}{2c}, 1; \frac{-b+2c+ia}{2c}; e^{2cz}\right)}{ia-b} + \frac{e^{(-b+2c-ia)z} {}_2F_1\left(\frac{-b+2c-ia}{2c}, 1; \frac{-b+4c-ia}{2c}; e^{2cz}\right)}{-b+2c-ia} + \frac{e^{(-b+2c+ia)z} {}_2F_1\left(\frac{-b+2c+ia}{2c}, 1; \frac{-b+4c+ia}{2c}; e^{2cz}\right)}{-b+2c+ia} - \frac{e^{(b-ia)z} {}_2F_1\left(\frac{b-ia}{2c}, 1; \frac{b+2c-ia}{2c}; e^{2cz}\right)}{b-ia} - \frac{e^{(b+ia)z} {}_2F_1\left(\frac{b+ia}{2c}, 1; \frac{b+2c+ia}{2c}; e^{2cz}\right)}{b+ia} - \frac{e^{(b+2c-ia)z} {}_2F_1\left(\frac{b+2c-ia}{2c}, 1; \frac{b+4c-ia}{2c}; e^{2cz}\right)}{b+2c-ia} - \frac{e^{(b+2c+ia)z} {}_2F_1\left(\frac{b+2c+ia}{2c}, 1; \frac{b+4c+ia}{2c}; e^{2cz}\right)}{b+2c+ia} \right)$$

Involving powers of cos and powers of sinh

Involving  $\cos^m(a z) \sinh^u(b z) \coth(c z)$

01.22.21.0178.01

$$\int \cos^m(a z) \sinh^u(b z) \coth(c z) dz =$$

$$\frac{i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \log(\sinh(c z)) (1 - m \bmod 2) (1 - u \bmod 2)}{c} - i^{u+1} 2^{-m-u} \binom{u}{\frac{u}{2}} (1 - u \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \left( e^{-i a(m-2k)z} \left( (2c - i a(m-2k)) {}_2F_1 \left( -\frac{i a(m-2k)}{2c}, 1; 1 - \frac{i a(m-2k)}{2c}; e^{2cz} \right) - i a e^{2cz} (m-2k) \right. \right. \right. \\ \left. \left. \left. {}_2F_1 \left( 1 - \frac{i a(m-2k)}{2c}, 1; 2 - \frac{i a(m-2k)}{2c}; e^{2cz} \right) \right) \right) / (a(m-2k)(2c - i a(m-2k))) -$$

$$\left( e^{i a(m-2k)z} \left( (2c + a i(m-2k)) {}_2F_1 \left( \frac{i a(m-2k)}{2c}, 1; \frac{a i(m-2k)}{2c} + 1; e^{2cz} \right) + a e^{2cz} i(m-2k) \right. \right. \\ \left. \left. {}_2F_1 \left( \frac{a i(m-2k)}{2c} + 1, 1; \frac{a i(m-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / (a(m-2k)(2c + a i(m-2k))) \Bigg) -$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( \left( e^{b(u-2k)z} \left( (2c + b(u-2k)) {}_2F_1 \left( \frac{b(u-2k)}{2c}, 1; \frac{b(u-2k)}{2c} + 1; e^{2cz} \right) + \right. \right. \right. \\ \left. \left. \left. b e^{2cz} (u-2k) {}_2F_1 \left( \frac{b(u-2k)}{2c} + 1, 1; \frac{b(u-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / (b(u-2k)(2c + b(u-2k))) -$$

$$\left( (-1)^u e^{-b(u-2k)z} \left( (2c - b(u-2k)) {}_2F_1 \left( -\frac{b(u-2k)}{2c}, 1; 1 - \frac{b(u-2k)}{2c}; e^{2cz} \right) - \right. \right. \\ \left. \left. b e^{2cz} (u-2k) {}_2F_1 \left( 1 - \frac{b(u-2k)}{2c}, 1; 2 - \frac{b(u-2k)}{2c}; e^{2cz} \right) \right) \right) / (b(u-2k)(2c - b(u-2k))) \Bigg) -$$

$$2^{-m-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^s \binom{m}{k} \binom{u}{s} \left( \left( e^{(b(u-2s)-i a(m-2k))z} \left( (2c - i a(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{b(u-2s) - i a(m-2k)}{2c}, \right. \right. \right. \right. \\ \left. \left. \left. 1; \frac{b(u-2s) - i a(m-2k)}{2c} + 1; e^{2cz} \right) + e^{2cz} (b(u-2s) - i a(m-2k)) {}_2F_1 \right. \right. \\ \left. \left. \left. \left( \frac{b(u-2s) - i a(m-2k)}{2c} + 1, 1; \frac{b(u-2s) - i a(m-2k)}{2c} + 2; e^{2cz} \right) \right) \right) \right) /$$

$$((b(u-2s) - i a(m-2k))(2c - i a(m-2k) + b(u-2s))) + \left( e^{(a i(m-2k)+b(u-2s))z}$$

$$\left( (2c + a i(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{a i(m-2k) + b(u-2s)}{2c}, 1; \frac{a i(m-2k) + b(u-2s)}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{2cz} (a i(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{a i(m-2k) + b(u-2s)}{2c} + 1, 1; \frac{a i(m-2k) + b(u-2s)}{2c} + 2; \right. \right.$$



$$\begin{aligned}
 & e^{2cz} \Big) \Big) / \left( (ai(m-2k) + b(u-2s))(2c + ai(m-2k) + b(u-2s)) \right) + \\
 & \left( (-1)^u e^{(-ia(m-2k) - b(u-2s))z} \left( (2c - ia(m-2k) - b(u-2s)) {}_2F_1 \left( \frac{-ia(m-2k) - b(u-2s)}{2c}, \right. \right. \right. \\
 & \quad \left. \left. \left. 1; \frac{-ia(m-2k) - b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (-ia(m-2k) - b(u-2s)) {}_2F_1 \right. \right. \\
 & \quad \left. \left. \left( \frac{-ia(m-2k) - b(u-2s)}{2c} + 1, 1; \frac{-ia(m-2k) - b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \right) / \\
 & \left( (-ia(m-2k) - b(u-2s))(2c - ia(m-2k) - b(u-2s)) \right) + \left( (-1)^u e^{(ia(m-2k) - b(u-2s))z} \right. \\
 & \quad \left. \left( (2c + ai(m-2k) - b(u-2s)) {}_2F_1 \left( \frac{ia(m-2k) - b(u-2s)}{2c}, 1; \frac{ia(m-2k) - b(u-2s)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \quad \left. \left. e^{2cz} (ia(m-2k) - b(u-2s)) {}_2F_1 \left( \frac{ia(m-2k) - b(u-2s)}{2c} + 1, 1; \frac{ia(m-2k) - b(u-2s)}{2c} + 2; \right. \right. \right. \\
 & \quad \left. \left. \left. e^{2cz} \right) \right) \right) / \left( (ia(m-2k) - b(u-2s))(2c + ai(m-2k) - b(u-2s)) \right) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

01.22.21.0179.01

$$\begin{aligned}
 \int \cos^m(az) \sinh^\mu(cz) \coth(cz) dz &= \frac{1}{c(\mu-2)\mu} 2^{-m-\mu} (-e^{-cz} + e^{cz})^\mu (-1)^{\left(\frac{m}{2}\right)} (m \bmod 2 - 1) \\
 & (1 - e^{2cz})^{-\mu} \left( (\mu-2) {}_2F_1 \left( -\frac{\mu}{2}, 1-\mu; 1-\frac{\mu}{2}; e^{2cz} \right) + e^{2cz} \mu {}_2F_1 \left( 1-\frac{\mu}{2}, 1-\mu; 2-\frac{\mu}{2}; e^{2cz} \right) \right) - \\
 & 2^{-m-\mu} (-e^{-cz} + e^{cz})^\mu (1 - e^{2cz})^{-\mu} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \frac{e^{(2c-ia(m-2k))z} {}_2F_1 \left( \frac{-(\mu-2)c-ia(m-2k)}{2c}, 1-\mu; \frac{-(\mu-4)c-ia(m-2k)}{2c}; e^{2cz} \right)}{ia(2k-m) - c(\mu-2)} + \right. \\
 & \frac{e^{(2c+ia(m-2k))z} {}_2F_1 \left( \frac{ia(m-2k)-c(\mu-2)}{2c}, 1-\mu; \frac{ia(m-2k)-c(\mu-4)}{2c}; e^{2cz} \right)}{ia(m-2k) - c(\mu-2)} + \\
 & \frac{e^{-ia(m-2k)z} {}_2F_1 \left( \frac{-ia(m-2k)-c\mu}{2c}, 1-\mu; \frac{-(\mu-2)c-ia(m-2k)}{2c}; e^{2cz} \right)}{-ia(m-2k) - c\mu} + \\
 & \left. \frac{e^{ia(m-2k)z} {}_2F_1 \left( \frac{ia(m-2k)-c\mu}{2c}, 1-\mu; \frac{ia(m-2k)-c(\mu-2)}{2c}; e^{2cz} \right)}{ia(m-2k) - c\mu} \right) /; m \in \mathbb{N}^+
 \end{aligned}$$

Involving sin and cosh

Involving sin(a z) cosh(b z) coth(c z)

01.22.21.0180.01

$$\int \sin(az) \cosh(bz) \coth(cz) dz = \frac{1}{4} i \left( -\frac{e^{(-b-ia)z} {}_2F_1\left(\frac{-b-ia}{2c}, 1; \frac{-b+2c-ia}{2c}; e^{2cz}\right)}{-b-ia} + \frac{e^{(ia-b)z} {}_2F_1\left(\frac{ia-b}{2c}, 1; \frac{-b+2c+ia}{2c}; e^{2cz}\right)}{ia-b} + \frac{e^{(b+ia)z} {}_2F_1\left(\frac{b+ia}{2c}, 1; \frac{b+2c+ia}{2c}; e^{2cz}\right)}{b+ia} + \frac{e^{(-b+2c+ia)z} {}_2F_1\left(\frac{-b+2c+ia}{2c}, 1; \frac{-b+4c+ia}{2c}; e^{2cz}\right)}{-b+2c+ia} + \frac{e^{(b+2c+ia)z} {}_2F_1\left(\frac{b+2c+ia}{2c}, 1; \frac{b+4c+ia}{2c}; e^{2cz}\right)}{b+2c+ia} - \frac{e^{(b-ia)z} {}_2F_1\left(\frac{b-ia}{2c}, 1; \frac{b+2c-ia}{2c}; e^{2cz}\right)}{b-ia} - \frac{e^{(-b+2c-ia)z} {}_2F_1\left(\frac{-b+2c-ia}{2c}, 1; \frac{-b+4c-ia}{2c}; e^{2cz}\right)}{-b+2c-ia} - \frac{e^{(b+2c-ia)z} {}_2F_1\left(\frac{b+2c-ia}{2c}, 1; \frac{b+4c-ia}{2c}; e^{2cz}\right)}{b+2c-ia} \right)$$

Involving powers of sin and powers of cosh

Involving  $\sin^m(az) \cosh^u(bz) \coth(cz)$

01.22.21.0181.01

$$\int \sin^m(az) \cosh^u(bz) \coth(cz) dz = -2^{-m-u} \left(\frac{u}{2}\right) (1 - u \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \left( i e^{\frac{im\pi}{2} - ia(m-2k)z} \left( (2c - ia(m-2k)) {}_2F_1\left(-\frac{ia(m-2k)}{2c}, 1; 1 - \frac{ia(m-2k)}{2c}; e^{2cz}\right) - ia e^{2cz} (m-2k) {}_2F_1\left(1 - \frac{ia(m-2k)}{2c}, 1; 2 - \frac{ia(m-2k)}{2c}; e^{2cz}\right) \right) / (a(m-2k)(2c - ia(m-2k))) - \left( i e^{ia(m-2k)z - \frac{im\pi}{2}} \left( (2c + ia(m-2k)) {}_2F_1\left(\frac{ia(m-2k)}{2c}, 1; \frac{ia(m-2k)}{2c} + 1; e^{2cz}\right) + a e^{2cz} i (m-2k) {}_2F_1\left(\frac{ia(m-2k)}{2c} + 1, 1; \frac{ia(m-2k)}{2c} + 2; e^{2cz}\right) \right) / (a(m-2k)(2c + ia(m-2k))) \right) \right)$$

$$\frac{2^{-m-u} \log(\sinh(cz)) (1 - m \bmod 2) (1 - u \bmod 2)}{c} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} - 2^{-m-u} \binom{m}{\frac{m}{2}} (1 - m \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{k} \left( \left( e^{b(u-2k)z} \left( (2c + b(u-2k)) {}_2F_1\left(\frac{b(u-2k)}{2c}, 1; \frac{b(u-2k)}{2c} + 1; e^{2cz}\right) + b e^{2cz} (u-2k) {}_2F_1\left(\frac{b(u-2k)}{2c} + 1, 1; \frac{b(u-2k)}{2c} + 2; e^{2cz}\right) \right) / (b(u-2k)(2c + b(u-2k))) - \left( e^{-b(u-2k)z} \left( (2c - b(u-2k)) {}_2F_1\left(-\frac{b(u-2k)}{2c}, 1; 1 - \frac{b(u-2k)}{2c}; e^{2cz}\right) - b e^{2cz} (u-2k) {}_2F_1\left(1 - \frac{b(u-2k)}{2c}, 1; 2 - \frac{b(u-2k)}{2c}; e^{2cz}\right) \right) / (b(u-2k)(2c - b(u-2k))) \right) \right)$$

$$2^{-m-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \binom{u}{s} \left( \left( e^{\frac{i\pi m}{2} + (b(u-2s) - ia(m-2k))z} \left( (2c - ia(m-2k) + b(u-2s)) {}_2F_1\left(\frac{2c - ia(m-2k) + b(u-2s)}{2c}, 1; \frac{2c - ia(m-2k) + b(u-2s)}{2c} + 1; e^{2cz}\right) + (2c - ia(m-2k) + b(u-2s)) e^{2cz} {}_2F_1\left(\frac{2c - ia(m-2k) + b(u-2s)}{2c} + 1, 1; \frac{2c - ia(m-2k) + b(u-2s)}{2c} + 2; e^{2cz}\right) \right) / (2c - ia(m-2k) + b(u-2s)) - \left( e^{-(b(u-2s) - ia(m-2k))z} \left( (2c + ia(m-2k) - b(u-2s)) {}_2F_1\left(\frac{2c + ia(m-2k) - b(u-2s)}{2c}, 1; \frac{2c + ia(m-2k) - b(u-2s)}{2c} + 1; e^{2cz}\right) + (2c + ia(m-2k) - b(u-2s)) e^{2cz} {}_2F_1\left(\frac{2c + ia(m-2k) - b(u-2s)}{2c} + 1, 1; \frac{2c + ia(m-2k) - b(u-2s)}{2c} + 2; e^{2cz}\right) \right) / (2c + ia(m-2k) - b(u-2s)) \right) \right)$$

$$\begin{aligned}
 & \left( \frac{b(u-2s) - ia(m-2k)}{2c}, 1; \frac{b(u-2s) - ia(m-2k)}{2c} + 1; e^{2cz} \right) + e^{2cz} (b(u-2s) - \\
 & ia(m-2k)) {}_2F_1 \left( \frac{b(u-2s) - ia(m-2k)}{2c} + 1, 1; \frac{b(u-2s) - ia(m-2k)}{2c} + 2; e^{2cz} \right) \Bigg) / \\
 & ((b(u-2s) - ia(m-2k)) (2c - ia(m-2k) + b(u-2s))) + \left( e^{(ia(m-2k) + b(u-2s))z - \frac{im\pi}{2}} \right. \\
 & \left. \left( (2c + ia(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{ia(m-2k) + b(u-2s)}{2c}, 1; \frac{ia(m-2k) + b(u-2s)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \left. \left. e^{2cz} (ia(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{ia(m-2k) + b(u-2s)}{2c} + 1, 1; \frac{ia(m-2k) + b(u-2s)}{2c} + 2; \right. \right. \right. \\
 & \left. \left. \left. e^{2cz} \right) \right) \right) / ((ia(m-2k) + b(u-2s)) (2c + ia(m-2k) + b(u-2s))) + \\
 & \left( e^{\frac{i\pi m}{2} + (-ia(m-2k) - b(u-2s))z} \left( (2c - ia(m-2k) - b(u-2s)) {}_2F_1 \left( \frac{-ia(m-2k) - b(u-2s)}{2c}, 1; \right. \right. \right. \\
 & \left. \left. \frac{-ia(m-2k) - b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (-ia(m-2k) - b(u-2s)) {}_2F_1 \right. \\
 & \left. \left( \frac{-ia(m-2k) - b(u-2s)}{2c} + 1, 1; \frac{-ia(m-2k) - b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \Bigg) / \\
 & ((-ia(m-2k) - b(u-2s)) (2c - ia(m-2k) - b(u-2s))) + \left( e^{(ia(m-2k) - b(u-2s))z - \frac{im\pi}{2}} \right. \\
 & \left. \left( (2c + ia(m-2k) - b(u-2s)) {}_2F_1 \left( \frac{ia(m-2k) - b(u-2s)}{2c}, 1; \frac{ia(m-2k) - b(u-2s)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \left. \left. e^{2cz} (ia(m-2k) - b(u-2s)) {}_2F_1 \left( \frac{ia(m-2k) - b(u-2s)}{2c} + 1, 1; \frac{ia(m-2k) - b(u-2s)}{2c} + 2; \right. \right. \right. \\
 & \left. \left. \left. e^{2cz} \right) \right) \right) / ((ia(m-2k) - b(u-2s)) (2c + ia(m-2k) - b(u-2s))) \Bigg); m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

01.22.21.0182.01

$$\int \sin^m(a z) \cosh^\mu(c z) \coth(c z) dz = 2^{-m} (1 + e^{-2cz})^{-\mu} \cosh^\mu(c z)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \frac{e^{(i a(m-2k)-2c)z - \frac{i m \pi}{2}} F_1\left(-\frac{(\mu-2)c + a i(m-2k)}{2c}; -\mu, 1; \frac{c(4-\mu) - i a(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{a i(m-2k) + c(\mu-2)} + \right.$$

$$\left. \frac{\left( e^{\frac{i \pi m}{2} + (-2c - i a(m-2k))z} F_1\left(-\frac{a i(2k-m) + c(\mu-2)}{2c}; -\mu, 1; \frac{a i(m-2k) + c(4-\mu)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right)}{(c(\mu-2) - i a(m-2k)) + \frac{e^{\frac{i m \pi}{2} - i a(m-2k)z} F_1\left(-\frac{a i(2k-m) + c \mu}{2c}; -\mu, 1; \frac{-\mu c + 2c - 2i a k + i a m}{2c}; -e^{-2cz}, e^{-2cz}\right)}{a i(2k-m) + c \mu}} + \right.$$

$$\left. \frac{e^{i a(m-2k)z - \frac{i m \pi}{2}} F_1\left(-\frac{a i(m-2k) + c \mu}{2c}; -\mu, 1; \frac{c(2-\mu) - i a(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{a i(m-2k) + c \mu} \right) - \frac{1}{c(\mu-2)\mu} 2^{-m} e^{-2cz}$$

$$(1 + e^{-2cz})^{-\mu} \left( e^{2cz} (\mu-2) F_1\left(-\frac{\mu}{2}; -\mu, 1; 1 - \frac{\mu}{2}; -e^{-2cz}, e^{-2cz}\right) + \mu F_1\left(1 - \frac{\mu}{2}; -\mu, 1; 2 - \frac{\mu}{2}; -e^{-2cz}, e^{-2cz}\right) \right)$$

$$\left( \frac{m}{2} \right) \cosh^\mu(c z) (m \bmod 2 - 1) /; m \in \mathbb{N}^+$$

Involving cos and cosh

### Involving cos(a z) cosh(b z) coth(c z)

01.22.21.0183.01

$$\int \cos(a z) \cosh(b z) \coth(c z) dz =$$

$$\frac{1}{4} \left( \frac{e^{(-b-ia)z} {}_2F_1\left(\frac{-b-ia}{2c}, 1; \frac{-b+2c-ia}{2c}; e^{2cz}\right)}{-b-ia} - \frac{e^{(ia-b)z} {}_2F_1\left(\frac{ia-b}{2c}, 1; \frac{-b+2c+ia}{2c}; e^{2cz}\right)}{ia-b} - \frac{e^{(b-ia)z} {}_2F_1\left(\frac{b-ia}{2c}, 1; \frac{b+2c-ia}{2c}; e^{2cz}\right)}{b-ia} - \right.$$

$$\frac{e^{(b+ia)z} {}_2F_1\left(\frac{b+ia}{2c}, 1; \frac{b+2c+ia}{2c}; e^{2cz}\right)}{b+ia} - \frac{e^{(-b+2c-ia)z} {}_2F_1\left(\frac{-b+2c-ia}{2c}, 1; \frac{-b+4c-ia}{2c}; e^{2cz}\right)}{-b+2c-ia} - \frac{e^{(-b+2c+ia)z} {}_2F_1\left(\frac{-b+2c+ia}{2c}, 1; \frac{-b+4c+ia}{2c}; e^{2cz}\right)}{-b+2c+ia} - \left.$$

$$\frac{e^{(b+2c-ia)z} {}_2F_1\left(\frac{b+2c-ia}{2c}, 1; \frac{b+4c-ia}{2c}; e^{2cz}\right)}{b+2c-ia} - \frac{e^{(b+2c+ia)z} {}_2F_1\left(\frac{b+2c+ia}{2c}, 1; \frac{b+4c+ia}{2c}; e^{2cz}\right)}{b+2c+ia} \right)$$

Involving powers of cos and powers of cosh

### Involving cos<sup>m</sup>(a z) cosh<sup>u</sup>(b z) coth(c z)

01.22.21.0184.01

$$\int \cos^m(a z) \cosh^u(b z) \coth(c z) dz =$$

$$\begin{aligned}
 & -2^{-m-u} \binom{u}{\frac{u}{2}} (1-u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \left( i e^{-i a(m-2k)z} \left( (2c - i a(m-2k)) {}_2F_1 \left( -\frac{i a(m-2k)}{2c}, 1; 1 - \frac{i a(m-2k)}{2c}; e^{2cz} \right) - \right. \right. \right. \\
 & \quad \left. \left. \left. i a e^{2cz} (m-2k) {}_2F_1 \left( 1 - \frac{i a(m-2k)}{2c}, 1; 2 - \frac{i a(m-2k)}{2c}; e^{2cz} \right) \right) \right) / (a(m-2k)(2c - i a(m-2k))) - \\
 & \quad \left( i e^{i a(m-2k)z} \left( (2c + a i(m-2k)) {}_2F_1 \left( \frac{i a(m-2k)}{2c}, 1; \frac{a i(m-2k)}{2c} + 1; e^{2cz} \right) + a e^{2cz} i(m-2k) \right. \right. \\
 & \quad \left. \left. {}_2F_1 \left( \frac{a i(m-2k)}{2c} + 1, 1; \frac{a i(m-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / (a(m-2k)(2c + a i(m-2k))) \Bigg) + \\
 & \frac{2^{-m-u} \log(\sinh(c z)) (1-m \bmod 2) (1-u \bmod 2)}{c} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} - 2^{-m-u} \binom{m}{\frac{m}{2}} (1-m \bmod 2) \\
 & \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{k} \left( \left( e^{b(u-2k)z} \left( (2c + b(u-2k)) {}_2F_1 \left( \frac{b(u-2k)}{2c}, 1; \frac{b(u-2k)}{2c} + 1; e^{2cz} \right) + \right. \right. \right. \\
 & \quad \left. \left. \left. b e^{2cz} (u-2k) {}_2F_1 \left( \frac{b(u-2k)}{2c} + 1, 1; \frac{b(u-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / (b(u-2k)(2c + b(u-2k))) - \\
 & \quad \left( e^{-b(u-2k)z} \left( (2c - b(u-2k)) {}_2F_1 \left( -\frac{b(u-2k)}{2c}, 1; 1 - \frac{b(u-2k)}{2c}; e^{2cz} \right) - b e^{2cz} (u-2k) \right. \right. \\
 & \quad \left. \left. {}_2F_1 \left( 1 - \frac{b(u-2k)}{2c}, 1; 2 - \frac{b(u-2k)}{2c}; e^{2cz} \right) \right) \right) / (b(u-2k)(2c - b(u-2k))) \Bigg) - \\
 & 2^{-m-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \binom{u}{s} \left( \left( e^{(b(u-2s)-i a(m-2k))z} \left( (2c - i a(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{b(u-2s) - i a(m-2k)}{2c}, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 1; \frac{b(u-2s) - i a(m-2k)}{2c} + 1; e^{2cz} \right) + e^{2cz} (b(u-2s) - i a(m-2k)) {}_2F_1 \right. \right. \\
 & \quad \left. \left. \left( \frac{b(u-2s) - i a(m-2k)}{2c} + 1, 1; \frac{b(u-2s) - i a(m-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / \\
 & \quad ((b(u-2s) - i a(m-2k))(2c - i a(m-2k) + b(u-2s))) + \left( e^{(a i(m-2k)+b(u-2s))z} \right. \\
 & \quad \left( (2c + a i(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{a i(m-2k) + b(u-2s)}{2c}, 1; \frac{a i(m-2k) + b(u-2s)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{2cz} (a i(m-2k) + b(u-2s)) {}_2F_1 \left( \frac{a i(m-2k) + b(u-2s)}{2c} + 1, 1; \frac{a i(m-2k) + b(u-2s)}{2c} + 2; \right. \right. \\
 & \quad \left. \left. e^{2cz} \right) \right) / ((a i(m-2k) + b(u-2s))(2c + a i(m-2k) + b(u-2s))) + \\
 & \quad \left( e^{(-i a(m-2k)-b(u-2s))z} \left( (2c - i a(m-2k) - b(u-2s)) {}_2F_1 \left( \frac{-i a(m-2k) - b(u-2s)}{2c}, 1; \right. \right. \right. \\
 & \quad \left. \left. \frac{-i a(m-2k) - b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (-i a(m-2k) - b(u-2s)) {}_2F_1 \right.
 \end{aligned}$$

$$\left( \frac{-i a (m-2 k)-b(u-2 s)}{2 c}+1, 1 ; \frac{-i a (m-2 k)-b(u-2 s)}{2 c}+2 ; e^{2 c z} \right) /$$

$$\left( (-i a (m-2 k)-b(u-2 s))(2 c-i a (m-2 k)-b(u-2 s)) \right)+\left( e^{i a (m-2 k)-b(u-2 s) z} \right.$$

$$\left. \left( (2 c+a i (m-2 k)-b(u-2 s)) {}_2 F_1\left(\frac{i a (m-2 k)-b(u-2 s)}{2 c}, 1 ; \frac{i a (m-2 k)-b(u-2 s)}{2 c}+1 ; e^{2 c z}\right)+\right.$$

$$\left. e^{2 c z} (i a (m-2 k)-b(u-2 s)) {}_2 F_1\left(\frac{i a (m-2 k)-b(u-2 s)}{2 c}+1, 1 ; \frac{i a (m-2 k)-b(u-2 s)}{2 c}+2 ;\right.\right.$$

$$\left. \left. e^{2 c z}\right)\right) / \left( (i a (m-2 k)-b(u-2 s))(2 c+a i (m-2 k)-b(u-2 s)) \right) ; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.22.21.0185.01

$$\int \cos^m(a z) \cosh^\mu(c z) \coth(c z) d z = \frac{2^{-m} \cosh^\mu(c z) \left(1+e^{2 c z}\right)^{-\mu} F_1\left(-\frac{\mu}{2} ; 1,-\mu-1 ; 1-\frac{\mu}{2} ; e^{2 c z},-e^{2 c z}\right)\left(\frac{m}{2}\right)\left(1-m \bmod 2\right)}{c \mu}$$

$$2^{-m} \cosh^\mu(c z) \left(1+e^{2 c z}\right)^{-\mu} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \frac{e^{i a(m-2 k) z} F_1\left(\frac{i a(m-2 k)-c \mu}{2 c} ; 1,-\mu-1 ; \frac{1}{2}\left(\frac{a i(m-2 k)}{c}-\mu+2\right) ; e^{2 c z},-e^{2 c z}\right)}{i a(m-2 k)-c \mu} - \frac{e^{-i a(m-2 k) z} F_1\left(\frac{-i a(m-2 k)-c \mu}{2 c} ; 1,-\mu-1 ; \frac{1}{2}\left(\frac{-i a(m-2 k)}{c}-\mu+2\right) ; e^{2 c z},-e^{2 c z}\right)}{a i(m-2 k)+c \mu} \right) ; m \in \mathbb{N}^+$$

Involving powers of sin and powers of tanh

**Involving  $\sin^m(a z) \tanh^\mu(c z) \coth(c z)$**

01.22.21.0186.01

$$\int \sin^m(a z) \tanh^\mu(c z) \coth(c z) d z = \frac{2^{-m} i \left(1-e^{-2 c z}\right)^{-\mu} \left(1+e^{-2 c z}\right)^\mu}{a} \tanh^\mu(c z)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{(-1)^k}{m-2 k} \binom{m}{k} \left( e^{\frac{i m \pi}{2}-i a(m-2 k) z} F_1\left(\frac{i a(m-2 k)}{2 c} ; \mu-1, 1-\mu ; \frac{a i(m-2 k)}{2 c}+1 ; -e^{-2 c z}, e^{-2 c z}\right) - \right.$$

$$\left. e^{i a(m-2 k) z-\frac{i m \pi}{2}} F_1\left(-\frac{i a(m-2 k)}{2 c} ; \mu-1, 1-\mu ; 1-\frac{i a(m-2 k)}{2 c} ; -e^{-2 c z}, e^{-2 c z}\right) \right) +$$

$$\frac{2^{-m} (1-m \bmod 2) \tanh^\mu(c z)}{c \mu} \left( \frac{m}{2} \right) {}_2 F_1\left(\frac{\mu}{2}, 1 ; \frac{\mu}{2}+1 ; \tanh^2(c z)\right) ; m \in \mathbb{N}^+$$

Involving powers of cos and powers of tanh

**Involving  $\cos^m(a z) \tanh^\mu(c z) \coth(c z)$**

01.22.21.0187.01

$$\int \cos^m(a z) \tanh^\mu(c z) \coth(c z) dz = \frac{2^{-m} i (1 - e^{-2cz})^{-\mu} (1 + e^{-2cz})^\mu \tanh^\mu(c z)}{a}$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{1}{m-2k} \binom{m}{k} \left( e^{-ia(m-2k)z} F_1 \left( \frac{ia(m-2k)}{2c}; \mu-1, 1-\mu; \frac{ai(m-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz} \right) - e^{ia(m-2k)z} F_1 \left( -\frac{ia(m-2k)}{2c}; \mu-1, 1-\mu; 1 - \frac{ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) + \frac{2^{-m} (1 - m \bmod 2) \tanh^\mu(c z)}{c \mu} \left( \frac{m}{2} \right)_2 F_1 \left( \frac{\mu}{2}, 1; \frac{\mu}{2} + 1; \tanh^2(c z) \right) /; m \in \mathbb{N}^+$$

### Involving hyperbolic, exponential and a power functions

Involving sinh, exp and power

### Involving $z^n e^{pz} \sinh(a + bz) \coth(cz)$

01.22.21.0188.01

$$\int z^n e^{pz} \sinh(a + bz) \coth(cz) dz =$$

$$-\frac{1}{2} n! \left( -e^{(p-b)z-a} \sum_{j=0}^n \frac{(-1)^j (p-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-b}{2c}, \dots, \frac{p-b}{2c}, 1; \frac{p-b}{2c} + 1, \dots, \frac{p-b}{2c} + 1; e^{2cz} \right) + e^{a+(b+p)z} \sum_{j=0}^n \frac{(-1)^j (b+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p}{2c}, \dots, \frac{b+p}{2c}, 1; \frac{b+p}{2c} + 1, \dots, \frac{b+p}{2c} + 1; e^{2cz} \right) - e^{(-b+2c+p)z-a} \sum_{j=0}^n \frac{(-1)^j (-b+2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c+p}{2c}, \dots, \frac{-b+2c+p}{2c}, 1; \frac{-b+2c+p}{2c} + 1, \dots, \frac{-b+2c+p}{2c} + 1; e^{2cz} \right) + e^{a+(b+2c+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c+p}{2c}, \dots, \frac{b+2c+p}{2c}, 1; \frac{b+2c+p}{2c} + 1, \dots, \frac{b+2c+p}{2c} + 1; e^{2cz} \right) \right) /; n \in \mathbb{N}$$

01.22.21.0189.01

$$\int z^n e^{pz} \sinh(bz) \coth(cz) dz = -\frac{1}{2} n! \left( -e^{(p-b)z} \sum_{j=0}^n \frac{(-1)^j (p-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-b}{2c}, \dots, \frac{p-b}{2c}, 1; \frac{p-b}{2c} + 1, \dots, \frac{p-b}{2c} + 1; e^{2cz} \right) + e^{(b+p)z} \sum_{j=0}^n \frac{(-1)^j (b+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p}{2c}, \dots, \frac{b+p}{2c}, 1; \frac{b+p}{2c} + 1, \dots, \frac{b+p}{2c} + 1; e^{2cz} \right) - e^{(-b+2c+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c+p}{2c}, \dots, \frac{-b+2c+p}{2c}, 1; \frac{-b+2c+p}{2c} + 1, \dots, \frac{-b+2c+p}{2c} + 1; e^{2cz} \right) + e^{(b+2c+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c+p}{2c}, \dots, \frac{b+2c+p}{2c}, 1; \frac{b+2c+p}{2c} + 1, \dots, \frac{b+2c+p}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

01.22.21.0190.01

$$\int z^n e^{bz} \sinh(bz) \coth(cz) dz = \frac{1}{2} \left( \frac{z^{n+1}}{n+1} + 2 e^{2cz} n! \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} (1, \dots, 1; 2, \dots, 2; e^{2cz}) - n! \left( e^{2bz} \sum_{j=0}^n \frac{(-1)^j (2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b}{c}, \dots, \frac{b}{c}, 1; \frac{b}{c} + 1, \dots, \frac{b}{c} + 1; e^{2cz} \right) + e^{2(b+c)z} \sum_{j=0}^n \frac{(-1)^j (2b+2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+c}{c}, \dots, \frac{b+c}{c}, 1; \frac{b+c}{c} + 1, \dots, \frac{b+c}{c} + 1; e^{2cz} \right) \right) \right); n \in \mathbb{N}$$

01.22.21.0191.01

$$\int z^n e^{-bz} \sinh(bz) \coth(cz) dz = -\frac{1}{2} \left( \frac{z^{n+1}}{n+1} + 2 e^{2cz} n! \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} (1, \dots, 1; 2, \dots, 2; e^{2cz}) - n! \left( e^{-2bz} \sum_{j=0}^n \frac{(-1)^j (-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{b}{c}, \dots, -\frac{b}{c}, 1; 1 - \frac{b}{c}, \dots, 1 - \frac{b}{c}; e^{2cz} \right) + e^{2(c-b)z} \sum_{j=0}^n \frac{(-1)^j (2c-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c-b}{c}, \dots, \frac{c-b}{c}, 1; \frac{c-b}{c} + 1, \dots, \frac{c-b}{c} + 1; e^{2cz} \right) \right) \right); n \in \mathbb{N}$$

Involving powers of sinh, exp and power

**Involving  $z^n e^{pz} \sinh^u(bz) \coth(cz)$**



01.22.21.0192.01

$$\int z^n e^{pz} \sinh^u(bz) \coth(cz) dz =$$

$$\begin{aligned}
 & -\left(\frac{i}{2}\right)^u \left(\frac{u}{2}\right) n! (1 - u \bmod 2) \left( e^{pz} \sum_{j=0}^n \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p}{2c}, \dots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \dots, \frac{p}{2c} + 1; e^{2cz}\right) + \right. \\
 & \left. e^{(2c+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+2c}{2c}, \dots, \frac{p+2c}{2c}, 1; \frac{p+2c}{2c} + 1, \dots, \frac{p+2c}{2c} + 1; e^{2cz}\right) \right) - \\
 & 2^{-u} n! \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( (-1)^u \left( e^{(p-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p-b(u-2k)}{2c}, \right. \right. \right. \\
 & \left. \left. \left. \dots, \frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c} + 1, \dots, \frac{p-b(u-2k)}{2c} + 1; e^{2cz}\right) + \right. \right. \\
 & \left. e^{(2c+p-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c+p-b(u-2k)}{2c}, \dots, \right. \right. \\
 & \left. \left. \left. \frac{2c+p-b(u-2k)}{2c}, 1; \frac{2c+p-b(u-2k)}{2c} + 1, \dots, \frac{2c+p-b(u-2k)}{2c} + 1; e^{2cz}\right) \right) + \right. \\
 & \left. e^{(p+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+b(u-2k)}{2c}, \dots, \frac{p+b(u-2k)}{2c}, \right. \right. \\
 & \left. \left. \left. 1; \frac{p+b(u-2k)}{2c} + 1, \dots, \frac{p+b(u-2k)}{2c} + 1; e^{2cz}\right) + \right. \right. \\
 & \left. e^{(2c+p+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c+p+b(u-2k)}{2c}, \dots, \right. \right. \\
 & \left. \left. \left. \frac{2c+p+b(u-2k)}{2c}, 1; \frac{2c+p+b(u-2k)}{2c} + 1, \dots, \frac{2c+p+b(u-2k)}{2c} + 1; e^{2cz}\right) \right) \Bigg) /; n \in \mathbb{N} \wedge u \in \mathbb{N}^+
 \end{aligned}$$

Involving cosh, exp and power

Involving  $z^n e^{pz} \cosh(a + bz) \coth(cz)$

01.22.21.0193.01

$$\int z^n e^{p z} \cosh(a + b z) \coth(c z) dz =$$

$$-\frac{1}{2} n! \left( e^{(p-b)z-a} \sum_{j=0}^n \frac{(-1)^j (p-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-b}{2c}, \dots, \frac{p-b}{2c}, 1; \frac{p-b}{2c} + 1, \dots, \frac{p-b}{2c} + 1; e^{2cz} \right) + \right.$$

$$e^{a+(b+p)z} \sum_{j=0}^n \frac{(-1)^j (b+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p}{2c}, \dots, \frac{b+p}{2c}, 1; \frac{b+p}{2c} + 1, \dots, \frac{b+p}{2c} + 1; e^{2cz} \right) +$$

$$e^{(-b+2c+p)z-a} \sum_{j=0}^n \frac{(-1)^j (-b+2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c+p}{2c}, \dots, \frac{-b+2c+p}{2c}, 1; \right.$$

$$\left. \frac{-b+2c+p}{2c} + 1, \dots, \frac{-b+2c+p}{2c} + 1; e^{2cz} \right) + e^{a+(b+2c+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{b+2c+p}{2c}, \dots, \frac{b+2c+p}{2c}, 1; \frac{b+2c+p}{2c} + 1, \dots, \frac{b+2c+p}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

01.22.21.0194.01

$$\int z^n e^{p z} \cosh(b z) \coth(c z) dz =$$

$$-\frac{1}{2} n! \left( e^{(p-b)z} \sum_{j=0}^n \frac{(-1)^j (p-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-b}{2c}, \dots, \frac{p-b}{2c}, 1; \frac{p-b}{2c} + 1, \dots, \frac{p-b}{2c} + 1; e^{2cz} \right) + \right.$$

$$e^{(b+p)z} \sum_{j=0}^n \frac{(-1)^j (b+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p}{2c}, \dots, \frac{b+p}{2c}, 1; \frac{b+p}{2c} + 1, \dots, \frac{b+p}{2c} + 1; e^{2cz} \right) +$$

$$e^{(-b+2c+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c+p}{2c}, \dots, \frac{-b+2c+p}{2c}, 1; \right.$$

$$\left. \frac{-b+2c+p}{2c} + 1, \dots, \frac{-b+2c+p}{2c} + 1; e^{2cz} \right) + e^{(b+2c+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{b+2c+p}{2c}, \dots, \frac{b+2c+p}{2c}, 1; \frac{b+2c+p}{2c} + 1, \dots, \frac{b+2c+p}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

01.22.21.0195.01

$$\int z^n e^{b z} \cosh(b z) \coth(c z) dz = -\frac{1}{2} \left( \frac{z^{n+1}}{n+1} + 2 e^{2cz} n! \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} (1, \dots, 1; 2, \dots, 2; e^{2cz}) + \right.$$

$$n! \left( e^{2bz} \sum_{j=0}^n \frac{(-1)^j (2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b}{c}, \dots, \frac{b}{c}, 1; \frac{b}{c} + 1, \dots, \frac{b}{c} + 1; e^{2cz} \right) + \right.$$

$$\left. \left. e^{2(b+c)z} \sum_{j=0}^n \frac{(-1)^j (2b+2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+c}{c}, \dots, \frac{b+c}{c}, 1; \frac{b+c}{c} + 1, \dots, \frac{b+c}{c} + 1; e^{2cz} \right) \right) \right); n \in \mathbb{N}$$

01.22.21.0196.01

$$\int z^n e^{-bz} \cosh(bz) \coth(cz) dz = -\frac{1}{2} \left( \frac{z^{n+1}}{n+1} + 2 e^{2cz} n! \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \dots, 1; 2, \dots, 2; e^{2cz}) + \right. \\ \left. n! \left( e^{-2bz} \sum_{j=0}^n \frac{(-1)^j (-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(-\frac{b}{c}, \dots, -\frac{b}{c}, 1; 1 - \frac{b}{c}, \dots, 1 - \frac{b}{c}; e^{2cz}\right) + \right. \right. \\ \left. \left. e^{2(c-b)z} \sum_{j=0}^n \frac{(-1)^j (2c-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c-b}{c}, \dots, \frac{c-b}{c}, 1; \frac{c-b}{c} + 1, \dots, \frac{c-b}{c} + 1; e^{2cz}\right) \right) \right); n \in \mathbb{N}$$

01.22.21.0197.01

$$\int z^n e^{pz} \cosh(cz) \coth(cz) dz = \\ -e^{(c+p)z} n! \sum_{j=0}^n \frac{(-1)^j (c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c+p}{2c}, \dots, \frac{c+p}{2c}, 1; \frac{c+p}{2c} + 1, \dots, \frac{c+p}{2c} + 1; e^{2cz}\right) - \\ \frac{1}{2} e^{cz} n! \left( e^{(p-2c)z} \sum_{j=0}^n \frac{(-1)^j (p-c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p-c}{2c}, \dots, \frac{p-c}{2c}, 1; \frac{p-c}{2c} + 1, \dots, \frac{p-c}{2c} + 1; e^{2cz}\right) + \right. \\ \left. e^{(2c+p)z} \sum_{j=0}^n \frac{(-1)^j (3c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{3c+p}{2c}, \dots, \frac{3c+p}{2c}, 1; \frac{3c+p}{2c} + 1, \dots, \frac{3c+p}{2c} + 1; e^{2cz}\right) \right); n \in \mathbb{N}$$

Involving powers of cosh, exp and power

Involving  $z^n e^{pz} \cosh^u(bz) \coth(cz)$

01.22.21.0198.01

$$\int z^n e^{pz} \cosh^u(bz) \coth(cz) dz =$$

$$-2^{-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \left( e^{pz} \sum_{j=0}^n \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p}{2c}, \dots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \dots, \frac{p}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(2c+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p}{2c}, \dots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c} + 1, \dots, \frac{2c+p}{2c} + 1; e^{2cz} \right) \right) -$$

$$2^{-u} n! \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{s} \left( e^{(p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1} \left( \frac{p+2bs-bu}{2c}, \dots, \frac{p+2bs-bu}{2c}, 1; \frac{p+2bs-bu}{2c} + 1, \dots, \frac{p+2bs-bu}{2c} + 1; e^{2cz} \right) +$$

$$e^{(2c+p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (2c+p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p+2bs-bu}{2c}, \dots, \right.$$

$$\left. \frac{2c+p+2bs-bu}{2c}, 1; \frac{2c+p+2bs-bu}{2c} + 1, \dots, \frac{2c+p+2bs-bu}{2c} + 1; e^{2cz} \right) +$$

$$e^{(p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-2bs+bu}{2c}, \dots, \frac{p-2bs+bu}{2c}, \right.$$

$$\left. 1; \frac{p-2bs+bu}{2c} + 1, \dots, \frac{p-2bs+bu}{2c} + 1; e^{2cz} \right) +$$

$$\left. e^{(2c+p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (2c+p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p-2bs+bu}{2c}, \dots, \right. \right.$$

$$\left. \left. \frac{2c+p-2bs+bu}{2c}, 1; \frac{2c+p-2bs+bu}{2c} + 1, \dots, \frac{2c+p-2bs+bu}{2c} + 1; e^{2cz} \right) \right) /; n \in \mathbb{N} \wedge u \in \mathbb{N}^+$$

01.22.21.0199.01

$$\int z^n e^{pz} \cosh^u(cz) \coth(cz) dz = -2^{-u} e^{(c+p)z} \left(\frac{u+1}{2}\right) n! (1 - (u+1) \bmod 2) \\ \sum_{j=0}^n \frac{(-1)^j (c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c+p}{2c}, \dots, \frac{c+p}{2c}, 1; \frac{c+p}{2c} + 1, \dots, \frac{c+p}{2c} + 1; e^{2cz}\right) - \\ 2^{-u} e^{cz} n! \sum_{s=0}^{\lfloor \frac{u}{2} \rfloor} \binom{u+1}{s} \left( e^{(p-c(-2s+u+1))z} \sum_{j=0}^n \frac{(-1)^j (p-c(u-2s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p-c(u-2s)}{2c}, \dots, \frac{p-c(u-2s)}{2c}, 1; \frac{p-c(u-2s)}{2c} + 1, \dots, \frac{p-c(u-2s)}{2c} + 1; e^{2cz}\right) + \right. \\ \left. e^{(p+c(-2s+u+1))z} \sum_{j=0}^n \frac{(-1)^j (p+c(-2s+u+2))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+c(-2s+u+2)}{2c}, \dots, \frac{p+c(-2s+u+2)}{2c}, 1; \frac{p+c(-2s+u+2)}{2c} + 1, \dots, \frac{p+c(-2s+u+2)}{2c} + 1; e^{2cz}\right) \right); n \in \mathbb{N} \wedge u \in \mathbb{N}^+$$

### Involving hyperbolic, exponential and trigonometric functions

Involving sin, sinh and exp

### Involving $e^{pz} \sin(az) \sinh(bz) \coth(cz)$

01.22.21.0200.01

$$\int e^{pz} \sin(az) \sinh(bz) \coth(cz) dz = \\ \frac{1}{4} i \left( \frac{e^{(-b-ia+p)z} {}_2F_1\left(\frac{-b-ia+p}{2c}, 1; \frac{-b+2c-ia+p}{2c}; e^{2cz}\right)}{-b-ia+p} + \frac{e^{(b+ia+p)z} {}_2F_1\left(\frac{b+ia+p}{2c}, 1; \frac{b+2c+ia+p}{2c}; e^{2cz}\right)}{b+ia+p} + \right. \\ \frac{e^{(-b+2c-ia+p)z} {}_2F_1\left(\frac{-b+2c-ia+p}{2c}, 1; \frac{-b+4c-ia+p}{2c}; e^{2cz}\right)}{-b+2c-ia+p} - \frac{e^{(b+2c-ia+p)z} {}_2F_1\left(\frac{b+2c-ia+p}{2c}, 1; \frac{b+4c-ia+p}{2c}; e^{2cz}\right)}{b+2c-ia+p} + \\ \frac{e^{(b+2c+ia+p)z} {}_2F_1\left(\frac{b+2c+ia+p}{2c}, 1; \frac{b+4c+ia+p}{2c}; e^{2cz}\right)}{b+2c+ia+p} - \frac{e^{(-b+ia+p)z} {}_2F_1\left(\frac{-b+ia+p}{2c}, 1; \frac{-b+2c+ia+p}{2c}; e^{2cz}\right)}{-b+ia+p} - \\ \left. \frac{e^{(b-ia+p)z} {}_2F_1\left(\frac{b-ia+p}{2c}, 1; \frac{b+2c-ia+p}{2c}; e^{2cz}\right)}{b-ia+p} - \frac{e^{(-b+2c+ia+p)z} {}_2F_1\left(\frac{-b+2c+ia+p}{2c}, 1; \frac{-b+4c+ia+p}{2c}; e^{2cz}\right)}{-b+2c+ia+p} \right)$$

Involving powers of sin, powers of sinh and exp

### Involving $e^{pz} \sin^m(az) \sinh^u(bz) \coth(cz)$

01.22.21.0201.01

$$\begin{aligned}
 & \int e^{pz} \sin^m(az) \sinh^u(bz) \coth(cz) dz = \\
 & -\frac{1}{p(2c+p)} \left( i^u 2^{-m-u} e^{pz} \left( \frac{m}{2} \right) \left( \frac{u}{2} \right) \left( (2c+p) {}_2F_1 \left( \frac{p}{2c}, 1; \frac{p}{2c} + 1; e^{2cz} \right) + e^{2cz} p {}_2F_1 \left( \frac{p}{2c} + 1, 1; \frac{p}{2c} + 2; e^{2cz} \right) \right) \right. \\
 & \quad \left. (1-m \bmod 2) (1-u \bmod 2) - i^u 2^{-m-u} \left( \frac{u}{2} \right) (1-u \bmod 2) \right. \\
 & \quad \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \left( e^{\frac{i\pi m}{2} + (p-ia(m-2k))z} \left( (2c-ia(m-2k)+p) {}_2F_1 \left( \frac{p-ia(m-2k)}{2c}, 1; \frac{p-ia(m-2k)}{2c} + 1; e^{2cz} \right) + \right. \right. \right. \\
 & \quad \left. \left. \left. e^{2cz} (p-ia(m-2k)) {}_2F_1 \left( \frac{p-ia(m-2k)}{2c} + 1, 1; \frac{p-ia(m-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / \right. \\
 & \quad \left. ((p-ia(m-2k))(2c-ia(m-2k)+p)) + \left( e^{(ai(m-2k)+p)z - \frac{i\pi m}{2}} \right. \right. \\
 & \quad \left. \left. \left( (2c+ai(m-2k)+p) {}_2F_1 \left( \frac{ai(m-2k)+p}{2c}, 1; \frac{ai(m-2k)+p}{2c} + 1; e^{2cz} \right) + \right. \right. \right. \\
 & \quad \left. \left. \left. e^{2cz} (ai(m-2k)+p) {}_2F_1 \left( \frac{ai(m-2k)+p}{2c} + 1, 1; \frac{ai(m-2k)+p}{2c} + 2; e^{2cz} \right) \right) \right) / \right. \\
 & \quad \left. ((ai(m-2k)+p)(2c+ai(m-2k)+p)) \right) - 2^{-m-u} \left( \frac{m}{2} \right) (1-m \bmod 2) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \\
 & \quad \left( \left( e^{(p+b(u-2k))z} \left( (2c+p+b(u-2k)) {}_2F_1 \left( \frac{p+b(u-2k)}{2c}, 1; \frac{p+b(u-2k)}{2c} + 1; e^{2cz} \right) + e^{2cz} (p+b(u-2k)) \right. \right. \right. \\
 & \quad \left. \left. \left. {}_2F_1 \left( \frac{p+b(u-2k)}{2c} + 1, 1; \frac{p+b(u-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / ((p+b(u-2k))(2c+p+b(u-2k))) + \right. \\
 & \quad \left. \left( (-1)^u e^{(p-b(u-2k))z} \left( (2c+p-b(u-2k)) {}_2F_1 \left( \frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c} + 1; e^{2cz} \right) + \right. \right. \right. \\
 & \quad \left. \left. \left. e^{2cz} (p-b(u-2k)) {}_2F_1 \left( \frac{p-b(u-2k)}{2c} + 1, 1; \frac{p-b(u-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / \right. \\
 & \quad \left. ((p-b(u-2k))(2c+p-b(u-2k))) \right) - 2^{-m-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{k+s} \binom{m}{k} \binom{u}{s} \\
 & \quad \left( (-1)^u \left( e^{(ai(m-2k)+p-b(u-2s))z - \frac{i\pi m}{2}} \left( (2c+ai(m-2k)+p-b(u-2s)) {}_2F_1 \left( \frac{ai(m-2k)+p-b(u-2s)}{2c}, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 1; \frac{ai(m-2k)+p-b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (ai(m-2k)+p-b(u-2s)) \right. \right. \right. \\
 & \quad \left. \left. \left. {}_2F_1 \left( \frac{ai(m-2k)+p-b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k)+p-b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \right) / \right. \\
 & \quad \left. ((ai(m-2k)+p-b(u-2s))(2c+ai(m-2k)+p-b(u-2s))) + \right. \\
 & \quad \left. \left( (-1)^u \left( e^{\frac{i\pi m}{2} + (-ia(m-2k)+p-b(u-2s))z} \left( (2c-ia(m-2k)+p-b(u-2s)) {}_2F_1 \left( \frac{-ia(m-2k)+p-b(u-2s)}{2c}, \right. \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 1; \frac{-i a(m-2k) + p - b(u-2s)}{2c} + 1; e^{2cz} \Big) + e^{2cz} (-i a(m-2k) + p - b(u-2s)) \\
 & {}_2F_1 \left( \frac{-i a(m-2k) + p - b(u-2s)}{2c} + 1, 1; \frac{-i a(m-2k) + p - b(u-2s)}{2c} + 2; e^{2cz} \right) \Big) \Big) / \\
 & ((-i a(m-2k) + p - b(u-2s)) (2c - i a(m-2k) + p - b(u-2s))) + \\
 & \left( e^{\frac{i\pi m}{2} + (-i a(m-2k) + p + b(u-2s))z} \left( (2c - i a(m-2k) + p + b(u-2s)) {}_2F_1 \left( \frac{-i a(m-2k) + p + b(u-2s)}{2c}, \right. \right. \right. \\
 & \left. \left. \left. 1; \frac{-i a(m-2k) + p + b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (-i a(m-2k) + p + b(u-2s)) {}_2F_1 \right. \right. \\
 & \left. \left. \left( \frac{-i a(m-2k) + p + b(u-2s)}{2c} + 1, 1; \frac{-i a(m-2k) + p + b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \Big) \Big) / \\
 & ((-i a(m-2k) + p + b(u-2s)) (2c - i a(m-2k) + p + b(u-2s))) + \\
 & \left( e^{(ai(m-2k) + p + b(u-2s))z - \frac{i\pi m}{2}} \left( (2c + ai(m-2k) + p + b(u-2s)) {}_2F_1 \left( \frac{ai(m-2k) + p + b(u-2s)}{2c}, \right. \right. \right. \\
 & \left. \left. \left. 1; \frac{ai(m-2k) + p + b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (ai(m-2k) + p + b(u-2s)) {}_2F_1 \right. \right. \\
 & \left. \left. \left( \frac{ai(m-2k) + p + b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k) + p + b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \Big) \Big) / \\
 & ((ai(m-2k) + p + b(u-2s)) (2c + ai(m-2k) + p + b(u-2s))) \Big) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

01.22.21.0202.01

$$\begin{aligned}
 \int e^{pz} \sin^m(az) \sinh^\mu(cz) \coth(cz) dz &= -2^{-m-\mu} (-e^{-cz} + e^{cz})^\mu (1 - e^{2cz})^{-\mu} \binom{m}{\frac{m}{2}} \\
 & \left( \frac{e^{pz} {}_2F_1 \left( \frac{p-c\mu}{2c}, 1-\mu; \frac{-\mu c+2c+p}{2c}; e^{2cz} \right)}{p-c\mu} + \frac{e^{(2c+p)z} {}_2F_1 \left( \frac{-\mu c+2c+p}{2c}, 1-\mu; \frac{-\mu c+4c+p}{2c}; e^{2cz} \right)}{-\mu c+2c+p} \right) (1-m \bmod 2) - 2^{-m-\mu} \\
 & (-e^{-cz} + e^{cz})^\mu (1 - e^{2cz})^{-\mu} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \frac{e^{\frac{i\pi m}{2} + (p-ia(m-2k))z} {}_2F_1 \left( \frac{-ia(m-2k) + p - c\mu}{2c}, 1-\mu; \frac{-\mu c+2c-ia(m-2k)+p}{2c}; e^{2cz} \right)}{-ia(m-2k) + p - c\mu} + \right. \\
 & \frac{e^{(ai(m-2k)+p)z - \frac{i\pi m}{2}} {}_2F_1 \left( \frac{ai(m-2k) + p - c\mu}{2c}, 1-\mu; \frac{-\mu c+2c+ai(m-2k)+p}{2c}; e^{2cz} \right)}{ai(m-2k) + p - c\mu} + \\
 & \frac{e^{(2c+ai(m-2k)+p)z - \frac{i\pi m}{2}} {}_2F_1 \left( \frac{-\mu c+2c+ai(m-2k)+p}{2c}, 1-\mu; \frac{-\mu c+4c+ai(m-2k)+p}{2c}; e^{2cz} \right)}{-\mu c+2c+ai(m-2k)+p} + \\
 & \left. \frac{e^{\frac{i\pi m}{2} + (2c-ia(m-2k)+p)z} {}_2F_1 \left( \frac{-\mu c+2c-ia(m-2k)+p}{2c}, 1-\mu; \frac{-\mu c+4c-ia(m-2k)+p}{2c}; e^{2cz} \right)}{-\mu c+2c-ia(m-2k)+p} \right) /; m \in \mathbb{N}^+
 \end{aligned}$$

Involving cos, sinh and exp

### Involving $e^{pz} \cos(az) \sinh(bz) \coth(cz)$

01.22.21.0203.01

$$\int e^{pz} \cos(az) \sinh(bz) \coth(cz) dz =$$

$$\frac{1}{4} \left( \frac{e^{(-b-ia+p)z} {}_2F_1\left(\frac{-b-ia+p}{2c}, 1; \frac{-b+2c-ia+p}{2c}; e^{2cz}\right)}{-b-ia+p} + \frac{e^{(-b+ia+p)z} {}_2F_1\left(\frac{-b+ia+p}{2c}, 1; \frac{-b+2c+ia+p}{2c}; e^{2cz}\right)}{-b+ia+p} + \right.$$

$$\frac{e^{(-b+2c-ia+p)z} {}_2F_1\left(\frac{-b+2c-ia+p}{2c}, 1; \frac{-b+4c-ia+p}{2c}; e^{2cz}\right)}{-b+2c-ia+p} + \frac{e^{(-b+2c+ia+p)z} {}_2F_1\left(\frac{-b+2c+ia+p}{2c}, 1; \frac{-b+4c+ia+p}{2c}; e^{2cz}\right)}{-b+2c+ia+p}$$

$$\left. - \frac{e^{(b+2c-ia+p)z} {}_2F_1\left(\frac{b+2c-ia+p}{2c}, 1; \frac{b+4c-ia+p}{2c}; e^{2cz}\right)}{b+2c-ia+p} - \frac{e^{(b-ia+p)z} {}_2F_1\left(\frac{b-ia+p}{2c}, 1; \frac{b+2c-ia+p}{2c}; e^{2cz}\right)}{b-ia+p} - \right.$$

$$\left. \frac{e^{(b+ia+p)z} {}_2F_1\left(\frac{b+ia+p}{2c}, 1; \frac{b+2c+ia+p}{2c}; e^{2cz}\right)}{b+ia+p} - \frac{e^{(b+2c+ia+p)z} {}_2F_1\left(\frac{b+2c+ia+p}{2c}, 1; \frac{b+4c+ia+p}{2c}; e^{2cz}\right)}{b+2c+ia+p} \right)$$

### Involving powers of cos, powers of sinh and exp

### Involving $e^{pz} \cos^m(az) \sinh^u(bz) \coth(cz)$

01.22.21.0204.01

$$\int e^{pz} \cos^m(az) \sinh^u(bz) \coth(cz) dz =$$

$$-\frac{1}{p(2c+p)} \left( i^u 2^{-m-u} e^{pz} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \left( (2c+p) {}_2F_1\left(\frac{p}{2c}, 1; \frac{p}{2c}+1; e^{2cz}\right) + e^{2cz} p {}_2F_1\left(\frac{p}{2c}+1, 1; \frac{p}{2c}+2; e^{2cz}\right) \right) \right.$$

$$\left. (1-m \bmod 2)(1-u \bmod 2) - i^u 2^{-m-u} \binom{u}{\frac{u}{2}} (1-u \bmod 2) \right)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \left( e^{(p-ia(m-2k))z} \left( (2c-ia(m-2k)+p) {}_2F_1\left(\frac{p-ia(m-2k)}{2c}, 1; \frac{p-ia(m-2k)}{2c}+1; e^{2cz}\right) + \right. \right. \right.$$

$$\left. \left. e^{2cz} (p-ia(m-2k)) {}_2F_1\left(\frac{p-ia(m-2k)}{2c}+1, 1; \frac{p-ia(m-2k)}{2c}+2; e^{2cz}\right) \right) \right) / ((p-$$

$$ia(m-2k))(2c-ia(m-2k)+p)) +$$

$$\left( e^{(ai(m-2k)+p)z} \left( (2c+ai(m-2k)+p) {}_2F_1\left(\frac{ai(m-2k)+p}{2c}, 1; \frac{ai(m-2k)+p}{2c}+1; e^{2cz}\right) + \right. \right.$$

$$\left. \left. e^{2cz} (ai(m-2k)+p) {}_2F_1\left(\frac{ai(m-2k)+p}{2c}+1, 1; \frac{ai(m-2k)+p}{2c}+2; e^{2cz}\right) \right) \right) /$$

$$\left( (ai(m-2k)+p)(2c+ai(m-2k)+p) \right) - 2^{-m-u} \binom{m}{\frac{m}{2}} (1-m \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( \left( (-1)^u \left( e^{(p-b(u-2k))z} \left( (2c+p-b(u-2k)) {}_2F_1\left(\frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c}+1; e^{2cz}\right) + \right. \right. \right.$$



$$\begin{aligned}
 & e^{2cz} (p-b(u-2k)) {}_2F_1\left(\frac{p-b(u-2k)}{2c} + 1, 1; \frac{p-b(u-2k)}{2c} + 2; e^{2cz}\right) \Big/ ((p-b(u-2k)) \\
 & (2c+p-b(u-2k))) + \left( e^{(p+b(u-2k))z} \left( (2c+p+b(u-2k)) {}_2F_1\left(\frac{p+b(u-2k)}{2c}, 1; \frac{p+b(u-2k)}{2c} + 1; \right. \right. \right. \\
 & \left. \left. \left. e^{2cz}\right) + e^{2cz} (p+b(u-2k)) {}_2F_1\left(\frac{p+b(u-2k)}{2c} + 1, 1; \frac{p+b(u-2k)}{2c} + 2; e^{2cz}\right) \right) \Big/ \right. \\
 & \left. ((p+b(u-2k))(2c+p+b(u-2k))) \right) - 2^{-m-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^s \binom{m}{k} \binom{u}{s} \\
 & \left( (-1)^u \left( e^{(-ia(m-2k)+p-b(u-2s))z} \left( (2c-ia(m-2k)+p-b(u-2s)) {}_2F_1\left(\frac{-ia(m-2k)+p-b(u-2s)}{2c}, \right. \right. \right. \right. \\
 & \left. \left. \left. 1; \frac{-ia(m-2k)+p-b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (-ia(m-2k)+p-b(u-2s)) \right. \right. \\
 & \left. \left. {}_2F_1\left(\frac{-ia(m-2k)+p-b(u-2s)}{2c} + 1, 1; \frac{-ia(m-2k)+p-b(u-2s)}{2c} + 2; e^{2cz}\right) \right) \right) \Big/ \\
 & ((-ia(m-2k)+p-b(u-2s))(2c-ia(m-2k)+p-b(u-2s))) + \\
 & \left( e^{(-ia(m-2k)+p+b(u-2s))z} \left( (2c-ia(m-2k)+p+b(u-2s)) {}_2F_1\left(\frac{-ia(m-2k)+p+b(u-2s)}{2c}, 1; \right. \right. \right. \\
 & \left. \left. \left. \frac{-ia(m-2k)+p+b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (-ia(m-2k)+p+b(u-2s)) {}_2F_1 \right. \right. \\
 & \left. \left. \left( \frac{-ia(m-2k)+p+b(u-2s)}{2c} + 1, 1; \frac{-ia(m-2k)+p+b(u-2s)}{2c} + 2; e^{2cz}\right) \right) \right) \Big/ \\
 & ((-ia(m-2k)+p+b(u-2s))(2c-ia(m-2k)+p+b(u-2s))) + \\
 & \left( e^{(ai(m-2k)+p+b(u-2s))z} \left( (2c+ai(m-2k)+p+b(u-2s)) {}_2F_1\left(\frac{ai(m-2k)+p+b(u-2s)}{2c}, 1; \right. \right. \right. \\
 & \left. \left. \left. \frac{ai(m-2k)+p+b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (ai(m-2k)+p+b(u-2s)) {}_2F_1 \right. \right. \\
 & \left. \left. \left( \frac{ai(m-2k)+p+b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k)+p+b(u-2s)}{2c} + 2; e^{2cz}\right) \right) \right) \Big/ \\
 & ((ai(m-2k)+p+b(u-2s))(2c+ai(m-2k)+p+b(u-2s))) + \\
 & \left( (-1)^u e^{(ai(m-2k)+p-b(u-2s))z} \left( (2c+ai(m-2k)+p-b(u-2s)) {}_2F_1\left(\frac{ai(m-2k)+p-b(u-2s)}{2c}, \right. \right. \right. \\
 & \left. \left. \left. 1; \frac{ai(m-2k)+p-b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (ai(m-2k)+p-b(u-2s)) {}_2F_1 \right. \right. \\
 & \left. \left. \left( \frac{ai(m-2k)+p-b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k)+p-b(u-2s)}{2c} + 2; e^{2cz}\right) \right) \right) \Big/ \\
 & ((ai(m-2k)+p-b(u-2s))(2c+ai(m-2k)+p-b(u-2s))) \Big/; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

01.22.21.0205.01

$$\int e^{pz} \cos^m(az) \sinh^\mu(cz) \coth(cz) dz = -2^{-m-\mu} (-e^{-cz} + e^{cz})^\mu (1 - e^{2cz})^{-\mu} \left( \frac{m}{2} \right) \left( \frac{e^{pz} {}_2F_1\left(\frac{p-c\mu}{2c}, 1-\mu; \frac{-\mu c+2c+p}{2c}; e^{2cz}\right)}{p-c\mu} + \frac{e^{(2c+p)z} {}_2F_1\left(\frac{-\mu c+2c+p}{2c}, 1-\mu; \frac{-\mu c+4c+p}{2c}; e^{2cz}\right)}{-\mu c+2c+p} \right) (1-m \bmod 2) - 2^{-m-\mu} (-e^{-cz} + e^{cz})^\mu (1 - e^{2cz})^{-\mu} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \frac{e^{(p-ia(m-2k))z} {}_2F_1\left(\frac{-ia(m-2k)+p-c\mu}{2c}, 1-\mu; \frac{-\mu c+2c-ia(m-2k)+p}{2c}; e^{2cz}\right)}{-ia(m-2k)+p-c\mu} + \frac{e^{(a i(m-2k)+p)z} {}_2F_1\left(\frac{a i(m-2k)+p-c\mu}{2c}, 1-\mu; \frac{-\mu c+2c+a i(m-2k)+p}{2c}; e^{2cz}\right)}{a i(m-2k)+p-c\mu} + \frac{e^{(2c+a i(m-2k)+p)z} {}_2F_1\left(\frac{-\mu c+2c+a i(m-2k)+p}{2c}, 1-\mu; \frac{-\mu c+4c+a i(m-2k)+p}{2c}; e^{2cz}\right)}{-\mu c+2c+a i(m-2k)+p} + \frac{e^{(2c-ia(m-2k)+p)z} {}_2F_1\left(\frac{-\mu c+2c-ia(m-2k)+p}{2c}, 1-\mu; \frac{-\mu c+4c-ia(m-2k)+p}{2c}; e^{2cz}\right)}{-\mu c+2c-ia(m-2k)+p} \right) /; m \in \mathbb{N}^+$$

Involving sin, cosh and exp

Involving  $e^{pz} \sin(az) \cosh(bz) \coth(cz)$

01.22.21.0206.01

$$\int e^{pz} \sin(az) \cosh(bz) \coth(cz) dz = \frac{1}{4} i \left( -\frac{e^{(-b-ia+p)z} {}_2F_1\left(\frac{-b-ia+p}{2c}, 1; \frac{-b+2c-ia+p}{2c}; e^{2cz}\right)}{-b-ia+p} + \frac{e^{(-b+ia+p)z} {}_2F_1\left(\frac{-b+ia+p}{2c}, 1; \frac{-b+2c+ia+p}{2c}; e^{2cz}\right)}{-b+ia+p} - \frac{e^{(b-ia+p)z} {}_2F_1\left(\frac{b-ia+p}{2c}, 1; \frac{b+2c-ia+p}{2c}; e^{2cz}\right)}{b-ia+p} + \frac{e^{(b+ia+p)z} {}_2F_1\left(\frac{b+ia+p}{2c}, 1; \frac{b+2c+ia+p}{2c}; e^{2cz}\right)}{b+ia+p} - \frac{e^{(-b+2c+ia+p)z} {}_2F_1\left(\frac{-b+2c+ia+p}{2c}, 1; \frac{-b+4c+ia+p}{2c}; e^{2cz}\right)}{-b+2c+ia+p} + \frac{e^{(b+2c+ia+p)z} {}_2F_1\left(\frac{b+2c+ia+p}{2c}, 1; \frac{b+4c+ia+p}{2c}; e^{2cz}\right)}{b+2c+ia+p} - \frac{e^{(-b+2c-ia+p)z} {}_2F_1\left(\frac{-b+2c-ia+p}{2c}, 1; \frac{-b+4c-ia+p}{2c}; e^{2cz}\right)}{-b+2c-ia+p} + \frac{e^{(b+2c-ia+p)z} {}_2F_1\left(\frac{b+2c-ia+p}{2c}, 1; \frac{b+4c-ia+p}{2c}; e^{2cz}\right)}{b+2c-ia+p} \right)$$

Involving powers of sin, powers of cosh and exp

Involving  $e^{pz} \sin^m(az) \cosh^u(bz) \coth(cz)$

01.22.21.0207.01

$$\int e^{pz} \sin^m(az) \cosh^u(bz) \coth(cz) dz = -2^{-m-u} \left(\frac{u}{2}\right) (1-u \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \left( e^{\frac{i\pi m}{2} + (p-ia(m-2k))z} \left( (2c-ia(m-2k)+p) {}_2F_1\left(\frac{p-ia(m-2k)}{2c}, 1; \frac{p-ia(m-2k)}{2c} + 1; e^{2cz}\right) + e^{2cz} (p-ia(m-2k)) {}_2F_1\left(\frac{p-ia(m-2k)}{2c} + 1, 1; \frac{p-ia(m-2k)}{2c} + 2; e^{2cz}\right) \right) / \right. \right.$$

$$\left. \left( (p-ia(m-2k))(2c-ia(m-2k)+p) + \left( e^{(ai(m-2k)+p)z - \frac{im\pi}{2}} \left( (2c+ai(m-2k)+p) {}_2F_1\left(\frac{ai(m-2k)+p}{2c}, 1; \frac{ai(m-2k)+p}{2c} + 1; e^{2cz}\right) + e^{2cz} (ai(m-2k)+p) {}_2F_1\left(\frac{ai(m-2k)+p}{2c} + 1, 1; \frac{ai(m-2k)+p}{2c} + 2; e^{2cz}\right) \right) \right) / \left( (ai(m-2k)+p)(2c+ai(m-2k)+p) \right) \right) -$$

$$\frac{1}{p(2c+p)} \left( 2^{-m-u} e^{pz} \left(\frac{m}{2}\right) \left(\frac{u}{2}\right) \left( (2c+p) {}_2F_1\left(\frac{p}{2c}, 1; \frac{p}{2c} + 1; e^{2cz}\right) + e^{2cz} p {}_2F_1\left(\frac{p}{2c} + 1, 1; \frac{p}{2c} + 2; e^{2cz}\right) \right) \right.$$

$$\left. (1-m \bmod 2)(1-u \bmod 2) \right) - 2^{-m-u} \left(\frac{m}{2}\right) (1-m \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{k} \left( \left( e^{(p+b(u-2k))z} \left( (2c+p+b(u-2k)) {}_2F_1\left(\frac{p+b(u-2k)}{2c}, 1; \frac{p+b(u-2k)}{2c} + 1; e^{2cz}\right) + e^{2cz} (p+b(u-2k)) {}_2F_1\left(\frac{p+b(u-2k)}{2c} + 1, 1; \frac{p+b(u-2k)}{2c} + 2; e^{2cz}\right) \right) / \left( (p+b(u-2k))(2c+p+b(u-2k)) \right) + \right.$$

$$\left( e^{(p-b(u-2k))z} \left( (2c+p-b(u-2k)) {}_2F_1\left(\frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c} + 1; e^{2cz}\right) + e^{2cz} (p-b(u-2k)) {}_2F_1\left(\frac{p-b(u-2k)}{2c} + 1, 1; \frac{p-b(u-2k)}{2c} + 2; e^{2cz}\right) \right) / \right.$$

$$\left. \left( (p-b(u-2k))(2c+p-b(u-2k)) \right) \right) - 2^{-m-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \binom{u}{s}$$

$$\left( \left( e^{\frac{i\pi m}{2} + (-ia(m-2k)+p+b(u-2s))z} \left( (2c-ia(m-2k)+p+b(u-2s)) {}_2F_1\left(\frac{-ia(m-2k)+p+b(u-2s)}{2c}, 1; \frac{-ia(m-2k)+p+b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (-ia(m-2k)+p+b(u-2s)) {}_2F_1\left(\frac{-ia(m-2k)+p+b(u-2s)}{2c} + 1, 1; \frac{-ia(m-2k)+p+b(u-2s)}{2c} + 2; e^{2cz}\right) \right) / \right.$$

$$\left. \left( (-ia(m-2k)+p+b(u-2s))(2c-ia(m-2k)+p+b(u-2s)) \right) + \left( e^{(ai(m-2k)+p+b(u-2s))z - \frac{im\pi}{2}} \left( (2c+ai(m-2k)+p+b(u-2s)) {}_2F_1\left(\frac{ai(m-2k)+p+b(u-2s)}{2c}, 1; \frac{ai(m-2k)+p+b(u-2s)}{2c} + 1; e^{2cz}\right) + e^{2cz} (ai(m-2k)+p+b(u-2s)) {}_2F_1\left(\frac{ai(m-2k)+p+b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k)+p+b(u-2s)}{2c} + 2; e^{2cz}\right) \right) \right)$$

$$\left( \frac{ai(m-2k)+p+b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k)+p+b(u-2s)}{2c} + 2; e^{2cz} \right) \Big/$$

$$((ai(m-2k)+p+b(u-2s))(2c+ai(m-2k)+p+b(u-2s))) +$$

$$\left( e^{\frac{i\pi m}{2}+(-ia(m-2k)+p-b(u-2s))z} \left( (2c-ia(m-2k)+p-b(u-2s)) {}_2F_1 \left( \frac{-ia(m-2k)+p-b(u-2s)}{2c}, \right. \right. \right.$$

$$\left. \left. 1; \frac{-ia(m-2k)+p-b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (-ia(m-2k)+p-b(u-2s)) {}_2F_1 \right.$$

$$\left. \left( \frac{-ia(m-2k)+p-b(u-2s)}{2c} + 1, 1; \frac{-ia(m-2k)+p-b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \Big/$$

$$((-ia(m-2k)+p-b(u-2s))(2c-ia(m-2k)+p-b(u-2s))) +$$

$$\left( e^{(ai(m-2k)+p-b(u-2s))z-\frac{i\pi m}{2}} \left( (2c+ai(m-2k)+p-b(u-2s)) {}_2F_1 \left( \frac{ai(m-2k)+p-b(u-2s)}{2c}, \right. \right. \right.$$

$$\left. \left. 1; \frac{ai(m-2k)+p-b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (ai(m-2k)+p-b(u-2s)) {}_2F_1 \right.$$

$$\left. \left( \frac{ai(m-2k)+p-b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k)+p-b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \Big/$$

$$((ai(m-2k)+p-b(u-2s))(2c+ai(m-2k)+p-b(u-2s))) \Big/; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.22.21.0208.01

$$\int e^{pz} \sin^m(az) \cosh^\mu(cz) \coth(cz) dz =$$

$$2^{-m} \binom{m}{\frac{m}{2}} (1-m \bmod 2) (1+e^{-2cz})^{-\mu} \cosh^\mu(cz) \left( \frac{e^{pz} {}_2F_1 \left( -\frac{p+c\mu}{2c}; -\mu, 1; \frac{(3-\mu)c-c-p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{(\mu-1)c+c+p} + \right.$$

$$\left. \frac{e^{(p-2c)z} {}_2F_1 \left( -\frac{\mu c-2c+p}{2c}; -\mu, 1; \frac{(3-\mu)c+c-p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{(\mu-1)c-c+p} \right) + 2^{-m} \cosh^\mu(cz) (1+e^{-2cz})^{-\mu} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k}$$

$$\left( \left( e^{\frac{i\pi m}{2}+(p-ia(m-2k))z} {}_2F_1 \left( -\frac{-ia(m-2k)+p+c\mu}{2c}; -\mu, 1; \frac{(3-\mu)c-c+ai(m-2k)-p}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) \Big/ \right.$$

$$\left( (\mu-1)c+c-ia(m-2k)+p \right) + \left( e^{(ai(m-2k)+p)z-\frac{i\pi m}{2}} {}_2F_1 \left( -\frac{ai(m-2k)+p+c\mu}{2c}; -\mu, 1; \right. \right.$$

$$\left. \left. \frac{(3-\mu)c-c-ia(m-2k)-p}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) \Big/ ((\mu-1)c+c+ai(m-2k)+p) +$$

$$\left( e^{(-2c+ai(m-2k)+p)z-\frac{i\pi m}{2}} {}_2F_1 \left( -\frac{\mu c-2c+ai(m-2k)+p}{2c}; -\mu, 1; \frac{(3-\mu)c+c-ia(m-2k)-p}{2c}; -e^{-2cz}, \right. \right.$$

$$\left. \left. e^{-2cz} \right) \right) \Big/ ((\mu-1)c-c+ai(m-2k)+p) + \left( e^{\frac{i\pi m}{2}+(-2c-ia(m-2k)+p)z} {}_2F_1 \left( -\frac{\mu c-2c-ia(m-2k)+p}{2c}; \right. \right.$$

$$\left. \left. -\mu, 1; \frac{(3-\mu)c+c+ai(m-2k)-p}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) \Big/ ((\mu-1)c-c-ia(m-2k)+p) \Big/; m \in \mathbb{N}^+$$

Involving cos, cosh and exp

### Involving $e^{pz} \cos(az) \cosh(bz) \coth(cz)$

01.22.21.0209.01

$$\int e^{pz} \cos(az) \cosh(bz) \coth(cz) dz =$$

$$\frac{1}{4} \left( \frac{e^{(-b-ia+p)z} {}_2F_1\left(\frac{-b-ia+p}{2c}, 1; \frac{-b+2c-ia+p}{2c}; e^{2cz}\right)}{-b-ia+p} - \frac{e^{(-b+ia+p)z} {}_2F_1\left(\frac{-b+ia+p}{2c}, 1; \frac{-b+2c+ia+p}{2c}; e^{2cz}\right)}{-b+ia+p} - \right.$$

$$\frac{e^{(b-ia+p)z} {}_2F_1\left(\frac{b-ia+p}{2c}, 1; \frac{b+2c-ia+p}{2c}; e^{2cz}\right)}{b-ia+p} - \frac{e^{(b+ia+p)z} {}_2F_1\left(\frac{b+ia+p}{2c}, 1; \frac{b+2c+ia+p}{2c}; e^{2cz}\right)}{b+ia+p} -$$

$$\frac{e^{(-b+2c-ia+p)z} {}_2F_1\left(\frac{-b+2c-ia+p}{2c}, 1; \frac{-b+4c-ia+p}{2c}; e^{2cz}\right)}{-b+2c-ia+p} - \frac{e^{(-b+2c+ia+p)z} {}_2F_1\left(\frac{-b+2c+ia+p}{2c}, 1; \frac{-b+4c+ia+p}{2c}; e^{2cz}\right)}{-b+2c+ia+p} -$$

$$\left. \frac{e^{(b+2c-ia+p)z} {}_2F_1\left(\frac{b+2c-ia+p}{2c}, 1; \frac{b+4c-ia+p}{2c}; e^{2cz}\right)}{b+2c-ia+p} - \frac{e^{(b+2c+ia+p)z} {}_2F_1\left(\frac{b+2c+ia+p}{2c}, 1; \frac{b+4c+ia+p}{2c}; e^{2cz}\right)}{b+2c+ia+p} \right)$$

### Involving powers of cos, powers of cosh and exp

### Involving $e^{pz} \cos^m(az) \cosh^u(bz) \coth(cz)$

01.22.21.0210.01

$$\int e^{pz} \cos^m(az) \cosh^u(bz) \coth(cz) dz = -2^{-m-u} \left(\frac{u}{2}\right) (1 - u \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(p-ia(m-2k))z} \left( (2c-ia(m-2k)+p) {}_2F_1\left(\frac{p-ia(m-2k)}{2c}, 1; \frac{p-ia(m-2k)}{2c} + 1; e^{2cz}\right) + \right.$$

$$\left. e^{2cz} (p-ia(m-2k)) {}_2F_1\left(\frac{p-ia(m-2k)}{2c} + 1, 1; \frac{p-ia(m-2k)}{2c} + 2; e^{2cz}\right) \right) \Big/$$

$$\left( (p-ia(m-2k))(2c-ia(m-2k)+p) + \left( e^{(ai(m-2k)+p)z} \left( (2c+ai(m-2k)+p) \right. \right.$$

$$\left. {}_2F_1\left(\frac{ai(m-2k)+p}{2c}, 1; \frac{ai(m-2k)+p}{2c} + 1; e^{2cz}\right) + e^{2cz} (ai(m-2k)+p) {}_2F_1\left(\frac{ai(m-2k)+p}{2c} + \right.$$

$$\left. \left. 1, 1; \frac{ai(m-2k)+p}{2c} + 2; e^{2cz}\right) \right) \Big/ ((ai(m-2k)+p)(2c+ai(m-2k)+p)) \right) -$$

$$\frac{1}{p(2c+p)} 2^{-m-u} e^{pz} \left(\frac{m}{2}\right) \left(\frac{u}{2}\right) \left( (2c+p) {}_2F_1\left(\frac{p}{2c}, 1; \frac{p}{2c} + 1; e^{2cz}\right) + e^{2cz} p {}_2F_1\left(\frac{p}{2c} + 1, 1; \frac{p}{2c} + 2; e^{2cz}\right) \right)$$

$$(1 - m \bmod 2)$$

$$(1 - u \bmod 2) - 2^{-m-u}$$

$$\left(\frac{m}{2}\right)$$

$$(1 - m \bmod 2)$$

$$\begin{aligned}
 & \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{k} \left( \left( e^{(p+b(u-2k))z} \left( (2c+p+b(u-2k)) {}_2F_1 \left( \frac{p+b(u-2k)}{2c}, 1; \frac{p+b(u-2k)}{2c} + 1; e^{2cz} \right) + e^{2cz} (p+b(u-2k)) \right. \right. \right. \\
 & \quad \left. \left. {}_2F_1 \left( \frac{p+b(u-2k)}{2c} + 1, 1; \frac{p+b(u-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / ((p+b(u-2k))(2c+p+b(u-2k))) + \\
 & \quad \left( e^{(p-b(u-2k))z} \left( (2c+p-b(u-2k)) {}_2F_1 \left( \frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c} + 1; e^{2cz} \right) + e^{2cz} (p-b(u-2k)) \right. \right. \\
 & \quad \left. \left. {}_2F_1 \left( \frac{p-b(u-2k)}{2c} + 1, 1; \frac{p-b(u-2k)}{2c} + 2; e^{2cz} \right) \right) \right) / ((p-b(u-2k))(2c+p-b(u-2k))) \Big) - \\
 & 2^{-m-u} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \binom{u}{s} \left( \left( e^{(-ia(m-2k)+p+b(u-2s))z} \left( (2c-ia(m-2k)+p+b(u-2s)) {}_2F_1 \left( \frac{-ia(m-2k)+p+b(u-2s)}{2c}, \right. \right. \right. \\
 & \quad \left. \left. 1; \frac{-ia(m-2k)+p+b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (-ia(m-2k)+p+b(u-2s)) {}_2F_1 \right. \\
 & \quad \left. \left( \frac{-ia(m-2k)+p+b(u-2s)}{2c} + 1, 1; \frac{-ia(m-2k)+p+b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \right) / \\
 & \quad ((-ia(m-2k)+p+b(u-2s))(2c-ia(m-2k)+p+b(u-2s))) + \\
 & \quad \left( e^{(ai(m-2k)+p+b(u-2s))z} \left( (2c+ai(m-2k)+p+b(u-2s)) {}_2F_1 \left( \frac{ai(m-2k)+p+b(u-2s)}{2c}, 1; \right. \right. \right. \\
 & \quad \left. \left. \frac{ai(m-2k)+p+b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (ai(m-2k)+p+b(u-2s)) {}_2F_1 \right. \\
 & \quad \left. \left( \frac{ai(m-2k)+p+b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k)+p+b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \right) / \\
 & \quad ((ai(m-2k)+p+b(u-2s))(2c+ai(m-2k)+p+b(u-2s))) + \\
 & \quad \left( e^{(-ia(m-2k)+p-b(u-2s))z} \left( (2c-ia(m-2k)+p-b(u-2s)) {}_2F_1 \left( \frac{-ia(m-2k)+p-b(u-2s)}{2c}, 1; \right. \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)+p-b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (-ia(m-2k)+p-b(u-2s)) {}_2F_1 \right. \\
 & \quad \left. \left( \frac{-ia(m-2k)+p-b(u-2s)}{2c} + 1, 1; \frac{-ia(m-2k)+p-b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \right) / \\
 & \quad ((-ia(m-2k)+p-b(u-2s))(2c-ia(m-2k)+p-b(u-2s))) + \\
 & \quad \left( e^{(ai(m-2k)+p-b(u-2s))z} \left( (2c+ai(m-2k)+p-b(u-2s)) {}_2F_1 \left( \frac{ai(m-2k)+p-b(u-2s)}{2c}, 1; \right. \right. \right. \\
 & \quad \left. \left. \frac{ai(m-2k)+p-b(u-2s)}{2c} + 1; e^{2cz} \right) + e^{2cz} (ai(m-2k)+p-b(u-2s)) {}_2F_1 \right. \\
 & \quad \left. \left( \frac{ai(m-2k)+p-b(u-2s)}{2c} + 1, 1; \frac{ai(m-2k)+p-b(u-2s)}{2c} + 2; e^{2cz} \right) \right) \right) / \\
 & \quad ((ai(m-2k)+p-b(u-2s))(2c+ai(m-2k)+p-b(u-2s))) \Big) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

01.22.21.0211.01

$$\int e^{pz} \cos^m(az) \cosh^\mu(cz) \coth(cz) dz =$$

$$2^{-m} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) (1 + e^{-2cz})^{-\mu} \cosh^\mu(cz) \left( \frac{e^{pz} F_1\left(-\frac{p+c\mu}{2c}; -\mu, 1; \frac{(3-\mu)c-c-p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{(\mu-1)c+c+p} + \frac{e^{(p-2c)z} F_1\left(-\frac{\mu c-2c+p}{2c}; -\mu, 1; \frac{(3-\mu)c+c-p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{(\mu-1)c-c+p} \right) + 2^{-m} \cosh^\mu(cz) (1 + e^{-2cz})^{-\mu} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \left( e^{(p-ia(m-2k))z} F_1\left(-\frac{-ia(m-2k)+p+c\mu}{2c}; -\mu, 1; \frac{(3-\mu)c-c+ai(m-2k)-p}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / \left( (\mu-1)c+c-ia(m-2k)+p \right) + \left( e^{ai(m-2k+p)z} F_1\left(-\frac{ai(m-2k)+p+c\mu}{2c}; -\mu, 1; \frac{(3-\mu)c-c-ia(m-2k)-p}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / \left( (\mu-1)c+c+ai(m-2k)+p \right) + \left( e^{(-2c+ai(m-2k+p))z} F_1\left(-\frac{\mu c-2c+ai(m-2k)+p}{2c}; -\mu, 1; \frac{(3-\mu)c+c-ia(m-2k)-p}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / \left( (\mu-1)c-c+ai(m-2k)+p \right) + \left( e^{(-2c-ia(m-2k+p))z} F_1\left(-\frac{\mu c-2c-ia(m-2k)+p}{2c}; -\mu, 1; \frac{(3-\mu)c+c+ai(m-2k)-p}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / \left( (\mu-1)c-c-ia(m-2k)+p \right) \right) /; m \in \mathbb{N}^+$$

Involving powers of sin, powers of tanh and exp

### Involving $e^{pZ} \sin^m(az) \tanh^\mu(cz) \coth(cz)$

01.22.21.0212.01

$$\int e^{pz} \sin^m(az) \tanh^\mu(cz) \coth(cz) dz = 2^{-m} (1 + e^{-2cz})^{\mu-1} \tanh^{\mu-1}(cz) (1 - e^{-2cz})^{1-\mu}$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \frac{e^{\frac{i\pi m}{2} + (p-ia(m-2k))z} F_1\left(-\frac{p-ia(m-2k)}{2c}; \mu-1, 1-\mu; 1 - \frac{p-ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p-ia(m-2k)} + \frac{e^{(ai(m-2k+p)z - \frac{i\pi m}{2})} F_1\left(-\frac{ai(m-2k+p)}{2c}; \mu-1, 1-\mu; 1 - \frac{ai(m-2k+p)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ai(m-2k)+p} \right) + \frac{1}{p} 2^{-m} e^{pz} (1 + e^{-2cz})^\mu F_1\left(-\frac{p}{2c}; \mu-1, 1-\mu; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz}\right) \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \tanh^\mu(cz) (1 - e^{-2cz})^{-\mu} /; m \in \mathbb{N}^+$$

Involving powers of cos, powers of tanh and exp

**Involving  $e^{pz} \cos^m(az) \tanh^\mu(cz) \coth(cz)$**

01.22.21.0213.01

$$\int e^{pz} \cos^m(az) \tanh^\mu(cz) \coth(cz) dz = 2^{-m} (1 + e^{-2cz})^{\mu-1} \tanh^{\mu-1}(cz) (1 - e^{-2cz})^{1-\mu}$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \frac{e^{(p-ia(m-2k))z} F_1\left(-\frac{p-ia(m-2k)}{2c}; \mu-1, 1-\mu; 1-\frac{p-ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p-ia(m-2k)} + \frac{e^{(a i(m-2k)+p)z} F_1\left(-\frac{a i(m-2k)+p}{2c}; \mu-1, 1-\mu; 1-\frac{a i(m-2k)+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{a i(m-2k)+p} \right) + \frac{1}{p} 2^{-m} e^{pz} (1 + e^{-2cz})^\mu$$

$$\left(\frac{m}{2}\right) (1 - m \bmod 2) \tanh^\mu(cz) (1 - e^{-2cz})^{-\mu} F_1\left(-\frac{p}{2c}; \mu-1, 1-\mu; 1-\frac{p}{2c}; -e^{-2cz}, e^{-2cz}\right); m \in \mathbb{N}^+$$

**Involving hyperbolic, trigonometric and a power functions**

Involving sin, sinh and power

**Involving  $z^n \sin(az) \sinh(bz) \coth(cz)$**



01.22.21.0214.01

$$\int z^n \sin(a z) \sinh(b z) \coth(c z) dz =$$

$$-\frac{1}{4} i n! \left( e^{(i a-b) z} \sum_{j=0}^n \frac{(-1)^j (i a-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i a-b}{2 c}, \dots, \frac{i a-b}{2 c}, 1; \frac{i a-b}{2 c} + 1, \dots, \frac{i a-b}{2 c} + 1; e^{2 c z} \right) + \right.$$

$$e^{(b-i a) z} \sum_{j=0}^n \frac{(-1)^j (b-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-i a}{2 c}, \dots, \frac{b-i a}{2 c}, 1; \frac{b-i a}{2 c} + 1, \dots, \frac{b-i a}{2 c} + 1; e^{2 c z} \right) +$$

$$e^{(-b+2 c+i a) z} \sum_{j=0}^n \frac{(-1)^j (-b+2 c+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2 c+i a}{2 c}, \dots, \frac{-b+2 c+i a}{2 c}, 1; \right.$$

$$\left. \frac{-b+2 c+i a}{2 c} + 1, \dots, \frac{-b+2 c+i a}{2 c} + 1; e^{2 c z} \right) + e^{(b+2 c-i a) z} \sum_{j=0}^n \frac{(-1)^j (b+2 c-i a)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{b+2 c-i a}{2 c}, \dots, \frac{b+2 c-i a}{2 c}, 1; \frac{b+2 c-i a}{2 c} + 1, \dots, \frac{b+2 c-i a}{2 c} + 1; e^{2 c z} \right) -$$

$$e^{(-b-i a) z} \sum_{j=0}^n \frac{(-1)^j (-b-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-i a}{2 c}, \dots, \frac{-b-i a}{2 c}, 1; \frac{-b-i a}{2 c} + 1, \dots, \frac{-b-i a}{2 c} + 1; e^{2 c z} \right) -$$

$$e^{(b+i a) z} \sum_{j=0}^n \frac{(-1)^j (b+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+i a}{2 c}, \dots, \frac{b+i a}{2 c}, 1; \frac{b+i a}{2 c} + 1, \dots, \frac{b+i a}{2 c} + 1; e^{2 c z} \right) -$$

$$e^{(-b+2 c-i a) z} \sum_{j=0}^n \frac{(-1)^j (-b+2 c-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2 c-i a}{2 c}, \dots, \frac{-b+2 c-i a}{2 c}, 1; \right.$$

$$\left. \frac{-b+2 c-i a}{2 c} + 1, \dots, \frac{-b+2 c-i a}{2 c} + 1; e^{2 c z} \right) - e^{(b+2 c+i a) z} \sum_{j=0}^n \frac{(-1)^j (b+2 c+i a)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{b+2 c+i a}{2 c}, \dots, \frac{b+2 c+i a}{2 c}, 1; \frac{b+2 c+i a}{2 c} + 1, \dots, \frac{b+2 c+i a}{2 c} + 1; e^{2 c z} \right) \Big/ ; n \in \mathbb{N}$$

Involving powers of sin, powers of sinh and power

Involving  $z^n \sin^m(a z) \sinh^u(b z) \coth(c z)$

01.22.21.0215.01

$$\int z^n \sin^m(a z) \sinh^u(b z) \coth(c z) dz = -i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)$$

$$(1 - u \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + 2 e^{2 c z} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \dots, 1; 2, \dots, 2; e^{2 c z}) \right) -$$

$$i^u 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{\frac{i m \pi}{2}} \left( e^{-i a(m-2 k) z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2 k))^{-j-1} z^{n-j}}{(n-j)!} \right. \right.$$

$$\begin{aligned}
 & {}_{j+2}F_{j+1}\left(-\frac{ia(m-2k)}{2c}, \dots, -\frac{ia(m-2k)}{2c}, 1; 1-\frac{ia(m-2k)}{2c}, \dots, 1-\frac{ia(m-2k)}{2c}; e^{2cz}\right) + \\
 & e^{(2c-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c-ia(m-2k)}{2c}, \dots, \right. \\
 & \left. \frac{2c-ia(m-2k)}{2c}, 1; \frac{2c-ia(m-2k)}{2c}+1, \dots, \frac{2c-ia(m-2k)}{2c}+1; e^{2cz}\right) + \\
 & e^{-\frac{1}{2}im\pi} \left( e^{ia(m-2k)z} \sum_{j=0}^n \frac{(-1)^j (ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{ia(m-2k)}{2c}, \dots, \frac{ia(m-2k)}{2c}, \right. \right. \\
 & \left. \left. 1; \frac{ia(m-2k)}{2c}+1, \dots, \frac{ia(m-2k)}{2c}+1; e^{2cz}\right) + \right. \\
 & \left. e^{(2c+ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c+ia(m-2k)}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{2c+ia(m-2k)}{2c}, 1; \frac{2c+ia(m-2k)}{2c}+1, \dots, \frac{2c+ia(m-2k)}{2c}+1; e^{2cz}\right) \right) - \\
 & i^u 2^{-m-u} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^i \binom{u}{i} \left( e^{\frac{i\pi u}{2}} \left( e^{-b(u-2i)z} \sum_{j=0}^n \frac{(-1)^j (-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \left. \left. {}_{j+2}F_{j+1}\left(-\frac{b(u-2i)}{2c}, \dots, -\frac{b(u-2i)}{2c}, 1; 1-\frac{b(u-2i)}{2c}, \dots, 1-\frac{b(u-2i)}{2c}; e^{2cz}\right) + \right. \right. \\
 & \left. \left. e^{(2c-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c-b(u-2i)}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{2c-b(u-2i)}{2c}, 1; \frac{2c-b(u-2i)}{2c}+1, \dots, \frac{2c-b(u-2i)}{2c}+1; e^{2cz}\right) + \right. \\
 & \left. e^{-\frac{1}{2}i\pi u} \left( e^{b(u-2i)z} \sum_{j=0}^n \frac{(-1)^j (b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b(u-2i)}{2c}, \dots, \frac{b(u-2i)}{2c}, 1; \frac{b(u-2i)}{2c}+1, \right. \right. \\
 & \left. \left. \dots, \frac{b(u-2i)}{2c}+1; e^{2cz}\right) + e^{(2c+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left. \left(\frac{2c+b(u-2i)}{2c}, \dots, \frac{2c+b(u-2i)}{2c}, 1; \frac{2c+b(u-2i)}{2c}+1, \dots, \frac{2c+b(u-2i)}{2c}+1; e^{2cz}\right) \right) \right) - \\
 & i^u 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{i+k} \binom{m}{k} \binom{u}{i} \left( e^{\frac{1}{2}i\pi(m+u)} \left( e^{(-ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \left. \left. {}_{j+2}F_{j+1}\left(\frac{-ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{-ia(m-2k)-b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{-ia(m-2k)-b(u-2i)}{2c}+1, \dots, \frac{-ia(m-2k)-b(u-2i)}{2c}+1; e^{2cz}\right) + \right.
 \end{aligned}$$

$$\begin{aligned}
 & e^{(2c-ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left( \frac{2c-ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{2c-ia(m-2k)-b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{2c-ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{2c-ia(m-2k)-b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{1}{2}i\pi(u-m)} \left( e^{(ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)-b(u-2i)}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{ia(m-2k)-b(u-2i)}{2c}, 1; \frac{ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)-b(u-2i)}{2c} + 1; \right. \right. \\
 & \left. \left. e^{2cz} \right) + e^{(2c+ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{2c+ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{2c+ia(m-2k)-b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{2c+ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{2c+ia(m-2k)-b(u-2i)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{\frac{1}{2}i\pi(m-u)} \left( e^{(b(u-2i)-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (b(u-2i)-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2i)-ia(m-2k)}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{b(u-2i)-ia(m-2k)}{2c}, 1; \frac{b(u-2i)-ia(m-2k)}{2c} + 1, \dots, \frac{b(u-2i)-ia(m-2k)}{2c} + 1; \right. \right. \\
 & \left. \left. e^{2cz} \right) + e^{(2c-ia(m-2k)+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{2c-ia(m-2k)+b(u-2i)}{2c}, \dots, \frac{2c-ia(m-2k)+b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{2c-ia(m-2k)+b(u-2i)}{2c} + 1, \dots, \frac{2c-ia(m-2k)+b(u-2i)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{-\frac{1}{2}i\pi(m+u)} \left( e^{(ai(m-2k)+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left( \frac{ia(m-2k)+b(u-2i)}{2c}, \dots, \frac{ia(m-2k)+b(u-2i)}{2c}, 1; \frac{ia(m-2k)+b(u-2i)}{2c} + 1, \right. \\
 & \left. \dots, \frac{ia(m-2k)+b(u-2i)}{2c} + 1; e^{2cz} \right) + e^{(2c+ia(m-2k)+b(u-2i))z} \\
 & \sum_{j=0}^n \frac{(-1)^j (2c+ia(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+ia(m-2k)+b(u-2i)}{2c}, \right. \\
 & \left. \dots, \frac{2c+ia(m-2k)+b(u-2i)}{2c}, 1; \frac{2c+ia(m-2k)+b(u-2i)}{2c} + 1, \right.
 \end{aligned}$$

Involving cos, sinh and power

**Involving  $z^n \cos(a z) \sinh(b z) \coth(c z)$**

01.22.21.0216.01

$$\int z^n \cos(a z) \sinh(b z) \coth(c z) dz =$$

$$-\frac{1}{4} n! \left( -e^{(i a-b) z} \sum_{j=0}^n \frac{(-1)^j (i a-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i a-b}{2 c}, \dots, \frac{i a-b}{2 c}, 1; \frac{i a-b}{2 c} + 1, \dots, \frac{i a-b}{2 c} + 1; e^{2 c z} \right) + \right.$$

$$e^{(b-i a) z} \sum_{j=0}^n \frac{(-1)^j (b-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-i a}{2 c}, \dots, \frac{b-i a}{2 c}, 1; \frac{b-i a}{2 c} + 1, \dots, \frac{b-i a}{2 c} + 1; e^{2 c z} \right) -$$

$$e^{(-b+2 c+i a) z} \sum_{j=0}^n \frac{(-1)^j (-b+2 c+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2 c+i a}{2 c}, \dots, \frac{-b+2 c+i a}{2 c}, 1; \right.$$

$$\left. \frac{-b+2 c+i a}{2 c} + 1, \dots, \frac{-b+2 c+i a}{2 c} + 1; e^{2 c z} \right) + e^{(b+2 c-i a) z} \sum_{j=0}^n \frac{(-1)^j (b+2 c-i a)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{b+2 c-i a}{2 c}, \dots, \frac{b+2 c-i a}{2 c}, 1; \frac{b+2 c-i a}{2 c} + 1, \dots, \frac{b+2 c-i a}{2 c} + 1; e^{2 c z} \right) -$$

$$e^{(-b-i a) z} \sum_{j=0}^n \frac{(-1)^j (-b-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-i a}{2 c}, \dots, \frac{-b-i a}{2 c}, 1; \frac{-b-i a}{2 c} + 1, \dots, \frac{-b-i a}{2 c} + 1; e^{2 c z} \right) +$$

$$e^{(b+i a) z} \sum_{j=0}^n \frac{(-1)^j (b+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+i a}{2 c}, \dots, \frac{b+i a}{2 c}, 1; \frac{b+i a}{2 c} + 1, \dots, \frac{b+i a}{2 c} + 1; e^{2 c z} \right) -$$

$$e^{(-b+2 c-i a) z} \sum_{j=0}^n \frac{(-1)^j (-b+2 c-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2 c-i a}{2 c}, \dots, \frac{-b+2 c-i a}{2 c}, 1; \right.$$

$$\left. \frac{-b+2 c-i a}{2 c} + 1, \dots, \frac{-b+2 c-i a}{2 c} + 1; e^{2 c z} \right) + e^{(b+2 c+i a) z} \sum_{j=0}^n \frac{(-1)^j (b+2 c+i a)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{b+2 c+i a}{2 c}, \dots, \frac{b+2 c+i a}{2 c}, 1; \frac{b+2 c+i a}{2 c} + 1, \dots, \frac{b+2 c+i a}{2 c} + 1; e^{2 c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of cos, powers of sinh and power

**Involving  $z^n \cos^m(a z) \sinh^u(b z) \coth(c z)$**

01.22.21.0217.01

$$\int z^n \cos^m(a z) \sinh^u(b z) \coth(c z) dz = -i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)$$

$$(1 - u \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + 2 e^{2 c z} \sum_{j=0}^n \frac{(-1)^j (2 c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} (1, \dots, 1; 2, \dots, 2; e^{2 c z}) \right) -$$

$$\begin{aligned}
 & i^u 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1-u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{-ia(m-2k)z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( -\frac{ia(m-2k)}{2c}, \dots, -\frac{ia(m-2k)}{2c}, 1; 1 - \frac{ia(m-2k)}{2c}, \dots, 1 - \frac{ia(m-2k)}{2c}; e^{2cz} \right) + \right. \\
 & \quad e^{ia(m-2k)z} \sum_{j=0}^n \frac{(-1)^j (ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)}{2c}, \dots, \frac{ia(m-2k)}{2c}, 1; \frac{ia(m-2k)}{2c} + 1, \right. \\
 & \quad \left. \dots, \frac{ia(m-2k)}{2c} + 1; e^{2cz} \right) + e^{(2c-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \quad \left( \frac{2c-ia(m-2k)}{2c}, \dots, \frac{2c-ia(m-2k)}{2c}, 1; \frac{2c-ia(m-2k)}{2c} + 1, \dots, \frac{2c-ia(m-2k)}{2c} + 1; e^{2cz} \right) + \\
 & \quad \left. e^{(2c+ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+ia(m-2k)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{2c+ia(m-2k)}{2c}, 1; \frac{2c+ia(m-2k)}{2c} + 1, \dots, \frac{2c+ia(m-2k)}{2c} + 1; e^{2cz} \right) \right) - \\
 & 2^{-m-u} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( e^{b(u-2k)z} \sum_{j=0}^n \frac{(-1)^j (b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{b(u-2k)}{2c}, \dots, \frac{b(u-2k)}{2c}, 1; \frac{b(u-2k)}{2c} + 1, \dots, \frac{b(u-2k)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. (-1)^u \left( e^{-b(u-2k)z} \sum_{j=0}^n \frac{(-1)^j (-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( -\frac{b(u-2k)}{2c}, \dots, -\frac{b(u-2k)}{2c}, 1; 1 - \frac{b(u-2k)}{2c}, \right. \right. \right. \\
 & \quad \left. \left. \dots, 1 - \frac{b(u-2k)}{2c}; e^{2cz} \right) + e^{(2c-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \quad \left. \left( \frac{2c-b(u-2k)}{2c}, \dots, \frac{2c-b(u-2k)}{2c}, 1; \frac{2c-b(u-2k)}{2c} + 1, \dots, \frac{2c-b(u-2k)}{2c} + 1; e^{2cz} \right) \right) + \\
 & \quad \left. e^{(2c+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+b(u-2k)}{2c}, \dots, \frac{2c+b(u-2k)}{2c}, \right. \right. \\
 & \quad \left. \left. 1; \frac{2c+b(u-2k)}{2c} + 1, \dots, \frac{2c+b(u-2k)}{2c} + 1; e^{2cz} \right) \right) - \\
 & 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^i \binom{m}{k} \binom{u}{i} \left( (-1)^u \left( e^{(-ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{-ia(m-2k)-b(u-2i)}{2c}, 1; \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{-i a(m-2 k)-b(u-2 i)}{2 c}+1, \dots, \frac{-i a(m-2 k)-b(u-2 i)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & e^{(2 c-i a(m-2 k)-b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j(2 c-i a(m-2 k)-b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \\
 & \left( \frac{2 c-i a(m-2 k)-b(u-2 i)}{2 c}, \dots, \frac{2 c-i a(m-2 k)-b(u-2 i)}{2 c}, 1 ; \right. \\
 & \left. \frac{2 c-i a(m-2 k)-b(u-2 i)}{2 c}+1, \dots, \frac{2 c-i a(m-2 k)-b(u-2 i)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & e^{(i a(m-2 k)-b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j(i a(m-2 k)-b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)-b(u-2 i)}{2 c}, \right. \\
 & \dots, \frac{i a(m-2 k)-b(u-2 i)}{2 c}, 1 ; \frac{i a(m-2 k)-b(u-2 i)}{2 c}+1, \dots, \frac{i a(m-2 k)-b(u-2 i)}{2 c}+1 ; \\
 & \left. e^{2 c z}\right)+ e^{(2 c+a i(m-2 k)-b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j(2 c+a i(m-2 k)-b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2} F_{j+1} \left( \frac{2 c+i a(m-2 k)-b(u-2 i)}{2 c}, \dots, \frac{2 c+i a(m-2 k)-b(u-2 i)}{2 c}, 1 ; \right. \\
 & \left. \frac{2 c+i a(m-2 k)-b(u-2 i)}{2 c}+1, \dots, \frac{2 c+i a(m-2 k)-b(u-2 i)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & e^{(b(u-2 i)-i a(m-2 k)) z} \sum_{j=0}^n \frac{(-1)^j(b(u-2 i)-i a(m-2 k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{b(u-2 i)-i a(m-2 k)}{2 c}, \dots, \right. \\
 & \left. \frac{b(u-2 i)-i a(m-2 k)}{2 c}, 1 ; \frac{b(u-2 i)-i a(m-2 k)}{2 c}+1, \dots, \frac{b(u-2 i)-i a(m-2 k)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & e^{(2 c-i a(m-2 k)+b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j(2 c-i a(m-2 k)+b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2} F_{j+1} \left( \frac{2 c-i a(m-2 k)+b(u-2 i)}{2 c}, \dots, \frac{2 c-i a(m-2 k)+b(u-2 i)}{2 c}, 1 ; \right. \\
 & \left. \frac{2 c-i a(m-2 k)+b(u-2 i)}{2 c}+1, \dots, \frac{2 c-i a(m-2 k)+b(u-2 i)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & e^{(a i(m-2 k)+b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j(a i(m-2 k)+b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)+b(u-2 i)}{2 c}, \dots, \right. \\
 & \left. \frac{i a(m-2 k)+b(u-2 i)}{2 c}, 1 ; \frac{i a(m-2 k)+b(u-2 i)}{2 c}+1, \dots, \frac{i a(m-2 k)+b(u-2 i)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & e^{(2 c+a i(m-2 k)+b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j(2 c+a i(m-2 k)+b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \\
 & \left( \frac{2 c+i a(m-2 k)+b(u-2 i)}{2 c}, \dots, \frac{2 c+i a(m-2 k)+b(u-2 i)}{2 c}, 1 ; \frac{2 c+i a(m-2 k)+b(u-2 i)}{2 c}+1, \dots, \right. \\
 & \left. \frac{2 c+i a(m-2 k)+b(u-2 i)}{2 c}+1 ; e^{2 c z}\right) .
 \end{aligned}$$

Involving sin, cosh and power

Involving  $z^n \sin(az) \cosh(bz) \coth(cz)$

01.22.21.0218.01

$$\int z^n \sin(az) \cosh(bz) \coth(cz) dz =$$

$$-\frac{i}{4} n! \left( -e^{(b+ia)z} \sum_{j=0}^n \frac{(-1)^j (b+ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+ia}{2c}, \dots, \frac{b+ia}{2c}, 1; \frac{b+ia}{2c} + 1, \dots, \frac{b+ia}{2c} + 1; e^{2cz} \right) + \right.$$

$$e^{(-b-ia)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-ia}{2c}, \dots, \frac{-b-ia}{2c}, 1; \frac{-b-ia}{2c} + 1, \dots, \frac{-b-ia}{2c} + 1; e^{2cz} \right) -$$

$$e^{(ia-b)z} \sum_{j=0}^n \frac{(-1)^j (ia-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b}{2c}, \dots, \frac{ia-b}{2c}, 1; \frac{ia-b}{2c} + 1, \dots, \frac{ia-b}{2c} + 1; e^{2cz} \right) +$$

$$e^{(b-ia)z} \sum_{j=0}^n \frac{(-1)^j (b-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-ia}{2c}, \dots, \frac{b-ia}{2c}, 1; \frac{b-ia}{2c} + 1, \dots, \frac{b-ia}{2c} + 1; e^{2cz} \right) +$$

$$e^{(-b+2c-ia)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c-ia}{2c}, \dots, \frac{-b+2c-ia}{2c}, 1; \right.$$

$$\left. \frac{-b+2c-ia}{2c} + 1, \dots, \frac{-b+2c-ia}{2c} + 1; e^{2cz} \right) - e^{(-b+2c+ia)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c+ia)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{-b+2c+ia}{2c}, \dots, \frac{-b+2c+ia}{2c}, 1; \frac{-b+2c+ia}{2c} + 1, \dots, \frac{-b+2c+ia}{2c} + 1; e^{2cz} \right) +$$

$$e^{(b+2c-ia)z} \sum_{j=0}^n \frac{(-1)^j (b+2c-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c-ia}{2c}, \dots, \frac{b+2c-ia}{2c}, 1; \frac{b+2c-ia}{2c} + 1, \right.$$

$$\left. \dots, \frac{b+2c-ia}{2c} + 1; e^{2cz} \right) - e^{(b+2c+ia)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+ia)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{b+2c+ia}{2c}, \dots, \frac{b+2c+ia}{2c}, 1; \frac{b+2c+ia}{2c} + 1, \dots, \frac{b+2c+ia}{2c} + 1; e^{2cz} \right) \right) /; n \in \mathbb{N}$$

01.22.21.0219.01

$$\int z^n \sin(a z) \cosh(c z) \coth(c z) dz =$$

$$-\frac{1}{2} e^{c z} i n! \left( e^{-i a z} \sum_{j=0}^n \frac{(-1)^j (c-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c-i a}{2 c}, \dots, \frac{c-i a}{2 c}, 1; \frac{c-i a}{2 c} + 1, \dots, \frac{c-i a}{2 c} + 1; e^{2 c z} \right) - \right.$$

$$\left. e^{i a z} \sum_{j=0}^n \frac{(-1)^j (c+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c+i a}{2 c}, \dots, \frac{c+i a}{2 c}, 1; \frac{c+i a}{2 c} + 1, \dots, \frac{c+i a}{2 c} + 1; e^{2 c z} \right) \right) - \frac{1}{4} i e^{c z} n!$$

$$\left( -e^{(i a-2 c) z} \sum_{j=0}^n \frac{(-1)^j (i a-c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i a-c}{2 c}, \dots, \frac{i a-c}{2 c}, 1; \frac{i a-c}{2 c} + 1, \dots, \frac{i a-c}{2 c} + 1; e^{2 c z} \right) + e^{(-2 c-i a) z} \right.$$

$$\sum_{j=0}^n \frac{(-1)^j (-c-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-c-i a}{2 c}, \dots, \frac{-c-i a}{2 c}, 1; \frac{-c-i a}{2 c} + 1, \dots, \frac{-c-i a}{2 c} + 1; e^{2 c z} \right) - e^{(2 c+i a) z}$$

$$\sum_{j=0}^n \frac{(-1)^j (3 c+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{3 c+i a}{2 c}, \dots, \frac{3 c+i a}{2 c}, 1; \frac{3 c+i a}{2 c} + 1, \dots, \frac{3 c+i a}{2 c} + 1; e^{2 c z} \right) + e^{(2 c-i a) z}$$

$$\left. \sum_{j=0}^n \frac{(-1)^j (3 c-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{3 c-i a}{2 c}, \dots, \frac{3 c-i a}{2 c}, 1; \frac{3 c-i a}{2 c} + 1, \dots, \frac{3 c-i a}{2 c} + 1; e^{2 c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of sin, powers of cosh and power

### Involving $z^n \sin^m(a z) \cosh^u(c z) \coth(c z)$

01.22.21.0220.01

$$\int z^n \sin^m(a z) \cosh^u(b z) \coth(c z) dz = -2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)$$

$$(1 - u \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + 2 e^{2 c z} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \dots, 1; 2, \dots, 2; e^{2 c z}) \right) -$$

$$2^{-m-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{\frac{i m \pi}{2}} \left( e^{-i a(m-2 k) z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2 k))^{-j-1} z^{n-j}}{(n-j)!} \right. \right.$$

$$\left. {}_{j+2}F_{j+1} \left( -\frac{i a(m-2 k)}{2 c}, \dots, -\frac{i a(m-2 k)}{2 c}, 1; 1 - \frac{i a(m-2 k)}{2 c}, \dots, 1 - \frac{i a(m-2 k)}{2 c}; e^{2 c z} \right) + \right.$$

$$\left. e^{(2 c-i a(m-2 k) z} \sum_{j=0}^n \frac{(-1)^j (2 c-i a(m-2 k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2 c-i a(m-2 k)}{2 c}, \dots, \right. \right.$$

$$\left. \left. \frac{2 c-i a(m-2 k)}{2 c}, 1; \frac{2 c-i a(m-2 k)}{2 c} + 1, \dots, \frac{2 c-i a(m-2 k)}{2 c} + 1; e^{2 c z} \right) \right) +$$

$$e^{-\frac{1}{2} i m \pi} \left( e^{i a(m-2 k) z} \sum_{j=0}^n \frac{(-1)^j (i a(m-2 k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i a(m-2 k)}{2 c}, \dots, \frac{i a(m-2 k)}{2 c}, \right. \right.$$



$$\begin{aligned}
 & \left. 1; \frac{ia(m-2k)}{2c} + 1, \dots, \frac{ia(m-2k)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c+ai(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+ai(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+ia(m-2k)}{2c}, \dots, \right. \\
 & \left. \frac{2c+ia(m-2k)}{2c}, 1; \frac{2c+ia(m-2k)}{2c} + 1, \dots, \frac{2c+ia(m-2k)}{2c} + 1; e^{2cz} \right) \Bigg) - \\
 & 2^{-m-u} \left( \frac{m}{2} \right) n! (1-m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{i} \left( e^{-b(u-2i)z} \sum_{j=0}^n \frac{(-1)^j (-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left( -\frac{b(u-2i)}{2c}, \dots, -\frac{b(u-2i)}{2c}, 1; 1 - \frac{b(u-2i)}{2c}, \dots, 1 - \frac{b(u-2i)}{2c}; e^{2cz} \right) + e^{b(u-2i)z} \\
 & \sum_{j=0}^n \frac{(-1)^j (b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2i)}{2c}, \dots, \frac{b(u-2i)}{2c}, 1; \frac{b(u-2i)}{2c} + 1, \dots, \frac{b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-b(u-2i)}{2c}, \dots, \frac{2c-b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{2c-b(u-2i)}{2c} + 1, \dots, \frac{2c-b(u-2i)}{2c} + 1; e^{2cz} \right) + e^{(2c+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{2c+b(u-2i)}{2c}, \dots, \frac{2c+b(u-2i)}{2c}, 1; \frac{2c+b(u-2i)}{2c} + 1, \dots, \frac{2c+b(u-2i)}{2c} + 1; e^{2cz} \right) \right) - \\
 & 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \binom{u}{i} \left( e^{\frac{i\pi m}{2}} \left( e^{(-ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{-ia(m-2k)-b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{-ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{-ia(m-2k)-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(2c-ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left( \frac{2c-ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{2c-ia(m-2k)-b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{2c-ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{2c-ia(m-2k)-b(u-2i)}{2c} + 1; e^{2cz} \right) \Bigg) + \\
 & e^{-\frac{1}{2}i\pi m} \left( e^{(ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)-b(u-2i)}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{ia(m-2k)-b(u-2i)}{2c}, 1; \frac{ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)-b(u-2i)}{2c} + 1; \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & e^{2cz} \Bigg) + e^{(2c+ai(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+ai(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2c+ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{2c+ia(m-2k)-b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{2c+ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{2c+ia(m-2k)-b(u-2i)}{2c} + 1; e^{2cz} \right) \Bigg) + \\
 & e^{\frac{i\pi m}{2}} \left( e^{(b(u-2i)-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (b(u-2i)-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2i)-ia(m-2k)}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{b(u-2i)-ia(m-2k)}{2c}, 1; \frac{b(u-2i)-ia(m-2k)}{2c} + 1, \dots, \frac{b(u-2i)-ia(m-2k)}{2c} + 1; \right. \right. \\
 & \left. \left. e^{2cz} \right) + e^{(2c-ia(m-2k)+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{2c-ia(m-2k)+b(u-2i)}{2c}, \dots, \frac{2c-ia(m-2k)+b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{2c-ia(m-2k)+b(u-2i)}{2c} + 1, \dots, \frac{2c-ia(m-2k)+b(u-2i)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{-\frac{1}{2}i\pi m} \left( e^{(ai(m-2k)+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+b(u-2i)}{2c}, \right. \right. \\
 & \left. \left. \frac{ia(m-2k)+b(u-2i)}{2c}, \dots, \frac{ia(m-2k)+b(u-2i)}{2c}, 1; \frac{ia(m-2k)+b(u-2i)}{2c} + 1, \right. \right. \\
 & \left. \left. \dots, \frac{ia(m-2k)+b(u-2i)}{2c} + 1; e^{2cz} \right) + e^{(2c+ai(m-2k)+b(u-2i))z} \right. \\
 & \left. \sum_{j=0}^n \frac{(-1)^j (2c+ai(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+ia(m-2k)+b(u-2i)}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{2c+ia(m-2k)+b(u-2i)}{2c}, 1; \frac{2c+ia(m-2k)+b(u-2i)}{2c} + 1, \right. \right. \\
 & \left. \left. \dots, \frac{2c+ia(m-2k)+b(u-2i)}{2c} + 1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

Involving cos, cosh and power

Involving  $z^n \cos(az) \cosh(bz) \coth(cz)$

01.22.21.0221.01

$$\int z^n \cos(az) \cosh(bz) \coth(cz) dz =$$

$$\begin{aligned}
 & -\frac{1}{4} n! \left( e^{(-b-ia)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-b}{2c}, \dots, \frac{-ia-b}{2c}, 1; \frac{-ia-b}{2c} + 1, \dots, \frac{-ia-b}{2c} + 1; e^{2cz} \right) + \right. \\
 & e^{(ia-b)z} \sum_{j=0}^n \frac{(-1)^j (ia-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b}{2c}, \dots, \frac{ia-b}{2c}, 1; \frac{ia-b}{2c} + 1, \dots, \frac{ia-b}{2c} + 1; e^{2cz} \right) + \\
 & e^{(b-ia)z} \sum_{j=0}^n \frac{(-1)^j (b-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-ia}{2c}, \dots, \frac{b-ia}{2c}, 1; \frac{b-ia}{2c} + 1, \dots, \frac{b-ia}{2c} + 1; e^{2cz} \right) + \\
 & e^{(b+ia)z} \sum_{j=0}^n \frac{(-1)^j (b+ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b}{2c}, \dots, \frac{ia+b}{2c}, 1; \frac{ia+b}{2c} + 1, \dots, \frac{ia+b}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-b+2c-ia)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-b+2c}{2c}, \dots, \frac{-ia-b+2c}{2c}, 1; \right. \\
 & \left. \frac{-ia-b+2c}{2c} + 1, \dots, \frac{-ia-b+2c}{2c} + 1; e^{2cz} \right) + e^{(-b+2c+ia)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c+ia)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{ia-b+2c}{2c}, \dots, \frac{ia-b+2c}{2c}, 1; \frac{ia-b+2c}{2c} + 1, \dots, \frac{ia-b+2c}{2c} + 1; e^{2cz} \right) + \\
 & e^{(b+2c-ia)z} \sum_{j=0}^n \frac{(-1)^j (b+2c-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia+b+2c}{2c}, \dots, \frac{-ia+b+2c}{2c}, 1; \right. \\
 & \left. \frac{-ia+b+2c}{2c} + 1, \dots, \frac{-ia+b+2c}{2c} + 1; e^{2cz} \right) + e^{(b+2c+ia)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+ia)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{ia+b+2c}{2c}, \dots, \frac{ia+b+2c}{2c}, 1; \frac{ia+b+2c}{2c} + 1, \dots, \frac{ia+b+2c}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N} \wedge v \in \mathbb{Z}
 \end{aligned}$$

01.22.21.0222.01

$$\int z^n \cos(az) \cosh(cz) \coth(cz) dz =$$

$$-\frac{1}{2} e^{cz} n! \left( e^{-iaz} \sum_{j=0}^n \frac{(-1)^j (c-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c-ia}{2c}, \dots, \frac{c-ia}{2c}, 1; \frac{c-ia}{2c} + 1, \dots, \frac{c-ia}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{iaz} \sum_{j=0}^n \frac{(-1)^j (c+ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+c}{2c}, \dots, \frac{ia+c}{2c}, 1; \frac{ia+c}{2c} + 1, \dots, \frac{ia+c}{2c} + 1; e^{2cz} \right) - \frac{1}{4} e^{cz} n! \right.$$

$$\left( e^{(-2c-ia)z} \sum_{j=0}^n \frac{(-1)^j (-c-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-c}{2c}, \dots, \frac{-ia-c}{2c}, 1; \frac{-ia-c}{2c} + 1, \dots, \frac{-ia-c}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(ia-2c)z} \sum_{j=0}^n \frac{(-1)^j (ia-c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-c}{2c}, \dots, \frac{ia-c}{2c}, 1; \frac{ia-c}{2c} + 1, \dots, \frac{ia-c}{2c} + 1; e^{2cz} \right) + e^{(2c-ia)z} \right.$$

$$\sum_{j=0}^n \frac{(-1)^j (3c-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{3c-ia}{2c}, \dots, \frac{3c-ia}{2c}, 1; \frac{3c-ia}{2c} + 1, \dots, \frac{3c-ia}{2c} + 1; e^{2cz} \right) + e^{(2c+ia)z}$$

$$\left. \sum_{j=0}^n \frac{(-1)^j (3c+ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+3c}{2c}, \dots, \frac{ia+3c}{2c}, 1; \frac{ia+3c}{2c} + 1, \dots, \frac{ia+3c}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

Involving powers of cos, powers of cosh and power

### Involving $z^n \cos^m(az) \cosh^u(cz) \coth(cz)$

01.22.21.0223.01

$$\int z^n \cos^m(az) \cosh^u(bz) \coth(cz) dz = -2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)$$

$$(1 - u \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + 2 e^{2cz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \dots, 1; 2, \dots, 2; e^{2cz}) \right) -$$

$$2^{-m-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{-ia(m-2k)z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$\left. {}_{j+2}F_{j+1} \left( -\frac{ia(m-2k)}{2c}, \dots, -\frac{ia(m-2k)}{2c}, 1; 1 - \frac{ia(m-2k)}{2c}, \dots, 1 - \frac{ia(m-2k)}{2c}; e^{2cz} \right) + \right.$$

$$\left. e^{ia(m-2k)z} \sum_{j=0}^n \frac{(-1)^j (ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)}{2c}, \dots, \frac{ia(m-2k)}{2c}, 1; \frac{ia(m-2k)}{2c} + 1, \dots, \frac{ia(m-2k)}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(2c-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-ia(m-2k)}{2c}, \dots, \frac{2c-ia(m-2k)}{2c}, 1; \frac{2c-ia(m-2k)}{2c} + 1, \dots, \frac{2c-ia(m-2k)}{2c} + 1; e^{2cz} \right) \right)$$

$$\begin{aligned}
 & e^{(2c+ai(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+ai(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+ia(m-2k)}{2c}, \dots, \right. \\
 & \quad \left. \frac{2c+ia(m-2k)}{2c}, 1; \frac{2c+ia(m-2k)}{2c} + 1, \dots, \frac{2c+ia(m-2k)}{2c} + 1; e^{2cz} \right) - \\
 & 2^{-m-u} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{i} \left( e^{-b(u-2i)z} \sum_{j=0}^n \frac{(-1)^j (-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( -\frac{b(u-2i)}{2c}, \dots, -\frac{b(u-2i)}{2c}, 1; 1 - \frac{b(u-2i)}{2c}, \dots, 1 - \frac{b(u-2i)}{2c}; e^{2cz} \right) + e^{b(u-2i)z} \right. \\
 & \quad \left. \sum_{j=0}^n \frac{(-1)^j (b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2i)}{2c}, \dots, \frac{b(u-2i)}{2c}, 1; \frac{b(u-2i)}{2c} + 1, \dots, \frac{b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(2c-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-b(u-2i)}{2c}, \dots, \frac{2c-b(u-2i)}{2c}, 1; \right. \right. \\
 & \quad \left. \left. \frac{2c-b(u-2i)}{2c} + 1, \dots, \frac{2c-b(u-2i)}{2c} + 1; e^{2cz} \right) + e^{(2c+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{2c+b(u-2i)}{2c}, \dots, \frac{2c+b(u-2i)}{2c}, 1; \frac{2c+b(u-2i)}{2c} + 1, \dots, \frac{2c+b(u-2i)}{2c} + 1; e^{2cz} \right) \right) - \\
 & 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \binom{u}{i} \left( e^{(-ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{-ia(m-2k)-b(u-2i)}{2c}, 1; \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{-ia(m-2k)-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(2c-ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{2c-ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{2c-ia(m-2k)-b(u-2i)}{2c}, 1; \right. \right. \\
 & \quad \left. \left. \frac{2c-ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{2c-ia(m-2k)-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(ia(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ia(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)-b(u-2i)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k)-b(u-2i)}{2c}, 1; \frac{ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(2c+ai(m-2k)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+ai(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right.
 \end{aligned}$$

$$\begin{aligned}
 & {}_{j+2}F_{j+1}\left(\frac{2c+ia(m-2k)-b(u-2i)}{2c}, \dots, \frac{2c+ia(m-2k)-b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{2c+ia(m-2k)-b(u-2i)}{2c} + 1, \dots, \frac{2c+ia(m-2k)-b(u-2i)}{2c} + 1; e^{2cz}\right) + \\
 & e^{(b(u-2i)-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (b(u-2i)-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b(u-2i)-ia(m-2k)}{2c}, \dots, \right. \\
 & \left. \frac{b(u-2i)-ia(m-2k)}{2c}, 1; \frac{b(u-2i)-ia(m-2k)}{2c} + 1, \dots, \frac{b(u-2i)-ia(m-2k)}{2c} + 1; e^{2cz}\right) + \\
 & e^{(2c-ia(m-2k)+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1}\left(\frac{2c-ia(m-2k)+b(u-2i)}{2c}, \dots, \frac{2c-ia(m-2k)+b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{2c-ia(m-2k)+b(u-2i)}{2c} + 1, \dots, \frac{2c-ia(m-2k)+b(u-2i)}{2c} + 1; e^{2cz}\right) + \\
 & e^{(ia(m-2k)+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ia(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{ia(m-2k)+b(u-2i)}{2c}, \dots, \right. \\
 & \left. \frac{ia(m-2k)+b(u-2i)}{2c}, 1; \frac{ia(m-2k)+b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)+b(u-2i)}{2c} + 1; e^{2cz}\right) + \\
 & e^{(2c+ia(m-2k)+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+ia(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left(\frac{2c+ia(m-2k)+b(u-2i)}{2c}, \dots, \frac{2c+ia(m-2k)+b(u-2i)}{2c}, 1; \frac{2c+ia(m-2k)+b(u-2i)}{2c} + \right. \\
 & \left. 1, \dots, \frac{2c+ia(m-2k)+b(u-2i)}{2c} + 1; e^{2cz}\right) \Bigg) ; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

**Involving hyperbolic, exponential, trigonometric and a power functions**

Involving sin, sinh, exp and power

**Involving  $z^n e^{Pz} \sin(az) \sinh(bz) \coth(cz)$**

01.22.21.0224.01

$$\int z^n e^{pz} \sin(az) \sinh(bz) \coth(cz) dz =$$

$$\begin{aligned}
 & -\frac{i}{4} n! \left( -e^{(-b-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-ia+p}{2c}, \dots, \frac{-b-ia+p}{2c}, 1; \frac{-b-ia+p}{2c} + 1, \right. \right. \\
 & \quad \left. \left. \dots, \frac{-b-ia+p}{2c} + 1; e^{2cz} \right) - e^{(-b+2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-b+2c-ia+p}{2c}, \dots, \frac{-b+2c-ia+p}{2c}, 1; \frac{-b+2c-ia+p}{2c} + 1, \dots, \frac{-b+2c-ia+p}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(-b+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+ia+p}{2c}, \dots, \frac{-b+ia+p}{2c}, \right. \right. \\
 & \quad \left. \left. 1; \frac{-b+ia+p}{2c} + 1, \dots, \frac{-b+ia+p}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(-b+2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c+ia+p}{2c}, \dots, \frac{-b+2c+ia+p}{2c}, \right. \right. \\
 & \quad \left. \left. 1; \frac{-b+2c+ia+p}{2c} + 1, \dots, \frac{-b+2c+ia+p}{2c} + 1; e^{2cz} \right) + e^{(b-ia+p)z} \right. \\
 & \quad \left. \sum_{j=0}^n \frac{(-1)^j (b-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-ia+p}{2c}, \dots, \frac{b-ia+p}{2c}, 1; \frac{b-ia+p}{2c} + 1, \dots, \frac{b-ia+p}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(b+2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c-ia+p}{2c}, \dots, \frac{b+2c-ia+p}{2c}, \right. \right. \\
 & \quad \left. \left. 1; \frac{b+2c-ia+p}{2c} + 1, \dots, \frac{b+2c-ia+p}{2c} + 1; e^{2cz} \right) - e^{(b+ia+p)z} \right. \\
 & \quad \left. \sum_{j=0}^n \frac{(-1)^j (b+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+ia+p}{2c}, \dots, \frac{b+ia+p}{2c}, 1; \frac{b+ia+p}{2c} + 1, \dots, \frac{b+ia+p}{2c} + 1; e^{2cz} \right) - \right. \\
 & \quad \left. e^{(b+2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c+ia+p}{2c}, \dots, \frac{b+2c+ia+p}{2c}, \right. \right. \\
 & \quad \left. \left. \frac{b+2c+ia+p}{2c} + 1, \dots, \frac{b+2c+ia+p}{2c} + 1; e^{2cz} \right) \right) /; n \in \mathbb{N}
 \end{aligned}$$

Involving powers of sin, powers of sinh, exp and power

**Involving  $z^n e^{pz} \sin^m(az) \sinh^u(bz) \coth(cz)$**

01.22.21.0225.01

$$\int z^n e^{pz} \sin^m(az) \sinh^u(bz) \coth(cz) dz = -i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)$$

$$(1 - u \bmod 2) \left( e^{pz} \sum_{j=0}^n \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p}{2c}, \dots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \dots, \frac{p}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(2c+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p}{2c}, \dots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c} + 1, \dots, \frac{2c+p}{2c} + 1; e^{2cz} \right) \right) -$$

$$i^u 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{\frac{im\pi}{2}} \left( e^{(p-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (p-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right.$$

$$\left( \frac{p-ia(m-2k)}{2c}, \dots, \frac{p-ia(m-2k)}{2c}, 1; \frac{p-ia(m-2k)}{2c} + 1, \dots, \frac{p-ia(m-2k)}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(2c-ia(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-ia(m-2k)+p}{2c}, \dots, \right.$$

$$\left. \frac{2c-ia(m-2k)+p}{2c}, 1; \frac{2c-ia(m-2k)+p}{2c} + 1, \dots, \frac{2c-ia(m-2k)+p}{2c} + 1; e^{2cz} \right) \right) +$$

$$e^{-\frac{1}{2}im\pi} \left( e^{(a i(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (a i(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{a i(m-2k)+p}{2c}, \dots, \right.$$

$$\left. \frac{a i(m-2k)+p}{2c}, 1; \frac{a i(m-2k)+p}{2c} + 1, \dots, \frac{a i(m-2k)+p}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(2c+a i(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+a i(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+a i(m-2k)+p}{2c}, \dots, \right.$$

$$\left. \dots, \frac{2c+a i(m-2k)+p}{2c}, 1; \frac{2c+a i(m-2k)+p}{2c} + 1, \dots, \frac{2c+a i(m-2k)+p}{2c} + 1; e^{2cz} \right) \right) -$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( (-1)^u \left( e^{(p-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right.$$

$$\left( \frac{p-b(u-2k)}{2c}, \dots, \frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c} + 1, \dots, \frac{p-b(u-2k)}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(2c+p-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p-b(u-2k)}{2c}, \dots, \right.$$

$$\left. \frac{2c+p-b(u-2k)}{2c}, 1; \frac{2c+p-b(u-2k)}{2c} + 1, \dots, \frac{2c+p-b(u-2k)}{2c} + 1; e^{2cz} \right) \right) +$$

$$e^{(p+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+b(u-2k)}{2c}, \dots, \frac{p+b(u-2k)}{2c}, \right.$$



$$\begin{aligned}
 & 1; \frac{p+b(u-2k)}{2c} + 1, \dots, \frac{p+b(u-2k)}{2c} + 1; e^{2cz} \Big) + \\
 & e^{(2c+p+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p+b(u-2k)}{2c}, \dots, \right. \\
 & \left. \frac{2c+p+b(u-2k)}{2c}, 1; \frac{2c+p+b(u-2k)}{2c} + 1, \dots, \frac{2c+p+b(u-2k)}{2c} + 1; e^{2cz} \right) - \\
 & 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{i+k} \binom{m}{k} \binom{u}{i} \left( (-1)^u e^{\frac{im\pi}{2}} \left( e^{(-ia(m-2k)+p-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \left. \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p-b(u-2i)}{2c}, \dots, \frac{-ia(m-2k)+p-b(u-2i)}{2c}, 1; \right. \right. \right. \\
 & \left. \left. \frac{-ia(m-2k)+p-b(u-2i)}{2c} + 1, \dots, \frac{-ia(m-2k)+p-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(2c-ia(m-2k)+p-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{2c-ia(m-2k)+p-b(u-2i)}{2c}, \dots, \frac{2c-ia(m-2k)+p-b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{2c-ia(m-2k)+p-b(u-2i)}{2c} + 1, \dots, \frac{2c-ia(m-2k)+p-b(u-2i)}{2c} + 1; e^{2cz} \right) \right) + \\
 & (-1)^u e^{-\frac{1}{2}im\pi} \left( e^{(ai(m-2k)+p-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{ia(m-2k)+p-b(u-2i)}{2c}, \dots, \frac{ia(m-2k)+p-b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{ia(m-2k)+p-b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)+p-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(2c+ai(m-2k)+p-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+ai(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{2c+ia(m-2k)+p-b(u-2i)}{2c}, \dots, \frac{2c+ia(m-2k)+p-b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{2c+ia(m-2k)+p-b(u-2i)}{2c} + 1, \dots, \frac{2c+ia(m-2k)+p-b(u-2i)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{\frac{im\pi}{2}} \left( e^{(-ia(m-2k)+p+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{-ia(m-2k)+p+b(u-2i)}{2c}, \dots, \frac{-ia(m-2k)+p+b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{-ia(m-2k)+p+b(u-2i)}{2c} + 1, \dots, \frac{-ia(m-2k)+p+b(u-2i)}{2c} + 1; e^{2cz} \right) + \right.
 \end{aligned}$$

$$\begin{aligned}
 & e^{(2c-ia(m-2k)+p+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left( \frac{2c-ia(m-2k)+p+b(u-2i)}{2c}, \dots, \frac{2c-ia(m-2k)+p+b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{2c-ia(m-2k)+p+b(u-2i)}{2c} + 1, \dots, \frac{2c-ia(m-2k)+p+b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & e^{-\frac{1}{2}im\pi} \left( e^{(ai(m-2k)+p+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left( \frac{ia(m-2k)+p+b(u-2i)}{2c}, \dots, \frac{ia(m-2k)+p+b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{ia(m-2k)+p+b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)+p+b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c+ia(m-2k)+p+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+ia(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left( \frac{2c+ia(m-2k)+p+b(u-2i)}{2c}, \dots, \frac{2c+ia(m-2k)+p+b(u-2i)}{2c}, \right. \\
 & \left. 1; \frac{2c+ia(m-2k)+p+b(u-2i)}{2c} + 1, \dots, \right. \\
 & \left. \left. \frac{2c+ia(m-2k)+p+b(u-2i)}{2c} + 1; e^{2cz} \right) \right) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

Involving cos, sinh, exp and power

Involving  $z^n e^{pz} \cos(az) \sinh(bz) \coth(cz)$

01.22.21.0226.01

$$\int z^n e^{pz} \cos(az) \sinh(bz) \coth(cz) dz =$$

$$\frac{n!}{4} \left( e^{(-b-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-ia+p}{2c}, \dots, \frac{-b-ia+p}{2c}, 1; \frac{-b-ia+p}{2c} + 1, \dots, \frac{-b-ia+p}{2c} + 1; e^{2cz} \right) + e^{(-b+2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c-ia+p}{2c}, \dots, \frac{-b+2c-ia+p}{2c}, 1; \frac{-b+2c-ia+p}{2c} + 1, \dots, \frac{-b+2c-ia+p}{2c} + 1; e^{2cz} \right) + e^{(-b+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+ia+p}{2c}, \dots, \frac{-b+ia+p}{2c}, 1; \frac{-b+ia+p}{2c} + 1, \dots, \frac{-b+ia+p}{2c} + 1; e^{2cz} \right) + e^{(-b+2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c+ia+p}{2c}, \dots, \frac{-b+2c+ia+p}{2c}, 1; \frac{-b+2c+ia+p}{2c} + 1, \dots, \frac{-b+2c+ia+p}{2c} + 1; e^{2cz} \right) - \sum_{j=0}^n \frac{(-1)^j (b-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-ia+p}{2c}, \dots, \frac{b-ia+p}{2c}, 1; \frac{b-ia+p}{2c} + 1, \dots, \frac{b-ia+p}{2c} + 1; e^{2cz} \right) - e^{(b+2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c-ia+p}{2c}, \dots, \frac{b+2c-ia+p}{2c}, 1; \frac{b+2c-ia+p}{2c} + 1, \dots, \frac{b+2c-ia+p}{2c} + 1; e^{2cz} \right) - e^{(b+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+ia+p}{2c}, \dots, \frac{b+ia+p}{2c}, 1; \frac{b+ia+p}{2c} + 1, \dots, \frac{b+ia+p}{2c} + 1; e^{2cz} \right) - e^{(b+2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c+ia+p}{2c}, \dots, \frac{b+2c+ia+p}{2c}, 1; \frac{b+2c+ia+p}{2c} + 1, \dots, \frac{b+2c+ia+p}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

Involving powers of cos, powers of sinh, exp and power

**Involving  $z^n e^{pz} \cos^m(az) \sinh^u(bz) \coth(cz)$**

01.22.21.0227.01

$$\int z^n e^{pz} \cos^m(az) \sinh^u(bz) \coth(cz) dz = -i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)$$

$$(1 - u \bmod 2) \left( e^{pz} \sum_{j=0}^n \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p}{2c}, \dots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \dots, \frac{p}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(2c+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p}{2c}, \dots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c} + 1, \dots, \frac{2c+p}{2c} + 1; e^{2cz} \right) \right) -$$

$$i^u 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(p-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (p-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1} \left( \frac{p-ia(m-2k)}{2c}, \dots, \frac{p-ia(m-2k)}{2c}, 1; \frac{p-ia(m-2k)}{2c} + 1, \dots, \frac{p-ia(m-2k)}{2c} + 1; e^{2cz} \right) +$$

$$e^{(2c-ia(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-ia(m-2k)+p}{2c}, \dots, \right.$$

$$\left. \frac{2c-ia(m-2k)+p}{2c}, 1; \frac{2c-ia(m-2k)+p}{2c} + 1, \dots, \frac{2c-ia(m-2k)+p}{2c} + 1; e^{2cz} \right) +$$

$$e^{(a i(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (a i(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p}{2c}, \dots, \frac{ia(m-2k)+p}{2c}, \right.$$

$$\left. 1; \frac{ia(m-2k)+p}{2c} + 1, \dots, \frac{ia(m-2k)+p}{2c} + 1; e^{2cz} \right) +$$

$$e^{(2c+ai(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+ai(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+ia(m-2k)+p}{2c}, \dots, \right.$$

$$\left. \frac{2c+ia(m-2k)+p}{2c}, 1; \frac{2c+ia(m-2k)+p}{2c} + 1, \dots, \frac{2c+ia(m-2k)+p}{2c} + 1; e^{2cz} \right) \Big) -$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( (-1)^u \left( e^{(p-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} \right. \right.$$

$${}_{j+2}F_{j+1} \left( \frac{p-b(u-2k)}{2c}, \dots, \frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c} + 1, \dots, \frac{p-b(u-2k)}{2c} + 1; e^{2cz} \right) +$$

$$\left. e^{(2c+p-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p-b(u-2k)}{2c}, \dots, \right. \right.$$

$$\left. \left. \frac{2c+p-b(u-2k)}{2c}, 1; \frac{2c+p-b(u-2k)}{2c} + 1, \dots, \frac{2c+p-b(u-2k)}{2c} + 1; e^{2cz} \right) \right) +$$

$$e^{(p+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+b(u-2k)}{2c}, \dots, \frac{p+b(u-2k)}{2c}, \right.$$

$$\begin{aligned}
 & \left. 1; \frac{p+b(u-2k)}{2c} + 1, \dots, \frac{p+b(u-2k)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c+p+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c+p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p+b(u-2k)}{2c}, \dots, \right. \\
 & \left. \frac{2c+p+b(u-2k)}{2c}, 1; \frac{2c+p+b(u-2k)}{2c} + 1, \dots, \frac{2c+p+b(u-2k)}{2c} + 1; e^{2cz} \right) - \\
 & 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^i \binom{m}{k} \binom{u}{i} \left( (-1)^u \left( e^{(-i a(m-2k)+p-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \left. \left. {}_{j+2}F_{j+1} \left( \frac{-i a(m-2k)+p-b(u-2i)}{2c}, \dots, \frac{-i a(m-2k)+p-b(u-2i)}{2c}, 1; \right. \right. \right. \\
 & \left. \left. \frac{-i a(m-2k)+p-b(u-2i)}{2c} + 1, \dots, \frac{-i a(m-2k)+p-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(2c-i a(m-2k)+p-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-i a(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{2c-i a(m-2k)+p-b(u-2i)}{2c}, \dots, \frac{2c-i a(m-2k)+p-b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{2c-i a(m-2k)+p-b(u-2i)}{2c} + 1, \dots, \frac{2c-i a(m-2k)+p-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(a i(m-2k)+p-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (a i(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{i a(m-2k)+p-b(u-2i)}{2c}, \dots, \frac{i a(m-2k)+p-b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{i a(m-2k)+p-b(u-2i)}{2c} + 1, \dots, \frac{i a(m-2k)+p-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(2c+a i(m-2k)+p-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c+a i(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{2c+a i(m-2k)+p-b(u-2i)}{2c}, \dots, \frac{2c+a i(m-2k)+p-b(u-2i)}{2c}, 1; \right. \right. \\
 & \left. \left. \frac{2c+a i(m-2k)+p-b(u-2i)}{2c} + 1, \dots, \frac{2c+a i(m-2k)+p-b(u-2i)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{(-i a(m-2k)+p+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left( \frac{-i a(m-2k)+p+b(u-2i)}{2c}, \dots, \frac{-i a(m-2k)+p+b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{-i a(m-2k)+p+b(u-2i)}{2c} + 1, \dots, \frac{-i a(m-2k)+p+b(u-2i)}{2c} + 1; e^{2cz} \right) +
 \end{aligned}$$

$$\begin{aligned}
 & e^{(2c-ia(m-2k)+p+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c-ia(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2c-ia(m-2k)+p+b(u-2i)}{2c}, \dots, \frac{2c-ia(m-2k)+p+b(u-2i)}{2c}, 1; \right. \\
 & \left. \frac{2c-ia(m-2k)+p+b(u-2i)}{2c} + 1, \dots, \frac{2c-ia(m-2k)+p+b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(ai(m-2k)+p+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left( \frac{ia(m-2k)+p+b(u-2i)}{2c}, \dots, \frac{ia(m-2k)+p+b(u-2i)}{2c}, 1; \frac{ia(m-2k)+p+b(u-2i)}{2c} + 1, \right. \\
 & \left. \dots, \frac{ia(m-2k)+p+b(u-2i)}{2c} + 1; e^{2cz} \right) + e^{(2c+ai(m-2k)+p+b(u-2i))z} \\
 & \sum_{j=0}^n \frac{(-1)^j (2c+ai(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+ia(m-2k)+p+b(u-2i)}{2c}, \right. \\
 & \dots, \frac{2c+ia(m-2k)+p+b(u-2i)}{2c}, 1; \frac{2c+ia(m-2k)+p+b(u-2i)}{2c} + 1, \\
 & \left. \dots, \frac{2c+ia(m-2k)+p+b(u-2i)}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

Involving sin, cosh, exp and power

**Involving  $z^n e^{pz} \sin(az) \cosh(bz) \coth(cz)$**

01.22.21.0228.01

$$\int z^n e^{pz} \sin(az) \cosh(bz) \coth(cz) dz =$$

$$\frac{i}{4} n! \left( -e^{(-b-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-ia+p}{2c}, \dots, \frac{-b-ia+p}{2c}, 1; \frac{-b-ia+p}{2c} + 1, \dots, \frac{-b-ia+p}{2c} + 1; e^{2cz} \right) - e^{(-b+2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c-ia+p}{2c}, \dots, \frac{-b+2c-ia+p}{2c}, 1; \frac{-b+2c-ia+p}{2c} + 1, \dots, \frac{-b+2c-ia+p}{2c} + 1; e^{2cz} \right) + e^{(-b+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+ia+p}{2c}, \dots, \frac{-b+ia+p}{2c}, 1; \frac{-b+ia+p}{2c} + 1, \dots, \frac{-b+ia+p}{2c} + 1; e^{2cz} \right) + e^{(-b+2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2c+ia+p}{2c}, \dots, \frac{-b+2c+ia+p}{2c}, 1; \frac{-b+2c+ia+p}{2c} + 1, \dots, \frac{-b+2c+ia+p}{2c} + 1; e^{2cz} \right) - \sum_{j=0}^n \frac{(-1)^j (b-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-ia+p}{2c}, \dots, \frac{b-ia+p}{2c}, 1; \frac{b-ia+p}{2c} + 1, \dots, \frac{b-ia+p}{2c} + 1; e^{2cz} \right) - e^{(b+2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c-ia+p}{2c}, \dots, \frac{b+2c-ia+p}{2c}, 1; \frac{b+2c-ia+p}{2c} + 1, \dots, \frac{b+2c-ia+p}{2c} + 1; e^{2cz} \right) + e^{(b+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+ia+p}{2c}, \dots, \frac{b+ia+p}{2c}, 1; \frac{b+ia+p}{2c} + 1, \dots, \frac{b+ia+p}{2c} + 1; e^{2cz} \right) + e^{(b+2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c+ia+p}{2c}, \dots, \frac{b+2c+ia+p}{2c}, 1; \frac{b+2c+ia+p}{2c} + 1, \dots, \frac{b+2c+ia+p}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

01.22.21.0229.01

$$\int z^n e^{pz} \sin(az) \cosh(cz) \coth(cz) dz =$$

$$-\frac{1}{2} e^{cz} i n! \left( e^{(-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c-ia+p}{2c}, \dots, \frac{c-ia+p}{2c}, 1; \right. \right.$$

$$\left. \frac{c-ia+p}{2c} + 1, \dots, \frac{c-ia+p}{2c} + 1; e^{2cz} \right) - e^{(ia+p)z} \sum_{j=0}^n \frac{(-1)^j (c+ia+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{c+ia+p}{2c}, \dots, \frac{c+ia+p}{2c}, 1; \frac{c+ia+p}{2c} + 1, \dots, \frac{c+ia+p}{2c} + 1; e^{2cz} \right) \right) -$$

$$\frac{1}{4} i e^{cz} n! \left( -e^{(-2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-c+ia+p}{2c}, \dots, \frac{-c+ia+p}{2c}, 1; \right. \right.$$

$$\left. \frac{-c+ia+p}{2c} + 1, \dots, \frac{-c+ia+p}{2c} + 1; e^{2cz} \right) + e^{(-2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-c-ia+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{-c-ia+p}{2c}, \dots, \frac{-c-ia+p}{2c}, 1; \frac{-c-ia+p}{2c} + 1, \dots, \frac{-c-ia+p}{2c} + 1; e^{2cz} \right) - \right.$$

$$e^{(2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (3c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{3c+ia+p}{2c}, \dots, \frac{3c+ia+p}{2c}, 1; \right.$$

$$\left. \frac{3c+ia+p}{2c} + 1, \dots, \frac{3c+ia+p}{2c} + 1; e^{2cz} \right) + e^{(2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (3c-ia+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{3c-ia+p}{2c}, \dots, \frac{3c-ia+p}{2c}, 1; \frac{3c-ia+p}{2c} + 1, \dots, \frac{3c-ia+p}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N}$$

Involving powers of sin, powers of cosh, exp and power

### Involving $z^n e^{pz} \sin^m(az) \cosh^u(bz) \coth(cz)$

01.22.21.0230.01

$$\int z^n e^{pz} \sin^m(az) \cosh^u(bz) \coth(cz) dz = -2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)$$

$$(1 - u \bmod 2) \left( e^{pz} \sum_{j=0}^n \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p}{2c}, \dots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \dots, \frac{p}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(2c+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p}{2c}, \dots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c} + 1, \dots, \frac{2c+p}{2c} + 1; e^{2cz} \right) \right) -$$

$$2^{-m-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{\frac{ixm}{2} + (2iak-iam+p)z} \sum_{j=0}^n \frac{(-1)^j (2iak-iam+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right.$$



$$\begin{aligned}
 & \left( \frac{2iak - iam + p}{2c}, \dots, \frac{2iak - iam + p}{2c}, 1; \frac{2iak - iam + p}{2c} + 1, \dots, \frac{2iak - iam + p}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{i\pi m}{2} + (2c + 2iak - iam + p)z} \sum_{j=0}^n \frac{(-1)^j (2c + 2iak - iam + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c + 2iak - iam + p}{2c}, \right. \\
 & \quad \left. \dots, \frac{2c + 2iak - iam + p}{2c}, 1; \frac{2c + 2iak - iam + p}{2c} + 1, \dots, \frac{2c + 2iak - iam + p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-2aik + iam + p)z - \frac{i\pi m}{2}} \sum_{j=0}^n \frac{(-1)^j (-2aik + iam + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-2iak + iam + p}{2c}, \dots, \right. \\
 & \quad \left. \frac{-2iak + iam + p}{2c}, 1; \frac{-2iak + iam + p}{2c} + 1, \dots, \frac{-2iak + iam + p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c - 2iak + iam + p)z - \frac{i\pi m}{2}} \sum_{j=0}^n \frac{(-1)^j (2c - 2iak + iam + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c - 2iak + iam + p}{2c}, \right. \\
 & \quad \left. \dots, \frac{2c - 2iak + iam + p}{2c}, 1; \frac{2c - 2iak + iam + p}{2c} + 1, \dots, \frac{2c - 2iak + iam + p}{2c} + 1; e^{2cz} \right) \Bigg) - \\
 & 2^{-m-u} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{s} \left( e^{(p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{p+2bs-bu}{2c}, \dots, \frac{p+2bs-bu}{2c}, 1; \frac{p+2bs-bu}{2c} + 1, \dots, \frac{p+2bs-bu}{2c} + 1; e^{2cz} \right) + \right. \\
 & e^{(2c+p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (2c+p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p+2bs-bu}{2c}, \dots, \right. \\
 & \quad \left. \frac{2c+p+2bs-bu}{2c}, 1; \frac{2c+p+2bs-bu}{2c} + 1, \dots, \frac{2c+p+2bs-bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-2bs+bu}{2c}, \dots, \frac{p-2bs+bu}{2c}, \right. \\
 & \quad \left. 1; \frac{p-2bs+bu}{2c} + 1, \dots, \frac{p-2bs+bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c+p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (2c+p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p-2bs+bu}{2c}, \dots, \right. \\
 & \quad \left. \frac{2c+p-2bs+bu}{2c}, 1; \frac{2c+p-2bs+bu}{2c} + 1, \dots, \frac{2c+p-2bs+bu}{2c} + 1; e^{2cz} \right) \Bigg) - \\
 & 2^{-m-u} n! \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{m}{k} \binom{u}{s} \left( e^{\frac{i\pi m}{2} + (2iak - iam + p + 2bs - bu)z} \sum_{j=0}^n \frac{(-1)^j (2iak - iam + p + 2bs - bu)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{2iak - iam + p + 2bs - bu}{2c}, \dots, \frac{2iak - iam + p + 2bs - bu}{2c}, 1; \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{2iak - iam + p + 2bs - bu}{2c} + 1, \dots, \frac{2iak - iam + p + 2bs - bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{i\pi m}{2} + (2c + 2iak - iam + p + 2bs - bu)z} \sum_{j=0}^n \frac{(-1)^j (2c + 2iak - iam + p + 2bs - bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2c + 2iak - iam + p + 2bs - bu}{2c}, \dots, \frac{2c + 2iak - iam + p + 2bs - bu}{2c}, 1; \right. \\
 & \left. \frac{2c + 2iak - iam + p + 2bs - bu}{2c} + 1, \dots, \frac{2c + 2iak - iam + p + 2bs - bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-2aik + iam + p + 2bs - bu)z - \frac{i\pi m}{2}} \sum_{j=0}^n \frac{(-1)^j (-2aik + iam + p + 2bs - bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-2iak + iam + p + 2bs - bu}{2c}, \dots, \frac{-2iak + iam + p + 2bs - bu}{2c}, 1; \right. \\
 & \left. \frac{-2iak + iam + p + 2bs - bu}{2c} + 1, \dots, \frac{-2iak + iam + p + 2bs - bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c - 2iak + iam + p + 2bs - bu)z - \frac{i\pi m}{2}} \sum_{j=0}^n \frac{(-1)^j (2c - 2iak + iam + p + 2bs - bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2c - 2iak + iam + p + 2bs - bu}{2c}, \dots, \frac{2c - 2iak + iam + p + 2bs - bu}{2c}, 1; \right. \\
 & \left. \frac{2c - 2iak + iam + p + 2bs - bu}{2c} + 1, \dots, \frac{2c - 2iak + iam + p + 2bs - bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{i\pi m}{2} + (2iak - iam + p - 2bs + bu)z} \sum_{j=0}^n \frac{(-1)^j (2iak - iam + p - 2bs + bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2iak - iam + p - 2bs + bu}{2c}, \dots, \frac{2iak - iam + p - 2bs + bu}{2c}, 1; \right. \\
 & \left. \frac{2iak - iam + p - 2bs + bu}{2c} + 1, \dots, \frac{2iak - iam + p - 2bs + bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{i\pi m}{2} + (2c + 2iak - iam + p - 2bs + bu)z} \sum_{j=0}^n \frac{(-1)^j (2c + 2iak - iam + p - 2bs + bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2c + 2iak - iam + p - 2bs + bu}{2c}, \dots, \frac{2c + 2iak - iam + p - 2bs + bu}{2c}, 1; \right. \\
 & \left. \frac{2c + 2iak - iam + p - 2bs + bu}{2c} + 1, \dots, \frac{2c + 2iak - iam + p - 2bs + bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-2aik + iam + p - 2bs + bu)z - \frac{i\pi m}{2}} \sum_{j=0}^n \frac{(-1)^j (-2aik + iam + p - 2bs + bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-2iak + iam + p - 2bs + bu}{2c}, \dots, \frac{-2iak + iam + p - 2bs + bu}{2c}, 1; \right. \\
 & \left. \frac{-2iak + iam + p - 2bs + bu}{2c} + 1, \dots, \frac{-2iak + iam + p - 2bs + bu}{2c} + 1; e^{2cz} \right) +
 \end{aligned}$$

$$e^{(2c-2iak+iam+p-2bs+bu)z-\frac{im\pi}{2}} \sum_{j=0}^n \frac{(-1)^j (2c-2iak+iam+p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{2c-2iak+iam+p-2bs+bu}{2c}, \dots, \frac{2c-2iak+iam+p-2bs+bu}{2c}, \right.$$

$$1; \frac{2c-2iak+iam+p-2bs+bu}{2c} + 1, \dots,$$

$$\left. \frac{2c-2iak+iam+p-2bs+bu}{2c} + 1; e^{2cz} \right); n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.22.21.0231.01

$$\int z^n e^{pz} \sin^m(az) \cosh^u(cz) \coth(cz) dz =$$

$$-2^{-m-u} e^{(c+p)z} \binom{m}{\frac{m}{2}} \binom{u+1}{\frac{u+1}{2}} n! (1-m \bmod 2) (1-(u+1) \bmod 2) \sum_{j=0}^n \frac{(-1)^j (c+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{c+p}{2c}, \dots, \frac{c+p}{2c}, 1; \frac{c+p}{2c} + 1, \dots, \frac{c+p}{2c} + 1; e^{2cz} \right) - 2^{-m-u} e^{cz} \binom{u+1}{\frac{u+1}{2}} n! (1-(u+1) \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{\frac{i\pi m}{2} + (p-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (c-ia(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c-ia(m-2k)+p}{2c}, \right.$$

$$\dots, \frac{c-ia(m-2k)+p}{2c}, 1; \frac{c-ia(m-2k)+p}{2c} + 1, \dots, \frac{c-ia(m-2k)+p}{2c} + 1; e^{2cz} \right) +$$

$$e^{(a(m-2k)+p)z-\frac{im\pi}{2}} \sum_{j=0}^n \frac{(-1)^j (c+ai(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c+ia(m-2k)+p}{2c}, \dots,$$

$$\left. \frac{c+ia(m-2k)+p}{2c}, 1; \frac{c+ia(m-2k)+p}{2c} + 1, \dots, \frac{c+ia(m-2k)+p}{2c} + 1; e^{2cz} \right) \right) +$$

$$2^{-m-u} e^{cz} \binom{m}{\frac{m}{2}} n! (m \bmod 2 - 1) \sum_{s=0}^{\lfloor \frac{u}{2} \rfloor} \binom{u+1}{s} \left( e^{(p-c(-2s+u+1))z} \sum_{j=0}^n \frac{(-1)^j (-(-2s+u+1)c+c+p)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1} \left( \frac{p-c(u-2s)}{2c}, \dots, \frac{p-c(u-2s)}{2c}, 1; \frac{p-c(u-2s)}{2c} + 1, \dots, \frac{p-c(u-2s)}{2c} + 1; e^{2cz} \right) +$$

$$e^{(p+c(-2s+u+1))z} \sum_{j=0}^n \frac{(-1)^j ((-2s+u+1)c+c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+c(-2s+u+2)}{2c}, \dots,$$

$$\left. \frac{p+c(-2s+u+2)}{2c}, 1; \frac{p+c(-2s+u+2)}{2c} + 1, \dots, \frac{p+c(-2s+u+2)}{2c} + 1; e^{2cz} \right) \right) - 2^{-m-u} e^{cz} n!$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u}{2} \rfloor} (-1)^k \binom{m}{k} \binom{u+1}{s} \left( e^{\frac{i\pi m}{2} + (-ia(m-2k)+p-c(-2s+u+1))z} \sum_{j=0}^n \frac{(-1)^j (-(-2s+u+1)c+c-ia(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p-c(u-2s)}{2c}, \dots, \frac{-ia(m-2k)+p-c(u-2s)}{2c}, 1;$$

$$\begin{aligned}
 & \left. \frac{-i a(m-2k)+p-c(u-2s)}{2c} + 1, \dots, \frac{-i a(m-2k)+p-c(u-2s)}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{i\pi m}{2} + (-i a(m-2k)+p-c(-2s+u+1))z - \frac{i\pi n}{2}} \sum_{j=0}^n \frac{(-1)^j (-(-2s+u+1)c+c+a i(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{i a(m-2k)+p-c(u-2s)}{2c}, \dots, \frac{i a(m-2k)+p-c(u-2s)}{2c}, 1; \right. \\
 & \left. \frac{i a(m-2k)+p-c(u-2s)}{2c} + 1, \dots, \frac{i a(m-2k)+p-c(u-2s)}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{i\pi m}{2} + (-i a(m-2k)+p+c(-2s+u+1))z} \sum_{j=0}^n \frac{(-1)^j ((-2s+u+1)c+c-i a(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-i a(m-2k)+p+c(-2s+u+2)}{2c}, \dots, \frac{-i a(m-2k)+p+c(-2s+u+2)}{2c}, 1; \right. \\
 & \left. \frac{-i a(m-2k)+p+c(-2s+u+2)}{2c} + 1, \dots, \frac{-i a(m-2k)+p+c(-2s+u+2)}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{i\pi m}{2} + (i a(m-2k)+p+c(-2s+u+1))z - \frac{i\pi n}{2}} \sum_{j=0}^n \frac{(-1)^j ((-2s+u+1)c+c+a i(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{i a(m-2k)+p+c(-2s+u+2)}{2c}, \dots, \frac{i a(m-2k)+p+c(-2s+u+2)}{2c}, \right. \\
 & \left. 1; \frac{i a(m-2k)+p+c(-2s+u+2)}{2c} + 1, \dots, \right. \\
 & \left. \left. \frac{i a(m-2k)+p+c(-2s+u+2)}{2c} + 1; e^{2cz} \right) \right) ; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$

Involving cos, cosh, exp and power

**Involving  $z^n e^{pz} \cos(az) \cosh(bz) \coth(cz)$**

01.22.21.0232.01

$$\begin{aligned}
 \int z^n e^{pz} \cos(az) \cosh(bz) \coth(cz) dz = & -\frac{1}{4} n! \left( e^{(-b-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+p)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia-b+p}{2c}, \dots, \frac{-ia-b+p}{2c}, 1; \frac{-ia-b+p}{2c} + 1, \dots, \frac{-ia-b+p}{2c} + 1; e^{2cz} \right) + e^{(-b+ia+p)z} \\
 & \sum_{j=0}^n \frac{(-1)^j (-b+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b+p}{2c}, \dots, \frac{ia-b+p}{2c}, 1; \frac{ia-b+p}{2c} + 1, \dots, \frac{ia-b+p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(b-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b-ia+p)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia+b+p}{2c}, \dots, \frac{-ia+b+p}{2c}, 1; \frac{-ia+b+p}{2c} + 1, \dots, \frac{-ia+b+p}{2c} + 1; e^{2cz} \right) + e^{(b+ia+p)z} \\
 & \sum_{j=0}^n \frac{(-1)^j (b+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b+p}{2c}, \dots, \frac{ia+b+p}{2c}, 1; \frac{ia+b+p}{2c} + 1, \dots, \frac{ia+b+p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-b+2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia-b+2c+p}{2c}, \dots, \frac{-ia-b+2c+p}{2c}, 1; \frac{-ia-b+2c+p}{2c} + 1, \dots, \frac{-ia-b+2c+p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-b+2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b+2c+p}{2c}, \dots, \frac{ia-b+2c+p}{2c}, 1; \right. \\
 & \left. \frac{ia-b+2c+p}{2c} + 1, \dots, \frac{ia-b+2c+p}{2c} + 1; e^{2cz} \right) + e^{(b+2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia+b+2c+p}{2c}, \dots, \frac{-ia+b+2c+p}{2c}, 1; \frac{-ia+b+2c+p}{2c} + 1, \dots, \frac{-ia+b+2c+p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(b+2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2c+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b+2c+p}{2c}, \dots, \frac{ia+b+2c+p}{2c}, \right. \\
 & \left. 1; \frac{ia+b+2c+p}{2c} + 1, \dots, \frac{ia+b+2c+p}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N}
 \end{aligned}$$

01.22.21.0233.01

$$\int z^n e^{pz} \cos(az) \cosh(cz) \coth(cz) dz =$$

$$-\frac{1}{2} e^{cz} n! \left( e^{(-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia+c+p}{2c}, \dots, \frac{-ia+c+p}{2c}, 1; \right. \right.$$

$$\left. \frac{-ia+c+p}{2c} + 1, \dots, \frac{-ia+c+p}{2c} + 1; e^{2cz} \right) + e^{(ia+p)z} \sum_{j=0}^n \frac{(-1)^j (c+ia+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{ia+p+c}{2c}, \dots, \frac{ia+p+c}{2c}, 1; \frac{ia+p+c}{2c} + 1, \dots, \frac{ia+p+c}{2c} + 1; e^{2cz} \right) -$$

$$\frac{1}{4} e^{cz} n! \left( e^{(-2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-c+p}{2c}, \dots, \frac{-ia-c+p}{2c}, 1; \right. \right.$$

$$\left. \frac{-ia-c+p}{2c} + 1, \dots, \frac{-ia-c+p}{2c} + 1; e^{2cz} \right) + e^{(-2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-c+ia+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{ia-c+p}{2c}, \dots, \frac{ia-c+p}{2c}, 1; \frac{ia-c+p}{2c} + 1, \dots, \frac{ia-c+p}{2c} + 1; e^{2cz} \right) +$$

$$e^{(2c-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (3c-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia+3c+p}{2c}, \dots, \frac{-ia+3c+p}{2c}, 1; \right.$$

$$\left. \frac{-ia+3c+p}{2c} + 1, \dots, \frac{-ia+3c+p}{2c} + 1; e^{2cz} \right) + e^{(2c+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (3c+ia+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{ia+3c+p}{2c}, \dots, \frac{ia+3c+p}{2c}, 1; \frac{ia+3c+p}{2c} + 1, \dots, \frac{ia+3c+p}{2c} + 1; e^{2cz} \right) \Bigg); n \in \mathbb{N}$$

Involving powers of cos, powers of cosh, exp and power

### Involving $z^n e^{pz} \cos^m(az) \cosh^u(bz) \coth(cz)$

01.22.21.0234.01

$$\int z^n e^{pz} \cos^m(az) \cosh^u(bz) \coth(cz) dz = -2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)$$

$$(1 - u \bmod 2) \left( e^{pz} \sum_{j=0}^n \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p}{2c}, \dots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \dots, \frac{p}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(2c+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p}{2c}, \dots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c} + 1, \dots, \frac{2c+p}{2c} + 1; e^{2cz} \right) \right) -$$

$$2^{-m-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(2iak-iam+p)z} \sum_{j=0}^n \frac{(-1)^j (2iak-iam+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right.$$

$$\begin{aligned}
 & \left( \frac{2iak - iam + p}{2c}, \dots, \frac{2iak - iam + p}{2c}, 1; \frac{2iak - iam + p}{2c} + 1, \dots, \frac{2iak - iam + p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c+2iak-iam+p)z} \sum_{j=0}^n \frac{(-1)^j (2c+2iak-iam+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+2iak-iam+p}{2c}, \dots, \right. \\
 & \quad \left. \frac{2c+2iak-iam+p}{2c}, 1; \frac{2c+2iak-iam+p}{2c} + 1, \dots, \frac{2c+2iak-iam+p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-2aik+iam+p)z} \sum_{j=0}^n \frac{(-1)^j (-2aik+iam+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-2iak+iam+p}{2c}, \dots, \right. \\
 & \quad \left. \frac{-2iak+iam+p}{2c}, 1; \frac{-2iak+iam+p}{2c} + 1, \dots, \frac{-2iak+iam+p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c-2iak+iam+p)z} \sum_{j=0}^n \frac{(-1)^j (2c-2iak+iam+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c-2iak+iam+p}{2c}, \dots, \right. \\
 & \quad \left. \frac{2c-2iak+iam+p}{2c}, 1; \frac{2c-2iak+iam+p}{2c} + 1, \dots, \frac{2c-2iak+iam+p}{2c} + 1; e^{2cz} \right) - \\
 & 2^{-m-u} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{s} \left( e^{(p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{p+2bs-bu}{2c}, \dots, \frac{p+2bs-bu}{2c}, 1; \frac{p+2bs-bu}{2c} + 1, \dots, \frac{p+2bs-bu}{2c} + 1; e^{2cz} \right) + \right. \\
 & e^{(2c+p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (2c+p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p+2bs-bu}{2c}, \dots, \right. \\
 & \quad \left. \frac{2c+p+2bs-bu}{2c}, 1; \frac{2c+p+2bs-bu}{2c} + 1, \dots, \frac{2c+p+2bs-bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-2bs+bu}{2c}, \dots, \frac{p-2bs+bu}{2c}, \right. \\
 & \quad \left. 1; \frac{p-2bs+bu}{2c} + 1, \dots, \frac{p-2bs+bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c+p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (2c+p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c+p-2bs+bu}{2c}, \dots, \right. \\
 & \quad \left. \frac{2c+p-2bs+bu}{2c}, 1; \frac{2c+p-2bs+bu}{2c} + 1, \dots, \frac{2c+p-2bs+bu}{2c} + 1; e^{2cz} \right) - \\
 & 2^{-m-u} n! \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{m}{k} \binom{u}{s} \left( e^{(2iak-iam+p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (2iak-iam+p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{2iak-iam+p+2bs-bu}{2c}, \dots, \frac{2iak-iam+p+2bs-bu}{2c}, 1; \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{2iak - iam + p + 2bs - bu}{2c} + 1, \dots, \frac{2iak - iam + p + 2bs - bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c+2iak-iam+p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (2c+2iak-iam+p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2c+2iak-iam+p+2bs-bu}{2c}, \dots, \frac{2c+2iak-iam+p+2bs-bu}{2c}, 1; \right. \\
 & \left. \frac{2c+2iak-iam+p+2bs-bu}{2c} + 1, \dots, \frac{2c+2iak-iam+p+2bs-bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-2aik+iam+p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (-2aik+iam+p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-2iak+iam+p+2bs-bu}{2c}, \dots, \frac{-2iak+iam+p+2bs-bu}{2c}, 1; \right. \\
 & \left. \frac{-2iak+iam+p+2bs-bu}{2c} + 1, \dots, \frac{-2iak+iam+p+2bs-bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c-2iak+iam+p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (2c-2iak+iam+p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2c-2iak+iam+p+2bs-bu}{2c}, \dots, \frac{2c-2iak+iam+p+2bs-bu}{2c}, 1; \right. \\
 & \left. \frac{2c-2iak+iam+p+2bs-bu}{2c} + 1, \dots, \frac{2c-2iak+iam+p+2bs-bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2iak-iam+p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (2iak-iam+p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2iak-iam+p-2bs+bu}{2c}, \dots, \frac{2iak-iam+p-2bs+bu}{2c}, 1; \right. \\
 & \left. \frac{2iak-iam+p-2bs+bu}{2c} + 1, \dots, \frac{2iak-iam+p-2bs+bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(2c+2iak-iam+p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (2c+2iak-iam+p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{2c+2iak-iam+p-2bs+bu}{2c}, \dots, \frac{2c+2iak-iam+p-2bs+bu}{2c}, 1; \right. \\
 & \left. \frac{2c+2iak-iam+p-2bs+bu}{2c} + 1, \dots, \frac{2c+2iak-iam+p-2bs+bu}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-2aik+iam+p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (-2aik+iam+p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-2iak+iam+p-2bs+bu}{2c}, \dots, \frac{-2iak+iam+p-2bs+bu}{2c}, 1; \right. \\
 & \left. \frac{-2iak+iam+p-2bs+bu}{2c} + 1, \dots, \frac{-2iak+iam+p-2bs+bu}{2c} + 1; e^{2cz} \right) +
 \end{aligned}$$



$$e^{(2c-2iak+iam+p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (2c-2iak+iam+p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{2c-2iak+iam+p-2bs+bu}{2c}, \dots, \frac{2c-2iak+iam+p-2bs+bu}{2c}, \right.$$

$$1; \frac{2c-2iak+iam+p-2bs+bu}{2c} + 1, \dots,$$

$$\left. \frac{2c-2iak+iam+p-2bs+bu}{2c} + 1; e^{2cz} \right); n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.22.21.0235.01

$$\int z^n e^{pz} \cos^m(az) \cosh^u(cz) \coth(cz) dz =$$

$$-2^{-m-u} e^{(c+p)z} \binom{m}{\frac{m}{2}} \binom{u+1}{\frac{u+1}{2}} n! (1-m \bmod 2) (1-(u+1) \bmod 2) \sum_{j=0}^n \frac{(-1)^j (c+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{c+p}{2c}, \dots, \frac{c+p}{2c}, 1; \frac{c+p}{2c} + 1, \dots, \frac{c+p}{2c} + 1; e^{2cz} \right) - 2^{-m-u} e^{cz} \binom{u+1}{\frac{u+1}{2}} n! (1-(u+1) \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(p-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (c-ia(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c-ia(m-2k)+p}{2c}, \dots, \frac{c-ia(m-2k)+p}{2c}, \right.$$

$$1; \frac{c-ia(m-2k)+p}{2c} + 1, \dots, \frac{c-ia(m-2k)+p}{2c} + 1; e^{2cz} \right) +$$

$$e^{(a(m-2k)+p)z} \sum_{j=0}^n \frac{(-1)^j (c+ai(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c+ia(m-2k)+p}{2c}, \dots,$$

$$\left. \frac{c+ia(m-2k)+p}{2c}, 1; \frac{c+ia(m-2k)+p}{2c} + 1, \dots, \frac{c+ia(m-2k)+p}{2c} + 1; e^{2cz} \right) \right) -$$

$$2^{-m-u} e^{cz} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \sum_{s=0}^{\lfloor \frac{u}{2} \rfloor} \binom{u+1}{s} \left( e^{(p-c(-2s+u+1))z} \sum_{j=0}^n \frac{(-1)^j (p-c(u-2s))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{p-c(u-2s)}{2c}, \dots, \frac{p-c(u-2s)}{2c}, 1; \frac{p-c(u-2s)}{2c} + 1, \dots, \frac{p-c(u-2s)}{2c} + 1; e^{2cz} \right) +$$

$$e^{(p+c(-2s+u+1))z} \sum_{j=0}^n \frac{(-1)^j (p+c(-2s+u+2))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+c(-2s+u+2)}{2c}, \dots,$$

$$\left. \frac{p+c(-2s+u+2)}{2c}, 1; \frac{p+c(-2s+u+2)}{2c} + 1, \dots, \frac{p+c(-2s+u+2)}{2c} + 1; e^{2cz} \right) \right) -$$

$$2^{-m-u} e^{cz} n! \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u}{2} \rfloor} \binom{m}{k} \binom{u+1}{s} \left( e^{(-ia(m-2k)+p-c(-2s+u+1))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-c(u-2s))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p-c(u-2s)}{2c}, \dots, \frac{-ia(m-2k)+p-c(u-2s)}{2c}, 1;$$

$$\begin{aligned}
 & \left. \frac{-i a(m-2 k)+p-c(u-2 s)}{2 c}+1, \dots, \frac{-i a(m-2 k)+p-c(u-2 s)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & e^{(a i(m-2 k)+p-c(-2 s+u+1)) z} \sum_{j=0}^n \frac{(-1)^j(a i(m-2 k)+p-c(u-2 s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \\
 & \left(\frac{i a(m-2 k)+p-c(u-2 s)}{2 c}, \dots, \frac{i a(m-2 k)+p-c(u-2 s)}{2 c}, 1 ; \frac{i a(m-2 k)+p-c(u-2 s)}{2 c}+1, \right. \\
 & \left. \dots, \frac{i a(m-2 k)+p-c(u-2 s)}{2 c}+1 ; e^{2 c z}\right)+e^{(-i a(m-2 k)+p+c(-2 s+u+1)) z} \\
 & \sum_{j=0}^n \frac{(-1)^j(-i a(m-2 k)+p+c(-2 s+u+2))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left(\frac{-i a(m-2 k)+p+c(-2 s+u+2)}{2 c}, \right. \\
 & \left. \dots, \frac{-i a(m-2 k)+p+c(-2 s+u+2)}{2 c}, 1 ; \frac{-i a(m-2 k)+p+c(-2 s+u+2)}{2 c}+1, \right. \\
 & \left. \dots, \frac{-i a(m-2 k)+p+c(-2 s+u+2)}{2 c}+1 ; e^{2 c z}\right)+e^{(a i(m-2 k)+p+c(-2 s+u+1)) z} \\
 & \sum_{j=0}^n \frac{(-1)^j(a i(m-2 k)+p+c(-2 s+u+2))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left(\frac{i a(m-2 k)+p+c(-2 s+u+2)}{2 c}, \right. \\
 & \left. \dots, \frac{i a(m-2 k)+p+c(-2 s+u+2)}{2 c}, 1 ; \frac{i a(m-2 k)+p+c(-2 s+u+2)}{2 c}+1, \right. \\
 & \left. \dots, \frac{i a(m-2 k)+p+c(-2 s+u+2)}{2 c}+1 ; e^{2 c z}\right) \Bigg) ; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

**Involving functions of the direct function**

**Involving powers of the direct function**

**Involving powers of coth**

**Linear argument**

01.22.21.0236.01

$$\int \coth^{\nu}(c z) d z = \frac{\coth^{\nu+1}(c z)}{\nu c+c} {}_2 F_1\left(\frac{\nu+1}{2}, 1 ; \frac{\nu+3}{2} ; \coth^2(c z)\right)$$

01.22.21.0237.01

$$\int \coth^2(c z) d z = z - \frac{\coth(c z)}{c}$$

01.22.21.0238.01

$$\int \coth^3(c z) d z = -\frac{\operatorname{csch}^2(c z)-2 \log(\sinh(c z))}{2 c}$$

01.22.21.0239.01

$$\int \coth^4(c z) dz = \frac{3 c z - \coth(c z) (\operatorname{csch}^2(c z) + 4)}{3 c}$$

01.22.21.0240.01

$$\int \coth^5(c z) dz = -\frac{\operatorname{csch}^4(c z) + 4 \operatorname{csch}^2(c z) - 4 \log(\sinh(c z))}{4 c}$$

01.22.21.0241.01

$$\int \coth^6(c z) dz = \frac{15 c z - \coth(c z) (3 \operatorname{csch}^4(c z) + 11 \operatorname{csch}^2(c z) + 23)}{15 c}$$

01.22.21.0242.01

$$\int \coth^7(c z) dz = -\frac{2 \operatorname{csch}^6(c z) + 9 \operatorname{csch}^4(c z) + 18 \operatorname{csch}^2(c z) - 12 \log(\sinh(c z))}{12 c}$$

01.22.21.0243.01

$$\int \coth^8(c z) dz = \frac{105 c z - \coth(c z) (15 \operatorname{csch}^6(c z) + 66 \operatorname{csch}^4(c z) + 122 \operatorname{csch}^2(c z) + 176)}{105 c}$$

01.22.21.0499.01

$$\int \coth^{2n}(a z) dz = z - \frac{\tanh(a z)}{a} \sum_{k=1}^n \frac{\coth^{2k}(a z)}{2k-1}; n \in \mathbb{N}$$

01.22.21.0500.01

$$\int \coth^{2n+1}(a z) dz = \frac{\log(\sinh(a z))}{a} - \frac{(-1)^n S_{n+1}^{(2)}}{2 a n!} - \frac{1}{2 a} \sum_{k=1}^n \frac{\coth^{2k}(a z)}{k}; n \in \mathbb{N}$$

01.22.21.0501.01

$$\int \coth^{2n}(a z) dz = \frac{a z - \tanh^{-1}(\coth(a z))}{a} + \frac{\coth^{2n+1}(a z)}{a(2n+1)} {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; \coth^2(a z)\right); n \in \mathbb{N}$$

01.22.21.0502.01

$$\int \coth^{2n+1}(a z) dz = \frac{\coth^{2n+2}(a z)}{2 a (n+1)} {}_2F_1(n+1, 1; n+2; \coth^2(a z)) + \frac{\log(-\operatorname{csch}^2(a z)) + 2 \log(\sinh(a z))}{2 a} - \frac{(-1)^n S_{n+1}^{(2)}}{2 a n!}; n \in \mathbb{N}$$

01.22.21.0244.01

$$\int \coth^{\frac{1}{2}}(c z) dz = -\frac{2 \tan^{-1}\left(\coth^{\frac{1}{2}}(c z)\right) + \log\left(\coth^{\frac{1}{2}}(c z) - 1\right) - \log\left(\coth^{\frac{1}{2}}(c z) + 1\right)}{2 c}$$

01.22.21.0245.01

$$\int \frac{1}{\coth^{\frac{1}{2}}(c z)} dz = \frac{2 \tan^{-1}\left(\coth^{\frac{1}{2}}(c z)\right) - \log\left(\coth^{\frac{1}{2}}(c z) - 1\right) + \log\left(\coth^{\frac{1}{2}}(c z) + 1\right)}{2 c}$$

### Involving products of the direct functions

01.22.21.0246.01

$$\int \coth(a z + b) \coth(a z) dz = \frac{a z + \coth(b) (\log(\sinh(a z)) - \log(\sinh(b + a z)))}{a}$$

01.22.21.0247.01

$$\int \coth(b - a z) \coth(a z) dz = \frac{\coth(b) (\log(-\sinh(a z)) - \log(\sinh(b - a z))) - a z}{a}$$

### Involving powers of products of the direct function

01.22.21.0248.01

$$\int \sqrt{\coth(c z) \coth(2 c z)} dz = \frac{1}{c \cosh^{\frac{1}{2}}(2 c z)} \left( \sqrt{\coth(c z) \coth(2 c z)} \left( \sqrt{2} \log \left( \sqrt{2} \cosh(c z) + \cosh^{\frac{1}{2}}(2 c z) \right) - \tanh^{-1} \left( \frac{\cosh(c z)}{\cosh^{\frac{1}{2}}(2 c z)} \right) \right) \sinh(c z) \right)$$

### Involving rational functions of the direct function

#### Involving $(a + b \coth(c z))^{-n}$

01.22.21.0249.01

$$\int \frac{1}{a + b \coth(c z)} dz = \frac{a c z - b \log(b \cosh(c z) + a \sinh(c z))}{a^2 c - b^2 c}$$

01.22.21.0250.01

$$\int \frac{1}{(a + b \coth(c z))^2} dz = (c z a^3 + b a^2 - 2 b \log(b \cosh(c z) + a \sinh(c z)) a^2 + b^2 c z a - b^3 + b \coth(c z) ((a^2 + b^2) c z - 2 a b \log(b \cosh(c z) + a \sinh(c z))) / ((a - b)^2 (a + b)^2 c (a + b \coth(c z)))$$

01.22.21.0251.01

$$\int \frac{A + B \coth(z)}{(a + b \coth(z))^2} dz = \left( (A + B \coth(z)) \operatorname{csch}(z) (b \cosh(z) + a \sinh(z)) \left( \frac{(A a^2 - 2 b B a + A b^2) z (b \cosh(z) + a \sinh(z))}{(a - b)^2 (a + b)^2} + \frac{(B a^2 - 2 A b a + b^2 B) \log(b \cosh(z) + a \sinh(z)) (b \cosh(z) + a \sinh(z))}{(a^2 - b^2)^2} + \frac{A b \sinh(z) - a B \sinh(z)}{a^2 - b^2} \right) \right) / ((a + b \coth(z))^2 (B \cosh(z) + A \sinh(z)))$$

01.22.21.0252.01

$$\int \frac{A + B \coth(z)}{(a + b \coth(z))^3} dz = \left( (A + B \coth(z)) \operatorname{csch}^2(z) (b \cosh(z) + a \sinh(z)) \right. \\ \left. \left( \frac{(A b - a B) b^2}{(a - b)^2 (a + b)^2} + \frac{2 (A a^3 - 3 b B a^2 + 3 A b^2 a - b^3 B) z (b \cosh(z) + a \sinh(z))^2}{(a - b)^3 (a + b)^3} + \right. \right. \\ \left. \frac{1}{(a^2 - b^2)^3} (2 (B a^3 - 3 A b a^2 + 3 b^2 B a - A b^3) \log(b \cosh(z) + a \sinh(z)) (b \cosh(z) + a \sinh(z))^2) - \right. \\ \left. \left. \frac{2 (2 B a^2 - 3 A b a + b^2 B) \sinh(z) (b \cosh(z) + a \sinh(z))}{(a - b)^2 (a + b)^2} \right) \right) / \left( (2 (a + b \coth(z))^3 (B \cosh(z) + A \sinh(z))) \right)$$

01.22.21.0253.01

$$\int \frac{A + B \coth(z) + C \coth^2(z)}{(a + b \coth(z))^3} dz = \left( (C \coth^2(z) + B \coth(z) + A) \operatorname{csch}(z) (b \cosh(z) + a \sinh(z)) \right. \\ \left( \frac{b (A b^2 + a (C - b B))}{(a - b)^2 (a + b)^2} + \frac{2 (C a^3 - 2 b B a^2 + b^2 (3 A + 2 C) a - b^3 B) \sinh(z) (b \cosh(z) + a \sinh(z))}{(a - b)^2 b (a + b)^2} + \right. \\ \left. \frac{2 ((A + C) a^3 - 3 b B a^2 + 3 b^2 (A + C) a - b^3 B) z (b \cosh(z) + a \sinh(z))^2}{(a - b)^3 (a + b)^3} + \right. \\ \left. \frac{1}{(a^2 - b^2)^3} (2 (B a^3 - 3 b (A + C) a^2 + 3 b^2 B a - b^3 (A + C)) \log(b \cosh(z) + a \sinh(z)) (b \cosh(z) + a \sinh(z))^2) \right) / \\ ((a + b \coth(z))^3 (-A + C + (A + C) \cosh(2 z) + B \sinh(2 z)))$$

Involving  $(a + b \coth^2(c z))^{-n}$

01.22.21.0254.01

$$\int \frac{1}{a + b \coth^2(c z)} dz = \frac{1}{a + b} \left( z - \frac{\sqrt{b}}{\sqrt{a} c} \tan^{-1} \left( \frac{\sqrt{a} \tanh(c z)}{\sqrt{b}} \right) \right)$$

01.22.21.0255.01

$$\int \frac{1}{(a + b \coth^2(c z))^2} dz = \left( (-a + b + (a + b) \cosh(2 c z)) \operatorname{csch}^4(c z) \left( 2 z (-a + b + (a + b) \cosh(2 c z)) - \right. \right. \\ \left. \frac{\sqrt{b} (3 a + b) \tan^{-1} \left( \frac{\sqrt{a} \tanh(c z)}{\sqrt{b}} \right) (-a + b + (a + b) \cosh(2 c z))}{a^{3/2} c} + \frac{b (a + b) \sinh(2 c z)}{a c} \right) \right) / \left( 8 (a + b)^2 (b \coth^2(c z) + a)^2 \right)$$

Involving algebraic functions of the direct function

Involving  $(a + b \operatorname{coth}(c z))^\beta$

01.22.21.0256.01

$$\int (a + b \operatorname{coth}(c z))^\beta dz = \frac{(a + b \operatorname{coth}(c z))^{\beta+1}}{2(b-a)(a+b)c(\beta+1)} \left( (a+b) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b \operatorname{coth}(c z)}{a-b}\right) + (b-a) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b \operatorname{coth}(c z)}{a+b}\right) \right)$$

01.22.21.0257.01

$$\int \sqrt{a + b \operatorname{coth}(c z)} dz = \left( \left( \sqrt{i(a-b)} (a+b) \tanh^{-1}\left(\frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{\sqrt{i(a+b)}}\right) - (a-b) \sqrt{i(a+b)} \tanh^{-1}\left(\frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{\sqrt{i(a-b)}}\right) \right) \sqrt{i(a+b \operatorname{coth}(c z))} \sqrt{a + b \operatorname{coth}(c z)} \sinh(c z) \right) / \left( \sqrt{i(a-b)} \sqrt{i(a+b)} c (b \cosh(c z) + a \sinh(c z)) \right)$$

01.22.21.0258.01

$$\int \frac{1}{\sqrt{a + b \operatorname{coth}(c z)}} dz = -\frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{c \sqrt{a + b \operatorname{coth}(c z)}} \left( \frac{\tanh^{-1}\left(\frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{\sqrt{i(a-b)}}\right)}{\sqrt{i(a-b)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{\sqrt{i(a+b)}}\right)}{\sqrt{i(a+b)}} \right)$$

01.22.21.0259.01

$$\int \operatorname{coth}(c z) (a + b \operatorname{coth}(c z))^\beta dz = \frac{(a + b \operatorname{coth}(c z))^{\beta+1}}{2(a-b)(a+b)c(\beta+1)} \left( (a+b) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b \operatorname{coth}(c z)}{a-b}\right) + (a-b) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b \operatorname{coth}(c z)}{a+b}\right) \right)$$

01.22.21.0260.01

$$\int \operatorname{coth}(c z) \sqrt{a + b \operatorname{coth}(c z)} dz = \left( \sqrt{a + b \operatorname{coth}(c z)} \left( (a-b) \sqrt{i(a+b)} \tanh^{-1}\left(\frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{\sqrt{i(a-b)}}\right) \sqrt{i(a+b \operatorname{coth}(c z))} \sinh(c z) + \sqrt{i(a-b)} \left( (a+b) \tanh^{-1}\left(\frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{\sqrt{i(a+b)}}\right) \sqrt{i(a+b \operatorname{coth}(c z))} \sinh(c z) - 2 \sqrt{i(a+b)} (b \cosh(c z) + a \sinh(c z)) \right) \right) \right) / \left( \sqrt{i(a-b)} \sqrt{i(a+b)} c (b \cosh(c z) + a \sinh(c z)) \right)$$

01.22.21.0261.01

$$\int \frac{\operatorname{coth}(c z)}{\sqrt{a + b \operatorname{coth}(c z)}} dz = \frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{c \sqrt{a + b \operatorname{coth}(c z)}} \left( \frac{\tanh^{-1}\left(\frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{\sqrt{i(a-b)}}\right)}{\sqrt{i(a-b)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{i(a+b \operatorname{coth}(c z))}}{\sqrt{i(a+b)}}\right)}{\sqrt{i(a+b)}} \right)$$

Involving  $((a + b \operatorname{coth}(c z))^n)^\beta$

01.22.21.0262.01

$$\int ((a + b \operatorname{coth}(c z))^n)^\beta dz = \frac{(a + b \operatorname{coth}(c z)) ((a + b \operatorname{coth}(c z))^n)^\beta}{2 (b - a) (a + b) c (n \beta + 1)}$$

$$\left( (a + b) {}_2F_1\left(n \beta + 1, 1; n \beta + 2; \frac{a + b \operatorname{coth}(c z)}{a - b}\right) + (b - a) {}_2F_1\left(n \beta + 1, 1; n \beta + 2; \frac{a + b \operatorname{coth}(c z)}{a + b}\right) \right)$$

01.22.21.0263.01

$$\int \sqrt{(a + b \operatorname{coth}(c z))^3} dz =$$

$$\left( \sqrt{(a + b \operatorname{coth}(c z))^3} \sinh(c z) \left( \sqrt{i(a - b)} \left( (a + b)^2 \tanh^{-1}\left(\frac{\sqrt{i(a + b \operatorname{coth}(c z))}}{\sqrt{i(a + b)}}\right) \sqrt{i(a + b \operatorname{coth}(c z))} \sinh(c z) - \right. \right. \right.$$

$$\left. \left. 2 b \sqrt{i(a + b)} (b \cosh(c z) + a \sinh(c z)) \right) - (a - b)^2 \sqrt{i(a + b)} \tanh^{-1}\left(\frac{\sqrt{i(a + b \operatorname{coth}(c z))}}{\sqrt{i(a - b)}}\right) \right.$$

$$\left. \left. \sqrt{i(a + b \operatorname{coth}(c z))} \sinh(c z) \right) \right) / \left( \sqrt{i(a - b)} \sqrt{i(a + b)} c (b \cosh(c z) + a \sinh(c z))^2 \right)$$

01.22.21.0264.01

$$\int \frac{1}{\sqrt{(a + b \operatorname{coth}(c z))^3}} dz =$$

$$\left( \sqrt{i(a + b)} (a + b \operatorname{coth}(c z)) \left( i \tanh^{-1}\left(\frac{\sqrt{i(a + b \operatorname{coth}(c z))}}{\sqrt{i(a - b)}}\right) \sqrt{i(a + b \operatorname{coth}(c z))} (i(a + b))^{3/2} + \sqrt{i(a - b)} \left( 2 \sqrt{i(a + b)} b + \right. \right. \right.$$

$$\left. \left. (a - b) \tanh^{-1}\left(\frac{\sqrt{i(a + b \operatorname{coth}(c z))}}{\sqrt{i(a + b)}}\right) \sqrt{i(a + b \operatorname{coth}(c z))} \right) \right) \right) / \left( (i(a - b))^{3/2} (a + b)^2 c \sqrt{(a + b \operatorname{coth}(c z))^3} \right)$$

01.22.21.0265.01

$$\int \operatorname{coth}(c z) ((a + b \operatorname{coth}(c z))^n)^\beta dz = \frac{1}{2 (a^2 - b^2) c (n \beta + 1)} \left( (a + b \operatorname{coth}(c z)) ((a + b \operatorname{coth}(c z))^n)^\beta \right.$$

$$\left. \left( (a + b) {}_2F_1\left(n \beta + 1, 1; n \beta + 2; \frac{a + b \operatorname{coth}(c z)}{a - b}\right) + (a - b) {}_2F_1\left(n \beta + 1, 1; n \beta + 2; \frac{a + b \operatorname{coth}(c z)}{a + b}\right) \right) \right)$$

01.22.21.0266.01

$$\int \operatorname{coth}(c z) \sqrt{(a + b \operatorname{coth}(c z))^3} dz = \frac{1}{3 c (a + b \operatorname{coth}(c z))^{3/2}} \left( \sqrt{(a + b \operatorname{coth}(c z))^3} \right.$$

$$\left. \left( 3 \tanh^{-1}\left(\frac{\sqrt{a + b \operatorname{coth}(c z)}}{\sqrt{a - b}}\right) (a - b)^{3/2} + 3 (a + b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a + b \operatorname{coth}(c z)}}{\sqrt{a + b}}\right) - 2 \sqrt{a + b \operatorname{coth}(c z)} (4 a + b \operatorname{coth}(c z)) \right) \right)$$

01.22.21.0267.01

$$\int \frac{\coth(cz)}{\sqrt{(a+b\coth(cz))^3}} dz =$$

$$-\left( \sqrt{i(a+b)} (a+b\coth(cz)) \left( i \tanh^{-1} \left( \frac{\sqrt{i(a+b\coth(cz))}}{\sqrt{i(a-b)}} \right) \sqrt{i(a+b\coth(cz))} (i(a+b))^{3/2} + \sqrt{i(a-b)} \right. \right.$$

$$\left. \left. \left( 2a\sqrt{i(a+b)} - (a-b) \tanh^{-1} \left( \frac{\sqrt{i(a+b\coth(cz))}}{\sqrt{i(a+b)}} \right) \sqrt{i(a+b\coth(cz))} \right) \right) \right) / \left( i \right.$$

$$\left. (a-b)^{3/2} (a+b)^2 c \sqrt{(a+b\coth(cz))^3} \right)$$

Involving  $(a+b\coth^2(cz))^\beta$

01.22.21.0268.01

$$\int (a+b\coth^2(cz))^\beta dz = \frac{\coth(cz) (b\coth^2(cz)+a)^\beta \left( \frac{b\coth^2(cz)}{a} + 1 \right)^{-\beta}}{c} F_1 \left( \frac{1}{2}; 1, -\beta; \frac{3}{2}; \coth^2(cz), -\frac{b\coth^2(cz)}{a} \right)$$



01.22.21.0269.01

$$\int \sqrt{a + b \coth^2(c z)} dz =$$

$$- \left( \sqrt{b \coth^2(c z) + a} \left( \sqrt{b} \log \left( \frac{4 \left( b + \frac{\sqrt{b} \sqrt{-a+b+(a+b) \cosh(2 c z)} \operatorname{sech}^4\left(\frac{c z}{2}\right)}{\sqrt{2}} \right) \coth^2\left(\frac{c z}{2}\right) + 2 a + b}{b^{3/2}} \right) + \sqrt{b} \right. \right.$$

$$\left. \log \left( \frac{2 b \tanh^2\left(\frac{c z}{2}\right) + 4 a + 2 b + \sqrt{2} \sqrt{b} \sqrt{-a+b+(a+b) \cosh(2 c z)} \operatorname{sech}^4\left(\frac{c z}{2}\right)}{\sqrt{b}} \right) - \right.$$

$$\left. \left. 2 \sqrt{a+b} \log \left( \frac{\cosh^2\left(\frac{c z}{2}\right) \left( (a+b) \tanh^2\left(\frac{c z}{2}\right) + a+b + \frac{\sqrt{a+b} \sqrt{-a+b+(a+b) \cosh(2 c z)} \operatorname{sech}^4\left(\frac{c z}{2}\right)}{\sqrt{2}} \right)}{(a+b)^{3/2}} \right) \right) \right)$$

$$\left. \tanh\left(\frac{c z}{2}\right) \right) / \left( \sqrt{2} c \sqrt{\frac{-a+b+(a+b) \cosh(2 c z)}{(\cosh(c z) + 1)^2}} \right)$$

01.22.21.0270.01

$$\int \frac{1}{\sqrt{a + b \coth^2(c z)}} dz =$$

$$\left( \sqrt{-a+b+(a+b) \cosh(2 c z)} \coth(c z) \log \left( \sqrt{2} \sqrt{(a+b) \cosh^2(c z)} + \sqrt{-a+b+(a+b) \cosh(2 c z)} \right) \right) /$$

$$\left( \sqrt{2} c \sqrt{(a+b) \cosh^2(c z)} \sqrt{b \coth^2(c z) + a} \right)$$

01.22.21.0271.01

$$\int \coth(c z) (a + b \coth^2(c z))^\beta dz = \frac{(b \coth^2(c z) + a)^{\beta+1}}{2(a+b)c(\beta+1)} {}_2F_1\left(\beta+1, 1; \beta+2; \frac{b \coth^2(c z) + a}{a+b}\right)$$

01.22.21.0272.01

$$\int \coth(c z) \sqrt{a + b \coth^2(c z)} dz = \frac{\sqrt{b \coth^2(c z) + a}}{c} \left( \frac{\sqrt{2} \log\left(\sqrt{-a + b + (a + b) \cosh(2 c z)} + \sqrt{2} \sqrt{(a + b) \sinh^2(c z)}\right) \sqrt{(a + b) \sinh^2(c z)}}{\sqrt{-a + b + (a + b) \cosh(2 c z)}} - 1 \right)$$

01.22.21.0273.01

$$\int \frac{\coth(c z)}{\sqrt{a + b \coth^2(c z)}} dz = \frac{\sqrt{-a + b + (a + b) \cosh(2 c z)} \log\left(\sqrt{-a + b + (a + b) \cosh(2 c z)} + \sqrt{2} \sqrt{(a + b) \sinh^2(c z)}\right)}{\sqrt{2} c \sqrt{b \coth^2(c z) + a} \sqrt{(a + b) \sinh^2(c z)}}$$

Involving  $\left((a + b \coth^2(c z))^n\right)^\beta$

01.22.21.0274.01

$$\int \left((a + b \coth^2(c z))^n\right)^\beta dz = \frac{\coth(c z) \left((b \coth^2(c z) + a)^n\right)^\beta \left(\frac{b \coth^2(c z)}{a} + 1\right)^{-n\beta}}{c} F_1\left(\frac{1}{2}; 1, -n\beta; \frac{3}{2}; \coth^2(c z), -\frac{b \coth^2(c z)}{a}\right)$$

01.22.21.0275.01

$$\int \sqrt{(a + b \coth^2(c z))^3} dz =$$

$$\frac{1}{c} \left( \sqrt{(b \coth^2(c z) + a)^3} \sinh(c z) \left( (\cosh(c z) + 1) \sqrt{\frac{-a + b + (a + b) \cosh(2 c z)}{(\cosh(c z) + 1)^2}} \right) \left( 4 \log \left( -\frac{1}{2 \sqrt{a + b}} \cosh^2\left(\frac{c z}{2}\right) \right. \right. \right. \right.$$

$$\left. \left. \left. \left( (a + b) \tanh^2\left(\frac{c z}{2}\right) + a + b + \frac{\sqrt{a + b} \sqrt{(-a + b + (a + b) \cosh(2 c z)) \operatorname{sech}^4\left(\frac{c z}{2}\right)}}{\sqrt{2}} \right) (a + b)^2 - \right. \right. \right.$$

$$\left. \left. \left. \sqrt{b} (3 a + 2 b) \log \left( -\frac{4 \left( b + \frac{\sqrt{b} \sqrt{(-a + b + (a + b) \cosh(2 c z)) \operatorname{sech}^4\left(\frac{c z}{2}\right)}}{\sqrt{2}} \right) \coth^2\left(\frac{c z}{2}\right) + 2 a + b}{\sqrt{b}} \right) \sqrt{a + b} - \sqrt{b} \right. \right. \right.$$

$$\left. \left. \left. (3 a + 2 b) \log \left( \frac{1}{\sqrt{b}} \left( 2 b \tanh^2\left(\frac{c z}{2}\right) + 4 a + 2 b + \sqrt{2} \sqrt{b} \sqrt{(-a + b + (a + b) \cosh(2 c z)) \operatorname{sech}^4\left(\frac{c z}{2}\right)} \right) \right) \right. \right. \right.$$

$$\left. \left. \left. \sqrt{a + b} \sinh^2(c z) \right) / \left( \sqrt{2} \sqrt{a + b} (-a + b + (a + b) \cosh(2 c z))^2 \right) - \frac{b \cosh(c z)}{-a + b + (a + b) \cosh(2 c z)} \right) \right)$$

01.22.21.0276.01

$$\int \frac{1}{\sqrt{(a + b \coth^2(c z))^3}} dz =$$

$$\left( \operatorname{csch}^2(c z) \left( \sqrt{2} \sqrt{a + b} \cosh^2(c z) \operatorname{csch}(2 c z) \log \left( \sqrt{2} \sqrt{a + b} \cosh^2(c z) + \sqrt{-a + b + (a + b) \cosh(2 c z)} \right) \right. \right.$$

$$\left. \left. (-a + b + (a + b) \cosh(2 c z))^{3/2} + \frac{b (a + b) \coth(c z) (-a + b + (a + b) \cosh(2 c z))}{a} \right) \right) / \left( 2 (a + b)^2 c \sqrt{(b \coth^2(c z) + a)^3} \right)$$

01.22.21.0277.01

$$\int \coth(cz) \left( (a + b \coth^2(cz))^n \right)^\beta dz = \frac{(b \coth^2(cz) + a) \left( (b \coth^2(cz) + a)^n \right)^\beta}{2(a+b)c(n\beta+1)} {}_2F_1 \left( n\beta+1, 1; n\beta+2; \frac{b \coth^2(cz) + a}{a+b} \right)$$

01.22.21.0278.01

$$\int \coth(cz) \sqrt{(a + b \coth^2(cz))^5} dz = \frac{1}{c} \left( \sqrt{(b \coth^2(cz) + a)^5} \sinh^5(cz) \right. \\ \left. \left( \left( 4\sqrt{2} \operatorname{csch}^5(cz) \log \left( \sqrt{-a+b+(a+b)\cosh(2cz)} + \sqrt{2} \sqrt{(a+b)\sinh^2(cz)} \right) \right) ((a+b)\sinh^2(cz))^{5/2} \right) / \right. \\ \left. \left. (-a+b+(a+b)\cosh(2cz))^{5/2} - \frac{4 \operatorname{csch}(cz) (3b^2 \operatorname{csch}^4(cz) + 11b(a+b)\operatorname{csch}^2(cz) + 23(a+b)^2)}{15(-a+b+(a+b)\cosh(2cz))^2} \right) \right)$$

01.22.21.0279.01

$$\int \coth(cz) \sqrt{(a + b \coth^2(cz))^3} dz = \\ - \left( 2 \sqrt{(b \coth^2(cz) + a)^3} \left( -3\sqrt{2} \log \left( \sqrt{-a+b+(a+b)\cosh(2cz)} + \sqrt{2} \sqrt{(a+b)\sinh^2(cz)} \right) \right) ((a+b)\sinh^2(cz))^{3/2} - \right. \\ \left. (2a+b) \sqrt{-a+b+(a+b)\cosh(2cz)} + \right. \\ \left. 2(a+b)\cosh(2cz) \sqrt{-a+b+(a+b)\cosh(2cz)} \right) / (3c(-a+b+(a+b)\cosh(2cz))^{3/2})$$

01.22.21.0280.01

$$\int \frac{\coth(cz)}{\sqrt{(a + b \coth^2(cz))^3}} dz = \\ - \left( (-a+b+(a+b)\cosh(2cz)) \operatorname{csch}^4(cz) \left( -a-b+(a+b)\cosh(2cz) - \sqrt{2} \sqrt{-a+b+(a+b)\cosh(2cz)} \right. \right. \\ \left. \left. \log \left( \sqrt{-a+b+(a+b)\cosh(2cz)} + \sqrt{2} \sqrt{(a+b)\sinh^2(cz)} \right) \right) \right. \\ \left. \sqrt{(a+b)\sinh^2(cz)} \right) / \left( 4(a+b)^2 c \sqrt{(b \coth^2(cz) + a)^3} \right)$$

01.22.21.0281.01

$$\int \frac{\coth(cz)}{\sqrt{(a+b\coth^2(cz))^5}} dz =$$

$$\left( (-a+b+(a+b)\cosh(2cz)) \operatorname{csch}^6(cz) \left( -2\cosh(4cz)(a+b)^2 + \cosh(2cz) \left( 8a+2b+3\sqrt{2}\sqrt{-a+b+(a+b)\cosh(2cz)} \right. \right. \right.$$

$$\left. \left. \log\left( \sqrt{-a+b+(a+b)\cosh(2cz)} + \sqrt{2}\sqrt{(a+b)\sinh^2(cz)} \right) \sqrt{(a+b)\sinh^2(cz)} \right) (a+b) - \right.$$

$$\left. 3 \left( 2a(a+b) + \sqrt{2}(a-b)\sqrt{-a+b+(a+b)\cosh(2cz)} \log\left( \sqrt{-a+b+(a+b)\cosh(2cz)} + \right. \right. \right.$$

$$\left. \left. \sqrt{2}\sqrt{(a+b)\sinh^2(cz)} \right) \sqrt{(a+b)\sinh^2(cz)} \right) \right) / \left( 24(a+b)^3 c \sqrt{(b\coth^2(cz)+a)^5} \right)$$

Involving  $(a+b\coth^{\frac{1}{2}}(cz))^\beta$

01.22.21.0282.01

$$\int (a+b\coth^{\frac{1}{2}}(cz))^\beta dz =$$

$$\frac{1}{2(a^4-b^4)c(\beta+1)} \left( (a+b\coth^{\frac{1}{2}}(cz))^{\beta+1} \left( (a^3+ba^2+b^2a+b^3) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b\coth^{\frac{1}{2}}(cz)}{a-b}\right) - \right. \right.$$

$$\left. (a-b) \left( (a-ib) \left( (a+b) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b\coth^{\frac{1}{2}}(cz)}{a+ib}\right) - (a+ib) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b\coth^{\frac{1}{2}}(cz)}{a+b}\right) \right) \right) \right.$$

$$\left. \left. \left. \left. \left. (a^2+b(1+i)a+b^2i) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b\coth^{\frac{1}{2}}(cz)}{a-ib}\right) \right) \right) \right) \right)$$

01.22.21.0283.01

$$\int \sqrt{a+b\coth^{\frac{1}{2}}(cz)} dz = \left( (a-b)\sqrt{a+b}\sqrt{a+ib}\sqrt{a-ib} \tanh^{-1}\left(\frac{\sqrt{a+b\coth^{\frac{1}{2}}(cz)}}{\sqrt{a-b}}\right) + \right.$$

$$\left. \sqrt{a-b} \left( \sqrt{a-ib} \left( \sqrt{a+ib} (a+b) \tanh^{-1}\left(\frac{\sqrt{a+b\coth^{\frac{1}{2}}(cz)}}{\sqrt{a+b}}\right) - (a+ib)\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b\coth^{\frac{1}{2}}(cz)}}{\sqrt{a+ib}}\right) \right) \right) \right.$$

$$\left. \left. \left. \left. \left. (a-ib)\sqrt{a+ib}\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b\coth^{\frac{1}{2}}(cz)}}{\sqrt{a-ib}}\right) \right) \right) \right) \right) / \left( \sqrt{a-b}\sqrt{a-ib}\sqrt{a+ib}\sqrt{a+b}c \right)$$

01.22.21.0284.01

$$\int \frac{1}{\sqrt{a + b \coth^{\frac{1}{2}}(c z)}} dz = \frac{1}{(a^4 - b^4)c} \left( \sqrt{a - b} (a^3 + b a^2 + b^2 a + b^3) \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(c z)}}{\sqrt{a - b}} \right) - \right.$$

$$(a - b) \left( (a - i b) \left( \sqrt{a + i b} (a + b) \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(c z)}}{\sqrt{a + i b}} \right) - (a + i b) \sqrt{a + b} \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(c z)}}{\sqrt{a + b}} \right) \right) + \right.$$

$$\left. \left. \left. \sqrt{a - i b} (a^2 + b(1 + i)a + b^2 i) \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(c z)}}{\sqrt{a - i b}} \right) \right) \right) \right)$$

01.22.21.0285.01

$$\int \coth(c z) \left( a + b \coth^{\frac{1}{2}}(c z) \right)^\beta dz =$$

$$\frac{1}{c(2(a^4 - b^4)(\beta + 1))} \left( \left( a + b \coth^{\frac{1}{2}}(c z) \right)^{\beta+1} \left( (a^3 + b a^2 + b^2 a + b^3) {}_2F_1 \left( \beta + 1, 1; \beta + 2; \frac{a + b \coth^{\frac{1}{2}}(c z)}{a - b} \right) + \right.$$

$$(a - b) \left( (a - i b) \left( (a + i b) {}_2F_1 \left( \beta + 1, 1; \beta + 2; \frac{a + b \coth^{\frac{1}{2}}(c z)}{a + b} \right) + (a + b) {}_2F_1 \left( \beta + 1, 1; \beta + 2; \frac{a + b \coth^{\frac{1}{2}}(c z)}{a + i b} \right) \right) + \right.$$

$$\left. \left. \left. (a^2 + b(1 + i)a + b^2 i) {}_2F_1 \left( \beta + 1, 1; \beta + 2; \frac{a + b \coth^{\frac{1}{2}}(c z)}{a - i b} \right) \right) \right) \right)$$

01.22.21.0286.01

$$\int \coth(cz) \sqrt{a + b \coth^{\frac{1}{2}}(cz)} dz = \left( (a-b) \sqrt{a+b} \sqrt{a+ib} \sqrt{a-ib} \sqrt{a + b \coth^{\frac{1}{2}}(cz)} \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(cz)}}{\sqrt{a-b}} \right) + \right.$$

$$\left. \sqrt{a-b} \left( \sqrt{a+b} \sqrt{a+ib} (a-ib) \sqrt{a + b \coth^{\frac{1}{2}}(cz)} \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(cz)}}{\sqrt{a-ib}} \right) + \right.$$

$$\left. \sqrt{a-ib} \left( \sqrt{a+b} (a+ib) \sqrt{a + b \coth^{\frac{1}{2}}(cz)} \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(cz)}}{\sqrt{a+ib}} \right) + \right.$$

$$\left. \left. \left. \left. \left. \sqrt{a+ib} \left( (a+b) \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(cz)}}{\sqrt{a+b}} \right) \sqrt{a + b \coth^{\frac{1}{2}}(cz)} - 4 \sqrt{a+b} (a + b \coth^{\frac{1}{2}}(cz)) \right) \right) \right) \right) \right) \right) /$$

$$\left( \sqrt{a-b} \sqrt{a-ib} \sqrt{a+ib} \sqrt{a+b} c \sqrt{a + b \coth^{\frac{1}{2}}(cz)} \right)$$

01.22.21.0287.01

$$\int \frac{\coth(cz)}{\sqrt{a + b \coth^{\frac{1}{2}}(cz)}} dz = \frac{1}{(a^4 - b^4)c} \left( \sqrt{a-b} (a^3 + b a^2 + b^2 a + b^3) \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(cz)}}{\sqrt{a-b}} \right) + \right.$$

$$\left. (a-b) \left( (a-ib) \left( \sqrt{a+b} (a+ib) \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(cz)}}{\sqrt{a+b}} \right) + (a+b) \sqrt{a+ib} \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(cz)}}{\sqrt{a+ib}} \right) \right) \right) + \right.$$

$$\left. \left. \left. \left. \left. \sqrt{a-ib} (a^2 + b(1+i)a + b^2 i) \tanh^{-1} \left( \frac{\sqrt{a + b \coth^{\frac{1}{2}}(cz)}}{\sqrt{a-ib}} \right) \right) \right) \right) \right) \right)$$

01.22.21.0288.01

$$\int \frac{\coth(cz)}{\left(a + b \coth^{\frac{1}{2}}(cz)\right)^2} dz =$$

$$-\frac{1}{4c} \left( \frac{8a^3}{(a^4 - b^4)\left(a + b \coth^{\frac{1}{2}}(cz)\right)} + \frac{2i \tan^{-1}\left(\coth^{\frac{1}{2}}(cz)\right)}{(a + ib)^2} + \frac{2 \log\left(\coth^{\frac{1}{2}}(cz) - 1\right)}{(a + b)^2} + \frac{2 \log\left(\coth^{\frac{1}{2}}(cz) + 1\right)}{(a - b)^2} + \right.$$

$$\left. \frac{\log(\coth(cz) + 1)}{(a + ib)^2} + \frac{\log(\coth(cz) - 1)}{(a - ib)^2} - \frac{2i \tan^{-1}\left(\coth^{\frac{1}{2}}(cz)\right)}{(a - ib)^2} - \frac{8(a^6 + 3b^4 a^2) \log\left(a + b \coth^{\frac{1}{2}}(cz)\right)}{(a^4 - b^4)^2} \right)$$

**Involving functions of the direct function and a power function**

**Involving powers of the direct function and a power function**

Involving powers of coth and power

**Involving  $z^n$  and linear arguments**

01.22.21.0289.01

$$\int z^n \coth^v(cz) dz = \frac{(-1)^v z^{n+1}}{n+1} + (-1)^v e^{2cvz} n! \sum_{j=0}^n \frac{(-1)^j (2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) +$$

$$(-1)^v e^{2cvz} v n! \sum_{j=0}^n \frac{(-1)^j (2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) +$$

$$(-1)^v e^{cvz} \left(\frac{v}{2}\right) n! (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2} + 1, \dots, \frac{v}{2} + 1; e^{2cz}\right) +$$

$$(-1)^v n! \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + e^{2c(v-s)z} \right.$$

$$\left. \sum_{j=0}^n \frac{(-1)^j (2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \right); n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$



01.22.21.0290.01

$$\int z \coth^v(cz) dz = \frac{1}{2} (-1)^v z^2 + \frac{(-1)^v e^{2cvz}}{4c^2 v^2} (2cv {}_2F_1(v, v; v+1; e^{2cz}) - {}_3F_2(v, v, v; v+1, v+1; e^{2cz})) + \frac{(-1)^v e^{2cvz}}{4c^2} (2c {}_3F_2(1, 1, v+1; 2, 2; e^{2cz}) - {}_4F_3(1, 1, 1, v+1; 2, 2, 2; e^{2cz})) + \frac{(-1)^v e^{cvz} (1-v \bmod 2)}{c^2 v^2} \left( \frac{v}{2} \right) \left( c {}_2F_1\left(\frac{v}{2}, v; \frac{v}{2} + 1; e^{2cz}\right) - {}_3F_2\left(\frac{v}{2}, \frac{v}{2}, v; \frac{v}{2} + 1, \frac{v}{2} + 1; e^{2cz}\right) \right) + \frac{1}{4c^2} (-1)^v \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( \frac{e^{2csz}}{s^2} (2cs {}_2F_1(s, v; s+1; e^{2cz}) - {}_3F_2(s, s, v; s+1, s+1; e^{2cz})) - \frac{1}{(s-v)^2} e^{2c(v-s)z} (2c(s-v) {}_2F_1(v, v-s; -s+v+1; e^{2cz}) + {}_3F_2(v, v-s, v-s; -s+v+1, -s+v+1; e^{2cz})) \right) /; v \in \mathbb{N}^+$$

01.22.21.0291.01

$$\int z \coth^2(cz) dz = \frac{z^2}{2} - \frac{\coth(cz) z}{c} + \frac{\log(\sinh(cz))}{c^2}$$

01.22.21.0292.01

$$\int z \coth^3(cz) dz = -\frac{\coth(cz) + cz(\operatorname{csch}^2(cz) - cz - 2\log(1 - e^{-2cz})) + \operatorname{Li}_2(e^{-2cz})}{2c^2}$$

01.22.21.0293.01

$$\int z \coth^4(cz) dz = -\frac{-3c^2 z^2 + 2c \coth(cz)(\operatorname{csch}^2(cz) + 4)z + \operatorname{csch}^2(cz) - 8\log(\sinh(cz))}{6c^2}$$

01.22.21.0294.01

$$\int z \coth^5(cz) dz = -\frac{1}{12c^2} (\coth(cz)(\operatorname{csch}^2(cz) + 10) + 3cz(\operatorname{csch}^4(cz) + 4\operatorname{csch}^2(cz) - 2cz - 4\log(1 - e^{-2cz})) + 6\operatorname{Li}_2(e^{-2cz}))$$

01.22.21.0295.01

$$\int z^2 \coth^2(cz) dz = \frac{cz(cz + 3) - 3cz \coth(cz) + 6\log(1 - e^{-2cz}) - 3\operatorname{Li}_2(e^{-2cz})}{3c^3}$$

01.22.21.0296.01

$$\int z^3 \coth^3(cz) dz = \frac{1}{64c^4} (-16c^4 z^4 - 32c^3 \operatorname{csch}^2(cz) z^3 + 64c^3 \log(1 - e^{2cz}) z^3 + 96c^2 z^2 - 96c^2 \coth(cz) z^2 + 96c^2 \operatorname{Li}_2(e^{2cz}) z^2 + 192c \log(1 - e^{-2cz}) z - 96c \operatorname{Li}_3(e^{2cz}) z + \pi^4 - 96\operatorname{Li}_2(e^{-2cz}) + 48\operatorname{Li}_4(e^{2cz}))$$

**Involving functions of the direct function and exponential function**

**Involving powers of the direct function and exponential function**

Involving exp

Involving  $e^{bz}$

01.22.21.0297.01

$$\int e^{bz} \coth^{\nu}(cz) dz = \frac{e^{bz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz)}{b} {}_2F_1\left(-\frac{b}{2c}; -\nu, \nu; 1 - \frac{b}{2c}; -e^{-2cz}, e^{-2cz}\right); b \neq 2c$$

01.22.21.0298.01

$$\int e^{2cz} \coth^{\nu}(cz) dz = \frac{2^{-\nu-1} (1 - e^{2cz})^{\nu} (1 + e^{2cz}) \coth^{\nu}(cz)}{c(\nu+1)} {}_2F_1\left(\nu+1, \nu; \nu+2; \frac{1}{2}(1 + e^{2cz})\right)$$

01.22.21.0299.01

$$\int e^{cz} \coth^2(cz) dz = \frac{1}{c} \left( e^{cz} \left( 1 - \frac{2}{-1 + e^{2cz}} \right) + \log(-1 + e^{cz}) - \log(1 + e^{cz}) \right)$$

01.22.21.0300.01

$$\int e^{2cz} \coth^2(cz) dz = \frac{1}{2c} \left( 4 \log(-1 + e^{2cz}) + e^{2cz} - \frac{4}{-1 + e^{2cz}} \right)$$

01.22.21.0301.01

$$\int e^{2cz} \coth^3(cz) dz = \frac{\frac{8-12e^{2cz}}{(-1+e^{2cz})^2} + e^{2cz} + 6 \log(-1 + e^{2cz})}{2c}$$

01.22.21.0302.01

$$\int e^{2cz} \coth^4(cz) dz = \frac{24 \log(-1 + e^{2cz})(-1 + e^{2cz})^3 + 93 e^{2cz} - 63 e^{4cz} - 9 e^{6cz} + 3 e^{8cz} - 40}{6c(-1 + e^{2cz})^3}$$

01.22.21.0303.01

$$\int e^{-2cz} \coth^4(cz) dz = -\frac{1}{6c} \left( \frac{8 e^{2cz} (9 - 12 e^{2cz} + 5 e^{4cz})}{(-1 + e^{2cz})^3} + 3 e^{-2cz} + 24 \log(-1 + e^{-2cz}) \right)$$

**Involving functions of the direct function, exponential and a power functions**

**Involving powers of the direct function, exponential and a power functions**

Involving exp and power

**Involving  $z^n e^{bz}$**

01.22.21.0304.01

$$\int z^n e^{bz} \coth^v(cz) dz = (-1)^v e^{(b+cv)z} \left(\frac{v}{2}\right) n! (1-v \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j (b+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+cv}{2c}, \dots, \frac{b+cv}{2c}, v; \frac{b+cv}{2c}+1, \dots, \frac{b+cv}{2c}+1; e^{2cz}\right) + (-1)^v n! \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s}$$

$$\left( e^{(b+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2cs}{2c}, \dots, \frac{b+2cs}{2c}, v; \frac{b+2cs}{2c}+1, \dots, \frac{b+2cs}{2c}+1; e^{2cz}\right) + e^{(b+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2c(v-s)}{2c}, \dots, \frac{b+2c(v-s)}{2c}, v; \frac{b+2c(v-s)}{2c}+1, \dots, \frac{b+2c(v-s)}{2c}+1; e^{2cz}\right) \right); n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

**Involving functions of the direct function and trigonometric functions**

**Involving powers of the direct function and trigonometric functions**

Involving sin

**Involving sin(bz)**

01.22.21.0305.01

$$\int \sin(bz) \coth^v(cz) dz = -\frac{1}{2b} e^{-ibz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz)$$

$$\left( F_1\left(\frac{ib}{2c}; -v, v; 1 + \frac{ib}{2c}; -e^{-2cz}, e^{-2cz}\right) + e^{2ibz} F_1\left(-\frac{ib}{2c}; -v, v; 1 - \frac{ib}{2c}; -e^{-2cz}, e^{-2cz}\right) \right)$$

Involving powers of sin

**Involving sin<sup>m</sup>(bz)**

01.22.21.0306.01

$$\int \sin^m(bz) \coth^v(cz) dz =$$

$$\frac{2^{-m} \coth^{v+1}(cz) (1 - m \bmod 2)}{c(v+1)} \left(\frac{m}{2}\right) {}_2F_1\left(\frac{v+1}{2}, 1; \frac{v+1}{2}+1; \coth^2(cz)\right) + \frac{2^{-m} i^{1-m} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz)}{b}$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{(-1)^k e^{-ib(m-2k)z}}{2k-m} \binom{m}{k} \left( e^{2ib(m-2k)z} F_1\left(-\frac{ib(m-2k)}{2c}; -v, v; 1 - \frac{ib(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) - (-1)^m F_1\left(\frac{ib(m-2k)}{2c}; -v, v; \frac{ib(m-2k)}{2c}+1; -e^{-2cz}, e^{-2cz}\right) \right); m \in \mathbb{N}^+$$

Involving cos

**Involving cos(b z)**

01.22.21.0307.01

$$\int \cos(b z) \coth^{\nu}(c z) dz = -\frac{1}{2b} \left( i e^{-ibz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \right. \\ \left. \left( e^{2ibz} F_1\left(-\frac{ib}{2c}; -\nu, \nu; 1 - \frac{ib}{2c}; -e^{-2cz}, e^{-2cz}\right) - F_1\left(\frac{ib}{2c}; -\nu, \nu; 1 + \frac{ib}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) \coth^{\nu}(c z) \right)$$

Involving powers of cos

**Involving cos<sup>m</sup>(b z)**

01.22.21.0308.01

$$\int \cos^m(b z) \coth^{\nu}(c z) dz = \\ \frac{2^{-m} \coth^{\nu+1}(c z) (1 - m \bmod 2)}{c (\nu + 1)} \left( \frac{m}{\frac{m}{2}} {}_2F_1\left(\frac{\nu + 1}{2}, 1; \frac{\nu + 1}{2} + 1; \coth^2(c z)\right) + \frac{2^{-m} i (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(c z)}{b} \right) \\ - \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{1}{m - 2k} \binom{m}{k} \left( e^{-ib(m-2k)z} F_1\left(\frac{ib(m-2k)}{2c}; -\nu, \nu; \frac{b i(m-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) - \right. \\ \left. e^{ib(m-2k)z} F_1\left(-\frac{ib(m-2k)}{2c}; -\nu, \nu; 1 - \frac{ib(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) /; m \in \mathbb{N}^+$$

**Involving functions of the direct function, trigonometric and a power functions**

**Involving powers of the direct function, trigonometric and a power functions**

Involving sin and power

**Involving z<sup>n</sup> sin(a + b z) coth<sup>ν</sup>(c z)**

01.22.21.0309.01

$$\int z^n \sin(a + b z) \coth^v(c z) dz =$$

$$\begin{aligned} & \frac{1}{2} i (-1)^{v-1} n! \left[ -e^{-ia + (-ib+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ib+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ib+cv}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-ib+cv}{2c}, v; \frac{-ib+cv}{2c} + 1, \dots, \frac{-ib+cv}{2c} + 1; e^{2cz} \right) + e^{ia + (ib+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \right. \\ & \quad \left. \sum_{j=0}^n \frac{(-1)^j (ib+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib+cv}{2c}, \dots, \frac{ib+cv}{2c}, v; \frac{ib+cv}{2c} + 1, \dots, \frac{ib+cv}{2c} + 1; e^{2cz} \right) - \right. \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{-ia + (-ib+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-ib+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ib+2cs}{2c}, \dots, \frac{-ib+2cs}{2c}, v; \frac{-ib+2cs}{2c} + 1, \right. \right. \\ & \quad \left. \left. \dots, \frac{-ib+2cs}{2c} + 1; e^{2cz} \right) + e^{-ia + (-ib+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ib+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\ & \quad \left. \left( \frac{-ib+2c(v-s)}{2c}, \dots, \frac{-ib+2c(v-s)}{2c}, v; \frac{-ib+2c(v-s)}{2c} + 1, \dots, \frac{-ib+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{ia + (ib+2cs)z} \sum_{j=0}^n \frac{(-1)^j (ib+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib+2cs}{2c}, \dots, \frac{ib+2cs}{2c}, v; \frac{ib+2cs}{2c} + 1, \dots, \right. \right. \\ & \quad \left. \left. \frac{ib+2cs}{2c} + 1; e^{2cz} \right) + e^{ia + (ib+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ib+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib+2c(v-s)}{2c}, \right. \right. \\ & \quad \left. \left. \dots, \frac{ib+2c(v-s)}{2c}, v; \frac{ib+2c(v-s)}{2c} + 1, \dots, \frac{ib+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \right] /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$

01.22.21.0310.01

$$\int z^n \sin(bz) \coth^v(cz) dz = -\frac{1}{2} i (-1)^v n! \left[ -e^{(-ib+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ib+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ \left. {}_{j+2}F_{j+1} \left( \frac{cv-ib}{2c}, \dots, \frac{cv-ib}{2c}, v; \frac{cv-ib}{2c} + 1, \dots, \frac{cv-ib}{2c} + 1; e^{2cz} \right) + e^{(ib+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \right. \\ \left. \sum_{j=0}^n \frac{(-1)^j (ib+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib+cv}{2c}, \dots, \frac{ib+cv}{2c}, v; \frac{ib+cv}{2c} + 1, \dots, \frac{ib+cv}{2c} + 1; e^{2cz} \right) - \right. \\ \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ib+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-ib+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-ib}{2c}, \dots, \frac{2cs-ib}{2c}, v; \frac{2cs-ib}{2c} + 1, \right. \right. \\ \left. \left. \dots, \frac{2cs-ib}{2c} + 1; e^{2cz} \right) + e^{(-ib+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ib+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\ \left. {}_{j+2}F_{j+1} \left( \frac{2c(v-s)-ib}{2c}, \dots, \frac{2c(v-s)-ib}{2c}, v; \frac{2c(v-s)-ib}{2c} + 1, \dots, \frac{2c(v-s)-ib}{2c} + 1; e^{2cz} \right) \right) + \\ \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ib+2cs)z} \sum_{j=0}^n \frac{(-1)^j (ib+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib+2cs}{2c}, \dots, \frac{ib+2cs}{2c}, v; \frac{ib+2cs}{2c} + 1, \right. \right. \\ \left. \left. \dots, \frac{ib+2cs}{2c} + 1; e^{2cz} \right) + e^{(ib+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ib+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib+2c(v-s)}{2c}, \right. \right. \\ \left. \left. \dots, \frac{ib+2c(v-s)}{2c}, v; \frac{ib+2c(v-s)}{2c} + 1, \dots, \frac{ib+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg] ; n \in \mathbb{N} \wedge u \in \mathbb{N}^+$$

Involving powers of sin and power

Involving  $z^n \operatorname{sinh}^m(bz) \coth^v(cz)$

01.22.21.0311.01

$$\int z^n \sin^m(bz) \coth^v(cz) dz = \\ n! \left[ \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{k+v} \binom{m}{k} \left( (-1)^m e^{(cv-ib(m-2k))z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv-ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \right. \\ \left. \left. \left( \frac{cv-ib(m-2k)}{2c}, \dots, \frac{cv-ib(m-2k)}{2c}, v; \frac{cv-ib(m-2k)}{2c} + 1, \dots, \frac{cv-ib(m-2k)}{2c} + 1; e^{2cz} \right) + \right. \right. \\ \left. \left. e^{(bi(m-2k)+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (bi(m-2k)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right) \right]$$

$$\begin{aligned}
 & \left( \frac{i b (m-2 k)+c v}{2 c}, \dots, \frac{i b (m-2 k)+c v}{2 c}, v ; \frac{i b (m-2 k)+c v}{2 c}+1, \dots, \frac{i b (m-2 k)+c v}{2 c}+1 ; e^{2 c z} \right) + \\
 & (-1)^m \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2 c s-i b (m-2 k)) z} \sum_{j=0}^n \frac{(-1)^j (2 c s-i b (m-2 k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{2 c s-i b (m-2 k)}{2 c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{2 c s-i b (m-2 k)}{2 c}, v ; \frac{2 c s-i b (m-2 k)}{2 c}+1, \dots, \frac{2 c s-i b (m-2 k)}{2 c}+1 ; e^{2 c z} \right) + \right. \\
 & \quad \left. e^{(2 c (v-s)-i b (m-2 k)) z} \sum_{j=0}^n \frac{(-1)^j (2 c (v-s)-i b (m-2 k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{2 c (v-s)-i b (m-2 k)}{2 c}, \dots, \frac{2 c (v-s)-i b (m-2 k)}{2 c}, v ; \right. \right. \\
 & \quad \left. \left. \frac{2 c (v-s)-i b (m-2 k)}{2 c}+1, \dots, \frac{2 c (v-s)-i b (m-2 k)}{2 c}+1 ; e^{2 c z} \right) \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(b i (m-2 k)+2 c s) z} \sum_{j=0}^n \frac{(-1)^j (b i (m-2 k)+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{i b (m-2 k)+2 c s}{2 c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{i b (m-2 k)+2 c s}{2 c}, v ; \frac{i b (m-2 k)+2 c s}{2 c}+1, \dots, \frac{i b (m-2 k)+2 c s}{2 c}+1 ; e^{2 c z} \right) + \right. \\
 & \quad \left. e^{(b i (m-2 k)+2 c (v-s)) z} \sum_{j=0}^n \frac{(-1)^j (b i (m-2 k)+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{i b (m-2 k)+2 c (v-s)}{2 c}, \dots, \frac{i b (m-2 k)+2 c (v-s)}{2 c}, v ; \right. \right. \\
 & \quad \left. \left. \frac{i b (m-2 k)+2 c (v-s)}{2 c}+1, \dots, \frac{i b (m-2 k)+2 c (v-s)}{2 c}+1 ; e^{2 c z} \right) \right) \left. \right) \left. \right) \left. \right) (2 i)^{-m} + (-1)^v 2^{-m} \binom{m}{\frac{m}{2}} n! \\
 & (1-m \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + e^{2 c v z} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} (v, \dots, v ; v+1, \dots, v+1 ; e^{2 c z}) + \right. \\
 & \quad e^{2 c z} v \\
 & \quad \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3} F_{j+2} (1, \dots, 1, v+1 ; 2, \dots, 2 ; e^{2 c z}) + \\
 & \quad e^{c v z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{v}{2}, \dots, \frac{v}{2}, v ; \frac{v}{2}+1, \dots, \frac{v}{2}+1 ; e^{2 c z} \right) + \\
 & \quad \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2 c s z} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} (s, \dots, s, v ; s+1, \dots, s+1 ; e^{2 c z}) + \right. \\
 & \quad \left. e^{2 c (v-s) z} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (c (v-s))^{-j-1} z^{n-j}}{(n-j)!} \right.
 \end{aligned}$$

Involving cos and power

Involving  $z^n \cos(a + b z) \coth^v(c z)$

01.22.21.0312.01

$$\int z^n \cos(a + b z) \coth^v(c z) dz =$$

$$\begin{aligned} & \frac{1}{2} (-1)^v n! \left( e^{-ia + (-ib + cv)z} \left(\frac{v}{2}\right) (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ib + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ib + cv}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-ib + cv}{2c}, v; \frac{-ib + cv}{2c} + 1, \dots, \frac{-ib + cv}{2c} + 1; e^{2cz} \right) + e^{ia + (ib + cv)z} \left(\frac{v}{2}\right) (1 - v \bmod 2) \right. \\ & \quad \left. \sum_{j=0}^n \frac{(-1)^j (ib + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib + cv}{2c}, \dots, \frac{ib + cv}{2c}, v; \frac{ib + cv}{2c} + 1, \dots, \frac{ib + cv}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{-ia + (-ib + 2cs)z} \sum_{j=0}^n \frac{(-1)^j (-ib + 2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ib + 2cs}{2c}, \dots, \frac{-ib + 2cs}{2c}, v; \frac{-ib + 2cs}{2c} + 1, \right. \right. \\ & \quad \left. \left. \dots, \frac{-ib + 2cs}{2c} + 1; e^{2cz} \right) + e^{-ia + (-ib + 2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ib + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\ & \quad \left. \left( \frac{-ib + 2c(v-s)}{2c}, \dots, \frac{-ib + 2c(v-s)}{2c}, v; \frac{-ib + 2c(v-s)}{2c} + 1, \dots, \frac{-ib + 2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{ia + (ib + 2cs)z} \sum_{j=0}^n \frac{(-1)^j (ib + 2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib + 2cs}{2c}, \dots, \frac{ib + 2cs}{2c}, v; \frac{ib + 2cs}{2c} + 1, \dots, \right. \right. \\ & \quad \left. \left. \frac{ib + 2cs}{2c} + 1; e^{2cz} \right) + e^{ia + (ib + 2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ib + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib + 2c(v-s)}{2c}, \right. \right. \\ & \quad \left. \left. \dots, \frac{ib + 2c(v-s)}{2c}, v; \frac{ib + 2c(v-s)}{2c} + 1, \dots, \frac{ib + 2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$



01.22.21.0313.01

$$\int z^n \cos(bz) \coth^v(cz) dz = \frac{1}{2} (-1)^v n! \left[ e^{(-ib+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ib+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ \left. {}_{j+2}F_{j+1} \left( \frac{cv-ib}{2c}, \dots, \frac{cv-ib}{2c}, v; \frac{cv-ib}{2c} + 1, \dots, \frac{cv-ib}{2c} + 1; e^{2cz} \right) + e^{(ib+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \right. \\ \left. \sum_{j=0}^n \frac{(-1)^j (ib+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib+cv}{2c}, \dots, \frac{ib+cv}{2c}, v; \frac{ib+cv}{2c} + 1, \dots, \frac{ib+cv}{2c} + 1; e^{2cz} \right) + \right. \\ \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ib+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-ib+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-ib}{2c}, \dots, \frac{2cs-ib}{2c}, v; \frac{2cs-ib}{2c} + 1, \right. \right. \right. \\ \left. \left. \dots, \frac{2cs-ib}{2c} + 1; e^{2cz} \right) + e^{(-ib+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ib+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\ \left. {}_{j+2}F_{j+1} \left( \frac{2c(v-s)-ib}{2c}, \dots, \frac{2c(v-s)-ib}{2c}, v; \frac{2c(v-s)-ib}{2c} + 1, \dots, \frac{2c(v-s)-ib}{2c} + 1; e^{2cz} \right) \right) + \\ \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ib+2cs)z} \sum_{j=0}^n \frac{(-1)^j (ib+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib+2cs}{2c}, \dots, \frac{ib+2cs}{2c}, v; \frac{ib+2cs}{2c} + 1, \right. \right. \right. \\ \left. \left. \dots, \frac{ib+2cs}{2c} + 1; e^{2cz} \right) + e^{(ib+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ib+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib+2c(v-s)}{2c}, \right. \right. \\ \left. \left. \dots, \frac{ib+2c(v-s)}{2c}, v; \frac{ib+2c(v-s)}{2c} + 1, \dots, \frac{ib+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg] ; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of cos and power

### Involving $z^n \cos^m(bz) \coth^v(cz)$

01.22.21.0314.01

$$\int z^n \cos^m(bz) \coth^v(cz) dz = \\ (-1)^v 2^{-m} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + \right. \\ \left. e^{2cvz} v \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) + \right. \\ \left. e^{cvz} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2} + 1, \dots, \frac{v}{2} + 1; e^{2cz} \right) + \right.$$

$$\begin{aligned}
 & \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + \right. \\
 & \left. e^{2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \right) + \\
 & (-1)^v 2^{-m} n! \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(cv-ib(m-2k))z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv-ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{cv-ib(m-2k)}{2c}, \dots, \frac{cv-ib(m-2k)}{2c}, v; \frac{cv-ib(m-2k)}{2c} + 1, \dots, \frac{cv-ib(m-2k)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(bi(m-2k)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (bi(m-2k)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{ib(m-2k)+cv}{2c}, \dots, \frac{ib(m-2k)+cv}{2c}, v; \frac{ib(m-2k)+cv}{2c} + 1, \dots, \frac{ib(m-2k)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-ib(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2cs-ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-ib(m-2k)}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{2cs-ib(m-2k)}{2c}, v; \frac{2cs-ib(m-2k)}{2c} + 1, \dots, \frac{2cs-ib(m-2k)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(2c(v-s)-ib(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-ib(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{2c(v-s)-ib(m-2k)}{2c}, \dots, \frac{2c(v-s)-ib(m-2k)}{2c}, v; \right. \right. \\
 & \left. \left. \frac{2c(v-s)-ib(m-2k)}{2c} + 1, \dots, \frac{2c(v-s)-ib(m-2k)}{2c} + 1; e^{2cz} \right) \right) + \\
 & \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(bi(m-2k)+2cs)z} \sum_{j=0}^n \frac{(-1)^j (bi(m-2k)+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib(m-2k)+2cs}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{ib(m-2k)+2cs}{2c}, v; \frac{ib(m-2k)+2cs}{2c} + 1, \dots, \frac{ib(m-2k)+2cs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(bi(m-2k)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (bi(m-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{ib(m-2k)+2c(v-s)}{2c}, \dots, \frac{ib(m-2k)+2c(v-s)}{2c}, v; \frac{ib(m-2k)+2c(v-s)}{2c} + 1, \right. \right. \\
 & \left. \left. \dots, \frac{ib(m-2k)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$

**Involving functions of the direct function, trigonometric and exponential functions**

**Involving powers of the direct function, trigonometric and exponential functions**

Involving sin and exp

**Involving  $e^{pz} \sin(az) \coth^{\nu}(cz)$**

01.22.21.0315.01

$$\int e^{pz} \sin(az) \coth^{\nu}(cz) dz = \frac{1}{2} i (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz) \left( \frac{e^{(-ia+p)z} F_1\left(-\frac{-ia+p}{2c}; -\nu, \nu; 1 - \frac{-ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{-ia+p} - \frac{e^{(ia+p)z} F_1\left(-\frac{ia+p}{2c}; -\nu, \nu; 1 - \frac{ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ia+p} \right)$$

01.22.21.0316.01

$$\int e^{iaz} \sin(az) \coth^{\nu}(cz) dz = \frac{i \coth^{\nu+1}(cz)}{2c(\nu+1)} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+3}{2}; \coth^2(cz)\right) - \frac{e^{2iaz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz)}{4a} F_1\left(-\frac{ia}{c}; -\nu, \nu; 1 - \frac{ia}{c}; -e^{-2cz}, e^{-2cz}\right)$$

01.22.21.0317.01

$$\int e^{-iaz} \sin(az) \coth^{\nu}(cz) dz = -\frac{e^{-2iaz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz)}{4a} F_1\left(\frac{ia}{c}; -\nu, \nu; 1 + \frac{ia}{c}; -e^{-2cz}, e^{-2cz}\right) - \frac{i \coth^{\nu+1}(cz)}{2c(\nu+1)} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+3}{2}; \coth^2(cz)\right)$$

Involving powers of sin and exp

**Involving  $e^{pz} \sin^m(az) \coth^{\nu}(cz)$**

01.22.21.0318.01

$$\int e^{pz} \sin^m(az) \coth^{\nu}(cz) dz = \frac{1}{p} 2^{-m} e^{pz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \binom{m}{\frac{m}{2}} \coth^{\nu}(cz) (1 - m \bmod 2) F_1\left(-\frac{p}{2c}; -\nu, \nu; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz}\right) + 2^{-m} i^{-m} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \frac{(-1)^m e^{(p-ia(m-2k))z} F_1\left(-\frac{p-ia(m-2k)}{2c}; -\nu, \nu; 1 - \frac{p-ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p - ia(m-2k)} + \frac{e^{(a i(m-2k)+p)z} F_1\left(-\frac{a i(m-2k)+p}{2c}; -\nu, \nu; 1 - \frac{a i(m-2k)+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{a i(m-2k) + p} \right) /; m \in \mathbb{N}^+$$

Involving cos and exp

### Involving $e^{pz} \cos(az) \coth^{\nu}(cz)$

01.22.21.0319.01

$$\int e^{pz} \cos(az) \coth^{\nu}(cz) dz = \frac{1}{2} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz) \left( \frac{e^{(-ia+p)z} F_1\left(-\frac{-ia+p}{2c}; -\nu, \nu; 1 - \frac{-ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{-ia+p} + \frac{e^{(ia+p)z} F_1\left(-\frac{ia+p}{2c}; -\nu, \nu; 1 - \frac{ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ia+p} \right)$$

01.22.21.0320.01

$$\int e^{iaz} \cos(az) \coth^{\nu}(cz) dz = \frac{\coth^{\nu+1}(cz)}{2c(\nu+1)} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+3}{2}; \coth^2(cz)\right) - \frac{i e^{2iaz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz)}{4a} F_1\left(-\frac{ia}{c}; -\nu, \nu; 1 - \frac{ia}{c}; -e^{-2cz}, e^{-2cz}\right)$$

01.22.21.0321.01

$$\int e^{-iaz} \cos(az) \coth^{\nu}(cz) dz = \frac{i e^{-2iaz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz)}{4a} F_1\left(\frac{ia}{c}; -\nu, \nu; 1 + \frac{ia}{c}; -e^{-2cz}, e^{-2cz}\right) + \frac{\coth^{\nu+1}(cz)}{2c(\nu+1)} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+3}{2}; \coth^2(cz)\right)$$

### Involving powers of cos and exp

### Involving $e^{pz} \cos^m(az) \coth^{\nu}(cz)$

01.22.21.0322.01

$$\int e^{pz} \cos^m(az) \coth^{\nu}(cz) dz = 2^{-m} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \frac{e^{(p-ia(m-2k))z} F_1\left(-\frac{p-ia(m-2k)}{2c}; -\nu, \nu; 1 - \frac{p-ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p-ia(m-2k)} + \frac{e^{(ai(m-2k)+p)z} F_1\left(-\frac{ai(m-2k)+p}{2c}; -\nu, \nu; 1 - \frac{ai(m-2k)+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ai(m-2k)+p} \right) + \frac{1}{p} 2^{-m} e^{pz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz) \left(\frac{m}{2}\right) (1 - m \bmod 2) F_1\left(-\frac{p}{2c}; -\nu, \nu; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz}\right) /; m \in \mathbb{N}^+$$

### Involving functions of the direct function, trigonometric, exponential and a power functions

### Involving powers of the direct function, trigonometric, exponential and a power functions

### Involving sin, exp and power

### Involving $z^n e^{pz} \sin(a+bz) \coth^{\nu}(cz)$

01.22.21.0323.01

$$\int z^n e^{p z} \sin(a + b z) \coth^v(c z) dz =$$

$$\begin{aligned} & \frac{i}{2} (-1)^v n! \left( e^{(-i b + p + c v) z - i a} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-i b + p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + p + c v}{2 c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-i b + p + c v}{2 c}, v; \frac{-i b + p + c v}{2 c} + 1, \dots, \frac{-i b + p + c v}{2 c} + 1; e^{2 c z} \right) - e^{i a + (i b + p + c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \right. \\ & \quad \left. \sum_{j=0}^n \frac{(-1)^j (i b + p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p + c v}{2 c}, \dots, \frac{i b + p + c v}{2 c}, v; \frac{i b + p + c v}{2 c} + 1, \dots, \right. \right. \\ & \quad \left. \left. \frac{i b + p + c v}{2 c} + 1; e^{2 c z} \right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-i b + p + 2 c s) z - i a} \sum_{j=0}^n \frac{(-1)^j (-i b + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-i b + p + 2 c s}{2 c}, \dots, \frac{-i b + p + 2 c s}{2 c}, v; \frac{-i b + p + 2 c s}{2 c} + 1, \dots, \frac{-i b + p + 2 c s}{2 c} + 1; e^{2 c z} \right) + \right. \\ & \quad \left. e^{(-i b + p + 2 c (v-s)) z - i a} \sum_{j=0}^n \frac{(-1)^j (-i b + p + 2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + p + 2 c (v-s)}{2 c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-i b + p + 2 c (v-s)}{2 c}, v; \frac{-i b + p + 2 c (v-s)}{2 c} + 1, \dots, \frac{-i b + p + 2 c (v-s)}{2 c} + 1; e^{2 c z} \right) \right) - \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{i a + (i b + p + 2 c s) z} \sum_{j=0}^n \frac{(-1)^j (i b + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p + 2 c s}{2 c}, \dots, \frac{i b + p + 2 c s}{2 c}, \right. \right. \\ & \quad \left. \left. v; \frac{i b + p + 2 c s}{2 c} + 1, \dots, \frac{i b + p + 2 c s}{2 c} + 1; e^{2 c z} \right) + \right. \\ & \quad \left. e^{i a + (i b + p + 2 c (v-s)) z} \sum_{j=0}^n \frac{(-1)^j (i b + p + 2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p + 2 c (v-s)}{2 c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{i b + p + 2 c (v-s)}{2 c}, v; \frac{i b + p + 2 c (v-s)}{2 c} + 1, \dots, \frac{i b + p + 2 c (v-s)}{2 c} + 1; e^{2 c z} \right) \right) \Bigg) ; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$

01.22.21.0324.01

$$\int z^n e^{p z} \sin(b z) \coth^v(c z) dz =$$

$$\frac{i}{2} (-1)^v \binom{v}{\frac{v}{2}} n! (1 - v \bmod 2) \left( e^{(-i b + p + c v) z} \sum_{j=0}^n \frac{(-1)^j (-i b + p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + p + c v}{2c}, \dots, \frac{-i b + p + c v}{2c}, v; \right. \right.$$

$$\left. \frac{-i b + p + c v}{2c} + 1, \dots, \frac{-i b + p + c v}{2c} + 1; e^{2c z} \right) - e^{(i b + p + c v) z} \sum_{j=0}^n \frac{(-1)^j (i b + p + c v)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{i b + p + c v}{2c}, \dots, \frac{i b + p + c v}{2c}, v; \frac{i b + p + c v}{2c} + 1, \dots, \frac{i b + p + c v}{2c} + 1; e^{2c z} \right) \right) +$$

$$\frac{i}{2} (-1)^v n! \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( e^{(-i b + 2c i + p) z} \sum_{j=0}^n \frac{(-1)^j (-i b + 2c i + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + 2c i + p}{2c}, \dots, \frac{-i b + 2c i + p}{2c}, \right. \right.$$

$$\left. v; \frac{-i b + 2c i + p}{2c} + 1, \dots, \frac{-i b + 2c i + p}{2c} + 1; e^{2c z} \right) - e^{(i b + 2c i + p) z} \sum_{j=0}^n \frac{(-1)^j (i b + 2c i + p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{i b + 2c i + p}{2c}, \dots, \frac{i b + 2c i + p}{2c}, v; \frac{i b + 2c i + p}{2c} + 1, \dots, \frac{i b + 2c i + p}{2c} + 1; e^{2c z} \right) + \right.$$

$$e^{(-i b - 2c i + p + 2c v) z} \sum_{j=0}^n \frac{(-1)^j (-i b - 2c i + p + 2c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b - 2c i + p + 2c v}{2c}, \dots, \right.$$

$$\left. \frac{-i b - 2c i + p + 2c v}{2c}, v; \frac{-i b - 2c i + p + 2c v}{2c} + 1, \dots, \frac{-i b - 2c i + p + 2c v}{2c} + 1; e^{2c z} \right) -$$

$$e^{(i b - 2c i + p + 2c v) z} \sum_{j=0}^n \frac{(-1)^j (i b - 2c i + p + 2c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b - 2c i + p + 2c v}{2c}, \dots, \frac{i b - 2c i + p + 2c v}{2c}, \right.$$

$$\left. v; \frac{i b - 2c i + p + 2c v}{2c} + 1, \dots, \frac{i b - 2c i + p + 2c v}{2c} + 1; e^{2c z} \right) \Big/; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of sin, exp and power

Involving  $z^n e^{p z} \sin^m(b z) \coth^v(c z)$

01.22.21.0325.01

$$\int z^n e^{p z} \sin^m(b z) \coth^v(c z) dz = 2^{-m} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) (-1)^v n! \left( e^{(p+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \right.$$

$$\sum_{j=0}^n \frac{(-1)^j (p+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+c v}{2c}, \dots, \frac{p+c v}{2c}, v; \frac{p+c v}{2c} + 1, \dots, \frac{p+c v}{2c} + 1; e^{2c z} \right) +$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2c s) z} \sum_{j=0}^n \frac{(-1)^j (p+2c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2c s}{2c}, \dots, \frac{p+2c s}{2c}, v; \frac{p+2c s}{2c} + \right. \right.$$

$$\begin{aligned}
 & 1, \dots, \frac{p+2cs}{2c} + 1; e^{2cz} \Big) + e^{(p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{p+2c(v-s)}{2c}, \dots, \frac{p+2c(v-s)}{2c}, v; \frac{p+2c(v-s)}{2c} + 1, \dots, \frac{p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) + \\
 & (-1)^v i^{-m} 2^{-m} n! \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{(bi(m-2k)+p+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (bi(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1} \left( \frac{ib(m-2k)+p+cv}{2c}, \dots, \frac{ib(m-2k)+p+cv}{2c}, v; \frac{ib(m-2k)+p+cv}{2c} + 1, \right. \\
 & \left. \dots, \frac{ib(m-2k)+p+cv}{2c} + 1; e^{2cz} \right) + (-1)^m \left( e^{(-ib(m-2k)+p+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \right. \\
 & \sum_{j=0}^n \frac{(-1)^j (-ib(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ib(m-2k)+p+cv}{2c}, \dots, \right. \\
 & \left. \frac{-ib(m-2k)+p+cv}{2c}, v; \frac{-ib(m-2k)+p+cv}{2c} + 1, \dots, \frac{-ib(m-2k)+p+cv}{2c} + 1; e^{2cz} \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ib(m-2k)+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-ib(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1} \left( \frac{-ib(m-2k)+p+2cs}{2c}, \dots, \frac{-ib(m-2k)+p+2cs}{2c}, v; \right. \\
 & \left. \frac{-ib(m-2k)+p+2cs}{2c} + 1, \dots, \frac{-ib(m-2k)+p+2cs}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-ib(m-2k)+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ib(m-2k)+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ib(m-2k)+p+2c(v-s)}{2c}, \dots, \frac{-ib(m-2k)+p+2c(v-s)}{2c}, v; \right. \\
 & \left. \frac{-ib(m-2k)+p+2c(v-s)}{2c} + 1, \dots, \frac{-ib(m-2k)+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \Bigg) \Bigg) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(bi(m-2k)+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (bi(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ib(m-2k)+p+2cs}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{ib(m-2k)+p+2cs}{2c}, v; \frac{ib(m-2k)+p+2cs}{2c} + 1, \right. \right. \\
 & \left. \left. \dots, \frac{ib(m-2k)+p+2cs}{2c} + 1; e^{2cz} \right) + e^{(bi(m-2k)+p+2c(v-s))z} \right)
 \end{aligned}$$

$$\sum_{j=0}^n \frac{(-1)^j (b i (m-2k) + p + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b (m-2k) + p + 2c(v-s)}{2c}, \right. \\ \left. \dots, \frac{i b (m-2k) + p + 2c(v-s)}{2c}, v; \frac{i b (m-2k) + p + 2c(v-s)}{2c} + 1, \dots, \right. \\ \left. \frac{i b (m-2k) + p + 2c(v-s)}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+$$

Involving cos, exp and power

**Involving  $z^n e^{pz} \cos(a + bz) \coth^v(cz)$**



01.22.21.0326.01

$$\int z^n e^{p z} \cos(a + b z) \coth^v(c z) dz =$$

$$\begin{aligned} & \frac{1}{2} (-1)^v n! \left( e^{(-i b + p + c v) z - i a} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-i b + p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + p + c v}{2 c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-i b + p + c v}{2 c}, v; \frac{-i b + p + c v}{2 c} + 1, \dots, \frac{-i b + p + c v}{2 c} + 1; e^{2 c z} \right) + e^{i a + (i b + p + c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \right. \\ & \quad \left. \sum_{j=0}^n \frac{(-1)^j (i b + p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p + c v}{2 c}, \dots, \frac{i b + p + c v}{2 c}, v; \frac{i b + p + c v}{2 c} + 1, \dots, \right. \right. \\ & \quad \left. \left. \frac{i b + p + c v}{2 c} + 1; e^{2 c z} \right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-i b + p + 2 c s) z - i a} \sum_{j=0}^n \frac{(-1)^j (-i b + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-i b + p + 2 c s}{2 c}, \dots, \frac{-i b + p + 2 c s}{2 c}, v; \frac{-i b + p + 2 c s}{2 c} + 1, \dots, \frac{-i b + p + 2 c s}{2 c} + 1; e^{2 c z} \right) + \right. \\ & \quad \left. e^{(-i b + p + 2 c (v-s)) z - i a} \sum_{j=0}^n \frac{(-1)^j (-i b + p + 2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + p + 2 c (v-s)}{2 c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-i b + p + 2 c (v-s)}{2 c}, v; \frac{-i b + p + 2 c (v-s)}{2 c} + 1, \dots, \frac{-i b + p + 2 c (v-s)}{2 c} + 1; e^{2 c z} \right) \right) + \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{i a + (i b + p + 2 c s) z} \sum_{j=0}^n \frac{(-1)^j (i b + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p + 2 c s}{2 c}, \dots, \frac{i b + p + 2 c s}{2 c}, \right. \right. \\ & \quad \left. \left. v; \frac{i b + p + 2 c s}{2 c} + 1, \dots, \frac{i b + p + 2 c s}{2 c} + 1; e^{2 c z} \right) + \right. \\ & \quad \left. e^{i a + (i b + p + 2 c (v-s)) z} \sum_{j=0}^n \frac{(-1)^j (i b + p + 2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b + p + 2 c (v-s)}{2 c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{i b + p + 2 c (v-s)}{2 c}, v; \frac{i b + p + 2 c (v-s)}{2 c} + 1, \dots, \frac{i b + p + 2 c (v-s)}{2 c} + 1; e^{2 c z} \right) \right) \Bigg) ; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$

01.22.21.0327.01

$$\int z^n e^{p z} \cos(b z) \coth^v(c z) dz =$$

$$\frac{1}{2} (-1)^v \binom{v}{\frac{v}{2}} n! (1 - v \bmod 2) \left( e^{(-i b + p + c v) z} \sum_{j=0}^n \frac{(-1)^j (-i b + p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + p + c v}{2c}, \dots, \frac{-i b + p + c v}{2c}, v; \right. \right.$$

$$\left. \frac{-i b + p + c v}{2c} + 1, \dots, \frac{-i b + p + c v}{2c} + 1; e^{2c z} \right) + e^{(i b + p + c v) z} \sum_{j=0}^n \frac{(-1)^j (i b + p + c v)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{i b + p + c v}{2c}, \dots, \frac{i b + p + c v}{2c}, v; \frac{i b + p + c v}{2c} + 1, \dots, \frac{i b + p + c v}{2c} + 1; e^{2c z} \right) \right) +$$

$$\frac{1}{2} (-1)^v n! \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( e^{(-i b + 2c i + p) z} \sum_{j=0}^n \frac{(-1)^j (-i b + 2c i + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b + 2c i + p}{2c}, \dots, \frac{-i b + 2c i + p}{2c}, \right. \right.$$

$$\left. v; \frac{-i b + 2c i + p}{2c} + 1, \dots, \frac{-i b + 2c i + p}{2c} + 1; e^{2c z} \right) + e^{(i b + 2c i + p) z} \sum_{j=0}^n \frac{(-1)^j (i b + 2c i + p)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{i b + 2c i + p}{2c}, \dots, \frac{i b + 2c i + p}{2c}, v; \frac{i b + 2c i + p}{2c} + 1, \dots, \frac{i b + 2c i + p}{2c} + 1; e^{2c z} \right) \right) +$$

$$e^{(-i b - 2c i + p + 2c v) z} \sum_{j=0}^n \frac{(-1)^j (-i b - 2c i + p + 2c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i b - 2c i + p + 2c v}{2c}, \dots, \right.$$

$$\left. \frac{-i b - 2c i + p + 2c v}{2c}, v; \frac{-i b - 2c i + p + 2c v}{2c} + 1, \dots, \frac{-i b - 2c i + p + 2c v}{2c} + 1; e^{2c z} \right) +$$

$$e^{(i b - 2c i + p + 2c v) z} \sum_{j=0}^n \frac{(-1)^j (i b - 2c i + p + 2c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i b - 2c i + p + 2c v}{2c}, \dots, \frac{i b - 2c i + p + 2c v}{2c}, \right.$$

$$\left. v; \frac{i b - 2c i + p + 2c v}{2c} + 1, \dots, \frac{i b - 2c i + p + 2c v}{2c} + 1; e^{2c z} \right) \Big/ ; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of cos, exp and power

**Involving  $z^n e^{p z} \cos^m(b z) \coth^v(c z)$**

01.22.21.0328.01

$$\int z^n e^{p z} \cos^m(b z) \coth^v(c z) dz = (-1)^v 2^{-m} e^{(p+c v) z} \binom{m}{\frac{m}{2}} \binom{v}{\frac{v}{2}} n! (1 - m \bmod 2) (1 - v \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j (p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p + c v}{2c}, \dots, \frac{p + c v}{2c}, v; \frac{p + c v}{2c} + 1, \dots, \frac{p + c v}{2c} + 1; e^{2c z} \right) + (-1)^v 2^{-m} \binom{v}{\frac{v}{2}} n!$$

$$(1 - v \bmod 2) \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{s} \left( e^{(-i b (m-2s) + p + c v) z} \sum_{j=0}^n \frac{(-1)^j (-i b (m-2s) + p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p - i b (m-2s) + c v}{2c}, \right. \right.$$

$$\begin{aligned}
 & \dots, \frac{p - ib(m - 2s) + cv}{2c}, v; \frac{p - ib(m - 2s) + cv}{2c} + 1, \dots, \frac{p - ib(m - 2s) + cv}{2c} + 1; e^{2cz} \Big) + \\
 & e^{(bi(m-2s)+p+cv)z} \sum_{j=0}^n \frac{(-1)^j (bi(m-2s) + p + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p + ib(m - 2s) + cv}{2c}, \dots, \right. \\
 & \left. \frac{p + ib(m - 2s) + cv}{2c}, v; \frac{p + ib(m - 2s) + cv}{2c} + 1, \dots, \frac{p + ib(m - 2s) + cv}{2c} + 1; e^{2cz} \right) \Big) + \\
 & (-1)^v 2^{-m} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( e^{(2ci+p)z} \sum_{j=0}^n \frac{(-1)^j (2ci+p)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{2ci+p}{2c}, \dots, \frac{2ci+p}{2c}, v; \frac{2ci+p}{2c} + 1, \dots, \frac{2ci+p}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(-2ci+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-2ci+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-2ci+p+2cv}{2c}, \dots, \frac{-2ci+p+2cv}{2c}, \right. \right. \\
 & \left. \left. v; \frac{-2ci+p+2cv}{2c} + 1, \dots, \frac{-2ci+p+2cv}{2c} + 1; e^{2cz} \right) \right) + (-1)^v 2^{-m} n! \\
 & \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{s} \binom{v}{i} \left( e^{(2ci+ibm+p-2ibs)z} \sum_{j=0}^n \frac{(-1)^j (2ci+ibm+p-2ibs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2ci+ibm+p-2ibs}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{2ci+ibm+p-2ibs}{2c}, v; \frac{2ci+ibm+p-2ibs}{2c} + 1, \dots, \frac{2ci+ibm+p-2ibs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(2ci-ibm+p+2ibs)z} \sum_{j=0}^n \frac{(-1)^j (2ci-ibm+p+2ibs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2ci-ibm+p+2ibs}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{2ci-ibm+p+2ibs}{2c}, v; \frac{2ci-ibm+p+2ibs}{2c} + 1, \dots, \frac{2ci-ibm+p+2ibs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(-2ci+ibm+p-2ibs+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-2ci+ibm+p-2ibs+2cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{-2ci+ibm+p-2ibs+2cv}{2c}, \dots, \frac{-2ci+ibm+p-2ibs+2cv}{2c}, v; \right. \right. \\
 & \left. \left. \frac{-2ci+ibm+p-2ibs+2cv}{2c} + 1, \dots, \frac{-2ci+ibm+p-2ibs+2cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(-2ci-ibm+p+2ibs+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-2ci-ibm+p+2ibs+2cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{-2ci-ibm+p+2ibs+2cv}{2c}, \dots, \frac{-2ci-ibm+p+2ibs+2cv}{2c}, \right. \right. \\
 & \left. \left. v; \frac{-2ci-ibm+p+2ibs+2cv}{2c} + 1, \dots, \frac{-2ci-ibm+p+2ibs+2cv}{2c} + 1; e^{2cz} \right) \right) \Big) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$

**Involving functions of the direct function and hyperbolic functions**

**Involving powers of the direct function and hyperbolic functions**

Involving sinh

**Involving sinh(b z)**

01.22.21.0329.01

$$\int \sinh(b z) \coth^{\nu}(c z) dz = \frac{1}{2b} \left( e^{-bz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \right. \\ \left. \left( F_1 \left( \frac{b}{2c}; -\nu, \nu; \frac{b}{2c} + 1; -e^{-2cz}, e^{-2cz} \right) + e^{2bz} F_1 \left( -\frac{b}{2c}; -\nu, \nu; 1 - \frac{b}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) \coth^{\nu}(c z) \right)$$

01.22.21.0330.01

$$\int \sinh(c z) \coth^{\nu}(c z) dz = \frac{\cosh(c z) \coth^{\nu}(c z) (-\sinh^2(c z))^{\nu/2}}{c(\nu + 1)} {}_2F_1 \left( \frac{\nu + 1}{2}, \frac{\nu}{2}; \frac{\nu + 3}{2}; \cosh^2(c z) \right)$$

01.22.21.0331.01

$$\int \sinh(c z) \coth^2(c z) dz = \frac{\cosh(c z) - \log(\cosh(\frac{cz}{2})) + \log(\sinh(\frac{cz}{2}))}{c}$$

01.22.21.0332.01

$$\int \sinh(c z) \coth^3(c z) dz = \frac{\sinh(c z) - \operatorname{csch}(c z)}{c}$$

01.22.21.0333.01

$$\int \sinh(4 c z) \coth^4(c z) dz = \frac{-8 \operatorname{csch}^2(c z) + 16 \cosh(2 c z) + \cosh(4 c z) + 64 \log(\sinh(c z))}{4 c}$$

Involving power of sinh

**Involving sinh<sup>u</sup>(b z) coth<sup>ν</sup>(c z)**

01.22.21.0334.01

$$\int \sinh^u(b z) \coth^{\nu}(c z) dz = \\ \left( \frac{u}{2} \right) \frac{\coth^{\nu+1}(c z) (1 - u \bmod 2)}{c(\nu + 1)} \left( \frac{i}{2} \right)^u {}_2F_1 \left( \frac{\nu + 1}{2}, 1; \frac{\nu + 1}{2} + 1; \coth^2(c z) \right) + \frac{2^{-u} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(c z)}{b} \\ \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \frac{(-1)^k}{u - 2k} \binom{u}{k} \left( e^{b(u-2k)z} F_1 \left( -\frac{b(u-2k)}{2c}; -\nu, \nu; 1 - \frac{b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) - \right. \\ \left. (-1)^u e^{-b(u-2k)z} F_1 \left( \frac{b(u-2k)}{2c}; -\nu, \nu; \frac{b(u-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz} \right) \right); u \in \mathbb{N}^+$$

01.22.21.0335.01

$$\int \sinh^\mu(cz) \coth^\nu(cz) dz = -\frac{(1 - e^{2cz})^{\nu-\mu} (1 + e^{2cz})^{-\nu} \coth^\nu(cz) \sinh^\mu(cz)}{c\mu} F_1\left(-\frac{\mu}{2}; \nu - \mu, -\nu; \frac{2-\mu}{2}; e^{2cz}, -e^{2cz}\right)$$

01.22.21.0336.01

$$\int \sinh^\mu(cz) \coth^\nu(cz) dz = -\frac{\cosh(cz) \coth^\nu(cz) \sinh^{\mu+1}(cz) (-\sinh^2(cz))^{\frac{1}{2}(-\mu+\nu-1)}}{c(\nu+1)} {}_2F_1\left(\frac{\nu+1}{2}, \frac{1}{2}(-\mu+\nu+1); \frac{\nu+3}{2}; \cosh^2(cz)\right)$$

01.22.21.0337.01

$$\int \frac{\coth^3(cz)}{\sinh^{\frac{1}{2}}(2cz)} dz = \frac{3 E\left(\frac{\pi}{4} - icz \mid 2\right) \sqrt{i \sinh(2cz)} - 2(3 \cosh^2(cz) + \coth^2(cz))}{5c \sinh^{\frac{1}{2}}(2cz)}$$

01.22.21.0338.01

$$\int \sqrt{\sinh^3(2cz)} \coth^5(cz) dz = \frac{\sqrt{\sinh^3(2cz)}}{15c} \left(-6 \operatorname{csch}^4(cz) - 78 \operatorname{csch}^2(cz) + 231 \operatorname{csch}^2(2cz) E\left(\frac{\pi}{4} - icz \mid 2\right) \sqrt{i \sinh(2cz)} + 5\right)$$

Involving powers of products with sinh

**Involving  $\sqrt{\sinh^m(cz) \coth(cz)}$**

01.22.21.0339.01

$$\int \sqrt{\sinh(cz) \coth(cz)} dz = -\frac{2i}{c} E\left(\frac{icz}{2} \mid 2\right)$$

01.22.21.0340.01

$$\int \sqrt{\sinh^4(cz) \coth(cz)} dz = \frac{\coth(cz) \sqrt{\cosh(cz) \sinh^3(cz)} \left(3 \sqrt[4]{-\sinh^2(cz)} + {}_2F_1\left(\frac{3}{4}, \frac{3}{4}; \frac{7}{4}; \cosh^2(cz)\right)\right)}{6c \sqrt[4]{-\sinh^2(cz)}}$$

Involving algebraic functions of sinh

01.22.21.0341.01

$$\int \sqrt{a + b \sinh(cz)} \coth^2(cz) dz =$$

$$\frac{1}{4c} \left( -4 \sqrt{a + b \sinh(cz)} \coth(cz) + \frac{1}{a \sqrt{\frac{1}{ib-a} b}} \left( 6 \left( a(a+ib) i E \left( i \sinh^{-1} \left( \sqrt{\frac{1}{ib-a}} \sqrt{a + b \sinh(cz)} \right) \right) \middle| \frac{a-ib}{a+ib} \right) + \right. \right.$$

$$b \left( a F \left( i \sinh^{-1} \left( \sqrt{\frac{1}{ib-a}} \sqrt{a + b \sinh(cz)} \right) \right) \middle| \frac{a-ib}{a+ib} \right) +$$

$$b i \Pi \left( 1 - \frac{ib}{a}; i \sinh^{-1} \left( \sqrt{\frac{1}{ib-a}} \sqrt{a + b \sinh(cz)} \right) \middle| \frac{a-ib}{a+ib} \right) \left. \right) \operatorname{sech}(cz) \sqrt{\frac{b - ib \sinh(cz)}{b + ia}}$$

$$\sqrt{\frac{i \sinh(cz) b + b}{b - ia}} + \frac{6 \left( (b + ia) E \left( \frac{1}{4} (\pi - 2ic z) \middle| -\frac{2ib}{a-ib} \right) - ia F \left( \frac{1}{4} (\pi - 2ic z) \middle| -\frac{2ib}{a-ib} \right) \right) \sqrt{\frac{a+b \sinh(cz)}{a-ib}}}{\sqrt{a + b \sinh(cz)}} +$$

$$\left. \frac{8a \sqrt{\frac{a+b \sinh(cz)}{a-ib}} i F \left( \frac{1}{4} (\pi - 2ic z) \middle| -\frac{2ib}{a-ib} \right) + 2b \sqrt{\frac{a+b \sinh(cz)}{a-ib}} \Pi \left( 2; \frac{1}{4} (\pi - 2ic z) \middle| -\frac{2ib}{a-ib} \right)}{\sqrt{a + b \sinh(cz)}} \right)$$

Involving cosh

**Involving cosh(bz)**

01.22.21.0342.01

$$\int \cosh(bz) \coth^\nu(cz) dz = \frac{1}{2b} \left( e^{-bz} (1 - e^{-2cz})^\nu (1 + e^{-2cz})^{-\nu} \right.$$

$$\left. \left( e^{2bz} F_1 \left( -\frac{b}{2c}; -\nu, \nu; 1 - \frac{b}{2c}; -e^{-2cz}, e^{-2cz} \right) - F_1 \left( \frac{b}{2c}; -\nu, \nu; \frac{b}{2c} + 1; -e^{-2cz}, e^{-2cz} \right) \right) \coth^\nu(cz) \right)$$

01.22.21.0343.01

$$\int \cosh(cz) \coth^\nu(cz) dz = \frac{\cosh(cz) \coth^{\nu+1}(cz) (-\sinh^2(cz))^{\frac{\nu+1}{2}}}{c(\nu+2)} {}_2F_1 \left( \frac{\nu+2}{2}, \frac{\nu+1}{2}; \frac{\nu+4}{2}; \cosh^2(cz) \right)$$

01.22.21.0344.01

$$\int \cosh(z) \coth^2(z) dz = \sinh(z) - \operatorname{csch}(z)$$

01.22.21.0345.01

$$\int \cosh(z) \coth^3(z) dz = \frac{1}{2} \left( 2 \cosh(z) - \coth(z) \operatorname{csch}(z) - 3 \log \left( \cosh \left( \frac{z}{2} \right) \right) + 3 \log \left( \sinh \left( \frac{z}{2} \right) \right) \right)$$

01.22.21.0346.01

$$\int \cosh(2z) \coth^3(z) dz = \frac{1}{2} (-\operatorname{csch}^2(z) + \cosh(2z) + 6 \log(\sinh(z)))$$

01.22.21.0347.01

$$\int \cosh(4z) \coth^5(z) dz = -\frac{1}{4} \operatorname{csch}^4(z) - 5 \operatorname{csch}^2(z) + 5 \cosh(2z) + \frac{1}{4} \cosh(4z) + 25 \log(\sinh(z))$$

01.22.21.0348.01

$$\int \cosh(5z) \coth^5(z) dz = -\frac{1}{4} \coth(z) \operatorname{csch}^3(z) - \frac{57}{8} \coth(z) \operatorname{csch}(z) + 50 \cosh(z) + \frac{10}{3} \cosh(3z) + \frac{1}{5} \cosh(5z) - \frac{383}{8} \log\left(\cosh\left(\frac{z}{2}\right)\right) + \frac{383}{8} \log\left(\sinh\left(\frac{z}{2}\right)\right)$$

Involving power of cosh

**Involving  $\cosh^u(bz) \coth^v(cz)$**

01.22.21.0349.01

$$\int \cosh^u(bz) \coth^v(cz) dz = \frac{2^{-u} \coth^{v+1}(cz) (1 - u \bmod 2)}{c(v+1)} \left(\frac{u}{2}\right) {}_2F_1\left(\frac{v+1}{2}, 1; \frac{v+1}{2} + 1; \coth^2(cz)\right) + \frac{1}{b} (2^{-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz)) - \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \frac{1}{u-2k} \binom{u}{k} \left( e^{b(u-2k)z} F_1\left(-\frac{b(u-2k)}{2c}; -v, v; 1 - \frac{b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) - e^{-b(u-2k)z} F_1\left(\frac{b(u-2k)}{2c}; -v, v; \frac{b(u-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) \right); u \in \mathbb{N}^+$$

01.22.21.0350.01

$$\int \cosh^\mu(cz) \coth^v(cz) dz = -\frac{(1 - e^{2cz})^v (1 + e^{2cz})^{-\mu-v} \cosh^\mu(cz) \coth^v(cz)}{c\mu} F_1\left(-\frac{\mu}{2}; v, -\mu - v; \frac{2-\mu}{2}; e^{2cz}, -e^{2cz}\right)$$

01.22.21.0351.01

$$\int \cosh^\mu(cz) \coth^v(cz) dz = -\frac{\cosh^{\mu+1}(cz) \coth^v(cz) \sinh(cz) (-\sinh^2(cz))^{\frac{v-1}{2}}}{c(\mu+v+1)} {}_2F_1\left(\frac{1}{2}(\mu+v+1), \frac{v+1}{2}; \frac{1}{2}(\mu+v+3); \cosh^2(cz)\right)$$

01.22.21.0352.01

$$\int \cosh(5z) \coth^5(z) dz = -\frac{1}{4} \coth(z) \operatorname{csch}^3(z) - \frac{57}{8} \coth(z) \operatorname{csch}(z) + 50 \cosh(z) + \frac{10}{3} \cosh(3z) + \frac{1}{5} \cosh(5z) - \frac{383}{8} \log\left(\cosh\left(\frac{z}{2}\right)\right) + \frac{383}{8} \log\left(\sinh\left(\frac{z}{2}\right)\right)$$

01.22.21.0353.01

$$\int \cosh^2(z) \coth^2(z) dz = \frac{1}{4} (6z - 4 \coth(z) + \sinh(2z))$$

01.22.21.0354.01

$$\int \sqrt{\cosh^3(2z)} \coth^3(z) dz = \frac{\sqrt{\cosh^3(2z)} \left( \cosh^{\frac{1}{2}}(2z) (-3 \operatorname{csch}^2(z) + 2 \cosh(2z) + 18) - 24 \tanh^{-1} \left( \cosh^{\frac{1}{2}}(2z) \right) \right)}{6 \cosh^{\frac{3}{2}}(2z)}$$

Involving powers of products with cosh

### Involving $\sqrt{\cosh^m(cz) \coth(cz)}$

01.22.21.0355.01

$$\int \sqrt{\cosh(cz) \coth(cz)} dz = \frac{2 \sqrt{\cosh(cz) \coth(cz)} \left( \sqrt[4]{-\sinh^2(cz)} - 1 \right) \tanh(cz)}{c \sqrt[4]{-\sinh^2(cz)}}$$

01.22.21.0356.01

$$\int \sqrt{\cosh^m(cz) \coth(cz)} dz = -\frac{\sqrt{\cosh^{m+1}(cz) \operatorname{csch}(cz) \sinh(2cz)}}{c(m+3) \sqrt[4]{-\sinh^2(cz)}} {}_2F_1 \left( \frac{m+3}{4}, \frac{3}{4}; \frac{m+7}{4}; \cosh^2(cz) \right)$$

Involving algebraic functions of cosh

### Involving $(a + b \cosh(cz))^\beta$

01.22.21.0357.01

$$\int (a + b \cosh(cz))^\beta \coth(cz) dz = -\frac{1}{2(a-b)(a+b)c(\beta+1)} \left( (a + b \cosh(cz))^{\beta+1} \left( (a+b) {}_2F_1 \left( \beta+1, 1; \beta+2; \frac{a+b \cosh(cz)}{a-b} \right) + (a-b) {}_2F_1 \left( \beta+1, 1; \beta+2; \frac{a+b \cosh(cz)}{a+b} \right) \right) \right)$$

01.22.21.0358.01

$$\int \sqrt{a + b \cosh(cz)} \coth(cz) dz = \frac{1}{\sqrt{-a-b} \sqrt{b-a} c} \left( \sqrt{b-a} (a+b) \tan^{-1} \left( \frac{\sqrt{a+b \cosh(cz)}}{\sqrt{-a-b}} \right) + \sqrt{-a-b} \left( (a-b) \tan^{-1} \left( \frac{\sqrt{a+b \cosh(cz)}}{\sqrt{b-a}} \right) + 2 \sqrt{b-a} \sqrt{a+b \cosh(cz)} \right) \right)$$

01.22.21.0359.01

$$\int \frac{\coth(cz)}{\sqrt{a+b \cosh(cz)}} dz = -\frac{1}{c} \left( \frac{1}{\sqrt{a-b}} \tanh^{-1} \left( \frac{\sqrt{a+b \cosh(cz)}}{\sqrt{a-b}} \right) + \frac{1}{\sqrt{a+b}} \tanh^{-1} \left( \frac{\sqrt{a+b \cosh(cz)}}{\sqrt{a+b}} \right) \right)$$

### Involving $(a + b \cosh(2cz))^\beta$



01.22.21.0360.01

$$\int (a + b \cosh(2cz))^\beta \coth^\nu(cz) dz = -\frac{1}{c(\nu-1)} \left( F_1 \left( \frac{1-\nu}{2}; \frac{1-\nu}{2}, -\beta; \frac{3-\nu}{2}; -\sinh^2(cz), -\frac{2b \sinh^2(cz)}{a+b} \right) \right. \\ \left. \cosh^2(cz)^{\frac{1-\nu}{2}} (a + b \cosh(2cz))^\beta \left( \frac{a + b \cosh(2cz)}{a+b} \right)^{-\beta} \coth^{\nu-1}(cz) \right)$$

01.22.21.0361.01

$$\int \sqrt{a + b \cosh(2cz)} \coth^2(cz) dz = \frac{1}{c \sqrt{a + b \cosh(2cz)}} \left( -(a + b \cosh(2cz)) \coth(cz) - \right. \\ \left. 2i(a+b) \sqrt{\frac{a + b \cosh(2cz)}{a+b}} E \left( icz \left| \frac{2b}{a+b} \right. \right) + (a-b) \sqrt{\frac{a + b \cosh(2cz)}{a+b}} iF \left( icz \left| \frac{2b}{a+b} \right. \right) \right)$$

01.22.21.0362.01

$$\int \sqrt{a - a \cosh(2cz)} \coth^2(cz) dz = \frac{\sqrt{a - a \cosh(2cz)} \operatorname{csch}(cz) (\cosh(cz) - \log(\cosh(\frac{cz}{2})) + \log(\sinh(\frac{cz}{2})))}{c}$$

01.22.21.0363.01

$$\int \sqrt{\cosh(2cz)a + a} \coth^2(cz) dz = \frac{(\cosh(2cz) - 3) \sqrt{\cosh(2cz)a + a} \operatorname{csch}(cz) \operatorname{sech}(cz)}{2c}$$

01.22.21.0364.01

$$\int \sqrt{a + b \cosh(2cz)} \coth^3(cz) dz = \frac{1}{2\sqrt{a+b}c} \left( -2(a+2b) \tanh^{-1} \left( \frac{\sqrt{a + b \cosh(2cz)}}{\sqrt{a+b}} \right) - \sqrt{a+b} \sqrt{a + b \cosh(2cz)} (\operatorname{csch}^2(cz) - 2) \right)$$

01.22.21.0365.01

$$\int \sqrt{a + b \cosh(2cz)} \coth^4(cz) dz = \frac{1}{6(a+b)c \sqrt{a + b \cosh(2cz)}} \left( -(-2a^2 - 2ba + 3b^2 + 4(a^2 + ba - b^2) \cosh(2cz) + b(2a + 3b) \cosh(4cz)) \coth(cz) \operatorname{csch}^2(cz) - \right. \\ \left. 2i(7a^2 + 16ba + 9b^2) \sqrt{\frac{a + b \cosh(2cz)}{a+b}} E \left( icz \left| \frac{2b}{a+b} \right. \right) + 8 \sqrt{\frac{a + b \cosh(2cz)}{a+b}} (a^2 - b^2) iF \left( icz \left| \frac{2b}{a+b} \right. \right) \right)$$

01.22.21.0366.01

$$\int \frac{\coth^2(cz)}{\sqrt{a + b \cosh(2cz)}} dz = \frac{1}{(a+b)c \sqrt{a + b \cosh(2cz)}} \left( -(a + b \cosh(2cz)) \coth(cz) - i(a+b) \sqrt{\frac{a + b \cosh(2cz)}{a+b}} E \left( icz \left| \frac{2b}{a+b} \right. \right) \right)$$

01.22.21.0367.01

$$\int \frac{\coth^3(cz)}{\sqrt{a+b \cosh(2cz)}} dz = \frac{1}{2(a+b)^{3/2}c} \left( -\sqrt{a+b} \sqrt{a+b \cosh(2cz)} \operatorname{csch}^2(cz) - 2a \tanh^{-1} \left( \frac{\sqrt{a+b \cosh(2cz)}}{\sqrt{a+b}} \right) \right)$$

01.22.21.0368.01

$$\int \frac{\coth^4(cz)}{\sqrt{a+b \cosh(2cz)}} dz = \frac{1}{3(a+b)^2c \sqrt{a+b \cosh(2cz)}} \left( -((2a^2 - ba + b^2) \cosh(2cz) + a(-a + 2b + b \cosh(4cz))) \coth(cz) \operatorname{csch}^2(cz) - 4ia(a+b) \sqrt{\frac{a+b \cosh(2cz)}{a+b}} E\left(icz \left| \frac{2b}{a+b} \right. \right) + \sqrt{\frac{a+b \cosh(2cz)}{a+b}} (a^2 - b^2) i F\left(icz \left| \frac{2b}{a+b} \right. \right) \right)$$

### Involving $\cosh(cz) (a + b \cosh(2cz))^\beta$

01.22.21.0369.01

$$\int \cosh(cz) (a + b \cosh(2cz))^\beta \coth^\nu(cz) dz = -\frac{1}{c(\nu-1)} \cosh^2(cz)^{\frac{\nu}{2}} (a + b \cosh(2cz))^\beta \left( \frac{a + b \cosh(2cz)}{a+b} \right)^{-\beta} \coth^\nu(cz) \sinh(cz) F_1\left(\frac{1-\nu}{2}; -\frac{\nu}{2}, -\beta; \frac{3-\nu}{2}; -\sinh^2(cz), -\frac{2b \sinh^2(cz)}{a+b}\right)$$

01.22.21.0370.01

$$\int \cosh(cz) \sqrt{a+b \cosh(2cz)} \coth^2(cz) dz = \frac{\sqrt{2} (a+5b) \tanh^{-1} \left( \frac{\sqrt{2} \sqrt{b} \sinh(cz)}{\sqrt{a+b \cosh(2cz)}} \right) + \sqrt{b} (\cosh(2cz) - 5) \sqrt{a+b \cosh(2cz)} \operatorname{csch}(cz)}{4\sqrt{b}c}$$

01.22.21.0371.01

$$\int \frac{\cosh(cz) \coth^2(cz)}{\sqrt{a+b \cosh(2cz)}} dz = \frac{1}{2c} \left( \frac{\sqrt{2}}{\sqrt{b}} \tanh^{-1} \left( \frac{\sqrt{2} \sqrt{b} \sinh(cz)}{\sqrt{a+b \cosh(2cz)}} \right) - \frac{2\sqrt{a+b \cosh(2cz)} \operatorname{csch}(cz)}{a+b} \right)$$

### Involving $\cosh(2cz) (a + b \cosh(2cz))^\beta$

01.22.21.0372.01

$$\int \cosh(2cz) \sqrt{a+b \cosh(2cz)} \coth^2(cz) dz = \frac{1}{3bc \sqrt{a+b \cosh(2cz)}} \left( \frac{1}{2} b (a + b \cosh(2cz)) (\cosh(3cz) - 7 \cosh(cz)) \operatorname{csch}(cz) - i(a^2 + 10ba + 9b^2) \sqrt{\frac{a+b \cosh(2cz)}{a+b}} E\left(icz \left| \frac{2b}{a+b} \right. \right) + \sqrt{\frac{a+b \cosh(2cz)}{a+b}} (a^2 + 3ba - 4b^2) i F\left(icz \left| \frac{2b}{a+b} \right. \right) \right)$$

$$01.22.21.0373.01 \quad \int \cosh(2cz) \sqrt{\cosh(2cz)a+a} \coth^2(cz) dz = \frac{\sqrt{\cosh(2cz)a+a} (14 \cosh(2cz) + \cosh(4cz) - 27) \operatorname{csch}(cz) \operatorname{sech}(cz)}{12c}$$

$$01.22.21.0374.01 \quad \int \cosh(2cz) \sqrt{a-a \cosh(2cz)} \coth^2(cz) dz = \frac{\sqrt{a-a \cosh(2cz)} \operatorname{csch}(cz) (9 \cosh(cz) + \cosh(3cz) - 6 \log(\cosh(\frac{cz}{2})) + 6 \log(\sinh(\frac{cz}{2})))}{6c}$$

Involving tanh

**Involving  $\tanh(cz) \coth^{\nu}(cz)$**

$$01.22.21.0375.01 \quad \int \tanh(cz) \coth^{\nu}(cz) dz = \frac{\coth^{\nu}(cz)}{c\nu} {}_2F_1\left(\frac{\nu}{2}, 1; \frac{\nu}{2} + 1; \coth^2(cz)\right)$$

$$01.22.21.0376.01 \quad \int \tanh(z) \coth^2(z) dz = \log(\sinh(z))$$

$$01.22.21.0377.01 \quad \int \tanh(z) \coth^3(z) dz = z - \coth(z)$$

Involving power of tanh

**Involving  $\tanh^{\mu}(cz) \coth^{\nu}(cz)$**

$$01.22.21.0378.01 \quad \int \tanh^{\mu}(cz) \coth^{\nu}(cz) dz = -\frac{\coth^{\nu+1}(cz) \tanh^{\mu}(cz)}{c(\mu-\nu-1)} {}_2F_1\left(\frac{1}{2}(-\mu+\nu+1), 1; \frac{1}{2}(-\mu+\nu+3); \coth^2(cz)\right)$$

$$01.22.21.0379.01 \quad \int \tanh^{\mu}(cz) \coth^{\nu}(cz) dz = \frac{1}{2c(\mu-\nu-1)} \left( \coth^{\nu}(cz) {}_2F_1\left(\frac{1}{2}(-\mu+\nu+1), \frac{1}{2}(-\mu+\nu+1); \frac{1}{2}(-\mu+\nu+3); \cosh^2(cz)\right) (-\sinh^2(cz))^{\frac{1}{2}(-\mu+\nu-1)} \sinh(2cz) \tanh^{\mu}(cz) \right)$$

$$01.22.21.0380.01 \quad \int \tanh^{\nu+1}(cz) \coth^{\nu}(cz) dz = \frac{\coth^{\nu}(cz) \log(\cosh(cz) \tanh^{\nu}(cz))}{c}$$

$$01.22.21.0381.01 \quad \int \tanh^2(z) \coth^2(z) dz = z$$

Involving powers of products with tanh

### Involving $\sqrt{\tanh^m(cz) \coth(dz)}$

01.22.21.0382.01

$$\int \sqrt{\tanh(cz) \coth(2cz)} dz = \frac{\left( \sqrt{2} \sinh^{-1}(\sqrt{2} \sinh(cz)) - \tanh^{-1}\left(\frac{\sinh(cz)}{\cosh^{\frac{1}{2}}(2cz)}\right) \right) \cosh(cz) \sqrt{\coth(2cz) \tanh(cz)}}{c \cosh^{\frac{1}{2}}(2cz)}$$

01.22.21.0383.01

$$\int \sqrt{\tanh(2cz) \coth(cz)} dz = \frac{\cosh^{\frac{1}{2}}(2cz) \operatorname{sech}(cz) \sqrt{\coth(cz) \tanh(2cz)}}{\sqrt{2} c} \tanh^{-1}\left(\frac{\sqrt{2} \sinh(cz)}{\cosh^{\frac{1}{2}}(2cz)}\right)$$

01.22.21.0384.01

$$\int \sqrt{\tanh^m(2cz) \coth(cz)} dz = \frac{1}{c(m+3)(\coth^2(cz)-1)} \left( {}_2F_1\left(\frac{m+3}{4}; 1, \frac{m}{2}; \frac{m+3}{4} + 1; \coth^2(cz), -\coth^2(cz)\right) \coth(cz) (\coth^2(cz)+1)^{m/2} \operatorname{csch}^2(cz) \sqrt{\coth(cz) \tanh^m(2cz)} \right)$$

### Involving rational functions of the direct function and hyperbolic functions

Involving rational functions of sinh

### Involving $(a \sinh(cz) + b \coth(cz))^{-n}$

01.22.21.0385.01

$$\int \frac{1}{a \sinh(cz) + b \coth(cz)} dz = \frac{\log\left(-b - 2a \cosh(cz) + \sqrt{4a^2 + b^2}\right) - \log\left(b + 2a \cosh(cz) + \sqrt{4a^2 + b^2}\right)}{\sqrt{4a^2 + b^2} c}$$

$$\begin{aligned}
 & \int \frac{1}{(a \sinh(cz) + b \coth(cz))^2} dz = \\
 & \left( \operatorname{csch}^2(cz) (a \sinh^2(cz) + b \cosh(cz)) \left( -\frac{2(b + 2a \cosh(cz)) \sinh(cz)}{4a^2 + b^2} + \left( 2\sqrt{2} (4a^2 + b(\sqrt{4a^2 + b^2} - b)) \tan^{-1} \left( \frac{(2a - b + \sqrt{4a^2 + b^2}) \tanh(\frac{cz}{2})}{\sqrt{2} \sqrt{b} \sqrt{\sqrt{4a^2 + b^2} - b}} \right) (a \sinh^2(cz) + b \cosh(cz)) \right) / \left( \sqrt{b} (4a^2 + b^2)^{3/2} \sqrt{\sqrt{4a^2 + b^2} - b} \right) - \right. \right. \\
 & \left. \left( 2\sqrt{2} (b(b + \sqrt{4a^2 + b^2}) - 4a^2) \tan^{-1} \left( \frac{(-2a + b + \sqrt{4a^2 + b^2}) \tanh(\frac{cz}{2})}{\sqrt{2} \sqrt{-b(b + \sqrt{4a^2 + b^2})}} \right) (a \sinh^2(cz) + b \cosh(cz)) \right) / \left( (4a^2 + b^2)^{3/2} \sqrt{-b(b + \sqrt{4a^2 + b^2})} \right) \right) \Bigg) / (2c(b \coth(cz) + a \sinh(cz))^2)
 \end{aligned}$$

Involving rational functions of cosh

Involving  $(a \cosh(cz) + b \coth(cz))^{-n}$

$$\int \frac{1}{a \cosh(cz) + b \coth(cz)} dz = \frac{2a \tan^{-1}(\tanh(\frac{cz}{2})) + b \log(\cosh(cz)) - b \log(b + a \sinh(cz))}{ca^2 + b^2c}$$

$$\begin{aligned}
 & \int \frac{1}{(a \cosh(cz) + b \coth(cz))^2} dz = \\
 & \left( \left( -a \cosh(cz) \coth(cz) b^2 + \frac{2b(b^2 - 2a^2) \tan^{-1}\left(\frac{a - b \tanh(\frac{cz}{2})}{\sqrt{-a^2 - b^2}}\right) \coth(cz) (b + a \sinh(cz))}{\sqrt{-a^2 - b^2}} + \operatorname{csch}(cz) \right. \right. \\
 & \left. \left. (b + a \sinh(cz)) ((a^2 - b^2) \sinh(cz) - 2ab) \tanh(cz) \right) / \left( (a^2 + b^2)^2 c (b + a \sinh(cz)) \right) \right)
 \end{aligned}$$

Involving rational functions of tanh

**Involving  $(a \tanh(c z) + b \coth(c z))^{-n}$**

$$\int \frac{1}{a \tanh(c z) + b \coth(c z)} dz = \frac{\log(-a + b + (a + b) \cosh(2 c z))}{2 a c + 2 b c}$$

$$\int \frac{1}{(a \tanh(c z) + b \coth(c z))^2} dz = \frac{\left( (-a + b + (a + b) \cosh(2 c z)) \operatorname{csch}^2(c z) \operatorname{sech}^2(c z) \left( -(a - b) \left( 2 \sqrt{a} \sqrt{b} c z + (a - b) \tan^{-1} \left( \frac{\sqrt{a} \tanh(c z)}{\sqrt{b}} \right) \right) + (a + b) \cosh(2 c z) \left( 2 \sqrt{a} \sqrt{b} c z + (a - b) \tan^{-1} \left( \frac{\sqrt{a} \tanh(c z)}{\sqrt{b}} \right) \right) - \sqrt{a} \sqrt{b} (a + b) \sinh(2 c z) \right)}{8 \sqrt{a} \sqrt{b} (a + b)^2 c (b \coth(c z) + a \tanh(c z))^2} \right)}{}$$

**Involving  $(a + b \tanh(d z) + c \coth(d z))^{-n}$**

$$\int \frac{1}{a + c \coth(d z) + b \tanh(d z)} dz = \frac{\left( \operatorname{csch}(d z) \left( 2 a (b - c) \tan^{-1} \left( \frac{a + 2 b \tanh(d z)}{\sqrt{4 b c - a^2}} \right) + \sqrt{4 b c - a^2} (2 a d z - (b + c) \log(-b + c + (b + c) \cosh(2 d z) + a \sinh(2 d z))) \right) \operatorname{sech}(d z) (-b + c + (b + c) \cosh(2 d z) + a \sinh(2 d z)) \right)}{4 (a - b - c) (a + b + c) \sqrt{4 b c - a^2} d (a + c \coth(d z) + b \tanh(d z))}$$

01.22.21.0392.01

$$\int \frac{1}{(a + b \tanh(dz) + c \coth(dz))^2} dz = \left( \operatorname{csch}^2(dz) \operatorname{sech}^2(dz) (-b + c + (b + c) \cosh(2dz) + a \sinh(2dz))^2 \right. \\ \left. - \frac{2(b-c) \tan^{-1}\left(\frac{a+2b \tanh(dz)}{\sqrt{4bc-a^2}}\right)}{(4bc-a^2)^{3/2}} + \frac{2(b-c)(-3a^4 - 2(b-c)^2 a^2 + (b+c)^2 (b^2 + 10cb + c^2)) \tan^{-1}\left(\frac{a+2b \tanh(dz)}{\sqrt{4bc-a^2}}\right)}{(-a+b+c)^2 (a+b+c)^2 (4bc-a^2)^{3/2}} + \right. \\ \left. \frac{a(c-b) + (a^2 - (b+c)^2) \sinh(2dz)}{(b+c)(4bc-a^2)(-b+c + (b+c) \cosh(2dz) + a \sinh(2dz))} + \right. \\ \left. (-4(a^4 + (b-c)^2 a^2 - 4bc(b+c)^2) dz \cosh(2dz)(b+c)^2 + (b-c)(a^5 + 4(b+c) dz a^4 - \right. \\ \left. 2(b^2 + 6cb + c^2) a^3 + 4(b-c)^2 (b+c) dz a^2 + (b+c)^2 (b^2 + 10cb + c^2) a - 16bc(b+c)^3 dz) - \right. \\ \left. (a^2 + (b+c)^2)(a^4 + 4(b+c) dz a^3 - 8bc a^2 - 16bc(b+c) dz a - (b+c)^2 (b^2 - 6cb + c^2)) \sinh(2dz) \right) / \\ \left. \frac{4a(b+c) \log(-b+c + (b+c) \cosh(2dz) + a \sinh(2dz))}{(-a+b+c)^2 (a+b+c)^2} \right) / (16d(a+c \coth(dz) + b \tanh(dz))^2)$$

**Involving  $(a \tanh^2(cz) + b \coth^2(cz))^{-n}$**

01.22.21.0393.01

$$\int \frac{1}{a \tanh^2(cz) + b \coth^2(cz)} dz = \frac{1}{2 \sqrt[4]{a} \sqrt[4]{b} (a+b)c} \\ \left( 2 \sqrt[4]{a} cz \sqrt[4]{b} + \sqrt[4]{-1} (\sqrt[4]{a} + i \sqrt[4]{b}) \tan^{-1}\left(\frac{\sqrt[4]{-1} \sqrt[4]{a} \tanh(cz)}{\sqrt[4]{b}}\right) + \sqrt[4]{-1} (i \sqrt[4]{a} + \sqrt[4]{b}) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt[4]{a} \tanh(cz)}{\sqrt[4]{b}}\right) \right)$$

$$\begin{aligned}
 & \int \frac{1}{(a \tanh^2(c z) + b \coth^2(c z))^2} dz = \\
 & \left( \frac{1024 c z}{(a+b)^2} + \frac{1}{a^{7/4} (\sqrt{a} - i \sqrt{b})^2 b^{7/4}} \right. \\
 & \left. \frac{\sqrt[4]{-1} (-i \sqrt{a} + \sqrt{b}) (20 \sqrt{b} i a^{3/2} + 3 a^2 + 174 b a - 20 i b^{3/2} \sqrt{a} + 3 b^2) \tan^{-1} \left( \frac{\sqrt[4]{-1} \sqrt[4]{a} \tanh(c z)}{\sqrt[4]{b}} \right)}{a^{7/4} (\sqrt{a} + i \sqrt{b})^2 b^{7/4}} \right. \\
 & \left. \frac{1}{a^{7/4} (\sqrt{a} + i \sqrt{b})^2 b^{7/4}} \left( (-1)^{3/4} (i \sqrt{a} + \sqrt{b}) (-20 i \sqrt{b} a^{3/2} + 3 a^2 + 174 b a + 20 b^{3/2} i \sqrt{a} + 3 b^2) \right. \right. \\
 & \left. \left. \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt[4]{a} \tanh(c z)}{\sqrt[4]{b}} \right) \right) + \frac{1}{a^{7/4} b^{7/4}} \left( 3 \left( \frac{8 a^{3/4} (-2 a + 2 b + (a+b) \cosh(2 c z)) \sinh(2 c z) b^{3/4}}{4 (a-b) \cosh(2 c z) - (a+b) (\cosh(4 c z) + 3)} - \right. \right. \right. \\
 & \left. \left. \frac{3 (-1)^{3/4} (a+b)^2 \tan^{-1} \left( \frac{\sqrt[4]{-1} \sqrt[4]{a} \tanh(c z)}{\sqrt[4]{b}} \right)}{\sqrt{a} + i \sqrt{b}} - \frac{3 \sqrt[4]{-1} (a+b)^2 \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt[4]{a} \tanh(c z)}{\sqrt[4]{b}} \right)}{\sqrt{a} - i \sqrt{b}} \right) \right) + \\
 & \left. \frac{1}{a^{7/4} b^{7/4}} \left( 4 \left( (-1)^{3/4} (\sqrt{a} + i \sqrt{b}) (3 a + 2 i \sqrt{b} \sqrt{a} - 3 b) \tan^{-1} \left( \frac{\sqrt[4]{-1} \sqrt[4]{a} \tanh(c z)}{\sqrt[4]{b}} \right) + \right. \right. \right. \\
 & \left. \left. \sqrt[4]{-1} (\sqrt{a} - i \sqrt{b}) (3 a - 2 i \sqrt{b} \sqrt{a} - 3 b) \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt[4]{a} \tanh(c z)}{\sqrt[4]{b}} \right) + \right. \right. \\
 & \left. \left. \frac{8 a^{3/4} b^{3/4} (-2 a + 2 b + 3 (a+b) \cosh(2 c z)) \sinh(2 c z)}{4 (a-b) \cosh(2 c z) - (a+b) (\cosh(4 c z) + 3)} \right) \right) + \\
 & \left. \frac{8 (14 (b^2 - a^2) + (15 a^2 - 98 b a + 15 b^2) \cosh(2 c z)) \sinh(2 c z)}{a b (a+b) ((a+b) (\cosh(4 c z) + 3) - 4 (a-b) \cosh(2 c z))} \right) \Bigg/ (65 536 c (b \coth^2(c z) + a \tanh^2(c z))^2)
 \end{aligned}$$

**Involving algebraic functions of the direct function and hyperbolic functions**

Involving sinh

**Involving  $\sinh(c z) (a + b \coth(c z))^\beta$**



01.22.21.0395.01

$$\int \sinh(c z) (a + b \coth(c z))^\beta dz = \frac{1}{\sqrt{2} c} \sqrt{\coth(c z) + 1} (a + b \coth(c z))^\beta \left( \frac{a + b \coth(c z)}{a + b} \right)^{-\beta} \sinh(c z) F_1 \left( -\frac{1}{2}; \frac{3}{2}, -\beta; \frac{1}{2}; \frac{1}{2} (1 - \coth(c z)), \frac{b - b \coth(c z)}{a + b} \right)$$

01.22.21.0396.01

$$\int \sinh(c z) \sqrt{a + b \coth(c z)} dz = \left[ \cosh(c z) \sqrt{\frac{(\cosh(c z) + \sinh(c z)) (b \cosh(c z) + a \sinh(c z))}{b}} \left( (a + b) i \sqrt{\coth(c z) - 1} \sqrt{\frac{a + b \coth(c z)}{b \coth(c z) - b}} \right. \right. \\ \left. \left. E \left( i \sinh^{-1} \left( \frac{\sqrt{\frac{a+b}{b}}}{\sqrt{\coth(c z) - 1}} \right) \middle| \frac{2b}{a+b} \right) \operatorname{sech}(c z) \sqrt{\cosh(2 c z) + \sinh(2 c z)} (\cosh(c z) - \sinh(c z)) + \right. \right. \\ \left. \left. a i \sqrt{\coth(c z) - 1} \sqrt{\frac{a + b \coth(c z)}{b \coth(c z) - b}} F \left( i \sinh^{-1} \left( \frac{\sqrt{\frac{a+b}{b}}}{\sqrt{\coth(c z) - 1}} \right) \middle| \frac{2b}{a+b} \right) \sqrt{\cosh(2 c z) + \sinh(2 c z)} \right. \right. \\ \left. \left. (\tanh(c z) - 1) + \sqrt{\frac{a+b}{b}} (b + a \tanh(c z)) \right) \right] / \left( \sqrt{\frac{a+b}{b}} c \sqrt{a + b \coth(c z)} \sqrt{\frac{a+b}{b \coth(c z) - b} + 1} \right)$$

01.22.21.0397.01

$$\int \frac{\sinh(cz)}{\sqrt{a+b \coth(cz)}} dz =$$

$$\left( \cosh(cz) \left( \sqrt{\frac{b(\coth(cz)+1)}{b-a}} \sqrt{\frac{a+b \coth(cz)}{a+b}} \sqrt{\coth(cz)+1} F\left(\sin^{-1}\left(\frac{\sqrt{1-\coth(cz)}}{\sqrt{2}}\right) \middle| \frac{2b}{a+b}\right) (\tanh(cz)-1)b^2 - \right. \right.$$

$$a \sqrt{\frac{a+b \coth(cz)}{a-b}} \sqrt{\frac{b-b \coth(cz)}{a+b}} (a+b) \sqrt{1-\coth(cz)} E\left(\sin^{-1}\left(\sqrt{\frac{a+b \coth(cz)}{a-b}}\right) \middle| \frac{a-b}{a+b}\right) \left. \right.$$

$$\operatorname{sech}(cz) (\cosh(cz) + \sinh(cz) + \sqrt{1-\coth(cz)}) \left( \sqrt{\frac{b(\coth(cz)+1)}{b-a}} (a^2 + b \operatorname{csch}(cz) \operatorname{sech}(cz) a - b^2) + \right.$$

$$a \sqrt{\frac{a+b \coth(cz)}{a-b}} b \sqrt{\frac{b-b \coth(cz)}{a+b}} F\left(\sin^{-1}\left(\sqrt{\frac{a+b \coth(cz)}{a-b}}\right) \middle| \frac{a-b}{a+b}\right) (\tanh(cz)+1) \left. \right) \left. \right) /$$

$$\left( (a-b)(a+b)c \sqrt{1-\coth(cz)} \sqrt{\frac{b(\coth(cz)+1)}{b-a}} \sqrt{a+b \coth(cz)} \right)$$

**Involving  $\sinh(cz) (a + b \coth^2(cz))^\beta$**

01.22.21.0398.01

$$\int \sinh(cz) (a + b \coth^2(cz))^\beta dz =$$

$$\frac{\cosh(cz) (b \coth^2(cz) + a)^\beta \left(\frac{b \coth^2(cz)}{a} + 1\right)^{-\beta} \sqrt{-\operatorname{csch}^2(cz)}}{c} F_1\left(\frac{1}{2}; \frac{3}{2}, -\beta; \frac{3}{2}; \coth^2(cz), -\frac{b \coth^2(cz)}{a}\right)$$

01.22.21.0399.01

$$\int \sinh(cz) \sqrt{a + b \coth^2(cz)} dz = \left( i \sqrt{2} \sqrt{\frac{1}{a}} b \cosh(cz) \sqrt{b \coth^2(cz) + a} \right.$$

$$\left( E\left(\sin^{-1}\left(\frac{\sqrt{\frac{1}{a}} \sqrt{-a+b+(a+b) \cosh(2cz)}}{\sqrt{2}}}\right) \middle| -\frac{a}{b}\right) - F\left(\sin^{-1}\left(\frac{\sqrt{\frac{1}{a}} \sqrt{-a+b+(a+b) \cosh(2cz)}}{\sqrt{2}}}\right) \middle| -\frac{a}{b}\right) \right)$$

$$\sqrt{-\frac{(a+b) \sinh^2(cz)}{b}} \left. \right) / \left( c \sqrt{\frac{(a+b) \cosh^2(cz)}{a}} \sqrt{-a+b+(a+b) \cosh(2cz)} \right)$$

01.22.21.0400.01

$$\int \frac{\sinh(c z)}{\sqrt{a + b \coth^2(c z)}} dz =$$

$$\left( (\cosh(c z) + 1) \sqrt{\frac{-a + b + (a + b) \cosh(2 c z)}{(\cosh(c z) + 1)^2}} \operatorname{csch}(c z) \sqrt{\frac{2 a - 2 \sqrt{a + b} \sqrt{a} + b}{b}} (-a + b + (a + b) \cosh(2 c z)) \tanh\left(\frac{c z}{2}\right) \right.$$

$$\operatorname{sech}^2\left(\frac{c z}{2}\right) + 4 \sqrt{\frac{(2 a + 2 \sqrt{a + b} \sqrt{a} + b) \tanh^2\left(\frac{c z}{2}\right) + b}{b}} \sqrt{\frac{(2 a - 2 \sqrt{a + b} \sqrt{a} + b) \tanh^2\left(\frac{c z}{2}\right) + b}{b}}$$

$$\left. (a - \sqrt{a + b} \sqrt{a} + b) i F\left(i \sinh^{-1}\left(\sqrt{\frac{2 a - 2 \sqrt{a + b} \sqrt{a} + b}{b}} \tanh\left(\frac{c z}{2}\right)\right) \left| \frac{2 a + 2 \sqrt{a + b} \sqrt{a} + b}{2 a - 2 \sqrt{a + b} \sqrt{a} + b} \right| - \right.$$

$$2 i (2 a - 2 \sqrt{a + b} \sqrt{a} + b) E\left(i \sinh^{-1}\left(\sqrt{\frac{2 a - 2 \sqrt{a + b} \sqrt{a} + b}{b}} \tanh\left(\frac{c z}{2}\right)\right) \left| \frac{2 a + 2 \sqrt{a + b} \sqrt{a} + b}{2 a - 2 \sqrt{a + b} \sqrt{a} + b} \right| \right.$$

$$\left. \left. \sqrt{\frac{(2 a - 2 \sqrt{a + b} \sqrt{a} + b) \tanh^2\left(\frac{c z}{2}\right) + b}{b}} \sqrt{\frac{(2 a + 2 \sqrt{a + b} \sqrt{a} + b) \tanh^2\left(\frac{c z}{2}\right) + b}{b}} \right) \right) /$$

$$\left( 2 \sqrt{2} (a + b) \sqrt{\frac{2 a - 2 \sqrt{a + b} \sqrt{a} + b}{b}} c \sqrt{b \coth^2(c z) + a} \sqrt{4 a \tanh^2\left(\frac{c z}{2}\right) + b \left(\tanh^2\left(\frac{c z}{2}\right) + 1\right)^2} \right)$$

Involving cosh

**Involving cosh(c z) (a + b coth(c z))<sup>β</sup>**

01.22.21.0401.01

$$\int \cosh(c z) (a + b \coth(c z))^\beta dz =$$

$$-\frac{1}{c(\beta - 2)(\beta - 1)} \left( \left( b + \frac{a + b}{\coth(c z) - 1} \right)^\beta \left( (\beta - 1) F_1\left(2 - \beta; \frac{3}{2}, -\beta; 3 - \beta; -\frac{2}{\coth(c z) - 1}, \frac{a + b}{b - b \coth(c z)}\right) + \right.$$

$$\left. (\beta - 2) F_1\left(1 - \beta; \frac{3}{2}, -\beta; 2 - \beta; -\frac{2}{\coth(c z) - 1}, \frac{a + b}{b - b \coth(c z)}\right) (\coth(c z) - 1) \right)$$

$$(a + b \coth(c z))^{\beta - 1} \left( \frac{a + b \coth(c z)}{\coth(c z) - 1} \right)^{1 - \beta} \left( \frac{a + b}{b \coth(c z) - b} + 1 \right)^{-\beta} \sinh(c z) \sqrt{\cosh(2 c z) + \sinh(2 c z)}$$

01.22.21.0402.01

$$\int \cosh(c z) \sqrt{a + b \coth(c z)} dz = \left( \cosh(c z) \sqrt{\frac{(\cosh(c z) + \sinh(c z))(b \cosh(c z) + a \sinh(c z))}{b}} \right. \\ \left. \left( \sqrt{2} b i \sqrt{\coth(c z) - 1} \sqrt{\frac{a + b \coth(c z)}{b \coth(c z) - b}} F\left(i \sinh^{-1}\left(\frac{\sqrt{2}}{\sqrt{\coth(c z) - 1}}\right) \middle| \frac{a + b}{2b}\right) \sqrt{\cosh(2 c z) + \sinh(2 c z)} \right. \right. \\ \left. \left. (\tanh(c z) - 1) + 2(b + a \tanh(c z)) \right) \right) / \left( 2 c \sqrt{a + b \coth(c z)} \sqrt{\frac{a + b}{b \coth(c z) - b} + 1} \right)$$

01.22.21.0403.01

$$\int \frac{\cosh(c z)}{\sqrt{a + b \coth(c z)}} dz = \left( \cosh(c z) \sqrt{a + b \coth(c z)} \sinh(c z) (\cosh(c z) + \sinh(c z)) \left( i b \sqrt{\coth(c z) - 1} \sqrt{\frac{a + b \coth(c z)}{b \coth(c z) - b}} \right. \right. \\ \left. \left. E\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{a+b}{b}}}{\sqrt{\coth(c z) - 1}}\right) \middle| \frac{2b}{a+b}\right) \sqrt{\cosh(2 c z) + \sinh(2 c z)} (\tanh(c z) - 1) - \sqrt{\frac{a+b}{b}} (b + a \tanh(c z)) \right) \right) / \\ \left( b(b-a) \sqrt{\frac{a+b}{b}} c \sqrt{\frac{a+b}{b \coth(c z) - b} + 1} \sqrt{\frac{(\cosh(c z) + \sinh(c z))(b \cosh(c z) + a \sinh(c z))}{b}} \right)$$

### Involving $\cosh(c z) (a + b \coth^2(c z))^\beta$

01.22.21.0404.01

$$\int \cosh(c z) (b \coth^2(c z) + a)^\beta dz = \frac{(b \coth^2(c z) + a)^{\beta+1} \sinh(c z)}{2(a+b)c(\beta+1)} \sqrt{-\frac{b \operatorname{csch}^2(c z)}{a+b}} {}_2F_1\left(\beta+1, \frac{3}{2}; \beta+2; \frac{b \coth^2(c z) + a}{a+b}\right)$$

01.22.21.0405.01

$$\int \cosh(c z) \sqrt{a + b \coth^2(c z)} dz = \\ \frac{\sinh(c z)}{c \sqrt{b \coth^2(c z) + a}} \left( b \coth^2(c z) + a - \sqrt{a+b} \sin^{-1}\left(\frac{\sqrt{b \coth^2(c z) + a}}{\sqrt{a+b}}\right) \sqrt{b \coth^2(c z) + a} \sqrt{-\frac{b \operatorname{csch}^2(c z)}{a+b}} \right)$$

01.22.21.0406.01

$$\int \frac{\cosh(c z)}{\sqrt{a + b \coth^2(c z)}} dz = -\frac{(a-b) \operatorname{csch}(c z)}{2(a+b)c \sqrt{b \coth^2(c z) + a}} \sqrt{\frac{(a+b) \cosh(2 c z)}{b-a} + 1} \left( \sqrt{\frac{(a+b) \cosh(2 c z)}{b-a} + 1} - 1 \right)$$

Involving tanh

Involving  $\tanh(cz)(a + b \coth(cz))^\beta$

01.22.21.0407.01

$$\int \tanh(cz)(a + b \coth(cz))^\beta dz = \frac{1}{2a(a-b)(a+b)c(\beta+1)} \left( (a + b \coth(cz))^{\beta+1} \left( (a-b) \left( a {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b \coth(cz)}{a+b}\right) - 2(a+b) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{b \coth(cz)}{a} + 1\right) \right) + a(a+b) {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b \coth(cz)}{a-b}\right) \right) \right)$$

01.22.21.0408.01

$$\int \tanh(cz) \sqrt{a + b \coth(cz)} dz = \frac{1}{c} \left( -2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b \coth(cz)}}{\sqrt{a}}\right) + \sqrt{a-b} \tanh^{-1}\left(\frac{\sqrt{a+b \coth(cz)}}{\sqrt{a-b}}\right) + \sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b \coth(cz)}}{\sqrt{a+b}}\right) \right)$$

01.22.21.0409.01

$$\int \frac{\tanh(cz)}{\sqrt{a + b \coth(cz)}} dz = \frac{1}{c} \left( -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b \coth(cz)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(cz)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(cz)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} \right)$$

Involving  $\tanh(cz)(a + b \coth^2(cz))^\beta$

01.22.21.0410.01

$$\int \tanh(cz)(a + b \coth^2(cz))^\beta dz = \frac{(b \coth^2(cz) + a)^\beta}{2c} \left( \frac{1}{\beta} {}_2F_1\left(-\beta, -\beta; 1-\beta; -\frac{a \tanh^2(cz)}{b}\right) \left(\frac{a \tanh^2(cz)}{b} + 1\right)^{-\beta} + \frac{(b \coth^2(cz) + a)}{(a+b)(\beta+1)} {}_2F_1\left(\beta+1, 1; \beta+2; \frac{b \coth^2(cz) + a}{a+b}\right) \right)$$

01.22.21.0411.01

$$\int \tanh(cz) \sqrt{a + b \coth^2(cz)} dz = \left( \sqrt{b} (a+b) \tanh^{-1}\left(\frac{\sqrt{b \coth^2(cz) + a}}{\sqrt{a+b}}\right) \sqrt{b \coth^2(cz) + a} \sqrt{\frac{a \tanh^2(cz)}{b} + 1} - \sqrt{a} \sqrt{a+b} \sinh^{-1}\left(\frac{\sqrt{a} \tanh(cz)}{\sqrt{b}}\right) (b \coth(cz) + a \tanh(cz)) \right) / \left( \sqrt{b} \sqrt{a+b} c \sqrt{b \coth^2(cz) + a} \sqrt{\frac{a \tanh^2(cz)}{b} + 1} \right)$$

01.22.21.0412.01

$$\int \frac{\tanh(cz)}{\sqrt{a+b \coth^2(cz)}} dz = \frac{1}{c} \left( \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth^2(cz)+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{b} \sinh^{-1}\left(\frac{\sqrt{a} \tanh(cz)}{\sqrt{b}}\right) \coth(cz) \sqrt{\frac{a \tanh^2(cz)}{b} + 1}}{\sqrt{a} \sqrt{b \coth^2(cz)+a}} \right)$$

**Involving sinh and tanh**

01.22.21.0413.01

$$\int \frac{d \coth^2(cz) + e \sinh(cz)}{\sqrt{(b \tanh^2(cz) + a)^3}} dz = \left( \operatorname{sech}(cz) (d \coth^2(cz) + e \sinh(cz)) \right. \\ \left. \left( (a-b+(a+b) \cosh(2cz))^{3/2} \left[ \sqrt{2} d \operatorname{csch}^3(cz) \log\left(\sqrt{a-b+(a+b) \cosh(2cz)} + \sqrt{2} \sqrt{(a+b) \sinh^2(cz)}\right) \right. \right. \right. \\ \left. \left. \left. ((a+b) \sinh^2(cz))^{3/2} + \frac{2(a+b)e \sqrt{a-b+(a+b) \cosh(2cz)} \sqrt{\frac{a-b+(a+b) \cosh(2cz)}{(\cosh(cz)+1)^2}}}{\sqrt{(a-b+(a+b) \cosh(2cz)) \operatorname{sech}^4\left(\frac{cz}{2}\right)}} \right] - \right. \right. \\ \left. \left. \frac{1}{a^2} ((a+b)(a-b+(a+b) \cosh(2cz)) \operatorname{csch}(cz) (2be \sinh(cz) a^2 + (a^3 + b a^2 - 2b^2 a - 2b^3) d + \right. \right. \\ \left. \left. (a^3 + 3b a^2 + 4b^2 a + 2b^3) d \cosh(2cz) \right) \right) \tanh^2(cz) \right) / \\ \left( (a+b)^3 c (2 \cosh(2cz) d + 2d - 3e \sinh(cz) + e \sinh(3cz)) \sqrt{(b \tanh^2(cz) + a)^3} \right)$$

**Involving functions of the direct function, hyperbolic and a power functions**

**Involving powers of the direct function, hyperbolic and a power functions**

**Involving sinh and power**

**Involving  $z^n \sinh(a + bz) \coth^v(cz)$**

01.22.21.0414.01

$$\int z^n \sinh(a + bz) \coth^v(cz) dz = \frac{1}{2} (-1)^v n! \left( -e^{(cv-b)z-a} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \right.$$

$$\sum_{j=0}^n \frac{(-1)^j (cv-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{cv-b}{2c}, \dots, \frac{cv-b}{2c}, v; \frac{cv-b}{2c} + 1, \dots, \frac{cv-b}{2c} + 1; e^{2cz} \right) + e^{a+(b+cv)z} \binom{v}{\frac{v}{2}}$$

$$(1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+cv}{2c}, \dots, \frac{b+cv}{2c}, v; \frac{b+cv}{2c} + 1, \dots, \frac{b+cv}{2c} + 1; e^{2cz} \right) -$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-b)z-a} \sum_{j=0}^n \frac{(-1)^j (2cs-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-b}{2c}, \dots, \frac{2cs-b}{2c}, v; \frac{2cs-b}{2c} + 1, \dots, \frac{2cs-b}{2c} + 1; e^{2cz} \right) + e^{(2c(v-s)-b)z-a} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c(v-s)-b}{2c}, \dots, \frac{2c(v-s)-b}{2c}, v; \frac{2c(v-s)-b}{2c} + 1, \dots, \frac{2c(v-s)-b}{2c} + 1; e^{2cz} \right) \right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s}$$

$$\left( e^{a+(b+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2cs}{2c}, \dots, \frac{b+2cs}{2c}, v; \frac{b+2cs}{2c} + 1, \dots, \frac{b+2cs}{2c} + 1; e^{2cz} \right) + e^{a+(b+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2c(v-s)}{2c}, \dots, \frac{b+2c(v-s)}{2c}, v; \frac{b+2c(v-s)}{2c} + 1, \dots, \frac{b+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

01.22.21.0415.01

$$\int z^n \sinh(bz) \coth^v(cz) dz = \frac{1}{2} (-1)^v \left(\frac{v}{2}\right) n! (1 - v \bmod 2) \left( e^{(b+cv)z} \sum_{j=0}^n \frac{(-1)^j (b+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+cv}{2c}, \dots, \frac{b+cv}{2c}, v; \frac{b+cv}{2c} + 1, \dots, \frac{b+cv}{2c} + 1; e^{2cz} \right) - e^{(c-b)z} \sum_{j=0}^n \frac{(-1)^j (c-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c-b}{2c}, \dots, \frac{c-b}{2c}, v; \frac{c-b}{2c} + 1, \dots, \frac{c-b}{2c} + 1; e^{2cz} \right) \right) + \frac{1}{2} (-1)^v n! \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( -e^{(2ci-b)z} \sum_{j=0}^n \frac{(-1)^j (2ci-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2ci-b}{2c}, \dots, \frac{2ci-b}{2c}, v; \frac{2ci-b}{2c} + 1, \dots, \frac{2ci-b}{2c} + 1; e^{2cz} \right) + e^{(b+2ci)z} \sum_{j=0}^n \frac{(-1)^j (b+2ci)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2ci}{2c}, \dots, \frac{b+2ci}{2c}, v; \frac{b+2ci}{2c} + 1, \dots, \frac{b+2ci}{2c} + 1; e^{2cz} \right) - e^{(-b-2ci+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-2ci+2cv}{2c}, \dots, \frac{-b-2ci+2cv}{2c}, v; \frac{-b-2ci+2cv}{2c} + 1, \dots, \frac{-b-2ci+2cv}{2c} + 1; e^{2cz} \right) + e^{(b-2ci+2cv)z} \sum_{j=0}^n \frac{(-1)^j (b-2ci+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-2ci+2cv}{2c}, \dots, \frac{b-2ci+2cv}{2c}, v; \frac{b-2ci+2cv}{2c} + 1, \dots, \frac{b-2ci+2cv}{2c} + 1; e^{2cz} \right) \right) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of sinh and power

### Involving $z^n \sinh^u(bz) \coth^v(cz)$

01.22.21.0416.01

$$\int z^n \sinh^u(bz) \coth^v(cz) dz = (-1)^v \left(\frac{u}{2}\right) n! (1 - u \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + e^{2cz} v \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) + e^{cvz} \left(\frac{v}{2}\right) (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2} + 1, \dots, \frac{v}{2} + 1; e^{2cz} \right) + \right)$$



$$\begin{aligned}
 & \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + e^{2c(v-s)z} \right. \\
 & \quad \left. \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \right) \left( \frac{i}{2} \right)^u + \\
 & 2^{-u} n! \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^{k+v} \binom{u}{k} \left( (-1)^u e^{(c v - b(u-2k))z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (c v - b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{c v - b(u-2k)}{2c}, \dots, \frac{c v - b(u-2k)}{2c}, v; \frac{c v - b(u-2k)}{2c} + 1, \dots, \frac{c v - b(u-2k)}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(b(u-2k)+c v)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b(u-2k)+c v)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{b(u-2k)+c v}{2c}, \dots, \frac{b(u-2k)+c v}{2c}, v; \frac{b(u-2k)+c v}{2c} + 1, \dots, \frac{b(u-2k)+c v}{2c} + 1; e^{2cz} \right) + \\
 & \quad (-1)^u \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2cs-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-b(u-2k)}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{2cs-b(u-2k)}{2c}, v; \frac{2cs-b(u-2k)}{2c} + 1, \dots, \frac{2cs-b(u-2k)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(2c(v-s)-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c(v-s)-b(u-2k)}{2c}, \dots, \frac{2c(v-s)-b(u-2k)}{2c}, v; \right. \right. \\
 & \quad \left. \left. \frac{2c(v-s)-b(u-2k)}{2c} + 1, \dots, \frac{2c(v-s)-b(u-2k)}{2c} + 1; e^{2cz} \right) \right) + \\
 & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2cs+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs+b(u-2k)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{2cs+b(u-2k)}{2c}, v; \frac{2cs+b(u-2k)}{2c} + 1, \dots, \frac{2cs+b(u-2k)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(b(u-2k)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b(u-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2k)+2c(v-s)}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{b(u-2k)+2c(v-s)}{2c}, v; \frac{b(u-2k)+2c(v-s)}{2c} + 1, \dots, \right. \right. \\
 & \quad \left. \left. \frac{b(u-2k)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg/ ; n \in \mathbb{N} \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$

Involving cosh and power

Involving  $z^n \cosh(a + b z) \coth^v(c z)$

01.22.21.0417.01

$$\int z^n \cosh(a + b z) \coth^v(c z) dz = \frac{1}{2} (-1)^v n! \left( e^{(-b+cv)z-a} \left( \frac{v}{2} \right) (1 - v \bmod 2) \right.$$

$$\sum_{j=0}^n \frac{(-1)^j (-b + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b + cv}{2c}, \dots, \frac{-b + cv}{2c}, v; \frac{-b + cv}{2c} + 1, \dots, \frac{-b + cv}{2c} + 1; e^{2cz} \right) + e^{a+(b+cv)z}$$

$$\left( \frac{v}{2} \right) (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b + cv}{2c}, \dots, \frac{b + cv}{2c}, v; \frac{b + cv}{2c} + 1, \dots, \frac{b + cv}{2c} + 1; e^{2cz} \right) +$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b+2cs)z-a} \sum_{j=0}^n \frac{(-1)^j (-b + 2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b + 2cs}{2c}, \dots, \frac{-b + 2cs}{2c}, v; \frac{-b + 2cs}{2c} + 1, \right.$$

$$\dots, \frac{-b + 2cs}{2c} + 1; e^{2cz} \right) + e^{(-b+2c(v-s))z-a} \sum_{j=0}^n \frac{(-1)^j (-b + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{-b + 2c(v-s)}{2c}, \dots, \frac{-b + 2c(v-s)}{2c}, v; \frac{-b + 2c(v-s)}{2c} + 1, \dots, \frac{-b + 2c(v-s)}{2c} + 1; e^{2cz} \right) \right) +$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{a+(b+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b + 2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b + 2cs}{2c}, \dots, \frac{b + 2cs}{2c}, v; \frac{b + 2cs}{2c} + 1, \dots, \right.$$

$$\frac{b + 2cs}{2c} + 1; e^{2cz} \right) + e^{a+(b+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b + 2c(v-s)}{2c}, \right.$$

$$\dots, \frac{b + 2c(v-s)}{2c}, v; \frac{b + 2c(v-s)}{2c} + 1, \dots, \frac{b + 2c(v-s)}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

01.22.21.0418.01

$$\int z^n \cosh(bz) \coth^v(cz) dz = \frac{1}{2} (-1)^v \binom{v}{\frac{v}{2}} n! (1 - v \bmod 2)$$

$$\left( e^{(cv-b)z} \sum_{j=0}^n \frac{(-1)^j (cv-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{cv-b}{2c}, \dots, \frac{cv-b}{2c}, v; \frac{cv-b}{2c} + 1, \dots, \frac{cv-b}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(b+cv)z} \sum_{j=0}^n \frac{(-1)^j (b+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+cv}{2c}, \dots, \frac{b+cv}{2c}, v; \frac{b+cv}{2c} + 1, \dots, \frac{b+cv}{2c} + 1; e^{2cz} \right) \right) + \frac{1}{2} (-1)^v n!$$

$$\sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( e^{(2ci-b)z} \sum_{j=0}^n \frac{(-1)^j (2ci-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2ci-b}{2c}, \dots, \frac{2ci-b}{2c}, v; \frac{2ci-b}{2c} + 1, \dots, \frac{2ci-b}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(b+2ci)z} \sum_{j=0}^n \frac{(-1)^j (b+2ci)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+2ci}{2c}, \dots, \frac{b+2ci}{2c}, v; \frac{b+2ci}{2c} + 1, \dots, \frac{b+2ci}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(-b-2ci+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-2ci+2cv}{2c}, \dots, \frac{-b-2ci+2cv}{2c}, v; \frac{-b-2ci+2cv}{2c} + 1, \dots, \frac{-b-2ci+2cv}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{(b-2ci+2cv)z} \sum_{j=0}^n \frac{(-1)^j (b-2ci+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-2ci+2cv}{2c}, \dots, \frac{b-2ci+2cv}{2c}, v; \frac{b-2ci+2cv}{2c} + 1, \dots, \frac{b-2ci+2cv}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

01.22.21.0419.01

$$\int z^n \cosh(cz) \coth^v(cz) dz =$$

$$\frac{1}{2} (-1)^v e^{cvz} \binom{v+1}{\frac{v+1}{2}} n! (1 - (v+1) \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2} + 1, \dots, \frac{v}{2} + 1; e^{2cz} \right) +$$

$$\frac{1}{2} (-1)^v e^{cvz} n! \sum_{s=0}^{\lfloor \frac{v}{2} \rfloor} \binom{v+1}{s} \left( e^{-c(-2s+v+1)z} \sum_{j=0}^n \frac{(-1)^j (-c(1-2s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{1}{2}(2s-1), \dots, \frac{1}{2}(2s-1), v; \frac{1}{2}(2s-1) + 1, \dots, \frac{1}{2}(2s-1) + 1; e^{2cz} \right) + \right.$$

$$\left. e^{c(-2s+v+1)z} \sum_{j=0}^n \frac{(-1)^j (c(-2s+2v+1))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{1}{2}(-2s+2v+1), \dots, \frac{1}{2}(-2s+2v+1), v; \frac{1}{2}(-2s+2v+1) + 1, \dots, \frac{1}{2}(-2s+2v+1) + 1; e^{2cz} \right) \right); n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of cosh and power

### Involving $z^n \cosh^u(bz) \coth^v(cz)$

01.22.21.0420.01

$$\int z^n \cosh^u(bz) \coth^v(cz) dz =$$

$$(-1)^v 2^{-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + \right.$$

$$e^{2cz} v \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) +$$

$$e^{cvz} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2} + 1, \dots, \frac{v}{2} + 1; e^{2cz}\right) +$$

$$\sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + \right.$$

$$\left. e^{2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j 2^{-j-1} (c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \right) +$$

$$(-1)^v 2^{-u} n! \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{k} \left( e^{(cv-b(u-2k))z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1}\left(\frac{cv-b(u-2k)}{2c}, \dots, \frac{cv-b(u-2k)}{2c}, v; \frac{cv-b(u-2k)}{2c} + 1, \dots, \frac{cv-b(u-2k)}{2c} + 1; e^{2cz}\right) +$$

$$e^{(b(u-2k)+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b(u-2k)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{b(u-2k)+cv}{2c}, \dots, \frac{b(u-2k)+cv}{2c}, v; \frac{b(u-2k)+cv}{2c} + 1, \dots, \frac{b(u-2k)+cv}{2c} + 1; e^{2cz}\right) +$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2cs-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2cs-b(u-2k)}{2c}, \dots,$$

$$\frac{2cs-b(u-2k)}{2c}, v; \frac{2cs-b(u-2k)}{2c} + 1, \dots, \frac{2cs-b(u-2k)}{2c} + 1; e^{2cz}\right) +$$

$$e^{(2c(v-s)-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c(v-s)-b(u-2k)}{2c}, \dots,$$

$$\frac{2c(v-s)-b(u-2k)}{2c}, v; \frac{2c(v-s)-b(u-2k)}{2c} + 1, \dots, \frac{2c(v-s)-b(u-2k)}{2c} + 1; e^{2cz}\right) +$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (2cs+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs+b(u-2k)}{2c}, \dots, \frac{2cs+b(u-2k)}{2c}, v; \frac{2cs+b(u-2k)}{2c} + 1, \dots, \frac{2cs+b(u-2k)}{2c} + 1; e^{2cz} \right) + e^{(b(u-2k)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b(u-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2k)+2c(v-s)}{2c}, \dots, \frac{b(u-2k)+2c(v-s)}{2c}, v; \frac{b(u-2k)+2c(v-s)}{2c} + 1, \dots, \frac{b(u-2k)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) ; n \in \mathbb{N} \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+$$

**Involving functions of the direct function, hyperbolic and exponential functions**

**Involving powers of the direct function, hyperbolic and exponential functions**

Involving sinh and exp

**Involving  $e^{pz} \sinh(bz) \coth^v(cz)$**

01.22.21.0421.01

$$\int e^{pz} \sinh(bz) \coth^v(cz) dz = \frac{1}{2} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \left( \frac{e^{(b+p)z}}{b+p} F_1 \left( -\frac{b+p}{2c}; -v, v; 1 - \frac{b+p}{2c}; -e^{-2cz}, e^{-2cz} \right) - \frac{e^{(p-b)z}}{p-b} F_1 \left( -\frac{p-b}{2c}; -v, v; 1 - \frac{p-b}{2c}; -e^{-2cz}, e^{-2cz} \right) \right)$$

01.22.21.0422.01

$$\int e^{bz} \sinh(bz) \coth^v(cz) dz = \frac{(e^{2bz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz))}{4b} F_1 \left( -\frac{b}{c}; -v, v; 1 - \frac{b}{c}; -e^{-2cz}, e^{-2cz} \right) - \frac{\coth^{v+1}(cz)}{2c(v+1)} {}_2F_1 \left( \frac{v+1}{2}, 1; \frac{v+3}{2}; \coth^2(cz) \right)$$

01.22.21.0423.01

$$\int e^{-bz} \sinh(bz) \coth^v(cz) dz = \frac{e^{-2bz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz)}{4b} F_1 \left( \frac{b}{c}; -v, v; \frac{b}{c} + 1; -e^{-2cz}, e^{-2cz} \right) + \frac{\coth^{v+1}(cz)}{2c(v+1)} {}_2F_1 \left( \frac{v+1}{2}, 1; \frac{v+3}{2}; \coth^2(cz) \right)$$

Involving powers of sinh and exp

**Involving  $e^{pz} \sinh^u(bz) \coth^v(cz)$**

01.22.21.0424.01

$$\int e^{pz} \sinh^u(bz) \coth^v(cz) dz = \frac{e^{pz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) (1 - u \bmod 2)}{p} \left(\frac{i}{2}\right)^u \left(\frac{u}{2}\right) F_1\left(-\frac{p}{2c}; -v, v; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz}\right) + 2^{-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( \frac{e^{(p+b(u-2k))z} F_1\left(-\frac{p+b(u-2k)}{2c}; -v, v; 1 - \frac{p+b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p+b(u-2k)} + \frac{(-1)^u e^{(p-b(u-2k))z} F_1\left(-\frac{p-b(u-2k)}{2c}; -v, v; 1 - \frac{p-b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p-b(u-2k)} \right) /; u \in \mathbb{N}^+$$

01.22.21.0425.01

$$\int e^{pz} \sinh^\mu(cz) \coth^v(cz) dz = \frac{e^{pz} (1 - e^{2cz})^{\nu-\mu} (1 + e^{2cz})^{-\nu} \coth^v(cz) \sinh^\mu(cz)}{p - c\mu} F_1\left(\frac{p - c\mu}{2c}; \nu - \mu, -\nu; \frac{1}{2}\left(\frac{p}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)$$

Involving cosh and exp

**Involving  $e^{pz} \cosh(bz) \coth^v(cz)$**

01.22.21.0426.01

$$\int e^{pz} \cosh(bz) \coth^v(cz) dz = \frac{1}{2} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \left( \frac{e^{(p-b)z}}{p-b} F_1\left(-\frac{p-b}{2c}; -v, v; 1 - \frac{p-b}{2c}; -e^{-2cz}, e^{-2cz}\right) + \frac{e^{(b+p)z}}{b+p} F_1\left(-\frac{b+p}{2c}; -v, v; 1 - \frac{b+p}{2c}; -e^{-2cz}, e^{-2cz}\right) \right)$$

01.22.21.0427.01

$$\int e^{bz} \cosh(bz) \coth^v(cz) dz = \frac{(e^{2bz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz))}{4b} F_1\left(-\frac{b}{c}; -v, v; 1 - \frac{b}{c}; -e^{-2cz}, e^{-2cz}\right) + \frac{\coth^{\nu+1}(cz)}{2c(\nu+1)} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+3}{2}; \coth^2(cz)\right)$$

01.22.21.0428.01

$$\int e^{-bz} \cosh(bz) \coth^v(cz) dz = \frac{\coth^{\nu+1}(cz)}{2c(\nu+1)} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+3}{2}; \coth^2(cz)\right) - \frac{e^{-2bz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz)}{4b} F_1\left(\frac{b}{c}; -v, v; \frac{b}{c} + 1; -e^{-2cz}, e^{-2cz}\right)$$

Involving powers of cosh and exp

**Involving  $e^{pz} \cosh^u(bz) \coth^v(cz)$**

01.22.21.0429.01

$$\int e^{pz} \cosh^u(bz) \coth^v(cz) dz = \frac{2^{-u} e^{pz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) (1 - u \bmod 2)}{p} \left( \frac{u}{2} \right) F_1 \left( -\frac{p}{2c}; -v, v; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz} \right) + 2^{-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{k} \left( \frac{e^{(p+b(u-2k))z} F_1 \left( -\frac{p+b(u-2k)}{2c}; -v, v; 1 - \frac{p+b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) + \frac{e^{(p-b(u-2k))z} F_1 \left( -\frac{p-b(u-2k)}{2c}; -v, v; 1 - \frac{p-b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p + b(u-2k)} \right) /; u \in \mathbb{N}^+$$

01.22.21.0430.01

$$\int e^{pz} \cosh^\mu(cz) \coth^v(cz) dz = \frac{e^{pz} (1 - e^{2cz})^v (1 + e^{2cz})^{-\mu-v} \cosh^\mu(cz) \coth^v(cz)}{p - c\mu} F_1 \left( \frac{p - c\mu}{2c}; v, -\mu - v; \frac{1}{2} \left( \frac{p}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right)$$

Involving tanh and exp

**Involving  $e^{pz} \tanh(cz) \coth^v(cz)$**

01.22.21.0431.01

$$\int e^{pz} \tanh(cz) \coth^v(cz) dz = \frac{e^{pz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz)}{p} F_1 \left( -\frac{p}{2c}; 1 - v, v - 1; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz} \right)$$

Involving powers of tanh and exp

**Involving  $e^{pz} \tanh^\mu(cz) \coth^v(cz)$**

01.22.21.0432.01

$$\int e^{pz} \tanh^\mu(cz) \coth^v(cz) dz = \frac{e^{pz} (1 - e^{-2cz})^{v-\mu} (1 + e^{-2cz})^{\mu-v} \coth^v(cz) \tanh^\mu(cz)}{p} F_1 \left( -\frac{p}{2c}; \mu - v, v - \mu; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz} \right)$$

**Involving functions of the direct function, hyperbolic and trigonometric functions**

**Involving powers of the direct function, hyperbolic and trigonometric functions**

Involving sin and sinh

**Involving  $\sin(az) \sinh(bz) \coth^v(cz)$**

01.22.21.0433.01

$$\int \sin(az) \sinh(bz) \coth^{\nu}(cz) dz = \frac{1}{4} i (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz) \left( -\frac{e^{(-b-ia)z} F_1\left(-\frac{b-ia}{2c}; -\nu, \nu; 1 - \frac{b-ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{-b-ia} + \frac{e^{(ia-b)z} F_1\left(-\frac{ia-b}{2c}; -\nu, \nu; 1 - \frac{ia-b}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ia-b} + \frac{e^{(b-ia)z} F_1\left(-\frac{b-ia}{2c}; -\nu, \nu; 1 - \frac{b-ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b-ia} - \frac{e^{(b+ia)z} F_1\left(-\frac{b+ia}{2c}; -\nu, \nu; 1 - \frac{b+ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b+ia} \right)$$

Involving powers of sin and powers of sinh

Involving  $\sin^m(az) \sinh^u(bz) \coth^{\nu}(cz)$



01.22.21.0434.01

$$\int \sin^m(a z) \sinh^u(b z) \coth^v(c z) dz =$$

$$\frac{i^u 2^{-m-u} \coth^{v+1}(c z) (1-m \bmod 2) (1-u \bmod 2)}{c (v+1)} \left(\frac{m}{2}\right) \left(\frac{u}{2}\right) {}_2F_1\left(\frac{v+1}{2}, 1; \frac{v+1}{2} + 1; \coth^2(c z)\right) +$$

$$\frac{i^{u+1} 2^{-m-u} (1-e^{-2cz})^v (1+e^{-2cz})^{-v} \coth^v(c z) (1-u \bmod 2)}{a} \left(\frac{u}{2}\right)$$

$$\sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{(-1)^s}{m-2s} \binom{m}{s} \left( e^{\frac{im\pi}{2} - ia(m-2s)z} F_1\left(\frac{ia(m-2s)}{2c}; -v, v; \frac{ai(m-2s)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) - \right.$$

$$\left. e^{ia(m-2s)z - \frac{im\pi}{2}} F_1\left(-\frac{ia(m-2s)}{2c}; -v, v; 1 - \frac{ia(m-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) +$$

$$\frac{2^{-m-u} (1-e^{-2cz})^v (1+e^{-2cz})^{-v} \coth^v(c z) (1-m \bmod 2)}{b} \left(\frac{m}{2}\right)$$

$$\sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \frac{(-1)^k}{u-2k} \binom{u}{k} \left( e^{b(u-2k)z} F_1\left(-\frac{b(u-2k)}{2c}; -v, v; 1 - \frac{b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) - \right.$$

$$\left. (-1)^u e^{-b(u-2k)z} F_1\left(\frac{b(u-2k)}{2c}; -v, v; \frac{b(u-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) \right) + 2^{-m-u} (1-e^{-2cz})^v$$

$$(1+e^{-2cz})^{-v} \coth^v(c z) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{k+s} \binom{m}{s} \binom{u}{k} \left( \left( e^{\frac{i\pi m}{2} + (b(u-2k) - ia(m-2s))z} F_1\left(-\frac{b(u-2k) - ia(m-2s)}{2c}; \right. \right. \right.$$

$$\left. \left. \left. -v, v; 1 - \frac{b(u-2k) - ia(m-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (b(u-2k) - ia(m-2s)) + \right.$$

$$\left. \left( e^{(ai(m-2s) + b(u-2k))z - \frac{im\pi}{2}} F_1\left(-\frac{ai(m-2s) + b(u-2k)}{2c}; -v, v; 1 - \frac{ai(m-2s) + b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / \right.$$

$$\left. (ai(m-2s) + b(u-2k)) + \left( (-1)^u e^{\frac{i\pi m}{2} + (-ia(m-2s) - b(u-2k))z} F_1\left(-\frac{-ia(m-2s) - b(u-2k)}{2c}; \right. \right. \right.$$

$$\left. \left. \left. -v, v; 1 - \frac{-ia(m-2s) - b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (-ia(m-2s) - b(u-2k)) + \right.$$

$$\left. \left( (-1)^u e^{(ia(m-2s) - b(u-2k))z - \frac{im\pi}{2}} F_1\left(-\frac{ia(m-2s) - b(u-2k)}{2c}; -v, v; 1 - \frac{ia(m-2s) - b(u-2k)}{2c}; \right. \right. \right.$$

$$\left. \left. \left. -e^{-2cz}, e^{-2cz}\right) \right) / (ia(m-2s) - b(u-2k)) \right) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.22.21.0435.01

$$\int \sin^m(a z) \sinh^\mu(c z) \coth^v(c z) dz = 2^{-m} (1 - e^{2cz})^{\nu-\mu} (1 + e^{2cz})^{-\nu} \coth^v(c z) \sinh^\mu(c z)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \frac{e^{\frac{im\pi}{2} - ia(m-2k)z} F_1\left(\frac{-ia(m-2k)-c\mu}{2c}; \nu-\mu, -\nu; \frac{1}{2}\left(-\frac{ia(m-2k)}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)}{-ia(m-2k) - c\mu} + \right.$$

$$\left. \frac{e^{ia(m-2k)z - \frac{im\pi}{2}} F_1\left(\frac{ia(m-2k)-c\mu}{2c}; \nu-\mu, -\nu; \frac{1}{2}\left(\frac{ia(m-2k)}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)}{ia(m-2k) - c\mu} \right) - \frac{1}{c\mu} 2^{-m} (1 - e^{2cz})^{\nu-\mu}$$

$$(1 + e^{2cz})^{-\nu} F_1\left(-\frac{\mu}{2}; \nu-\mu, -\nu; \frac{2-\mu}{2}; e^{2cz}, -e^{2cz}\right) \binom{m}{\frac{m}{2}} \coth^v(c z) (1 - m \bmod 2) \sinh^\mu(c z); m \in \mathbb{N}^+$$

Involving cos and sinh

**Involving cos(a z) sinh(b z) coth<sup>v</sup>(c z)**

01.22.21.0436.01

$$\int \cos(a z) \sinh(b z) \coth^v(c z) dz = \frac{1}{4} (1 - e^{-2cz})^\nu (1 + e^{-2cz})^{-\nu} \coth^v(c z)$$

$$\left( -\frac{e^{(-b-ia)z} F_1\left(-\frac{b-ia}{2c}; -\nu, \nu; 1 - \frac{b-ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{-b-ia} + \frac{e^{(b-ia)z} F_1\left(-\frac{b-ia}{2c}; -\nu, \nu; 1 - \frac{b-ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b-ia} + \right.$$

$$\left. \frac{e^{(b+ia)z} F_1\left(-\frac{b+ia}{2c}; -\nu, \nu; 1 - \frac{b+ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b+ia} - \frac{e^{(i a-b)z} F_1\left(-\frac{ia-b}{2c}; -\nu, \nu; 1 - \frac{ia-b}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ia-b} \right)$$

Involving powers of cos and powers of sinh

**Involving cos<sup>m</sup>(a z) sinh<sup>u</sup>(b z) coth<sup>v</sup>(c z)**

01.22.21.0437.01

$$\int \cos^m(a z) \sinh^u(b z) \coth^v(c z) dz =$$

$$\frac{i^u 2^{-m-u} \coth^{v+1}(c z) (1 - m \bmod 2) (1 - u \bmod 2)}{c (v + 1)} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} {}_2F_1\left(\frac{v+1}{2}, 1; \frac{v+1}{2} + 1; \coth^2(c z)\right) +$$

$$\frac{i^{u+1} 2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(c z) (1 - u \bmod 2)}{a} \binom{u}{\frac{u}{2}}$$

$$\sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{1}{m-2s} \binom{m}{s} \left( e^{-ia(m-2s)z} F_1\left(\frac{ia(m-2s)}{2c}; -v, v; \frac{ia(m-2s)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) - \right.$$

$$\left. e^{ia(m-2s)z} F_1\left(-\frac{ia(m-2s)}{2c}; -v, v; 1 - \frac{ia(m-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) +$$

$$\frac{2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(c z) (1 - m \bmod 2)}{b} \binom{m}{\frac{m}{2}}$$

$$\sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \frac{(-1)^k}{u-2k} \binom{u}{k} \left( e^{b(u-2k)z} F_1\left(-\frac{b(u-2k)}{2c}; -v, v; 1 - \frac{b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) - \right.$$

$$\left. (-1)^u e^{-b(u-2k)z} F_1\left(\frac{b(u-2k)}{2c}; -v, v; \frac{b(u-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) \right) + 2^{-m-u} (1 - e^{-2cz})^v$$

$$(1 + e^{-2cz})^{-v} \coth^v(c z) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{s} \binom{u}{k} \left( \left( e^{(b(u-2k)-ia(m-2s))z} F_1\left(-\frac{b(u-2k)-ia(m-2s)}{2c}; -v, \right. \right. \right.$$

$$\left. \left. \left. v; 1 - \frac{b(u-2k)-ia(m-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (b(u-2k) - ia(m-2s)) + \right.$$

$$\left. \left( e^{(a i(m-2s)+b(u-2k))z} F_1\left(-\frac{a i(m-2s)+b(u-2k)}{2c}; -v, v; 1 - \frac{a i(m-2s)+b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / \right.$$

$$\left. (a i(m-2s) + b(u-2k)) + \left( (-1)^u e^{(-ia(m-2s)-b(u-2k))z} F_1\left(-\frac{-ia(m-2s)-b(u-2k)}{2c}; -v, \right. \right. \right.$$

$$\left. \left. \left. v; 1 - \frac{-ia(m-2s)-b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (-ia(m-2s) - b(u-2k)) + \right.$$

$$\left. \left( (-1)^u e^{(ia(m-2s)-b(u-2k))z} F_1\left(-\frac{ia(m-2s)-b(u-2k)}{2c}; -v, v; 1 - \frac{ia(m-2s)-b(u-2k)}{2c}; \right. \right. \right.$$

$$\left. \left. \left. -e^{-2cz}, e^{-2cz}\right) \right) / (ia(m-2s) - b(u-2k)) \right) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.22.21.0438.01

$$\int \cos^m(a z) \sinh^\mu(c z) \coth^\nu(c z) dz = 2^{-m} (1 - e^{2cz})^{\nu-\mu} (1 + e^{2cz})^{-\nu} \coth^\nu(c z) \sinh^\mu(c z)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \frac{e^{-ia(m-2k)z} F_1\left(\frac{-ia(m-2k)-c\mu}{2c}; \nu-\mu, -\nu; \frac{1}{2}\left(\frac{-ia(m-2k)}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)}{-ia(m-2k)-c\mu} + \frac{e^{ia(m-2k)z} F_1\left(\frac{ia(m-2k)-c\mu}{2c}; \nu-\mu, -\nu; \frac{1}{2}\left(\frac{ia(m-2k)}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)}{ia(m-2k)-c\mu} \right) - \frac{1}{c\mu} 2^{-m} (1 - e^{2cz})^{\nu-\mu}$$

$$(1 + e^{2cz})^{-\nu} \coth^\nu(c z) \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \sinh^\mu(c z) F_1\left(-\frac{\mu}{2}; \nu-\mu, -\nu; \frac{2-\mu}{2}; e^{2cz}, -e^{2cz}\right) /; m \in \mathbb{N}^+$$

Involving sin and cosh

### Involving sin(a z) cosh(b z) coth<sup>ν</sup>(c z)

01.22.21.0439.01

$$\int \sin(a z) \cosh(b z) \coth^\nu(c z) dz = -\frac{1}{4} i (1 - e^{-2cz})^\nu (1 + e^{-2cz})^{-\nu} \coth^\nu(c z)$$

$$\left( -\frac{e^{(-b-ia)z} F_1\left(-\frac{b-ia}{2c}; -\nu, \nu; 1 - \frac{b-ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{-b-ia} + \frac{e^{(ia-b)z} F_1\left(-\frac{ia-b}{2c}; -\nu, \nu; 1 - \frac{ia-b}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ia-b} \right) +$$

$$\left( \frac{e^{(b+ia)z} F_1\left(-\frac{b+ia}{2c}; -\nu, \nu; 1 - \frac{b+ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b+ia} - \frac{e^{(b-ia)z} F_1\left(-\frac{b-ia}{2c}; -\nu, \nu; 1 - \frac{b-ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b-ia} \right)$$

Involving powers of sin and powers of cosh

### Involving sin<sup>m</sup>(a z) cosh<sup>u</sup>(b z) coth<sup>ν</sup>(c z)

01.22.21.0440.01

$$\int \sin^m(a z) \cosh^u(b z) \coth^\nu(c z) dz =$$

$$2^{-m-u} (1 - e^{-2cz})^\nu \coth^\nu(c z) \left( \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^s \left( \left( e^{\frac{i\pi m}{2} + (b(u-2k) - ia(m-2s))z} F_1\left(-\frac{b(u-2k) - ia(m-2s)}{2c}; -\nu, \nu; 1 - \frac{b(u-2k) - ia(m-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (b(u-2k) - ia(m-2s)) + \left( e^{(ai(m-2s) + b(u-2k))z - \frac{im\pi}{2}} F_1\left(-\frac{ai(m-2s) + b(u-2k)}{2c}; -\nu, \nu; 1 - \frac{ai(m-2s) + b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (ai(m-2s) + b(u-2k)) + \left( e^{\frac{i\pi m}{2} + (-ia(m-2s) - b(u-2k))z} F_1\left(-\frac{-ia(m-2s) - b(u-2k)}{2c}; -\nu, \nu; 1 - \frac{-ia(m-2s) - b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (-ia(m-2s) - b(u-2k)) \right)$$

$$\begin{aligned}
 & \left. v; 1 - \frac{-i a (m - 2 s) - b (u - 2 k)}{2 c}; -e^{-2 c z}, e^{-2 c z} \right) / (-i a (m - 2 s) - b (u - 2 k)) + \\
 & \left( e^{(i a (m - 2 s) - b (u - 2 k)) z - \frac{i m \pi}{2}} F_1 \left( -\frac{i a (m - 2 s) - b (u - 2 k)}{2 c}; -v, v; 1 - \frac{i a (m - 2 s) - b (u - 2 k)}{2 c}; \right. \right. \\
 & \left. \left. -e^{-2 c z}, e^{-2 c z} \right) / (i a (m - 2 s) - b (u - 2 k)) \binom{m}{s} \binom{u}{k} \right) (1 + e^{-2 c z})^{-v} + \\
 & \frac{2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \coth^{v+1}(c z) {}_2F_1 \left( \frac{v+1}{2}, 1; \frac{v+1}{2} + 1; \coth^2(c z) \right) (1 - m \bmod 2) (1 - u \bmod 2)}{c (v + 1)} + \\
 & \frac{1}{a} \\
 & \left( i 2^{-m-u} (1 - e^{-2 c z})^v (1 + e^{-2 c z})^{-v} \binom{u}{\frac{u}{2}} \coth^v(c z) (1 - u \bmod 2) \right) \\
 & \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{1}{m-2s} \left( (-1)^s \left( e^{\frac{i m \pi}{2} - i a (m-2s) z} F_1 \left( \frac{i a (m-2s)}{2c}; -v, v; \frac{a i (m-2s)}{2c} + 1; -e^{-2cz}, e^{-2cz} \right) - \right. \right. \\
 & \left. \left. e^{i a (m-2s) z - \frac{i m \pi}{2}} F_1 \left( -\frac{i a (m-2s)}{2c}; -v, v; 1 - \frac{i a (m-2s)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) \binom{m}{s} \right) + \\
 & \frac{1}{b} \left( \left( 2^{-m-u} (1 - e^{-2 c z})^v (1 + e^{-2 c z})^{-v} \binom{m}{\frac{m}{2}} \coth^v(c z) (1 - m \bmod 2) \right) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \frac{1}{u-2k} \right. \\
 & \left. \left( \left( e^{b(u-2k)z} F_1 \left( -\frac{b(u-2k)}{2c}; -v, v; 1 - \frac{b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) - \right. \right. \right. \\
 & \left. \left. \left. e^{-b(u-2k)z} F_1 \left( \frac{b(u-2k)}{2c}; -v, v; \frac{b(u-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz} \right) \right) \binom{u}{k} \right) \right); m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

01.22.21.0441.01

$$\int \sin^m(a z) \cosh^\mu(c z) \coth^\nu(c z) dz = 2^{-m} (1 - e^{2cz})^\nu (1 + e^{2cz})^{-\mu-\nu} \cosh^\mu(c z) \coth^\nu(c z)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \frac{e^{\frac{im\pi}{2} - ia(m-2k)z} F_1\left(\frac{-ia(m-2k)-c\mu}{2c}; \nu, -\mu-\nu; \frac{1}{2}\left(-\frac{ia(m-2k)}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)}{-ia(m-2k) - c\mu} + \right.$$

$$\left. \frac{e^{ia(m-2k)z - \frac{im\pi}{2}} F_1\left(\frac{ia(m-2k)-c\mu}{2c}; \nu, -\mu-\nu; \frac{1}{2}\left(\frac{ai(m-2k)}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)}{ia(m-2k) - c\mu} \right) - \frac{1}{c\mu} 2^{-m} (1 - e^{2cz})^\nu$$

$$(1 + e^{2cz})^{-\mu-\nu} \binom{m}{\frac{m}{2}} \cosh^\mu(c z) \coth^\nu(c z) (1 - m \bmod 2) F_1\left(-\frac{\mu}{2}; \nu, -\mu-\nu; \frac{2-\mu}{2}; e^{2cz}, -e^{2cz}\right) /; m \in \mathbb{N}^+$$

Involving cos and cosh

### Involving cos(a z) cosh(b z) coth<sup>ν</sup>(c z)

01.22.21.0442.01

$$\int \cos(a z) \cosh(b z) \coth^\nu(c z) dz = \frac{1}{4} (1 - e^{-2cz})^\nu (1 + e^{-2cz})^{-\nu} \coth^\nu(c z)$$

$$\left( \frac{e^{(-b-ia)z} F_1\left(-\frac{-b-ia}{2c}; -\nu, \nu; 1 - \frac{-b-ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{-b-ia} + \frac{e^{(i a-b)z} F_1\left(-\frac{ia-b}{2c}; -\nu, \nu; 1 - \frac{ia-b}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ia-b} + \right.$$

$$\left. \frac{e^{(b-ia)z} F_1\left(-\frac{b-ia}{2c}; -\nu, \nu; 1 - \frac{b-ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b-ia} + \frac{e^{(b+ia)z} F_1\left(-\frac{b+ia}{2c}; -\nu, \nu; 1 - \frac{b+ia}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b+ia} \right)$$

Involving powers of cos and powers of cosh

### Involving cos<sup>m</sup>(a z) cosh<sup>u</sup>(b z) coth<sup>ν</sup>(c z)

01.22.21.0443.01

$$\int \cos^m(az) \cosh^u(bz) \coth^v(cz) dz = \frac{2^{-m-u} i (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) (1 - u \bmod 2)}{a}$$

$$\left(\frac{u}{2}\right) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{1}{m-2k} \binom{m}{k} \left( e^{-ia(m-2k)z} F_1\left(-\frac{ia(m-2k)}{2c}; -v, v; \frac{ia(m-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) - e^{ia(m-2k)z} F_1\left(-\frac{ia(m-2k)}{2c}; -v, v; 1 - \frac{ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) +$$

$$\frac{2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) (1 - m \bmod 2)}{b} \binom{m}{\frac{m}{2}}$$

$$\sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \frac{1}{u-2s} \binom{u}{s} \left( e^{b(u-2s)z} F_1\left(-\frac{b(u-2s)}{2c}; -v, v; 1 - \frac{b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) - e^{-b(u-2s)z} F_1\left(\frac{b(u-2s)}{2c}; -v, v; \frac{b(u-2s)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) \right) +$$

$$2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{m}{k} \binom{u}{s} \left( \left( e^{(b(u-2s)-ia(m-2k))z} F_1\left(-\frac{b(u-2s)-ia(m-2k)}{2c}; -v, v; 1 - \frac{b(u-2s)-ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (b(u-2s) - ia(m-2k)) + \right.$$

$$\left. \left( e^{(ia(m-2k)+b(u-2s))z} F_1\left(-\frac{ia(m-2k)+b(u-2s)}{2c}; -v, v; 1 - \frac{ia(m-2k)+b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (ia(m-2k) + b(u-2s)) + \right.$$

$$\left. \left( e^{(-ia(m-2k)-b(u-2s))z} F_1\left(-\frac{-ia(m-2k)-b(u-2s)}{2c}; -v, v; 1 - \frac{-ia(m-2k)-b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (-ia(m-2k) - b(u-2s)) + \right.$$

$$\left. \left( e^{(ia(m-2k)-b(u-2s))z} F_1\left(-\frac{ia(m-2k)-b(u-2s)}{2c}; -v, v; 1 - \frac{ia(m-2k)-b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (ia(m-2k) - b(u-2s)) \right) +$$

$$\frac{2^{-m-u} \coth^{v+1}(cz) (1 - m \bmod 2) (1 - u \bmod 2)}{c(v+1)} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} {}_2F_1\left(\frac{v+1}{2}, 1; \frac{v+1}{2} + 1; \coth^2(cz)\right); m \in$$

$\mathbb{N}^+ \wedge u \in \mathbb{N}^+$

01.22.21.0444.01

$$\int \cos^m(a z) \cosh^\mu(c z) \coth^\nu(c z) dz = 2^{-m} (1 - e^{2cz})^\nu (1 + e^{2cz})^{-\mu-\nu} \cosh^\mu(c z) \coth^\nu(c z)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \frac{e^{-ia(m-2k)z}}{-ia(m-2k)-c\mu} F_1\left(\frac{-ia(m-2k)-c\mu}{2c}; \nu, -\mu-\nu; \frac{1}{2}\left(\frac{-ia(m-2k)}{c}-\mu+2\right); e^{2cz}, -e^{2cz}\right) + \right.$$

$$\left. \frac{e^{ia(m-2k)z}}{ia(m-2k)-c\mu} F_1\left(\frac{ia(m-2k)-c\mu}{2c}; \nu, -\mu-\nu; \frac{1}{2}\left(\frac{ia(m-2k)}{c}-\mu+2\right); e^{2cz}, -e^{2cz}\right) \right) -$$

$$\frac{1}{c\mu} 2^{-m} (1 - e^{2cz})^\nu (1 + e^{2cz})^{-\mu-\nu} F_1\left(-\frac{\mu}{2}; \nu, -\mu-\nu; \frac{2-\mu}{2}; e^{2cz}, -e^{2cz}\right) \binom{m}{\frac{m}{2}} \cosh^\mu(c z)$$

$$\coth^\nu(c z) (1 - m \bmod 2) /; m \in \mathbb{N}^+$$

Involving sin and tanh

### Involving sin(a z) tanh(c z) coth^\nu(c z)

01.22.21.0445.01

$$\int \sin(a z) \tanh(c z) \coth^\nu(c z) dz = -\frac{1}{2a} e^{-iaz} (1 - e^{-2cz})^\nu (1 + e^{-2cz})^{-\nu} \coth^\nu(c z)$$

$$\left( F_1\left(\frac{ia}{2c}; 1-\nu, \nu-1; 1+\frac{ia}{2c}; -e^{-2cz}, e^{-2cz}\right) + e^{2iaz} F_1\left(-\frac{ia}{2c}; 1-\nu, \nu-1; 1-\frac{ia}{2c}; -e^{-2cz}, e^{-2cz}\right) \right)$$

Involving powers of sin and powers of tanh

### Involving sin^m(a z) tanh^\mu(c z) coth^\nu(c z)

01.22.21.0446.01

$$\int \sin^m(a z) \tanh^\mu(c z) \coth^\nu(c z) dz = \frac{2^{-m} i (1 - e^{-2cz})^{\nu-\mu} (1 + e^{-2cz})^{\mu-\nu} \coth^\nu(c z) \tanh^\mu(c z)}{a}$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{(-1)^k}{m-2k} \binom{m}{k} \left( e^{\frac{im\pi}{2}-ia(m-2k)z} F_1\left(\frac{ia(m-2k)}{2c}; \mu-\nu, \nu-\mu; \frac{a i (m-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) - \right.$$

$$\left. e^{ia(m-2k)z-\frac{im\pi}{2}} F_1\left(-\frac{ia(m-2k)}{2c}; \mu-\nu, \nu-\mu; 1-\frac{ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) -$$

$$\frac{2^{-m} (1 - m \bmod 2) \coth^{\nu+1}(c z) \tanh^\mu(c z)}{c(\mu-\nu-1)} \binom{m}{\frac{m}{2}} {}_2F_1\left(\frac{1}{2}(-\mu+\nu+1), 1; \frac{1}{2}(-\mu+\nu+3); \coth^2(c z)\right) /; m \in \mathbb{N}^+$$

Involving cos and tanh

### Involving cos(a z) tanh(c z) coth^\nu(c z)



01.22.21.0447.01

$$\int \cos(az) \tanh(cz) \coth^{\nu}(cz) dz = -\frac{1}{2a} i e^{-iaz} (1 - e^{-2cz})^{\nu} (1 + e^{-2cz})^{-\nu} \coth^{\nu}(cz) \\ \left( e^{2iaz} F_1\left(-\frac{ia}{2c}; 1 - \nu, \nu - 1; 1 - \frac{ia}{2c}; -e^{-2cz}, e^{-2cz}\right) - F_1\left(\frac{ia}{2c}; 1 - \nu, \nu - 1; 1 + \frac{ia}{2c}; -e^{-2cz}, e^{-2cz}\right) \right)$$

Involving powers of cos and powers of tanh

**Involving  $\cos^m(az) \tanh^{\mu}(cz) \coth^{\nu}(cz)$**

01.22.21.0448.01

$$\int \cos^m(az) \tanh^{\mu}(cz) \coth^{\nu}(cz) dz = \frac{2^{-m} i (1 - e^{-2cz})^{\nu-\mu} (1 + e^{-2cz})^{\mu-\nu} \coth^{\nu}(cz) \tanh^{\mu}(cz)}{a} \\ \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{1}{m-2k} \binom{m}{k} \left( e^{-ia(m-2k)z} F_1\left(\frac{ia(m-2k)}{2c}; \mu - \nu, \nu - \mu; \frac{ia(m-2k)}{2c} + 1; -e^{-2cz}, e^{-2cz}\right) - \right. \\ \left. e^{ia(m-2k)z} F_1\left(-\frac{ia(m-2k)}{2c}; \mu - \nu, \nu - \mu; 1 - \frac{ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) - \\ \frac{2^{-m} (1 - m \bmod 2) \coth^{\nu+1}(cz) \tanh^{\mu}(cz)}{c(\mu - \nu - 1)} \left(\frac{m}{2}\right) {}_2F_1\left(\frac{1}{2}(-\mu + \nu + 1), 1; \frac{1}{2}(-\mu + \nu + 3); \coth^2(cz)\right); m \in \mathbb{N}^+$$

**Involving functions of the direct function, hyperbolic, exponential and a power functions**

**Involving powers of the direct function, hyperbolic, exponential and a power functions**

Involving sinh, exp and power

**Involving  $z^n e^{pz} \sinh(a + bz) \coth^{\nu}(cz)$**

01.22.21.0449.01

$$\int z^n e^{p z} \sinh(a + b z) \coth^v(c z) dz =$$

$$\begin{aligned} & \frac{1}{2} (-1)^v n! \left( -e^{(-b+p+cv)z-a} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-b+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+p+cv}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-b+p+cv}{2c}, v; \frac{-b+p+cv}{2c} + 1, \dots, \frac{-b+p+cv}{2c} + 1; e^{2cz} \right) + e^{a+(b+p+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \right. \\ & \quad \left. \sum_{j=0}^n \frac{(-1)^j (b+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p+cv}{2c}, \dots, \frac{b+p+cv}{2c}, v; \frac{b+p+cv}{2c} + 1, \dots, \frac{b+p+cv}{2c} + 1; e^{2cz} \right) - \right. \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b+p+2cs)z-a} \sum_{j=0}^n \frac{(-1)^j (-b+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\ & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{-b+p+2cs}{2c}, \dots, \frac{-b+p+2cs}{2c}, v; \frac{-b+p+2cs}{2c} + 1, \dots, \frac{-b+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \right. \\ & \quad \left. \left. e^{(-b+p+2c(v-s))z-a} \sum_{j=0}^n \frac{(-1)^j (-b+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+p+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-b+p+2c(v-s)}{2c}, v; \frac{-b+p+2c(v-s)}{2c} + 1, \dots, \frac{-b+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \right. \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{a+(b+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p+2cs}{2c}, \dots, \frac{b+p+2cs}{2c}, \right. \right. \\ & \quad \left. \left. v; \frac{b+p+2cs}{2c} + 1, \dots, \frac{b+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. e^{a+(b+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{b+p+2c(v-s)}{2c}, v; \frac{b+p+2c(v-s)}{2c} + 1, \dots, \frac{b+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$

01.22.21.0450.01

$$\int z^n e^{p z} \sinh(b z) \coth^v(c z) dz =$$

$$\frac{1}{2} (-1)^v \binom{v}{\frac{v}{2}} n! (1 - v \bmod 2) \left( -e^{(-b+p+cv)z} \sum_{j=0}^n \frac{(-1)^j (-b+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+p+cv}{2c}, \dots, \frac{-b+p+cv}{2c}, \right. \right.$$

$$\left. \left. v; \frac{-b+p+cv}{2c} + 1, \dots, \frac{-b+p+cv}{2c} + 1; e^{2cz} \right) + e^{(b+p+cv)z} \sum_{j=0}^n \frac{(-1)^j (b+p+cv)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{b+p+cv}{2c}, \dots, \frac{b+p+cv}{2c}, v; \frac{b+p+cv}{2c} + 1, \dots, \frac{b+p+cv}{2c} + 1; e^{2cz} \right) \right) +$$

$$\frac{1}{2} (-1)^v n! \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( -e^{(-b+2ci+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2ci+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2ci+p}{2c}, \dots, \frac{-b+2ci+p}{2c}, \right. \right.$$

$$\left. \left. v; \frac{-b+2ci+p}{2c} + 1, \dots, \frac{-b+2ci+p}{2c} + 1; e^{2cz} \right) + e^{(b+2ci+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2ci+p)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{b+2ci+p}{2c}, \dots, \frac{b+2ci+p}{2c}, v; \frac{b+2ci+p}{2c} + 1, \dots, \frac{b+2ci+p}{2c} + 1; e^{2cz} \right) - \right.$$

$$e^{(-b-2ci+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-2ci+p+2cv}{2c}, \dots, \right.$$

$$\left. \frac{-b-2ci+p+2cv}{2c}, v; \frac{-b-2ci+p+2cv}{2c} + 1, \dots, \frac{-b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) +$$

$$e^{(b-2ci+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (b-2ci+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-2ci+p+2cv}{2c}, \dots, \right.$$

$$\left. \frac{b-2ci+p+2cv}{2c}, v; \frac{b-2ci+p+2cv}{2c} + 1, \dots, \frac{b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

01.22.21.0451.01

$$\int z^n e^{bz} \sinh(bz) \coth^v(cz) dz =$$

$$\begin{aligned} & \frac{1}{2} ((-1)^v n!) \left( -\frac{z^{n+1}}{(n+1)!} - e^{2cvz} \sum_{j=0}^n \frac{(-1)^j (2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) - \right. \\ & v e^{2cz} \sum_{j=0}^n \frac{(-1)^j (2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) - e^{cvz} \left( \frac{v}{2} \right) (1-v \bmod 2) \\ & \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2}+1, \dots, \frac{v}{2}+1; e^{2cz}\right) + e^{(2b+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \\ & \sum_{j=0}^n \frac{(-1)^j (2b+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2b+cv}{2c}, \dots, \frac{2b+cv}{2c}, v; \frac{2b+cv}{2c}+1, \dots, \frac{2b+cv}{2c}+1; e^{2cz}\right) - \\ & \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + \right. \\ & \left. e^{2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \right) + \\ & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2(b+cs)z} \sum_{j=0}^n \frac{(-1)^j (2b+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+cs}{c}, \dots, \frac{b+cs}{c}, v; \frac{b+cs}{c}+1, \dots, \frac{b+cs}{c}+1; e^{2cz}\right) + \right. \\ & \left. e^{2(b+c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (2b+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\ & \left. \left( \frac{b+c(v-s)}{c}, \dots, \frac{b+c(v-s)}{c}, v; \frac{b+c(v-s)}{c}+1, \dots, \frac{b+c(v-s)}{c}+1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$

01.22.21.0452.01

$$\int z^n e^{-bz} \sinh(bz) \coth^v(cz) dz =$$

$$\begin{aligned} & \frac{1}{2} ((-1)^v n!) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j (2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + \right. \\ & v e^{2cz} \sum_{j=0}^n \frac{(-1)^j (2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) + e^{cvz} \left( \frac{v}{2} \right) (1-v \bmod 2) \\ & \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2}+1, \dots, \frac{v}{2}+1; e^{2cz}\right) - e^{(cv-2b)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \\ & \sum_{j=0}^n \frac{(-1)^j (cv-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{cv-2b}{2c}, \dots, \frac{cv-2b}{2c}, v; \frac{cv-2b}{2c}+1, \dots, \frac{cv-2b}{2c}+1; e^{2cz}\right) + \\ & \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + \right. \\ & \left. e^{2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \right) - \\ & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2(cs-b)z} \sum_{j=0}^n \frac{(-1)^j (2cs-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{cs-b}{c}, \dots, \frac{cs-b}{c}, v; \frac{cs-b}{c}+1, \dots, \frac{cs-b}{c}+1; e^{2cz}\right) + \right. \\ & \left. e^{2(c(v-s)-b)z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c(v-s)-b}{c}, \dots, \frac{c(v-s)-b}{c}, v; \frac{c(v-s)-b}{c}+1, \dots, \frac{c(v-s)-b}{c}+1; e^{2cz}\right) \right) \Bigg) ; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$

Involving powers of sinh, exp and power

**Involving  $z^n e^{pz} \sinh^u(bz) \coth^v(cz)$**

01.22.21.0453.01

$$\int z^n e^{pz} \sinh^u(bz) \coth^v(cz) dz =$$

$$\begin{aligned} & (-1)^v \left( \frac{u}{2} \right) n! (1-u \bmod 2) \left( e^{(p+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+cv}{2c}, \right. \right. \\ & \left. \left. \dots, \frac{p+cv}{2c}, v; \frac{p+cv}{2c}+1, \dots, \frac{p+cv}{2c}+1; e^{2cz}\right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs)z} \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \sum_{j=0}^n \frac{(-1)^j (p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs}{2c}, \dots, \frac{p+2cs}{2c}, v; \frac{p+2cs}{2c} + 1, \dots, \frac{p+2cs}{2c} + 1; e^{2cz} \right) + \\
 & e^{(p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2c(v-s)}{2c}, \dots, \frac{p+2c(v-s)}{2c}, v; \right. \\
 & \left. \frac{p+2c(v-s)}{2c} + 1, \dots, \frac{p+2c(v-s)}{2c} + 1; e^{2cz} \right) \left( \left( \frac{i}{2} \right)^u + (-1)^v 2^{-u} n! \right) \\
 & \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( e^{(p+b(u-2k)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+b(u-2k)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+b(u-2k)+cv}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{p+b(u-2k)+cv}{2c}, v; \frac{p+b(u-2k)+cv}{2c} + 1, \dots, \frac{p+b(u-2k)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. (-1)^u \left( e^{(p-b(u-2k)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p-b(u-2k)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-b(u-2k)+cv}{2c}, \right. \right. \right. \\
 & \left. \left. \dots, \frac{p-b(u-2k)+cv}{2c}, v; \frac{p-b(u-2k)+cv}{2c} + 1, \dots, \frac{p-b(u-2k)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs-b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (p+2cs-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs-b(u-2k)}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{p+2cs-b(u-2k)}{2c}, v; \frac{p+2cs-b(u-2k)}{2c} + 1, \dots, \frac{p+2cs-b(u-2k)}{2c} + 1; \right. \right. \\
 & \left. \left. e^{2cz} \right) + e^{(p-b(u-2k)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p-b(u-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{p-b(u-2k)+2c(v-s)}{2c}, \dots, \frac{p-b(u-2k)+2c(v-s)}{2c}, v; \right. \right. \\
 & \left. \left. \frac{p-b(u-2k)+2c(v-s)}{2c} + 1, \dots, \frac{p-b(u-2k)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \left. \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs+b(u-2k))z} \sum_{j=0}^n \frac{(-1)^j (p+2cs+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs+b(u-2k)}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{p+2cs+b(u-2k)}{2c}, v; \frac{p+2cs+b(u-2k)}{2c} + 1, \dots, \frac{p+2cs+b(u-2k)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(p+b(u-2k)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+b(u-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+b(u-2k)+2c(v-s)}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{p+b(u-2k)+2c(v-s)}{2c}, v; \frac{p+b(u-2k)+2c(v-s)}{2c} + 1, \dots, \frac{p+b(u-2k)+2c(v-s)}{2c} + 1; \right. \right. \\
 & \left. \left. e^{2cz} \right) \right) +
 \end{aligned}$$

Involving cosh, exp and power

Involving  $z^n e^{pz} \cosh(a + bz) \coth^v(cz)$

01.22.21.0454.01

$$\int z^n e^{pz} \cosh(a + bz) \coth^v(cz) dz =$$

$$\begin{aligned} & \frac{1}{2} (-1)^v n! \left( e^{(-b+p+cv)z-a} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-b+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+p+cv}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-b+p+cv}{2c}, v; \frac{-b+p+cv}{2c} + 1, \dots, \frac{-b+p+cv}{2c} + 1; e^{2cz} \right) + e^{a+(b+p+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \right. \\ & \quad \left. \sum_{j=0}^n \frac{(-1)^j (b+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p+cv}{2c}, \dots, \frac{b+p+cv}{2c}, v; \frac{b+p+cv}{2c} + 1, \dots, \frac{b+p+cv}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b+p+2cs)z-a} \sum_{j=0}^n \frac{(-1)^j (-b+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\ & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{-b+p+2cs}{2c}, \dots, \frac{-b+p+2cs}{2c}, v; \frac{-b+p+2cs}{2c} + 1, \dots, \frac{-b+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \right. \\ & \quad \left. \left. e^{(-b+p+2c(v-s))z-a} \sum_{j=0}^n \frac{(-1)^j (-b+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+p+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-b+p+2c(v-s)}{2c}, v; \frac{-b+p+2c(v-s)}{2c} + 1, \dots, \frac{-b+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \right. \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{a+(b+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p+2cs}{2c}, \dots, \frac{b+p+2cs}{2c}, \right. \right. \right. \\ & \quad \left. \left. v; \frac{b+p+2cs}{2c} + 1, \dots, \frac{b+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. e^{a+(b+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+p+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{b+p+2c(v-s)}{2c}, v; \frac{b+p+2c(v-s)}{2c} + 1, \dots, \frac{b+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$

01.22.21.0455.01

$$\int z^n e^{p z} \cosh(b z) \coth^v(c z) dz =$$

$$\begin{aligned} & \frac{1}{2} (-1)^v \binom{v}{\frac{v}{2}} n! (1 - v \bmod 2) \left( e^{(-b+p+cv)z} \sum_{j=0}^n \frac{(-1)^j (-b+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+p+cv}{2c}, \dots, \frac{-b+p+cv}{2c}, \right. \right. \\ & \quad \left. \left. v; \frac{-b+p+cv}{2c} + 1, \dots, \frac{-b+p+cv}{2c} + 1; e^{2cz} \right) + e^{(b+p+cv)z} \sum_{j=0}^n \frac{(-1)^j (b+p+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad \left. {}_{j+2}F_{j+1} \left( \frac{b+p+cv}{2c}, \dots, \frac{b+p+cv}{2c}, v; \frac{b+p+cv}{2c} + 1, \dots, \frac{b+p+cv}{2c} + 1; e^{2cz} \right) \right) + \\ & \frac{1}{2} (-1)^v n! \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( e^{(-b+2ci+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2ci+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+2ci+p}{2c}, \dots, \frac{-b+2ci+p}{2c}, \right. \right. \\ & \quad \left. \left. v; \frac{-b+2ci+p}{2c} + 1, \dots, \frac{-b+2ci+p}{2c} + 1; e^{2cz} \right) + e^{(b+2ci+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2ci+p)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad \left. {}_{j+2}F_{j+1} \left( \frac{b+2ci+p}{2c}, \dots, \frac{b+2ci+p}{2c}, v; \frac{b+2ci+p}{2c} + 1, \dots, \frac{b+2ci+p}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. e^{(-b-2ci+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-2ci+p+2cv}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-b-2ci+p+2cv}{2c}, v; \frac{-b-2ci+p+2cv}{2c} + 1, \dots, \frac{-b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. e^{(b-2ci+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (b-2ci+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-2ci+p+2cv}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{b-2ci+p+2cv}{2c}, v; \frac{b-2ci+p+2cv}{2c} + 1, \dots, \frac{b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) \right) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$



01.22.21.0456.01

$$\int z^n e^{bz} \cosh(bz) \coth^v(cz) dz =$$

$$\frac{1}{2} ((-1)^v n!) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j (2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + \right.$$

$$e^{2cz} \sum_{j=0}^n \frac{(-1)^j (2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) + e^{cvz} \left( \frac{v}{2} \right) (1-v \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2}+1, \dots, \frac{v}{2}+1; e^{2cz}\right) + e^{(2b+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j (2b+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2b+cv}{2c}, \dots, \frac{2b+cv}{2c}, v; \frac{2b+cv}{2c}+1, \dots, \frac{2b+cv}{2c}+1; e^{2cz}\right) +$$

$$\sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + \right.$$

$$e^{2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \Bigg) +$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2(b+cs)z} \sum_{j=0}^n \frac{(-1)^j (2b+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+cs}{c}, \dots, \frac{b+cs}{c}, v; \frac{b+cs}{c}+1, \dots, \frac{b+cs}{c}+1; e^{2cz}\right) + \right.$$

$$e^{2(b+c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (2b+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left. \left( \frac{b+c(v-s)}{c}, \dots, \frac{b+c(v-s)}{c}, v; \frac{b+c(v-s)}{c}+1, \dots, \frac{b+c(v-s)}{c}+1; e^{2cz} \right) \right) \Bigg) ; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

01.22.21.0457.01

$$\int z^n e^{-bz} \cosh(bz) \coth^v(cz) dz =$$

$$\frac{1}{2} ((-1)^v n!) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j (2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + \right.$$

$$e^{2cz} \sum_{j=0}^n \frac{(-1)^j (2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) + e^{cvz} \left( \frac{v}{2} \right) (1-v \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2}+1, \dots, \frac{v}{2}+1; e^{2cz}\right) + e^{(cv-2b)z} \left( \frac{v}{2} \right) (1-v \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j (cv-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{cv-2b}{2c}, \dots, \frac{cv-2b}{2c}, v; \frac{cv-2b}{2c}+1, \dots, \frac{cv-2b}{2c}+1; e^{2cz}\right) +$$

$$\sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + \right.$$

$$e^{2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \Bigg) +$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2(cs-b)z} \sum_{j=0}^n \frac{(-1)^j (2cs-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{cs-b}{c}, \dots, \frac{cs-b}{c}, v; \frac{cs-b}{c}+1, \dots, \frac{cs-b}{c}+1; e^{2cz}\right) + \right.$$

$$e^{2(c(v-s)-b)z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-2b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left. \left( \frac{c(v-s)-b}{c}, \dots, \frac{c(v-s)-b}{c}, v; \frac{c(v-s)-b}{c}+1, \dots, \frac{c(v-s)-b}{c}+1; e^{2cz} \right) \right) \Bigg) ; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

01.22.21.0458.01

$$\int z^n e^{pz} \cosh(cz) \coth^v(cz) dz = \frac{1}{2} (-1)^v e^{(p+c)vz} \left( \frac{v+1}{2} \right) n! (1 - (v+1) \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j (p+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+c v}{2c}, \dots, \frac{p+c v}{2c}, v; \frac{p+c v}{2c} + 1, \dots, \frac{p+c v}{2c} + 1; e^{2cz} \right) +$$

$$\frac{1}{2} (-1)^v e^{c v z} n! \sum_{s=0}^{\lfloor \frac{v}{2} \rfloor} \binom{v+1}{s} \left( e^{(p-c(-2s+v+1))z} \sum_{j=0}^n \frac{(-1)^j (p-c(1-2s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right.$$

$$\left. \left( \frac{p-c(1-2s)}{2c}, \dots, \frac{p-c(1-2s)}{2c}, v; \frac{p-c(1-2s)}{2c} + 1, \dots, \frac{p-c(1-2s)}{2c} + 1; e^{2cz} \right) + e^{(p+c(-2s+v+1))z} \right.$$

$$\left. \sum_{j=0}^n \frac{(-1)^j (p+c(-2s+2v+1))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+c(-2s+2v+1)}{2c}, \dots, \frac{p+c(-2s+2v+1)}{2c}, \right.$$

$$\left. v; \frac{p+c(-2s+2v+1)}{2c} + 1, \dots, \frac{p+c(-2s+2v+1)}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of cosh, exp and power

### Involving $z^n e^{pz} \cosh^u(bz) \coth^v(cz)$

01.22.21.0459.01

$$\int z^n e^{pz} \cosh^u(bz) \coth^v(cz) dz = (-1)^v 2^{-u} e^{(p+c)vz} \left( \frac{u}{2} \right) \left( \frac{v}{2} \right) n! (1 - u \bmod 2) (1 - v \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j (p+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+c v}{2c}, \dots, \frac{p+c v}{2c}, v; \frac{p+c v}{2c} + 1, \dots, \frac{p+c v}{2c} + 1; e^{2cz} \right) + (-1)^v 2^{-u} \left( \frac{v}{2} \right)$$

$$n! (1 - v \bmod 2) \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{s} \left( e^{(p-b(u-2s)+cv)z} \sum_{j=0}^n \frac{(-1)^j (p-b(u-2s)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-b(u-2s)+cv}{2c}, \right.$$

$$\dots, \frac{p-b(u-2s)+cv}{2c}, v; \frac{p-b(u-2s)+cv}{2c} + 1, \dots, \frac{p-b(u-2s)+cv}{2c} + 1; e^{2cz} \right) +$$

$$e^{(p+b(u-2s)+cv)z} \sum_{j=0}^n \frac{(-1)^j (p+b(u-2s)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+b(u-2s)+cv}{2c}, \dots, \right.$$

$$\left. \frac{p+b(u-2s)+cv}{2c}, v; \frac{p+b(u-2s)+cv}{2c} + 1, \dots, \frac{p+b(u-2s)+cv}{2c} + 1; e^{2cz} \right) \right) +$$

$$(-1)^v 2^{-u} \left( \frac{u}{2} \right) n! (1 - u \bmod 2) \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( e^{(2ci+p)z} \sum_{j=0}^n \frac{(-1)^j (2ci+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2ci+p}{2c}, \dots, \frac{2ci+p}{2c}, \right.$$

$$\left. v; \frac{2ci+p}{2c} + 1, \dots, \frac{2ci+p}{2c} + 1; e^{2cz} \right) + e^{(-2ci+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-2ci+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\begin{aligned}
 & \left( \frac{-2ci+p+2cv}{2c}, \dots, \frac{-2ci+p+2cv}{2c}, v; \frac{-2ci+p+2cv}{2c} + 1, \dots, \frac{-2ci+p+2cv}{2c} + 1; e^{2cz} \right) + \\
 & (-1)^v 2^{-u} n! \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{s} \binom{v}{i} \left( e^{(2ci+p+2bs-bu)z} \sum_{j=0}^n \frac{(-1)^j (2ci+p+2bs-bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2ci+p+2bs-bu}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{2ci+p+2bs-bu}{2c}, v; \frac{2ci+p+2bs-bu}{2c} + 1, \dots, \frac{2ci+p+2bs-bu}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(2ci+p-2bs+bu)z} \sum_{j=0}^n \frac{(-1)^j (2ci+p-2bs+bu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2ci+p-2bs+bu}{2c}, \dots, \right. \\
 & \quad \left. \frac{2ci+p-2bs+bu}{2c}, v; \frac{2ci+p-2bs+bu}{2c} + 1, \dots, \frac{2ci+p-2bs+bu}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(-2ci+p+2bs-bu+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-2ci+p+2bs-bu+2cv)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{-2ci+p+2bs-bu+2cv}{2c}, \dots, \frac{-2ci+p+2bs-bu+2cv}{2c}, v; \right. \\
 & \quad \left. \frac{-2ci+p+2bs-bu+2cv}{2c} + 1, \dots, \frac{-2ci+p+2bs-bu+2cv}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(-2ci+p-2bs+bu+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-2ci+p-2bs+bu+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \quad \left( \frac{-2ci+p-2bs+bu+2cv}{2c}, \dots, \frac{-2ci+p-2bs+bu+2cv}{2c}, v; \frac{-2ci+p-2bs+bu+2cv}{2c} + \right. \\
 & \quad \left. 1, \dots, \frac{-2ci+p-2bs+bu+2cv}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N} \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$

01.22.21.0460.01

$$\int z^n e^{pz} \cosh^u(cz) \coth^v(cz) dz = (-1)^v 2^{-u} e^{(p+c)vz} \left( \frac{u+v}{2} \right) n! (1 - (u+v) \bmod 2)$$

$$\begin{aligned}
 & \sum_{j=0}^n \frac{(-1)^j (p+c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{cv+p}{2c}, \dots, \frac{cv+p}{2c}, v; \frac{cv+p}{2c} + 1, \dots, \frac{cv+p}{2c} + 1; e^{2cz} \right) + \\
 & (-1)^v 2^{-u} e^{cvz} n! \sum_{s=0}^{\lfloor \frac{u+v-1}{2} \rfloor} \binom{u+v}{s} \left( e^{(p-c(-2s+u+v))z} \sum_{j=0}^n \frac{(-1)^j (p-c(u-2s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \quad \left( \frac{p-c(u-2s)}{2c}, \dots, \frac{p-c(u-2s)}{2c}, v; \frac{p-c(u-2s)}{2c} + 1, \dots, \frac{p-c(u-2s)}{2c} + 1; e^{2cz} \right) + e^{(p+c(-2s+u+v))z} \\
 & \quad \sum_{j=0}^n \frac{(-1)^j (p+c(-2s+u+2v))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+c(-2s+u+2v)}{2c}, \dots, \frac{p+c(-2s+u+2v)}{2c}, \right. \\
 & \quad \left. v; \frac{p+c(-2s+u+2v)}{2c} + 1, \dots, \frac{p+c(-2s+u+2v)}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N} \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$

**Involving functions of the direct function, hyperbolic, exponential and trigonometric functions**

**Involving powers of the direct function, hyperbolic, exponential and trigonometric functions**

Involving sin, sinh and exp

**Involving  $e^{pz} \sin(az) \sinh(bz) \coth^v(cz)$**

01.22.21.0461.01

$$\int e^{pz} \sin(az) \sinh(bz) \coth^v(cz) dz =$$

$$\frac{1}{4} i (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \left( -\frac{e^{(-b-ia+p)z} F_1\left(-\frac{-b-ia+p}{2c}; -v, v; 1 - \frac{-b-ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{-b-ia+p} + \frac{e^{(-b+ia+p)z} F_1\left(-\frac{-b+ia+p}{2c}; -v, v; 1 - \frac{-b+ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{-b+ia+p} + \frac{e^{(b-ia+p)z} F_1\left(-\frac{b-ia+p}{2c}; -v, v; 1 - \frac{b-ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b-ia+p} - \frac{e^{(b+ia+p)z} F_1\left(-\frac{b+ia+p}{2c}; -v, v; 1 - \frac{b+ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{b+ia+p} \right)$$

Involving powers of sin, powers of sinh and exp

**Involving  $e^{pz} \sin^m(az) \sinh^u(bz) \coth^v(cz)$**

01.22.21.0462.01

$$\int e^{pz} \sin^m(az) \sinh^u(bz) \coth^v(cz) dz =$$

$$\frac{1}{p} \left( i^u 2^{-m-u} e^{pz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} F_1 \left( -\frac{p}{2c}; -v, v; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz} \right) \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \right)$$

$$\coth^v(cz) (1 - m \bmod 2) (1 - u \bmod 2) - i^u 2^{-m-u} \binom{u}{\frac{u}{2}} (u \bmod 2 - 1) (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v}$$

$$\coth^v(cz) \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^s \binom{m}{s} \left( \frac{e^{(p+ai(m-2s))z - \frac{im\pi}{2}} F_1 \left( -\frac{p+ai(m-2s)}{2c}; -v, v; 1 - \frac{p+ai(m-2s)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p + ai(m-2s)} + \right.$$

$$\left. \frac{e^{\frac{i\pi m}{2} + (p-ia(m-2s))z} F_1 \left( -\frac{p-ia(m-2s)}{2c}; -v, v; 1 - \frac{p-ia(m-2s)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p - ia(m-2s)} \right) + 2^{-m-u} \binom{m}{\frac{m}{2}} (1 - m \bmod 2)$$

$$(1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( \frac{e^{(p+b(u-2k))z} F_1 \left( -\frac{p+b(u-2k)}{2c}; -v, v; 1 - \frac{p+b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p + b(u-2k)} + \right.$$

$$\left. \frac{(-1)^u e^{(p-b(u-2k))z} F_1 \left( -\frac{p-b(u-2k)}{2c}; -v, v; 1 - \frac{p-b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p - b(u-2k)} \right) +$$

$$2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{k+s} \binom{m}{s} \binom{u}{k}$$

$$\left( \left( e^{(p+ai(m-2s)+b(u-2k))z - \frac{im\pi}{2}} F_1 \left( -\frac{p+ai(m-2s)+b(u-2k)}{2c}; -v, v; 1 - \frac{p+ai(m-2s)+b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) / (p + ai(m-2s) + b(u-2k)) + \left( e^{\frac{i\pi m}{2} + (p-ia(m-2s)+b(u-2k))z} \right.$$

$$\left. F_1 \left( -\frac{p-ia(m-2s)+b(u-2k)}{2c}; -v, v; 1 - \frac{p-ia(m-2s)+b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) /$$

$$(p - ia(m-2s) + b(u-2k)) + \left( (-1)^u e^{(p+ai(m-2s)-b(u-2k))z - \frac{im\pi}{2}} F_1 \left( -\frac{p+ai(m-2s)-b(u-2k)}{2c}; -v, v; 1 - \frac{p+ai(m-2s)-b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) / (p + ai(m-2s) - b(u-2k)) +$$

$$\left( (-1)^u e^{\frac{i\pi m}{2} + (p-ia(m-2s)-b(u-2k))z} F_1 \left( -\frac{p-ia(m-2s)-b(u-2k)}{2c}; -v, v; 1 - \frac{p-ia(m-2s)-b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) / (p - ia(m-2s) - b(u-2k)) ; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.22.21.0463.01

$$\int e^{pz} \sin^m(az) \sinh^\mu(cz) \coth^v(cz) dz =$$

$$2^{-m} (1 - e^{2cz})^{v-\mu} (1 + e^{2cz})^{-v} \coth^v(cz) \sinh^\mu(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \left( e^{\frac{i\pi m}{2} + (p-ia(m-2k))z} F_1 \left( \frac{-ia(m-2k) + p - c\mu}{2c}; \right. \right. \right.$$

$$\left. \left. v - \mu, -v; \frac{1}{2} \left( \frac{p - ia(m-2k)}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) \right) / (-ia(m-2k) + p - c\mu) +$$

$$\left( e^{(ai(m-2k)+p)z - \frac{i\pi m}{2}} F_1 \left( \frac{ai(m-2k) + p - c\mu}{2c}; v - \mu, -v; \frac{1}{2} \left( \frac{ai(m-2k) + p}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) \right) /$$

$$(ai(m-2k) + p - c\mu) +$$

$$\frac{1}{p - c\mu} 2^{-m} e^{pz} (1 - e^{2cz})^{v-\mu} (1 + e^{2cz})^{-v} F_1 \left( \frac{p - c\mu}{2c}; v - \mu, -v; \frac{1}{2} \left( \frac{p}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right)$$

$$\left( \frac{m}{2} \right) \coth^v(cz) (1 - m \bmod 2) \sinh^\mu(cz) /; m \in \mathbb{N}^+$$

Involving cos, sinh and exp

### Involving $e^{pz} \cos(az) \sinh(bz) \coth^v(cz)$

01.22.21.0464.01

$$\int e^{pz} \cos(az) \sinh(bz) \coth^v(cz) dz =$$

$$\frac{1}{4} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \left( - \frac{e^{(-b-ia+p)z} F_1 \left( -\frac{-b-ia+p}{2c}; -v, v; 1 - \frac{-b-ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{-b - ia + p} - \right.$$

$$\frac{e^{(-b+ia+p)z} F_1 \left( -\frac{-b+ia+p}{2c}; -v, v; 1 - \frac{-b+ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{-b + ia + p} +$$

$$\left. \frac{e^{(b-ia+p)z} F_1 \left( -\frac{b-ia+p}{2c}; -v, v; 1 - \frac{b-ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{b - ia + p} + \frac{e^{(b+ia+p)z} F_1 \left( -\frac{b+ia+p}{2c}; -v, v; 1 - \frac{b+ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{b + ia + p} \right)$$

Involving powers of cos, powers of sinh and exp

### Involving $e^{pz} \cos^m(az) \sinh^u(bz) \coth^v(cz)$

01.22.21.0465.01

$$\int e^{pz} \cos^m(az) \sinh^u(bz) \coth^v(cz) dz =$$

$$\frac{1}{p} \left( i^u 2^{-m-u} e^{pz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} F_1 \left( -\frac{p}{2c}; -v, v; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz} \right) \binom{m}{\frac{m}{2}} \right.$$

$$\left. \binom{u}{\frac{u}{2}} \coth^v(cz) (1 - m \bmod 2) (1 - u \bmod 2) \right) - i^u 2^{-m-u} \binom{u}{\frac{u}{2}} (u \bmod 2 - 1) (1 - e^{-2cz})^v$$

$$(1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{s} \left( \frac{e^{(p+ai(m-2s))z} F_1 \left( -\frac{p+ai(m-2s)}{2c}; -v, v; 1 - \frac{p+ai(m-2s)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p + ai(m-2s)} + \right.$$

$$\left. \frac{e^{(p-ia(m-2s))z} F_1 \left( -\frac{p-ia(m-2s)}{2c}; -v, v; 1 - \frac{p-ia(m-2s)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p - ia(m-2s)} \right) + 2^{-m-u} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) (1 - e^{-2cz})^v$$

$$(1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \left( \frac{e^{(p+b(u-2k))z} F_1 \left( -\frac{p+b(u-2k)}{2c}; -v, v; 1 - \frac{p+b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p + b(u-2k)} + \right.$$

$$\left. \frac{(-1)^u e^{(p-b(u-2k))z} F_1 \left( -\frac{p-b(u-2k)}{2c}; -v, v; 1 - \frac{p-b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p - b(u-2k)} \right) +$$

$$2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{s} \binom{u}{k}$$

$$\left( \left( e^{(p+ai(m-2s)+b(u-2k))z} F_1 \left( -\frac{p+ai(m-2s)+b(u-2k)}{2c}; -v, v; 1 - \frac{p+ai(m-2s)+b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) / (p + ai(m-2s) + b(u-2k)) + \left( e^{(p-ia(m-2s)+b(u-2k))z} \right. \right.$$

$$F_1 \left( -\frac{p-ia(m-2s)+b(u-2k)}{2c}; -v, v; 1 - \frac{p-ia(m-2s)+b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) \left. \right) /$$

$$(p - ia(m-2s) + b(u-2k)) + \left( (-1)^u e^{(p+ai(m-2s)-b(u-2k))z} F_1 \left( -\frac{p+ai(m-2s)-b(u-2k)}{2c}; -v, v; 1 - \frac{p+ai(m-2s)-b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) / (p + ai(m-2s) - b(u-2k)) +$$

$$\left( (-1)^u e^{(p-ia(m-2s)-b(u-2k))z} F_1 \left( -\frac{p-ia(m-2s)-b(u-2k)}{2c}; -v, v; 1 - \frac{p-ia(m-2s)-b(u-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) / (p - ia(m-2s) - b(u-2k)) \Bigg) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$



01.22.21.0466.01

$$\int e^{pz} \cos^m(az) \sinh^\mu(cz) \coth^\nu(cz) dz = 2^{-m} (1 - e^{2cz})^{\nu-\mu} (1 + e^{2cz})^{-\nu} \coth^\nu(cz) \sinh^\mu(cz)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(p-ia(m-2k))z} F_1 \left( \frac{-ia(m-2k)+p-c\mu}{2c}; \nu-\mu, -\nu; \frac{1}{2} \left( \frac{p-ia(m-2k)}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) \right) /$$

$$(-ia(m-2k)+p-c\mu) + \left( e^{(a i(m-2k)+p)z} F_1 \left( \frac{a i(m-2k)+p-c\mu}{2c}; \nu-\mu, -\nu; \right. \right.$$

$$\left. \left. \frac{1}{2} \left( \frac{a i(m-2k)+p}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) \right) / (a i(m-2k)+p-c\mu) +$$

$$\frac{1}{p-c\mu} 2^{-m} e^{pz} (1 - e^{2cz})^{\nu-\mu} (1 + e^{2cz})^{-\nu} F_1 \left( \frac{p-c\mu}{2c}; \nu-\mu, -\nu; \frac{1}{2} \left( \frac{p}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right)$$

$$\left( \frac{m}{2} \right) \coth^\nu(cz) (1 - m \bmod 2) \sinh^\mu(cz) /; m \in \mathbb{N}^+$$

Involving sin, cosh and exp

**Involving  $e^{pz} \sin(az) \cosh(bz) \coth^\nu(cz)$**

01.22.21.0467.01

$$\int e^{pz} \sin(az) \cosh(bz) \coth^\nu(cz) dz =$$

$$-\frac{1}{4} i (1 - e^{-2cz})^\nu (1 + e^{-2cz})^{-\nu} \coth^\nu(cz) \left( -\frac{e^{(-b-ia+p)z} F_1 \left( -\frac{-b-ia+p}{2c}; -\nu, \nu; 1 - \frac{-b-ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{-b-ia+p} + \right.$$

$$\frac{e^{(-b+ia+p)z} F_1 \left( -\frac{-b+ia+p}{2c}; -\nu, \nu; 1 - \frac{-b+ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{-b+ia+p} +$$

$$\left. \frac{e^{(b+ia+p)z} F_1 \left( -\frac{b+ia+p}{2c}; -\nu, \nu; 1 - \frac{b+ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{b+ia+p} - \frac{e^{(b-ia+p)z} F_1 \left( -\frac{b-ia+p}{2c}; -\nu, \nu; 1 - \frac{b-ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{b-ia+p} \right)$$

Involving powers of sin, powers of cosh and exp

**Involving  $e^{pz} \sin^m(az) \cosh^u(bz) \coth^\nu(cz)$**

01.22.21.0468.01

$$\begin{aligned}
 & \int e^{pz} \sin^m(az) \cosh^u(bz) \coth^v(cz) dz = \\
 & \frac{1}{p} \left( 2^{-m-u} e^{pz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} F_1 \left( -\frac{p}{2c}; -v, v; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz} \right) \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \coth^v(cz) \right. \\
 & \quad \left. (1 - m \bmod 2) (1 - u \bmod 2) \right) + 2^{-m-u} i^{-m} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \binom{u}{\frac{u}{2}} \coth^v(cz) (1 - u \bmod 2) \\
 & \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \frac{(-1)^m e^{(p-ia(m-2k))z} F_1 \left( -\frac{p-ia(m-2k)}{2c}; -v, v; 1 - \frac{p-ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p - ia(m-2k)} + \right. \\
 & \quad \left. \frac{e^{(ai(m-2k)+p)z} F_1 \left( -\frac{ai(m-2k)+p}{2c}; -v, v; 1 - \frac{ai(m-2k)+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{ai(m-2k) + p} \right) - 2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \\
 & \binom{m}{\frac{m}{2}} \coth^v(cz) (m \bmod 2 - 1) \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{s} \left( \frac{e^{(p+b(u-2s))z} F_1 \left( -\frac{p+b(u-2s)}{2c}; -v, v; 1 - \frac{p+b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p + b(u-2s)} + \right. \\
 & \quad \left. \frac{e^{(p-b(u-2s))z} F_1 \left( -\frac{p-b(u-2s)}{2c}; -v, v; 1 - \frac{p-b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz} \right)}{p - b(u-2s)} \right) + \\
 & i^{-m} 2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{m}{k} \binom{u}{s} \\
 & \left( \left( (-1)^m e^{(-ia(m-2k)+p+b(u-2s))z} F_1 \left( -\frac{-ia(m-2k) + p + b(u-2s)}{2c}; -v, v; 1 - \frac{-ia(m-2k) + p + b(u-2s)}{2c}; \right. \right. \right. \\
 & \quad \left. \left. \left. -e^{-2cz}, e^{-2cz} \right) \right) / (-ia(m-2k) + p + b(u-2s)) + \left( e^{(ai(m-2k)+p+b(u-2s))z} \right. \right. \\
 & \quad \left. \left. F_1 \left( -\frac{ai(m-2k) + p + b(u-2s)}{2c}; -v, v; 1 - \frac{ai(m-2k) + p + b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) / \right. \\
 & \quad \left. (ai(m-2k) + p + b(u-2s)) + \left( (-1)^m e^{(-ia(m-2k)+p-b(u-2s))z} F_1 \left( -\frac{-ia(m-2k) + p - b(u-2s)}{2c}; \right. \right. \right. \\
 & \quad \left. \left. \left. -v, v; 1 - \frac{-ia(m-2k) + p - b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz} \right) \right) / (-ia(m-2k) + p - b(u-2s)) + \right. \\
 & \quad \left. \left( e^{(ai(m-2k)+p-b(u-2s))z} F_1 \left( -\frac{ai(m-2k) + p - b(u-2s)}{2c}; -v, v; 1 - \frac{ai(m-2k) + p - b(u-2s)}{2c}; \right. \right. \right. \\
 & \quad \left. \left. \left. -e^{-2cz}, e^{-2cz} \right) \right) / (ai(m-2k) + p - b(u-2s)) \right) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+
 \end{aligned}$$

01.22.21.0469.01

$$\int e^{pz} \sin^m(az) \cosh^\mu(cz) \coth^\nu(cz) dz =$$

$$2^{-m} (1 - e^{2cz})^\nu (1 + e^{2cz})^{-\mu-\nu} \coth^\nu(cz) \cosh^\mu(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \left( e^{\frac{i\pi m}{2} + (p-ia(m-2k))z} F_1 \left( \frac{-ia(m-2k) + p - c\mu}{2c}; \right. \right. \right.$$

$$\left. \left. \nu, -\mu - \nu; \frac{1}{2} \left( \frac{p - ia(m-2k)}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) \right) / (-ia(m-2k) + p - c\mu) +$$

$$\left( e^{(ai(m-2k)+p)z - \frac{i\pi m}{2}} F_1 \left( \frac{ai(m-2k) + p - c\mu}{2c}; \nu, -\mu - \nu; \frac{1}{2} \left( \frac{ai(m-2k) + p}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) \right) /$$

$$(ai(m-2k) + p - c\mu) + \frac{1}{p - c\mu} 2^{-m} e^{pz} (1 - e^{2cz})^\nu (1 + e^{2cz})^{-\mu-\nu} \binom{m}{\frac{m}{2}}$$

$$\cosh^\mu(cz) \coth^\nu(cz) F_1 \left( \frac{p - c\mu}{2c}; \nu, -\mu - \nu; \frac{1}{2} \left( \frac{p}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) (1 - m \bmod 2) /; m \in \mathbb{N}^+$$

Involving cos, cosh and exp

**Involving  $e^{pz} \cos(az) \cosh(bz) \coth^\nu(cz)$**

01.22.21.0470.01

$$\int e^{pz} \cos(az) \cosh(bz) \coth^\nu(cz) dz =$$

$$\frac{1}{4} (1 - e^{-2cz})^\nu (1 + e^{-2cz})^{-\nu} \coth^\nu(cz) \left( \frac{e^{(-b-ia+p)z} F_1 \left( -\frac{-b-ia+p}{2c}; -\nu, \nu; 1 - \frac{-b-ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{-b - ia + p} + \right.$$

$$\frac{e^{(-b+ia+p)z} F_1 \left( -\frac{-b+ia+p}{2c}; -\nu, \nu; 1 - \frac{-b+ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{-b + ia + p} +$$

$$\left. \frac{e^{(b-ia+p)z} F_1 \left( -\frac{b-ia+p}{2c}; -\nu, \nu; 1 - \frac{b-ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{b - ia + p} + \frac{e^{(b+ia+p)z} F_1 \left( -\frac{b+ia+p}{2c}; -\nu, \nu; 1 - \frac{b+ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{b + ia + p} \right)$$

Involving powers of cos, powers of cosh and exp

**Involving  $e^{pz} \cos^m(az) \cosh^u(bz) \coth^\nu(cz)$**

01.22.21.0471.01

$$\int e^{pz} \cos^m(az) \cosh^u(bz) \coth^v(cz) dz = 2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \left(\frac{u}{2}\right) \coth^v(cz)$$

$$(1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \frac{e^{(p-ia(m-2k))z} F_1\left(-\frac{p-ia(m-2k)}{2c}; -v, v; 1 - \frac{p-ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p - ia(m-2k)} + \right.$$

$$\left. \frac{e^{(ai(m-2k)+p)z} F_1\left(-\frac{ai(m-2k)+p}{2c}; -v, v; 1 - \frac{ai(m-2k)+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ai(m-2k) + p} \right) + 2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v}$$

$$\left(\frac{m}{2}\right) \coth^v(cz) (1 - m \bmod 2) \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{s} \left( \frac{e^{(p+b(u-2s))z} F_1\left(-\frac{p+b(u-2s)}{2c}; -v, v; 1 - \frac{p+b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p + b(u-2s)} + \right.$$

$$\left. \frac{e^{(p-b(u-2s))z} F_1\left(-\frac{p-b(u-2s)}{2c}; -v, v; 1 - \frac{p-b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p - b(u-2s)} \right) +$$

$$2^{-m-u} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} \coth^v(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{m}{k} \binom{u}{s} \left( \left( e^{(-ia(m-2k)+p+b(u-2s))z} F_1\left(-\frac{-ia(m-2k) + p + b(u-2s)}{2c}; \right. \right. \right.$$

$$\left. \left. \left. -v, v; 1 - \frac{-ia(m-2k) + p + b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (-ia(m-2k) + p + b(u-2s)) + \right.$$

$$\left( e^{(ai(m-2k)+p+b(u-2s))z} F_1\left(-\frac{ai(m-2k) + p + b(u-2s)}{2c}; -v, v; 1 - \frac{ai(m-2k) + p + b(u-2s)}{2c}; \right. \right.$$

$$\left. \left. -e^{-2cz}, e^{-2cz}\right) \right) / (ai(m-2k) + p + b(u-2s)) + \left( e^{(-ia(m-2k)+p-b(u-2s))z} \right.$$

$$F_1\left(-\frac{-ia(m-2k) + p - b(u-2s)}{2c}; -v, v; 1 - \frac{-ia(m-2k) + p - b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \Big/$$

$$(-ia(m-2k) + p - b(u-2s)) + \left( e^{(ai(m-2k)+p-b(u-2s))z} F_1\left(-\frac{ai(m-2k) + p - b(u-2s)}{2c}; \right. \right.$$

$$\left. \left. -v, v; 1 - \frac{ai(m-2k) + p - b(u-2s)}{2c}; -e^{-2cz}, e^{-2cz}\right) \right) / (ai(m-2k) + p - b(u-2s)) \Big) +$$

$$\frac{1}{p} 2^{-m-u} e^{pz} (1 - e^{-2cz})^v (1 + e^{-2cz})^{-v} F_1\left(-\frac{p}{2c}; -v, v; 1 - \frac{p}{2c}; -e^{-2cz}, e^{-2cz}\right)$$

$$\left(\frac{m}{2}\right)$$

$$\left(\frac{u}{2}\right)$$

$$\coth^v(cz)$$

$$(1 - m \bmod 2)$$

$$(1 - u \bmod 2) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.22.21.0472.01

$$\int e^{pz} \cos^m(az) \cosh^\mu(cz) \coth^\nu(cz) dz = 2^{-m} (1 - e^{2cz})^\nu (1 + e^{2cz})^{-\mu-\nu} \coth^\nu(cz) \cosh^\mu(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(p-ia(m-2k))z} F_1 \left( \frac{-ia(m-2k)+p-c\mu}{2c}; \nu, -\mu-\nu; \frac{1}{2} \left( \frac{p-ia(m-2k)}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) \right) / \left( (-ia(m-2k)+p-c\mu) + \left( e^{(ai(m-2k)+p)z} F_1 \left( \frac{ai(m-2k)+p-c\mu}{2c}; \nu, -\mu-\nu; \frac{1}{2} \left( \frac{ai(m-2k)+p}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) \right) / (ai(m-2k)+p-c\mu) \right) + \frac{1}{p-c\mu} 2^{-m} e^{pz} (1 - e^{2cz})^\nu (1 + e^{2cz})^{-\mu-\nu} \left( \frac{m}{2} \right) \cosh^\mu(cz) \coth^\nu(cz) F_1 \left( \frac{p-c\mu}{2c}; \nu, -\mu-\nu; \frac{1}{2} \left( \frac{p}{c} - \mu + 2 \right); e^{2cz}, -e^{2cz} \right) (1 - m \bmod 2) /; m \in \mathbb{N}^+$$

Involving sin, tanh and exp

**Involving  $e^{pz} \sin(az) \tanh(cz) \coth^\nu(cz)$**

01.22.21.0473.01

$$\int e^{pz} \sin(az) \tanh(cz) \coth^\nu(cz) dz = \frac{1}{2} (1 - e^{-2cz})^\nu (1 + e^{-2cz})^{-\nu} \coth^\nu(cz) \left( \frac{e^{\frac{i\pi}{2} + (-ia+p)z} F_1 \left( -\frac{ia+p}{2c}; 1-\nu, \nu-1; 1 - \frac{ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{-ia+p} + \frac{e^{-\frac{1}{2}(i\pi+ia+p)z} F_1 \left( -\frac{ia+p}{2c}; 1-\nu, \nu-1; 1 - \frac{ia+p}{2c}; -e^{-2cz}, e^{-2cz} \right)}{ia+p} \right)$$

Involving powers of sin, powers of tanh and exp

**Involving  $e^{pz} \sin^m(az) \tanh^\mu(cz) \coth^\nu(cz)$**

01.22.21.0474.01

$$\int e^{pz} \sin^m(az) \tanh^\mu(cz) \coth^\nu(cz) dz = \frac{1}{p} 2^{-m} e^{pz} (1 - e^{-2cz})^{\nu-\mu} (1 + e^{-2cz})^{\mu-\nu} \left(\frac{m}{2}\right) \coth^\nu(cz) \tanh^\mu(cz)$$

$$F_1\left(-\frac{P}{2c}; \mu - \nu, \nu - \mu; 1 - \frac{P}{2c}; -e^{-2cz}, e^{-2cz}\right) (1 - m \bmod 2) + 2^{-m} (1 - e^{-2cz})^{\nu-\mu} (1 + e^{-2cz})^{\mu-\nu} \coth^\nu(cz)$$

$$\tanh^\mu(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( \frac{e^{\frac{i\pi m}{2} + (p-ia(m-2k))z} F_1\left(-\frac{p-ia(m-2k)}{2c}; \mu - \nu, \nu - \mu; 1 - \frac{p-ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p - ia(m-2k)} + \frac{e^{(ai(m-2k)+p)z - \frac{i\pi m}{2}} F_1\left(-\frac{ai(m-2k)+p}{2c}; \mu - \nu, \nu - \mu; 1 - \frac{ai(m-2k)+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ai(m-2k) + p} \right); m \in \mathbb{N}^+$$

Involving cos, tanh and exp

**Involving  $e^{pZ} \cos(az) \tanh(cz) \coth^\nu(cz)$**

01.22.21.0475.01

$$\int e^{pz} \cos(az) \tanh(cz) \coth^\nu(cz) dz = \frac{1}{2} (1 - e^{-2cz})^\nu (1 + e^{-2cz})^{-\nu} \coth^\nu(cz)$$

$$\left( \frac{e^{(-ia+p)z} F_1\left(-\frac{-ia+p}{2c}; 1 - \nu, \nu - 1; 1 - \frac{-ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{-ia + p} + \frac{e^{(ia+p)z} F_1\left(-\frac{ia+p}{2c}; 1 - \nu, \nu - 1; 1 - \frac{ia+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ia + p} \right)$$

Involving powers of cos, powers of tanh and exp

**Involving  $e^{pZ} \cos^m(az) \tanh^\mu(cz) \coth^\nu(cz)$**

01.22.21.0476.01

$$\int e^{pz} \cos^m(az) \tanh^\mu(cz) \coth^\nu(cz) dz = \frac{1}{p} 2^{-m} e^{pz} (1 - e^{-2cz})^{\nu-\mu} (1 + e^{-2cz})^{\mu-\nu} \left(\frac{m}{2}\right) \coth^\nu(cz) \tanh^\mu(cz)$$

$$F_1\left(-\frac{P}{2c}; \mu - \nu, \nu - \mu; 1 - \frac{P}{2c}; -e^{-2cz}, e^{-2cz}\right) (1 - m \bmod 2) + 2^{-m} (1 - e^{-2cz})^{\nu-\mu} (1 + e^{-2cz})^{\mu-\nu} \coth^\nu(cz)$$

$$\tanh^\mu(cz) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( \frac{e^{(p-ia(m-2k))z} F_1\left(-\frac{p-ia(m-2k)}{2c}; \mu - \nu, \nu - \mu; 1 - \frac{p-ia(m-2k)}{2c}; -e^{-2cz}, e^{-2cz}\right)}{p - ia(m-2k)} + \frac{e^{(ai(m-2k)+p)z} F_1\left(-\frac{ai(m-2k)+p}{2c}; \mu - \nu, \nu - \mu; 1 - \frac{ai(m-2k)+p}{2c}; -e^{-2cz}, e^{-2cz}\right)}{ai(m-2k) + p} \right); m \in \mathbb{N}^+$$

**Involving functions of the direct function, hyperbolic, trigonometric and a power functions**

**Involving powers of the direct function, hyperbolic, trigonometric and a power functions**

Involving sin, sinh and power

Involving  $z^n \sin(a z) \sinh(b z) \coth^v(c z)$

01.22.21.0477.01

$$\int z^n \sin(a z) \sinh(b z) \coth^v(c z) dz =$$

$$\frac{1}{4} i (-1)^v n! \left( -e^{(-b-ia+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-b-ia+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-b+cv}{2c}, \dots, \right.$$

$$\left. \frac{-ia-b+cv}{2c}, v; \frac{-ia-b+cv}{2c} + 1, \dots, \frac{-ia-b+cv}{2c} + 1; e^{2cz} \right) + e^{(-b+ia+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j (-b+ia+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b+cv}{2c}, \dots, \frac{ia-b+cv}{2c}, v; \frac{ia-b+cv}{2c} + 1, \dots, \right.$$

$$\left. \frac{ia-b+cv}{2c} + 1; e^{2cz} \right) + e^{(b-ia+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b-ia+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{-ia+b+cv}{2c}, \dots, \frac{-ia+b+cv}{2c}, v; \frac{-ia+b+cv}{2c} + 1, \dots, \frac{-ia+b+cv}{2c} + 1; e^{2cz} \right) -$$

$$e^{(b+ia+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b+ia+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1} \left( \frac{ia+b+cv}{2c}, \dots, \frac{ia+b+cv}{2c}, v; \frac{ia+b+cv}{2c} + 1, \dots, \frac{ia+b+cv}{2c} + 1; e^{2cz} \right) -$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b-ia+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-b+2cs}{2c}, \dots, \right.$$

$$\left. \frac{-ia-b+2cs}{2c}, v; \frac{-ia-b+2cs}{2c} + 1, \dots, \frac{-ia-b+2cs}{2c} + 1; e^{2cz} \right) +$$

$$e^{(-b-ia+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-b+2c(v-s)}{2c}, \dots, \right.$$

$$\left. \frac{-ia-b+2c(v-s)}{2c}, v; \frac{-ia-b+2c(v-s)}{2c} + 1, \dots, \frac{-ia-b+2c(v-s)}{2c} + 1; e^{2cz} \right) \Bigg) +$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b+ia+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b+2cs}{2c}, \dots, \frac{ia-b+2cs}{2c}, \right.$$

$$\left. v; \frac{ia-b+2cs}{2c} + 1, \dots, \frac{ia-b+2cs}{2c} + 1; e^{2cz} \right) +$$

$$e^{(-b+ia+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b+2c(v-s)}{2c}, \dots, \right.$$

$$\left. \begin{aligned} & \frac{ia-b+2c(v-s)}{2c}, v; \frac{ia-b+2c(v-s)}{2c} + 1, \dots, \frac{ia-b+2c(v-s)}{2c} + 1; e^{2cz} \right) + \\ & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(b-ia+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b-ia+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia+b+2cs}{2c}, \dots, \frac{-ia+b+2cs}{2c}, \right. \right. \\ & \quad \left. \left. v; \frac{-ia+b+2cs}{2c} + 1, \dots, \frac{-ia+b+2cs}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. e^{(b-ia+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b-ia+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia+b+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-ia+b+2c(v-s)}{2c}, v; \frac{-ia+b+2c(v-s)}{2c} + 1, \dots, \frac{-ia+b+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) - \\ & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(b+ia+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b+ia+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b+2cs}{2c}, \dots, \frac{ia+b+2cs}{2c}, \right. \right. \\ & \quad \left. \left. v; \frac{ia+b+2cs}{2c} + 1, \dots, \frac{ia+b+2cs}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. e^{(b+ia+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b+ia+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{ia+b+2c(v-s)}{2c}, v; \frac{ia+b+2c(v-s)}{2c} + 1, \dots, \frac{ia+b+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+ \end{aligned}$$

Involving powers of sin, powers of sinh and power

### Involving $z^n \sin^m(az) \sinh^u(bz) \coth^v(cz)$

01.22.21.0478.01

$$\int z^n \sin^m(az) \sinh^u(bz) \coth^v(cz) dz = (-1)^v n! i^u 2^{-m-u} \binom{u}{\frac{u}{2}} \binom{m}{\frac{m}{2}} (1-u \bmod 2) \\ (1-m \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j (2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + \right. \\ \left. e^{2cvz} v \sum_{j=0}^n \frac{(-1)^j (2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) + \right. \\ \left. e^{cvz} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2} + 1, \dots, \frac{v}{2} + 1; e^{2cz}\right) \right)$$



$$\begin{aligned}
 & \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + \right. \\
 & \left. e^{2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \right) + \\
 & (-1)^v i^u 2^{-m-u} \left( \frac{u}{2} \right) n! (1-u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{\frac{im\pi}{2}} \left( e^{(cv-ia(m-2k))z} \left( \frac{v}{2} \right) (1-v \bmod 2) \right. \right. \\
 & \left. \sum_{j=0}^n \frac{(-1)^j (cv-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{cv-ia(m-2k)}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{cv-ia(m-2k)}{2c}, v; \frac{cv-ia(m-2k)}{2c} + 1, \dots, \frac{cv-ia(m-2k)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2cs-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-ia(m-2k)}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{2cs-ia(m-2k)}{2c}, v; \frac{2cs-ia(m-2k)}{2c} + 1, \dots, \frac{2cs-ia(m-2k)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(2c(v-s)-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{2c(v-s)-ia(m-2k)}{2c}, \dots, \frac{2c(v-s)-ia(m-2k)}{2c}, v; \right. \right. \\
 & \left. \left. \frac{2c(v-s)-ia(m-2k)}{2c} + 1, \dots, \frac{2c(v-s)-ia(m-2k)}{2c} + 1; e^{2cz} \right) \right) \Bigg) + \\
 & e^{-\frac{1}{2}im\pi} \left( e^{(ai(m-2k)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+cv}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{ia(m-2k)+cv}{2c}, v; \frac{ia(m-2k)+cv}{2c} + 1, \dots, \frac{ia(m-2k)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+2cs)z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+2cs}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{ia(m-2k)+2cs}{2c}, v; \frac{ia(m-2k)+2cs}{2c} + 1, \dots, \frac{ia(m-2k)+2cs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(ai(m-2k)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{ia(m-2k)+2c(v-s)}{2c}, \dots, \frac{ia(m-2k)+2c(v-s)}{2c}, v; \frac{ia(m-2k)+2c(v-s)}{2c} + 1, \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \dots, \frac{ia(m-2k) + 2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \right) + (-1)^v i^u 2^{-m-u} \left(\frac{m}{2}\right) n! \\
 (1-m \bmod 2) & \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^i \binom{u}{i} \left( e^{\frac{i\pi u}{2}} \left( e^{(cv-b(u-2i))z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \left. \left. {}_{j+2}F_{j+1} \left( \frac{cv-b(u-2i)}{2c}, \dots, \frac{cv-b(u-2i)}{2c}, v; \frac{cv-b(u-2i)}{2c} + 1, \dots, \frac{cv-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \left. \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-b(u-2i)}{2c}, \dots, \right. \right. \right. \\
 & \left. \left. \left. \frac{2cs-b(u-2i)}{2c}, v; \frac{2cs-b(u-2i)}{2c} + 1, \dots, \frac{2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \left. \left. e^{(2c(v-s)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c(v-s)-b(u-2i)}{2c}, \dots, \right. \right. \\
 & \left. \left. \left. \frac{2c(v-s)-b(u-2i)}{2c}, v; \frac{2c(v-s)-b(u-2i)}{2c} + 1, \dots, \frac{2c(v-s)-b(u-2i)}{2c} + 1; e^{2cz} \right) \right) \right) \right) + \\
 e^{-\frac{1}{2}i\pi u} & \left( e^{(b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2i)+cv}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{b(u-2i)+cv}{2c}, v; \frac{b(u-2i)+cv}{2c} + 1, \dots, \frac{b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs+b(u-2i)}{2c}, \dots, \right. \right. \right. \\
 & \left. \left. \left. \frac{2cs+b(u-2i)}{2c}, v; \frac{2cs+b(u-2i)}{2c} + 1, \dots, \frac{2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \left. \left. e^{(b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2i)+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \left. \left. \left. \frac{b(u-2i)+2c(v-s)}{2c}, v; \frac{b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & (-1)^v i^u 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{i+k} \binom{m}{k} \binom{u}{i} \left( e^{\frac{1}{2} i \pi (m+u)} \left( e^{(-i a (m-2k) - b (u-2i) + c v) z} \left( \frac{v}{2} \right) (1 - v \bmod 2) \right. \right. \\
 & \sum_{j=0}^n \frac{(-1)^j (-i a (m-2k) - b (u-2i) + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i a (m-2k) - b (u-2i) + c v}{2c}, \right. \\
 & \left. \dots, \frac{-i a (m-2k) - b (u-2i) + c v}{2c}, v; \frac{-i a (m-2k) - b (u-2i) + c v}{2c} + 1, \dots, \right. \\
 & \left. \frac{-i a (m-2k) - b (u-2i) + c v}{2c} + 1; e^{2cz} \right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-i a (m-2k) + 2cs - b (u-2i)) z} \right. \\
 & \sum_{j=0}^n \frac{(-1)^j (-i a (m-2k) + 2cs - b (u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i a (m-2k) + 2cs - b (u-2i)}{2c}, \right. \\
 & \left. \dots, \frac{-i a (m-2k) + 2cs - b (u-2i)}{2c}, v; \frac{-i a (m-2k) + 2cs - b (u-2i)}{2c} + 1, \dots, \right. \\
 & \left. \left. \dots, \frac{-i a (m-2k) + 2cs - b (u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(-i a (m-2k) - b (u-2i) + 2c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j (-i a (m-2k) - b (u-2i) + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \left. {}_{j+2}F_{j+1} \left( \frac{-i a (m-2k) - b (u-2i) + 2c(v-s)}{2c}, \dots, \right. \right. \\
 & \left. \frac{-i a (m-2k) - b (u-2i) + 2c(v-s)}{2c}, v; \frac{-i a (m-2k) - b (u-2i) + 2c(v-s)}{2c} + 1, \dots, \right. \\
 & \left. \left. \dots, \frac{-i a (m-2k) - b (u-2i) + 2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{\frac{1}{2} i \pi (u-m)} \left( e^{(i a (m-2k) - b (u-2i) + c v) z} \left( \frac{v}{2} \right) (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (i a (m-2k) - b (u-2i) + c v)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1} \left( \frac{i a (m-2k) - b (u-2i) + c v}{2c}, \dots, \frac{i a (m-2k) - b (u-2i) + c v}{2c}, v; \right. \\
 & \left. \frac{i a (m-2k) - b (u-2i) + c v}{2c} + 1, \dots, \frac{i a (m-2k) - b (u-2i) + c v}{2c} + 1; e^{2cz} \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(i a (m-2k) + 2cs - b (u-2i)) z} \sum_{j=0}^n \frac{(-1)^j (i a (m-2k) + 2cs - b (u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1} \left( \frac{i a (m-2k) + 2cs - b (u-2i)}{2c}, \dots, \frac{i a (m-2k) + 2cs - b (u-2i)}{2c}, v; \right. \\
 & \left. \frac{i a (m-2k) + 2cs - b (u-2i)}{2c} + 1, \dots, \frac{i a (m-2k) + 2cs - b (u-2i)}{2c} + 1; e^{2cz} \right) + \left. \right)
 \end{aligned}$$

$$\begin{aligned}
 & e^{(a i(m-2 k)-b(u-2 i)+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)-b(u-2 i)+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)-b(u-2 i)+2 c(v-s)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{i a(m-2 k)-b(u-2 i)+2 c(v-s)}{2 c}, v; \frac{i a(m-2 k)-b(u-2 i)+2 c(v-s)}{2 c} + 1, \right. \\
 & \quad \left. \dots, \frac{i a(m-2 k)-b(u-2 i)+2 c(v-s)}{2 c} + 1; e^{2 c z} \right) \Bigg) + \\
 & e^{\frac{1}{2} i \pi(m-u)} \left( e^{(-i a(m-2 k)+b(u-2 i)+c v) z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-i a(m-2 k)+b(u-2 i)+c v)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{-i a(m-2 k)+b(u-2 i)+c v}{2 c}, \dots, \frac{-i a(m-2 k)+b(u-2 i)+c v}{2 c}, v; \right. \\
 & \quad \left. \frac{-i a(m-2 k)+b(u-2 i)+c v}{2 c} + 1, \dots, \frac{-i a(m-2 k)+b(u-2 i)+c v}{2 c} + 1; e^{2 c z} \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-i a(m-2 k)+2 c s+b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2 k)+2 c s+b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{-i a(m-2 k)+2 c s+b(u-2 i)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{-i a(m-2 k)+2 c s+b(u-2 i)}{2 c}, v; \frac{-i a(m-2 k)+2 c s+b(u-2 i)}{2 c} + 1, \right. \\
 & \quad \left. \dots, \frac{-i a(m-2 k)+2 c s+b(u-2 i)}{2 c} + 1; e^{2 c z} \right) + \\
 & e^{(-i a(m-2 k)+b(u-2 i)+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2 k)+b(u-2 i)+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{-i a(m-2 k)+b(u-2 i)+2 c(v-s)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{-i a(m-2 k)+b(u-2 i)+2 c(v-s)}{2 c}, v; \frac{-i a(m-2 k)+b(u-2 i)+2 c(v-s)}{2 c} + 1, \right. \\
 & \quad \left. \dots, \frac{-i a(m-2 k)+b(u-2 i)+2 c(v-s)}{2 c} + 1; e^{2 c z} \right) \Bigg) + \\
 & e^{-\frac{1}{2} i \pi(m+u)} \left( e^{(a i(m-2 k)+b(u-2 i)+c v) z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)+b(u-2 i)+c v)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}, \dots, \frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}, v; \right.
 \end{aligned}$$

$$\begin{aligned} & \frac{ia(m-2k)+b(u-2i)+cv}{2c} + 1, \dots, \frac{ia(m-2k)+b(u-2i)+cv}{2c} + 1; e^{2cz} \Big) + \\ & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+2cs+b(u-2i)}{2c}, \dots, \frac{ia(m-2k)+2cs+b(u-2i)}{2c}, v; \right. \\ & \quad \left. \left. \frac{ia(m-2k)+2cs+b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)+2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. e^{(ai(m-2k)+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+b(u-2i)+2c(v-s)}{2c}, \dots, \frac{ia(m-2k)+b(u-2i)+2c(v-s)}{2c}, v; \right. \\ & \quad \left. \left. \frac{ia(m-2k)+b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{ia(m-2k)+b(u-2i)+2c(v-s)}{2c} + 1; \right. \right. \\ & \quad \left. \left. e^{2cz} \right) \right) \Big) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+ \end{aligned}$$

Involving cos, sinh and power

### Involving $z^n \cos(az) \sinh(bz) \coth^v(cz)$

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$$\int z^n \cos(az) \sinh(bz) \coth^v(cz) dz =$$

$$\begin{aligned} & \frac{1}{4} (-1)^{v-1} n! \left( e^{(-b-ia+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-b-ia+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad {}_{j+2}F_{j+1} \left( \frac{-ia-b+cv}{2c}, \dots, \right. \\ & \quad \left. \frac{-ia-b+cv}{2c}, v; \frac{-ia-b+cv}{2c} + 1, \dots, \frac{-ia-b+cv}{2c} + 1; e^{2cz} \right) + e^{(-b+ia+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \\ & \quad \sum_{j=0}^n \frac{(-1)^j (-b+ia+cv)^{-j-1} z^{n-j}}{(n-j)!} \left( \frac{ia-b+cv}{2c}, \dots, \frac{ia-b+cv}{2c}, v; \frac{ia-b+cv}{2c} + 1, \dots, \right. \\ & \quad \left. \frac{ia-b+cv}{2c} + 1; e^{2cz} \right) - e^{(b-ia+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b-ia+cv)^{-j-1} z^{n-j}}{(n-j)!} \\ & \quad {}_{j+2}F_{j+1} \left( \frac{-ia+b+cv}{2c}, \dots, \frac{-ia+b+cv}{2c}, v; \frac{-ia+b+cv}{2c} + 1, \dots, \frac{-ia+b+cv}{2c} + 1; e^{2cz} \right) - \\ & \quad \left. e^{(b+ia+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b+ia+cv)^{-j-1} z^{n-j}}{(n-j)!} \right) \end{aligned}$$

$$\begin{aligned}
 & {}_{j+2}F_{j+1}\left(\frac{ia+b+cv}{2c}, \dots, \frac{ia+b+cv}{2c}, v; \frac{ia+b+cv}{2c}+1, \dots, \frac{ia+b+cv}{2c}+1; e^{2cz}\right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b+ia+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{ia-b+2cs}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia-b+2cs}{2c}, v; \frac{ia-b+2cs}{2c}+1, \dots, \frac{ia-b+2cs}{2c}+1; e^{2cz}\right) + \right. \\
 & \quad \left. e^{(-b+ia+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{ia-b+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia-b+2c(v-s)}{2c}, v; \frac{ia-b+2c(v-s)}{2c}+1, \dots, \frac{ia-b+2c(v-s)}{2c}+1; e^{2cz}\right) \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b-ia+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-ia-b+2cs}{2c}, \dots, \frac{-ia-b+2cs}{2c}, \right. \right. \\
 & \quad \left. \left. v; \frac{-ia-b+2cs}{2c}+1, \dots, \frac{-ia-b+2cs}{2c}+1; e^{2cz}\right) + \right. \\
 & \quad \left. e^{(-b-ia+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-ia-b+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{-ia-b+2c(v-s)}{2c}, v; \frac{-ia-b+2c(v-s)}{2c}+1, \dots, \frac{-ia-b+2c(v-s)}{2c}+1; e^{2cz}\right) \right) - \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(b-ia+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b-ia+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-ia+b+2cs}{2c}, \dots, \frac{-ia+b+2cs}{2c}, \right. \right. \\
 & \quad \left. \left. v; \frac{-ia+b+2cs}{2c}+1, \dots, \frac{-ia+b+2cs}{2c}+1; e^{2cz}\right) + \right. \\
 & \quad \left. e^{(b-ia+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b-ia+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-ia+b+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{-ia+b+2c(v-s)}{2c}, v; \frac{-ia+b+2c(v-s)}{2c}+1, \dots, \frac{-ia+b+2c(v-s)}{2c}+1; e^{2cz}\right) \right) - \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(b+ia+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b+ia+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{ia+b+2cs}{2c}, \dots, \frac{ia+b+2cs}{2c}, \right. \right. \\
 & \quad \left. \left. v; \frac{ia+b+2cs}{2c}+1, \dots, \frac{ia+b+2cs}{2c}+1; e^{2cz}\right) + \right. \\
 & \quad \left. e^{(b+ia+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b+ia+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{ia+b+2c(v-s)}{2c}, \dots, \frac{ia+b+2c(v-s)}{2c}, \right. \right. \\
 & \quad \left. \left. v; \frac{ia+b+2c(v-s)}{2c}+1, \dots, \frac{ia+b+2c(v-s)}{2c}+1; e^{2cz}\right) \right) \Bigg| \Bigg| : n \in \mathbb{N} \wedge v \in \mathbb{Z} \wedge v > 0
 \end{aligned}$$

Involving powers of cos, powers of sinh and power

Involving  $z^n \cos^m(a z) \sinh^u(b z) \coth^v(c z)$

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$$\int z^n \cos^m(a z) \sinh^u(b z) \coth^v(c z) dz = (-1)^v n! i^u 2^{-m-u} \left(\frac{u}{2}\right) \left(\frac{m}{2}\right) (1 - u \bmod 2)$$

$$(1 - m \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j (2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + \right.$$

$$e^{2cz} v \sum_{j=0}^n \frac{(-1)^j (2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) +$$

$$e^{cvz} \left(\frac{v}{2}\right) (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \dots, \frac{v}{2}; \frac{v}{2} + 1, \dots, \frac{v}{2} + 1; e^{2cz}\right) +$$

$$\left. \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v-1}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + \right.$$

$$e^{2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \left. \right) + (-1)^v$$

$$i^u 2^{-m-u} \left(\frac{u}{2}\right) n! (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(cv-ia(m-2k))z} \left(\frac{v}{2}\right) (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv - ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right.$$

$$\left( \frac{cv - ia(m-2k)}{2c}, \dots, \frac{cv - ia(m-2k)}{2c}, v; \frac{cv - ia(m-2k)}{2c} + 1, \dots, \frac{cv - ia(m-2k)}{2c} + 1; e^{2cz} \right) +$$

$$e^{(ai(m-2k)+cv)z} \left(\frac{v}{2}\right) (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k) + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left( \frac{ia(m-2k) + cv}{2c}, \dots, \frac{ia(m-2k) + cv}{2c}, v; \frac{ia(m-2k) + cv}{2c} + 1, \dots, \frac{ia(m-2k) + cv}{2c} + 1; e^{2cz} \right) +$$

$$\left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v-1}{s} \left( e^{(2cs-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2cs - ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs - ia(m-2k)}{2c}, \dots, \right.$$

$$\frac{2cs - ia(m-2k)}{2c}, v; \frac{2cs - ia(m-2k)}{2c} + 1, \dots, \frac{2cs - ia(m-2k)}{2c} + 1; e^{2cz} \right) +$$

$$e^{(2c(v-s)-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s) - ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left( \frac{2c(v-s) - ia(m-2k)}{2c}, \dots, \frac{2c(v-s) - ia(m-2k)}{2c}, v; \right.$$

$$\begin{aligned}
 & \left. \frac{2c(v-s) - ia(m-2k)}{2c} + 1, \dots, \frac{2c(v-s) - ia(m-2k)}{2c} + 1; e^{2cz} \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+2cs)z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k) + 2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k) + 2cs}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{ia(m-2k) + 2cs}{2c}, v; \frac{ia(m-2k) + 2cs}{2c} + 1, \dots, \frac{ia(m-2k) + 2cs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(ai(m-2k)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k) + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \quad \left( \frac{ia(m-2k) + 2c(v-s)}{2c}, \dots, \frac{ia(m-2k) + 2c(v-s)}{2c}, v; \frac{ia(m-2k) + 2c(v-s)}{2c} + 1, \right. \\
 & \quad \left. \left. \dots, \frac{ia(m-2k) + 2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + (-1)^v i^u 2^{-m-u} \binom{m}{\frac{m}{2}} n! \\
 & (1 - m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^i \binom{u}{i} \left( e^{\frac{i\pi u}{2}} \left( e^{(cv-b(u-2i))z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{cv-b(u-2i)}{2c}, \dots, \frac{cv-b(u-2i)}{2c}, v; \frac{cv-b(u-2i)}{2c} + 1, \dots, \frac{cv-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \quad \left. \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-b(u-2i)}{2c}, \dots, \right. \right. \right. \\
 & \quad \left. \left. \frac{2cs-b(u-2i)}{2c}, v; \frac{2cs-b(u-2i)}{2c} + 1, \dots, \frac{2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(2c(v-s)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \quad \left( \frac{2c(v-s)-b(u-2i)}{2c}, \dots, \frac{2c(v-s)-b(u-2i)}{2c}, v; \right. \\
 & \quad \left. \left. \frac{2c(v-s)-b(u-2i)}{2c} + 1, \dots, \frac{2c(v-s)-b(u-2i)}{2c} + 1; e^{2cz} \right) \right) \right) + \\
 & e^{-\frac{1}{2}i\pi u} \left( e^{(b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b(u-2i) + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2i) + cv}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{b(u-2i) + cv}{2c}, v; \frac{b(u-2i) + cv}{2c} + 1, \dots, \frac{b(u-2i) + cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs+b(u-2i)}{2c}, \dots, \right. \right. \right.
 \end{aligned}$$



$$\begin{aligned}
 & \left. \left. \left. \frac{2cs+b(u-2i)}{2c}, v; \frac{2cs+b(u-2i)}{2c} + 1, \dots, \frac{2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & e^{(b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left( \frac{b(u-2i)+2c(v-s)}{2c}, \dots, \frac{b(u-2i)+2c(v-s)}{2c}, v; \right. \\
 & \left. \left. \left. \frac{b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \right) + \\
 & (-1)^v i^u 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^i \binom{m}{k} \binom{u}{i} \left( e^{\frac{i\pi u}{2}} \left( e^{(-ia(m-2k)-b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \right. \right. \right. \\
 & \left. \left. \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)-b(u-2i)+cv}{2c}, \right. \right. \right. \\
 & \left. \left. \left. \dots, \frac{-ia(m-2k)-b(u-2i)+cv}{2c}, v; \frac{-ia(m-2k)-b(u-2i)+cv}{2c} + 1, \dots, \right. \right. \right. \\
 & \left. \left. \left. \frac{-ia(m-2k)-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+2cs-b(u-2i))z} \right. \right. \right. \\
 & \left. \left. \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+2cs-b(u-2i)}{2c}, \right. \right. \right. \\
 & \left. \left. \left. \dots, \frac{-ia(m-2k)+2cs-b(u-2i)}{2c}, v; \frac{-ia(m-2k)+2cs-b(u-2i)}{2c} + 1, \right. \right. \right. \\
 & \left. \left. \left. \dots, \frac{-ia(m-2k)+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \left. \left. e^{(-ia(m-2k)-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \left. \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c}, \dots, \right. \right. \right. \\
 & \left. \left. \left. \frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c}, v; \frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c} + 1, \right. \right. \right. \\
 & \left. \left. \left. \dots, \frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \right) + \\
 & e^{\frac{i\pi u}{2}} \left( e^{(ai(m-2k)-b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right.
 \end{aligned}$$

$$\begin{aligned}
 & {}_{j+2}F_{j+1}\left(\frac{ia(m-2k)-b(u-2i)+cv}{2c}, \dots, \frac{ia(m-2k)-b(u-2i)+cv}{2c}, v; \right. \\
 & \left. \frac{ia(m-2k)-b(u-2i)+cv}{2c} + 1, \dots, \frac{ia(m-2k)-b(u-2i)+cv}{2c} + 1; e^{2cz}\right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1}\left(\frac{ia(m-2k)+2cs-b(u-2i)}{2c}, \dots, \frac{ia(m-2k)+2cs-b(u-2i)}{2c}, v; \right. \\
 & \left. \frac{ia(m-2k)+2cs-b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)+2cs-b(u-2i)}{2c} + 1; e^{2cz}\right) + \\
 & e^{(ai(m-2k)-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1}\left(\frac{ia(m-2k)-b(u-2i)+2c(v-s)}{2c}, \dots, \right. \\
 & \left. \frac{ia(m-2k)-b(u-2i)+2c(v-s)}{2c}, v; \frac{ia(m-2k)-b(u-2i)+2c(v-s)}{2c} + 1, \right. \\
 & \left. \dots, \frac{ia(m-2k)-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz}\right) \Bigg) + \\
 & e^{-\frac{1}{2}i\pi u} \left( e^{(-ia(m-2k)+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1}\left(\frac{-ia(m-2k)+b(u-2i)+cv}{2c}, \dots, \frac{-ia(m-2k)+b(u-2i)+cv}{2c}, v; \right. \\
 & \left. \frac{-ia(m-2k)+b(u-2i)+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+b(u-2i)+cv}{2c} + 1; e^{2cz}\right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1}\left(\frac{-ia(m-2k)+2cs+b(u-2i)}{2c}, \dots, \right. \\
 & \left. \frac{-ia(m-2k)+2cs+b(u-2i)}{2c}, v; \frac{-ia(m-2k)+2cs+b(u-2i)}{2c} + 1, \right. \\
 & \left. \dots, \frac{-ia(m-2k)+2cs+b(u-2i)}{2c} + 1; e^{2cz}\right) + \\
 & e^{(-ia(m-2k)+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1}\left(\frac{-ia(m-2k)+b(u-2i)+2c(v-s)}{2c}, \dots, \right.
 \end{aligned}$$





$$e^{-\frac{1}{2}(i\pi+(b-2ci+ia+2cv)z)} \sum_{j=0}^n \frac{(-1)^j (b-2ci+ia+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b-2ci+2cv}{2c}, \dots, \frac{ia+b-2ci+2cv}{2c}, v; \frac{ia+b-2ci+2cv}{2c} + 1, \dots, \frac{ia+b-2ci+2cv}{2c} + 1; e^{2cz} \right); n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

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$$\int z^n \sin(az) \cosh(cz) \coth^v(cz) dz = \frac{1}{4} (-1)^v i e^{czv} \left( \frac{v+1}{2} \right) n! (1 - (v+1) \bmod 2) \left( e^{-iaz} \sum_{j=0}^n \frac{(-1)^j (-ia+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{cv-ia}{2c}, \dots, \frac{cv-ia}{2c}, v; \frac{cv-ia}{2c} + 1, \dots, \frac{cv-ia}{2c} + 1; e^{2cz} \right) - e^{iaz} \sum_{j=0}^n \frac{(-1)^j (ia+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+cv}{2c}, \dots, \frac{ia+cv}{2c}, v; \frac{ia+cv}{2c} + 1, \dots, \frac{ia+cv}{2c} + 1; e^{2cz} \right) \right) + \frac{1}{4} (-1)^v i e^{czv} n! \sum_{s=0}^{\lfloor \frac{v}{2} \rfloor} \binom{v+1}{s} \left( e^{(-ia-c(-2s+v+1))z} \sum_{j=0}^n \frac{(-1)^j (-ia-c(1-2s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-c(1-2s)}{2c}, \dots, \frac{-ia-c(1-2s)}{2c}, v; \frac{-ia-c(1-2s)}{2c} + 1, \dots, \frac{-ia-c(1-2s)}{2c} + 1; e^{2cz} \right) - e^{(ia-c(-2s+v+1))z} \sum_{j=0}^n \frac{(-1)^j (ia-c(1-2s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-c(1-2s)}{2c}, \dots, \frac{ia-c(1-2s)}{2c}, v; \frac{ia-c(1-2s)}{2c} + 1, \dots, \frac{ia-c(1-2s)}{2c} + 1; e^{2cz} \right) + e^{(-ia+c(-2s+v+1))z} \sum_{j=0}^n \frac{(-1)^j (-ia+c(-2s+2v+1))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c(-2s+2v+1)-ia}{2c}, \dots, \frac{c(-2s+2v+1)-ia}{2c}, v; \frac{c(-2s+2v+1)-ia}{2c} + 1, \dots, \frac{c(-2s+2v+1)-ia}{2c} + 1; e^{2cz} \right) - e^{(ia+c(-2s+v+1))z} \sum_{j=0}^n \frac{(-1)^j (ia+c(-2s+2v+1))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+c(-2s+2v+1)}{2c}, \dots, \frac{ia+c(-2s+2v+1)}{2c}, v; \frac{ia+c(-2s+2v+1)}{2c} + 1, \dots, \frac{ia+c(-2s+2v+1)}{2c} + 1; e^{2cz} \right) \right); n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of sin, powers of cosh and power

**Involving  $z^n \sin^m(az) \cosh^u(bz) \coth^v(cz)$**

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$$\int z^n \sin^m(a z) \cosh^u(b z) \coth^v(c z) dz = (-1)^v n! 2^{-m-u} \binom{u}{\frac{u}{2}} \binom{m}{\frac{m}{2}} (1-u \bmod 2)$$

$$(1-m \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j (2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + \right.$$

$$e^{2cz} \sum_{j=0}^n \frac{(-1)^j (2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) +$$

$$e^{cvz} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2}+1, \dots, \frac{v}{2}+1; e^{2cz}\right) +$$

$$\left. \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2csz} \sum_{j=0}^n \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \dots, s, v; s+1, \dots, s+1; e^{2cz}) + \right. \right.$$

$$\left. \left. e^{2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2cz}) \right) \right) +$$

$$(-1)^v 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1-u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{\frac{im\pi}{2}} \left( e^{(cv-ia(m-2k))z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \right. \right.$$

$$\sum_{j=0}^n \frac{(-1)^j (cv-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{cv-ia(m-2k)}{2c}, \dots, \right.$$

$$\left. \left. \frac{cv-ia(m-2k)}{2c}, v; \frac{cv-ia(m-2k)}{2c}+1, \dots, \frac{cv-ia(m-2k)}{2c}+1; e^{2cz}\right) + \right.$$

$$\left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2cs-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2cs-ia(m-2k)}{2c}, \right. \right.$$

$$\left. \left. \dots, \frac{2cs-ia(m-2k)}{2c}, v; \frac{2cs-ia(m-2k)}{2c}+1, \dots, \frac{2cs-ia(m-2k)}{2c}+1; e^{2cz}\right) + \right.$$

$$\left. e^{(2c(v-s)-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-ia(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right.$$

$$\left( \frac{2c(v-s)-ia(m-2k)}{2c}, \dots, \frac{2c(v-s)-ia(m-2k)}{2c}, v; \right.$$

$$\left. \left. \frac{2c(v-s)-ia(m-2k)}{2c}+1, \dots, \frac{2c(v-s)-ia(m-2k)}{2c}+1; e^{2cz}\right) \right) \right) +$$

$$e^{-\frac{1}{2}im\pi} \left( e^{(ai(m-2k)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{ia(m-2k)+cv}{2c}, \right. \right.$$

$$\begin{aligned}
 & \dots, \frac{ia(m-2k)+cv}{2c}, v; \frac{ia(m-2k)+cv}{2c} + 1, \dots, \frac{ia(m-2k)+cv}{2c} + 1; e^{2cz} \Big) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+2cs)z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+2cs}{2c}, \right. \right. \\
 & \quad \dots, \frac{ia(m-2k)+2cs}{2c}, v; \frac{ia(m-2k)+2cs}{2c} + 1, \dots, \left. \left. \frac{ia(m-2k)+2cs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(ai(m-2k)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \quad \left( \frac{ia(m-2k)+2c(v-s)}{2c}, \dots, \frac{ia(m-2k)+2c(v-s)}{2c}, v; \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+2c(v-s)}{2c} + 1, \dots, \frac{ia(m-2k)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Big) + \\
 & (-1)^v 2^{-m-u} \left( \frac{m}{2} \right) n! (1-m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{i} \left( e^{(cv-b(u-2i))z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{cv-b(u-2i)}{2c}, \dots, \frac{cv-b(u-2i)}{2c}, v; \frac{cv-b(u-2i)}{2c} + 1, \dots, \frac{cv-b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{b(u-2i)+cv}{2c}, \dots, \frac{b(u-2i)+cv}{2c}, v; \frac{b(u-2i)+cv}{2c} + 1, \dots, \frac{b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\
 & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-b(u-2i)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{2cs-b(u-2i)}{2c}, v; \frac{2cs-b(u-2i)}{2c} + 1, \dots, \frac{2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(2c(v-s)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c(v-s)-b(u-2i)}{2c}, \dots, \right. \\
 & \quad \left. \left. \frac{2c(v-s)-b(u-2i)}{2c}, v; \frac{2c(v-s)-b(u-2i)}{2c} + 1, \dots, \frac{2c(v-s)-b(u-2i)}{2c} + 1; e^{2cz} \right) \right) + \\
 & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs+b(u-2i)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{2cs+b(u-2i)}{2c}, v; \frac{2cs+b(u-2i)}{2c} + 1, \dots, \frac{2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \right.
 \end{aligned}$$

$$\begin{aligned}
 & e^{(b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2i)+2c(v-s)}{2c}, \dots, \right. \\
 & \left. \frac{b(u-2i)+2c(v-s)}{2c}, v; \frac{b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \Bigg) + \\
 & (-1)^v 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \binom{u}{i} \left( e^{\frac{i\pi m}{2}} \left( e^{(-ia(m-2k)-b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \right. \right. \\
 & \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)-b(u-2i)+cv}{2c}, \right. \\
 & \dots, \frac{-ia(m-2k)-b(u-2i)+cv}{2c}, v; \frac{-ia(m-2k)-b(u-2i)+cv}{2c} + 1, \dots, \\
 & \left. \frac{-ia(m-2k)-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+2cs-b(u-2i))z} \right. \\
 & \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+2cs-b(u-2i)}{2c}, \right. \\
 & \dots, \frac{-ia(m-2k)+2cs-b(u-2i)}{2c}, v; \frac{-ia(m-2k)+2cs-b(u-2i)}{2c} + 1, \\
 & \left. \frac{-ia(m-2k)+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-ia(m-2k)-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c}, \dots, \right. \\
 & \left. \frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c}, v; \frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c} + 1, \right. \\
 & \left. \dots, \frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \Bigg) + \\
 & e^{-\frac{1}{2}i\pi m} \left( e^{(ai(m-2k)-b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)-b(u-2i)+cv}{2c}, \dots, \frac{ia(m-2k)-b(u-2i)+cv}{2c}, v; \right. \\
 & \left. \frac{ia(m-2k)-b(u-2i)+cv}{2c} + 1, \dots, \frac{ia(m-2k)-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) +
 \end{aligned}$$





$$\begin{aligned}
 & \left. \left. \left. \dots, \frac{-i a(m-2 k)+b(u-2 i)+2 c(v-s)}{2 c}+1 ; e^{2 c z}\right)\right)\right) + \\
 & e^{-\frac{1}{2} i \pi m}\left(e^{(a i(m-2 k)+b(u-2 i)+c v) z}\left(\frac{v}{2}\right)(1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j(a i(m-2 k)+b(u-2 i)+c v)^{-j-1} z^{n-j}}{(n-j)!}\right. \\
 & \quad \left. {}_{j+2} F_{j+1}\left(\frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}, \dots, \frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}, v ;\right.\right. \\
 & \quad \left.\left. \frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}+1, \dots, \frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}+1 ; e^{2 c z}\right)+\right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2}\rfloor}\binom{v}{s}\left(e^{(a i(m-2 k)+2 c s+b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j(a i(m-2 k)+2 c s+b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!}\right.\right. \\
 & \quad \left. {}_{j+2} F_{j+1}\left(\frac{i a(m-2 k)+2 c s+b(u-2 i)}{2 c}, \dots, \frac{i a(m-2 k)+2 c s+b(u-2 i)}{2 c}, v ;\right.\right. \\
 & \quad \left.\left. \frac{i a(m-2 k)+2 c s+b(u-2 i)}{2 c}+1, \dots, \frac{i a(m-2 k)+2 c s+b(u-2 i)}{2 c}+1 ; e^{2 c z}\right)+\right. \\
 & \quad \left. e^{(a i(m-2 k)+b(u-2 i)+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j(a i(m-2 k)+b(u-2 i)+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!}\right. \\
 & \quad \left. {}_{j+2} F_{j+1}\left(\frac{i a(m-2 k)+b(u-2 i)+2 c(v-s)}{2 c}, \dots, \frac{i a(m-2 k)+b(u-2 i)+2 c(v-s)}{2 c}, v ;\right.\right. \\
 & \quad \left.\left. \frac{i a(m-2 k)+b(u-2 i)+2 c(v-s)}{2 c}+1, \dots, \frac{i a(m-2 k)+b(u-2 i)+2 c(v-s)}{2 c}+1 ;\right.\right. \\
 & \quad \left. \left. e^{2 c z}\right)\right)\right) ; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$

Involving cos, cosh and power

### Involving $z^n \cos(a z) \cosh(b z) \coth^v(c z)$

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$$\begin{aligned}
 & \int z^n \cos(a z) \cosh(b z) \coth^v(c z) dz = \\
 & \frac{1}{4}(-1)^v\left(\frac{v}{2}\right) n!(1-v \bmod 2)\left(e^{(-b-i a+c v) z} \sum_{j=0}^n \frac{(-1)^j(-b-i a+c v)^{-j-1} z^{n-j}}{(n-j)!}\right. \\
 & \quad \left. {}_{j+2} F_{j+1}\left(\frac{-i a-b+c v}{2 c}, \dots, \frac{-i a-b+c v}{2 c}, v ;\right.\right. \\
 & \quad \left.\left. \frac{-i a-b+c v}{2 c}+1, \dots, \frac{-i a-b+c v}{2 c}+1 ; e^{2 c z}\right)+e^{(-b+i a+c v) z} \sum_{j=0}^n \frac{(-1)^j(-b+i a+c v)^{-j-1} z^{n-j}}{(n-j)!}\right. \\
 & \quad \left. {}_{j+2} F_{j+1}\left(\frac{i a-b+c v}{2 c}, \dots, \frac{i a-b+c v}{2 c}, v ;\right.\right. \\
 & \quad \left.\left. \frac{i a-b+c v}{2 c}+1, \dots, \frac{i a-b+c v}{2 c}+1 ; e^{2 c z}\right)+\right.
 \end{aligned}$$

$$\begin{aligned}
 & e^{(b+ia+cv)z} \sum_{j=0}^n \frac{(-1)^j (b+ia+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b+cv}{2c}, \dots, \frac{ia+b+cv}{2c}, v; \frac{ia+b+cv}{2c} + 1, \right. \\
 & \quad \left. \dots, \frac{ia+b+cv}{2c} + 1; e^{2cz} \right) + e^{(b-ia+cv)z} \sum_{j=0}^n \frac{(-1)^j (b-ia+cv)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{-ia+b+cv}{2c}, \dots, \frac{-ia+b+cv}{2c}, v; \frac{-ia+b+cv}{2c} + 1, \dots, \frac{-ia+b+cv}{2c} + 1; e^{2cz} \right) + \\
 & \frac{1}{4} (-1)^v n! \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( e^{(-b+2ci-ia)z} \sum_{j=0}^n \frac{(-1)^j (-b+2ci-ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-b+2ci}{2c}, \dots, \frac{-ia-b+2ci}{2c}, \right. \right. \\
 & \quad \left. \left. v; \frac{-ia-b+2ci}{2c} + 1, \dots, \frac{-ia-b+2ci}{2c} + 1; e^{2cz} \right) + e^{(b+2ci-ia)z} \sum_{j=0}^n \frac{(-1)^j (b+2ci-ia)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia+b+2ci}{2c}, \dots, \frac{-ia+b+2ci}{2c}, v; \frac{-ia+b+2ci}{2c} + 1, \dots, \frac{-ia+b+2ci}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(b+2ci+ia)z} \sum_{j=0}^n \frac{(-1)^j (b+2ci+ia)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b+2ci}{2c}, \dots, \frac{ia+b+2ci}{2c}, v; \right. \\
 & \quad \left. \frac{ia+b+2ci}{2c} + 1, \dots, \frac{ia+b+2ci}{2c} + 1; e^{2cz} \right) + e^{(-b+2ci+ia)z} \sum_{j=0}^n \frac{(-1)^j (-b+2ci+ia)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{ia+2ci-b}{2c}, \dots, \frac{ia+2ci-b}{2c}, v; \frac{ia+2ci-b}{2c} + 1, \dots, \frac{ia+2ci-b}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(-b-2ci-ia+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci-ia+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-b-2ci+2cv}{2c}, \dots, \right. \\
 & \quad \left. \frac{-ia-b-2ci+2cv}{2c}, v; \frac{-ia-b-2ci+2cv}{2c} + 1, \dots, \frac{-ia-b-2ci+2cv}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(-b-2ci+ia+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci+ia+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b-2ci+2cv}{2c}, \dots, \right. \\
 & \quad \left. \frac{ia-b-2ci+2cv}{2c}, v; \frac{ia-b-2ci+2cv}{2c} + 1, \dots, \frac{ia-b-2ci+2cv}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(b-2ci-ia+2cv)z} \sum_{j=0}^n \frac{(-1)^j (b-2ci-ia+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia+b-2ci+2cv}{2c}, \dots, \right. \\
 & \quad \left. \frac{-ia+b-2ci+2cv}{2c}, v; \frac{-ia+b-2ci+2cv}{2c} + 1, \dots, \frac{-ia+b-2ci+2cv}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(b-2ci+ia+2cv)z} \sum_{j=0}^n \frac{(-1)^j (b-2ci+ia+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b-2ci+2cv}{2c}, \dots, \frac{ia+b-2ci+2cv}{2c}, \right. \\
 & \quad \left. v; \frac{ia+b-2ci+2cv}{2c} + 1, \dots, \frac{ia+b-2ci+2cv}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{Z} \wedge v > 0
 \end{aligned}$$

01.22.21.0485.01

$$\int z^n \cos(az) \cosh(cz) \coth^v(cz) dz = \frac{1}{4} (-1)^v e^{czv} \binom{v+1}{\frac{v+1}{2}} n! (1 - (v+1) \bmod 2)$$

$$\left( e^{-iaz} \sum_{j=0}^n \frac{(-1)^j (-ia + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{cv-ia}{2c}, \dots, \frac{cv-ia}{2c}, v; \frac{cv-ia}{2c} + 1, \dots, \frac{cv-ia}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{iaz} \sum_{j=0}^n \frac{(-1)^j (ia + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+cv}{2c}, \dots, \frac{ia+cv}{2c}, v; \frac{ia+cv}{2c} + 1, \dots, \frac{ia+cv}{2c} + 1; e^{2cz} \right) \right)$$

$$\frac{1}{4} (-1)^v e^{czv} n! \sum_{s=0}^{\lfloor \frac{v}{2} \rfloor} \binom{v+1}{s} \left( e^{(-ia-c(-2s+v+1))z} \sum_{j=0}^n \frac{(-1)^j (-ia-c(1-2s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right.$$

$$\left( \frac{-ia-c(1-2s)}{2c}, \dots, \frac{-ia-c(1-2s)}{2c}, v; \frac{-ia-c(1-2s)}{2c} + 1, \dots, \frac{-ia-c(1-2s)}{2c} + 1; e^{2cz} \right) +$$

$$e^{(ia-c(-2s+v+1))z} \sum_{j=0}^n \frac{(-1)^j (ia-c(1-2s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-c(1-2s)}{2c}, \dots, \frac{ia-c(1-2s)}{2c}, \right.$$

$$\left. v; \frac{ia-c(1-2s)}{2c} + 1, \dots, \frac{ia-c(1-2s)}{2c} + 1; e^{2cz} \right) + e^{(-ia+c(-2s+v+1))z}$$

$$\sum_{j=0}^n \frac{(-1)^j (-ia+c(-2s+2v+1))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c(-2s+2v+1)-ia}{2c}, \dots, \frac{c(-2s+2v+1)-ia}{2c}, \right.$$

$$\left. v; \frac{c(-2s+2v+1)-ia}{2c} + 1, \dots, \frac{c(-2s+2v+1)-ia}{2c} + 1; e^{2cz} \right) + e^{(ia+c(-2s+v+1))z}$$

$$\sum_{j=0}^n \frac{(-1)^j (ia+c(-2s+2v+1))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+c(-2s+2v+1)}{2c}, \dots, \frac{ia+c(-2s+2v+1)}{2c}, \right.$$

$$\left. v; \frac{ia+c(-2s+2v+1)}{2c} + 1, \dots, \frac{ia+c(-2s+2v+1)}{2c} + 1; e^{2cz} \right) \Big/; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of cos, powers of cosh and power

### Involving $z^n \cos^m(az) \cosh^u(bz) \coth^v(cz)$

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$$\int z^n \cos^m(az) \cosh^u(bz) \coth^v(cz) dz = (-1)^v 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)$$

$$(1 - u \bmod 2) \left( \frac{z^{n+1}}{(n+1)!} + e^{2cvz} \sum_{j=0}^n \frac{(-1)^j (2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(v, \dots, v; v+1, \dots, v+1; e^{2cz}) + \right.$$

$$\left. e^{2cvz} \sum_{j=0}^n \frac{(-1)^j (2c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \dots, 1, v+1; 2, \dots, 2; e^{2cz}) + \right)$$

$$\begin{aligned}
 & e^{c v z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{v}{2}, \dots, \frac{v}{2}, v; \frac{v}{2} + 1, \dots, \frac{v}{2} + 1; e^{2 c z} \right) + \\
 & \sum_{s=1}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{2 c s z} \sum_{j=0}^n \frac{(-1)^j (2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} (s, \dots, s, v; s+1, \dots, s+1; e^{2 c z}) + \right. \\
 & \left. e^{2 c (v-s) z} \sum_{j=0}^n \frac{(-1)^j (2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} (v-s, \dots, v-s, v; -s+v+1, \dots, -s+v+1; e^{2 c z}) \right) + (-1)^v \\
 & 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(c v - i a (m-2k)) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (c v - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{c v - i a (m-2k)}{2c}, \dots, \frac{c v - i a (m-2k)}{2c}, v; \frac{c v - i a (m-2k)}{2c} + 1, \dots, \frac{c v - i a (m-2k)}{2c} + 1; e^{2 c z} \right) + \right. \\
 & e^{(a i (m-2k) + c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (a i (m-2k) + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left. \left( \frac{i a (m-2k) + c v}{2c}, \dots, \frac{i a (m-2k) + c v}{2c}, v; \frac{i a (m-2k) + c v}{2c} + 1, \dots, \frac{i a (m-2k) + c v}{2c} + 1; e^{2 c z} \right) + \right. \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2 c s - i a (m-2k)) z} \sum_{j=0}^n \frac{(-1)^j (2 c s - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2 c s - i a (m-2k)}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{2 c s - i a (m-2k)}{2c}, v; \frac{2 c s - i a (m-2k)}{2c} + 1, \dots, \frac{2 c s - i a (m-2k)}{2c} + 1; e^{2 c z} \right) + \right. \\
 & e^{(2 c (v-s) - i a (m-2k)) z} \sum_{j=0}^n \frac{(-1)^j (2 c (v-s) - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left. \left( \frac{2 c (v-s) - i a (m-2k)}{2c}, \dots, \frac{2 c (v-s) - i a (m-2k)}{2c}, v; \right. \right. \\
 & \left. \left. \frac{2 c (v-s) - i a (m-2k)}{2c} + 1, \dots, \frac{2 c (v-s) - i a (m-2k)}{2c} + 1; e^{2 c z} \right) \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(a i (m-2k) + 2 c s) z} \sum_{j=0}^n \frac{(-1)^j (a i (m-2k) + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i a (m-2k) + 2 c s}{2c}, \right. \right. \\
 & \left. \left. \dots, \frac{i a (m-2k) + 2 c s}{2c}, v; \frac{i a (m-2k) + 2 c s}{2c} + 1, \dots, \frac{i a (m-2k) + 2 c s}{2c} + 1; e^{2 c z} \right) + \right. \\
 & e^{(a i (m-2k) + 2 c (v-s)) z} \sum_{j=0}^n \frac{(-1)^j (a i (m-2k) + 2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left. \left( \frac{i a (m-2k) + 2 c (v-s)}{2c}, \dots, \frac{i a (m-2k) + 2 c (v-s)}{2c}, v; \right. \right.
 \end{aligned}$$

$$\left. \left. \left. \left. \left. \frac{ia(m-2k)+2c(v-s)}{2c} + 1, \dots, \frac{ia(m-2k)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \right) + \right.$$

$$(-1)^v 2^{-m-u} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{i} \left( e^{(cv-b(u-2i))z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (cv-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{cv-b(u-2i)}{2c}, \dots, \frac{cv-b(u-2i)}{2c}, v; \frac{cv-b(u-2i)}{2c} + 1, \dots, \frac{cv-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right.$$

$$e^{(b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{b(u-2i)+cv}{2c}, \dots, \frac{b(u-2i)+cv}{2c}, v; \frac{b(u-2i)+cv}{2c} + 1, \dots, \frac{b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right.$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs-b(u-2i)}{2c}, \dots, \right. \right.$$

$$\left. \left. \frac{2cs-b(u-2i)}{2c}, v; \frac{2cs-b(u-2i)}{2c} + 1, \dots, \frac{2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right.$$

$$e^{(2c(v-s)-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2c(v-s)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2c(v-s)-b(u-2i)}{2c}, \dots, \right.$$

$$\left. \left. \frac{2c(v-s)-b(u-2i)}{2c}, v; \frac{2c(v-s)-b(u-2i)}{2c} + 1, \dots, \frac{2c(v-s)-b(u-2i)}{2c} + 1; e^{2cz} \right) \right) +$$

$$\sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{2cs+b(u-2i)}{2c}, \dots, \right. \right.$$

$$\left. \left. \frac{2cs+b(u-2i)}{2c}, v; \frac{2cs+b(u-2i)}{2c} + 1, \dots, \frac{2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + e^{(b(u-2i)+2c(v-s))z} \right.$$

$$\left. \sum_{j=0}^n \frac{(-1)^j (b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b(u-2i)+2c(v-s)}{2c}, \dots, \frac{b(u-2i)+2c(v-s)}{2c}, \right. \right.$$

$$\left. \left. v; \frac{b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + (-1)^v 2^{-m-u} n!$$

$$\sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \binom{u}{i} \left( e^{(-ia(m-2k)-b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)-b(u-2i)+cv}{2c}, \dots, \frac{-ia(m-2k)-b(u-2i)+cv}{2c}, v; \right. \right.$$

$$\left. \left. \frac{-ia(m-2k)-b(u-2i)+cv}{2c} + 1, \dots, \frac{-ia(m-2k)-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right.$$

$$\begin{aligned}
 & e^{(a i(m-2 k)-b(u-2 i)+c v) z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)-b(u-2 i)+c v)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)-b(u-2 i)+c v}{2 c}, \dots, \frac{i a(m-2 k)-b(u-2 i)+c v}{2 c}, v; \right. \\
 & \quad \left. \frac{i a(m-2 k)-b(u-2 i)+c v}{2 c}+1, \dots, \frac{i a(m-2 k)-b(u-2 i)+c v}{2 c}+1; e^{2 c z} \right) + \\
 & e^{(-i a(m-2 k)+b(u-2 i)+c v) z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-i a(m-2 k)+b(u-2 i)+c v)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{-i a(m-2 k)+b(u-2 i)+c v}{2 c}, \dots, \frac{-i a(m-2 k)+b(u-2 i)+c v}{2 c}, v; \right. \\
 & \quad \left. \frac{-i a(m-2 k)+b(u-2 i)+c v}{2 c}+1, \dots, \frac{-i a(m-2 k)+b(u-2 i)+c v}{2 c}+1; e^{2 c z} \right) + \\
 & e^{(a i(m-2 k)+b(u-2 i)+c v) z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)+b(u-2 i)+c v)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}, \dots, \frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}, v; \right. \\
 & \quad \left. \frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}+1, \dots, \frac{i a(m-2 k)+b(u-2 i)+c v}{2 c}+1; e^{2 c z} \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-i a(m-2 k)+2 c s-b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2 k)+2 c s-b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{-i a(m-2 k)+2 c s-b(u-2 i)}{2 c}, \dots, \frac{-i a(m-2 k)+2 c s-b(u-2 i)}{2 c}, v; \right. \\
 & \quad \left. \frac{-i a(m-2 k)+2 c s-b(u-2 i)}{2 c}+1, \dots, \frac{-i a(m-2 k)+2 c s-b(u-2 i)}{2 c}+1; e^{2 c z} \right) + \\
 & e^{(-i a(m-2 k)-b(u-2 i)+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2 k)-b(u-2 i)+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{-i a(m-2 k)-b(u-2 i)+2 c(v-s)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{-i a(m-2 k)-b(u-2 i)+2 c(v-s)}{2 c}, v; \frac{-i a(m-2 k)-b(u-2 i)+2 c(v-s)}{2 c}+1, \right. \\
 & \quad \left. \dots, \frac{-i a(m-2 k)-b(u-2 i)+2 c(v-s)}{2 c}+1; e^{2 c z} \right) \Bigg) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(a i(m-2 k)+2 c s-b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)+2 c s-b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)+2 c s-b(u-2 i)}{2 c}, \dots, \frac{i a(m-2 k)+2 c s-b(u-2 i)}{2 c}, v; \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{ia(m-2k)+2cs-b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(ai(m-2k)-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)-b(u-2i)+2c(v-s)}{2c}, \dots, \right. \\
 & \left. \frac{ia(m-2k)-b(u-2i)+2c(v-s)}{2c}, v; \frac{ia(m-2k)-b(u-2i)+2c(v-s)}{2c} + 1, \right. \\
 & \left. \dots, \frac{ia(m-2k)-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+2cs+b(u-2i)}{2c}, \dots, \frac{-ia(m-2k)+2cs+b(u-2i)}{2c}, v; \right. \\
 & \left. \frac{-ia(m-2k)+2cs+b(u-2i)}{2c} + 1, \dots, \frac{-ia(m-2k)+2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-ia(m-2k)+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+b(u-2i)+2c(v-s)}{2c}, \dots, \right. \\
 & \left. \frac{-ia(m-2k)+b(u-2i)+2c(v-s)}{2c}, v; \frac{-ia(m-2k)+b(u-2i)+2c(v-s)}{2c} + 1, \right. \\
 & \left. \dots, \frac{-ia(m-2k)+b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+2cs+b(u-2i)}{2c}, \dots, \frac{ia(m-2k)+2cs+b(u-2i)}{2c}, v; \right. \\
 & \left. \frac{ia(m-2k)+2cs+b(u-2i)}{2c} + 1, \dots, \frac{ia(m-2k)+2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(ai(m-2k)+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+b(u-2i)+2c(v-s)}{2c}, \dots, \frac{ia(m-2k)+b(u-2i)+2c(v-s)}{2c}, \right. \\
 & \left. v; \frac{ia(m-2k)+b(u-2i)+2c(v-s)}{2c} + 1, \dots, \right. \\
 & \left. \dots, \frac{ia(m-2k)+b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) +
 \end{aligned}$$



Involving functions of the direct function, hyperbolic, exponential, trigonometric and a power functions

Involving powers of the direct function, hyperbolic, exponential, trigonometric and a power functions

Involving sin, sinh, exp and power

Involving  $z^n e^{pz} \sin(az) \sinh(bz) \coth^v(cz)$

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$$\int z^n e^{pz} \sin(az) \sinh(bz) \coth^v(cz) dz =$$

$$\begin{aligned} & \frac{1}{4} i (-1)^v n! \left( -e^{(-b-ia+pcv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-b-ia+pcv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\ & \quad \left( \frac{-b-ia+pcv}{2c}, \dots, \frac{-b-ia+pcv}{2c}, v; \frac{-b-ia+pcv}{2c} + 1, \dots, \frac{-b-ia+pcv}{2c} + 1; e^{2cz} \right) + \\ & \quad e^{(-b+ia+pcv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-b+ia+pcv)^{-j-1} z^{n-j}}{(n-j)!} \\ & \quad {}_{j+2}F_{j+1} \left( \frac{-b+ia+pcv}{2c}, \dots, \frac{-b+ia+pcv}{2c}, v; \frac{-b+ia+pcv}{2c} + 1, \dots, \frac{-b+ia+pcv}{2c} + 1; e^{2cz} \right) + \\ & \quad e^{(b-ia+pcv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b-ia+pcv)^{-j-1} z^{n-j}}{(n-j)!} \\ & \quad {}_{j+2}F_{j+1} \left( \frac{b-ia+pcv}{2c}, \dots, \frac{b-ia+pcv}{2c}, v; \frac{b-ia+pcv}{2c} + 1, \dots, \frac{b-ia+pcv}{2c} + 1; e^{2cz} \right) - \\ & \quad e^{(b+ia+pcv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b+ia+pcv)^{-j-1} z^{n-j}}{(n-j)!} \\ & \quad {}_{j+2}F_{j+1} \left( \frac{b+ia+pcv}{2c}, \dots, \frac{b+ia+pcv}{2c}, v; \frac{b+ia+pcv}{2c} + 1, \dots, \frac{b+ia+pcv}{2c} + 1; e^{2cz} \right) - \\ & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b-ia+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-ia+p+2cs}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-b-ia+p+2cs}{2c}, v; \frac{-b-ia+p+2cs}{2c} + 1, \dots, \frac{-b-ia+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. e^{(-b-ia+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-ia+p+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-b-ia+p+2c(v-s)}{2c}, v; \frac{-b-ia+p+2c(v-s)}{2c} + 1, \dots, \frac{-b-ia+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \end{aligned}$$

$$\begin{aligned}
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b+ia+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+ia+p+2cs}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{-b+ia+p+2cs}{2c}, v; \frac{-b+ia+p+2cs}{2c} + 1, \dots, \frac{-b+ia+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(-b+ia+p+2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+ia+p+2c(v-s)}{2c}, \dots, \frac{-b+ia+p+2c(v-s)}{2c}, v; \right. \right. \\
 & \quad \left. \left. \frac{-b+ia+p+2c(v-s)}{2c} + 1, \dots, \frac{-b+ia+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(b-ia+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b-ia+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-ia+p+2cs}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{b-ia+p+2cs}{2c}, v; \frac{b-ia+p+2cs}{2c} + 1, \dots, \frac{b-ia+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(b-ia+p+2c(v-s)z} \sum_{j=0}^n \frac{(-1)^j (b-ia+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-ia+p+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{b-ia+p+2c(v-s)}{2c}, v; \frac{b-ia+p+2c(v-s)}{2c} + 1, \dots, \frac{b-ia+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) - \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(b+ia+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b+ia+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+ia+p+2cs}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{b+ia+p+2cs}{2c}, v; \frac{b+ia+p+2cs}{2c} + 1, \dots, \frac{b+ia+p+2cs}{2c} + 1; e^{2cz} \right) + e^{(b+ia+p+2c(v-s)z} \right. \\
 & \quad \left. \sum_{j=0}^n \frac{(-1)^j (b+ia+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+ia+p+2c(v-s)}{2c}, \dots, \frac{b+ia+p+2c(v-s)}{2c}, \right. \right. \\
 & \quad \left. \left. v; \frac{b+ia+p+2c(v-s)}{2c} + 1, \dots, \frac{b+ia+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+
 \end{aligned}$$

Involving powers of sin, powers of sinh, exp and power

**Involving  $z^n e^{pz} \sin^m(az) \sinh^u(bz) \coth^v(cz)$**

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$$\int z^n e^{p z} \sin^m(a z) \sinh^u(b z) \coth^v(c z) dz =$$

$$\begin{aligned} & (-1)^v n! i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} (1-m \bmod 2) (1-u \bmod 2) \left( e^{(p+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad {}_{j+2}F_{j+1} \left( \frac{p+cv}{2c}, \dots, \frac{p+cv}{2c}, v; \frac{p+cv}{2c} + 1, \dots, \frac{p+cv}{2c} + 1; e^{2cz} \right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs)z} \right. \\ & \quad \sum_{j=0}^n \frac{(-1)^j (p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs}{2c}, \dots, \frac{p+2cs}{2c}, v; \frac{p+2cs}{2c} + 1, \dots, \frac{p+2cs}{2c} + 1; e^{2cz} \right) + \\ & \quad \left. e^{(p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{p+2c(v-s)}{2c}, v; \frac{p+2c(v-s)}{2c} + 1, \dots, \frac{p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) + \\ & (-1)^v n! i^u 2^{-m-u} \binom{u}{\frac{u}{2}} (1-u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{\frac{im\pi}{2}} \left( e^{-ia(m-2k)+p+cv} z \binom{v}{\frac{v}{2}} (1-v \bmod 2) \right. \right. \\ & \quad \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+cv}{2c}, \dots, \right. \\ & \quad \left. \frac{-ia(m-2k)+p+cv}{2c}, v; \frac{-ia(m-2k)+p+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p+cv}{2c} + 1; e^{2cz} \right) + \\ & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\ & \quad {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2cs}{2c}, \dots, \frac{-ia(m-2k)+p+2cs}{2c}, v; \right. \\ & \quad \left. \frac{-ia(m-2k)+p+2cs}{2c} + 1, \dots, \frac{-ia(m-2k)+p+2cs}{2c} + 1; e^{2cz} \right) + \\ & \quad \left. e^{(-ia(m-2k)+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2c(v-s)}{2c}, \dots, \frac{-ia(m-2k)+p+2c(v-s)}{2c}, v; \right. \right. \\ & \quad \left. \left. \frac{-ia(m-2k)+p+2c(v-s)}{2c} + 1, \dots, \frac{-ia(m-2k)+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) \Bigg) + \end{aligned}$$

$$\begin{aligned}
 & e^{-\frac{1}{2} i m \pi} \left( e^{(a i(m-2 k)+p+c v) z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)+p+c v)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)+p+c v}{2 c}, \dots, \frac{i a(m-2 k)+p+c v}{2 c}, v; \right. \right. \\
 & \quad \left. \left. \frac{i a(m-2 k)+p+c v}{2 c}+1, \dots, \frac{i a(m-2 k)+p+c v}{2 c}+1; e^{2 c z} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(a i(m-2 k)+p+2 c s) z} \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)+p+2 c s}{2 c}, \right. \right. \right. \\
 & \quad \left. \left. \dots, \frac{i a(m-2 k)+p+2 c s}{2 c}, v; \frac{i a(m-2 k)+p+2 c s}{2 c}+1, \dots, \frac{i a(m-2 k)+p+2 c s}{2 c}+1; e^{2 c z} \right) + \right. \\
 & \quad \left. e^{(a i(m-2 k)+p+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)+p+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)+p+2 c(v-s)}{2 c}, \dots, \frac{i a(m-2 k)+p+2 c(v-s)}{2 c}, v; \right. \right. \\
 & \quad \left. \left. \frac{i a(m-2 k)+p+2 c(v-s)}{2 c}+1, \dots, \frac{i a(m-2 k)+p+2 c(v-s)}{2 c}+1; e^{2 c z} \right) \right) \Bigg) + \\
 & (-1)^v n! i^u 2^{-m-u} \binom{m}{2} (1-m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^i \binom{u}{i} \left( e^{\frac{i \pi u}{2}} \left( e^{(p-b(u-2 i)+c v) z} \left( \frac{v}{2} \right) (1-v \bmod 2) \right. \right. \\
 & \quad \left. \sum_{j=0}^n \frac{(-1)^j (p-b(u-2 i)+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{p-b(u-2 i)+c v}{2 c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{p-b(u-2 i)+c v}{2 c}, v; \frac{p-b(u-2 i)+c v}{2 c}+1, \dots, \frac{p-b(u-2 i)+c v}{2 c}+1; e^{2 c z} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2 c s-b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j (p+2 c s-b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{p+2 c s-b(u-2 i)}{2 c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{p+2 c s-b(u-2 i)}{2 c}, v; \frac{p+2 c s-b(u-2 i)}{2 c}+1, \dots, \frac{p+2 c s-b(u-2 i)}{2 c}+1; e^{2 c z} \right) + \right. \\
 & \quad \left. e^{(p-b(u-2 i)+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j (p-b(u-2 i)+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{p-b(u-2 i)+2 c(v-s)}{2 c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{p-b(u-2 i)+2 c(v-s)}{2 c}, v; \right. \right. \\
 & \quad \left. \left. \frac{p-b(u-2 i)+2 c(v-s)}{2 c}+1, \dots, \frac{p-b(u-2 i)+2 c(v-s)}{2 c}+1; e^{2 c z} \right) \right) \Bigg) +
 \end{aligned}$$

$$\begin{aligned}
 & e^{-\frac{1}{2}i\pi u} \left( e^{(p+b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+b(u-2i)+cv}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{p+b(u-2i)+cv}{2c}, v; \frac{p+b(u-2i)+cv}{2c} + 1, \dots, \frac{p+b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs+b(u-2i)}{2c}, \dots, \right. \right. \right. \\
 & \quad \left. \left. \frac{p+2cs+b(u-2i)}{2c}, v; \frac{p+2cs+b(u-2i)}{2c} + 1, \dots, \frac{p+2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(p+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \quad \left. \left( \frac{p+b(u-2i)+2c(v-s)}{2c}, \dots, \frac{p+b(u-2i)+2c(v-s)}{2c}, v; \right. \right. \\
 & \quad \left. \left. \frac{p+b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{p+b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) + \\
 & (-1)^v n! i^u 2^{-m-u} \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{i+k} \binom{m}{k} \binom{u}{i} \left( e^{\frac{1}{2}i\pi(m+u)} \left( e^{(-ia(m-2k)+p-b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \right. \right. \\
 & \quad \left. \left. \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \right. \\
 & \quad \left. \left. \left( \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c}, \dots, \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c}, v; \right. \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c}, \dots, \right. \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c}, v; \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1, \right. \right. \\
 & \quad \left. \left. \dots, \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(-ia(m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \begin{aligned}
 & \dots, \frac{-i a(m-2k) + p - b(u-2i) + 2c(v-s)}{2c}, v; \\
 & \frac{-i a(m-2k) + p - b(u-2i) + 2c(v-s)}{2c} + 1, \dots, \\
 & \left. \left. \frac{-i a(m-2k) + p - b(u-2i) + 2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + e^{\frac{1}{2}i\pi(u-m)} \\
 & \left( e^{(ai(m-2k)+p-b(u-2i)+cv)z} \left( \frac{v}{\frac{v}{2}} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k) + p - b(u-2i) + cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k) + p - b(u-2i) + cv}{2c}, \dots, \frac{ia(m-2k) + p - b(u-2i) + cv}{2c}, v; \right. \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k) + p - b(u-2i) + cv}{2c} + 1, \dots, \frac{ia(m-2k) + p - b(u-2i) + cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k) + p + 2cs - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k) + p + 2cs - b(u-2i)}{2c}, \dots, \right. \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k) + p + 2cs - b(u-2i)}{2c}, v; \frac{ia(m-2k) + p + 2cs - b(u-2i)}{2c} + 1, \right. \right. \\
 & \quad \left. \left. \dots, \frac{ia(m-2k) + p + 2cs - b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(ai(m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k) + p - b(u-2i) + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k) + p - b(u-2i) + 2c(v-s)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k) + p - b(u-2i) + 2c(v-s)}{2c}, v; \frac{ia(m-2k) + p - b(u-2i) + 2c(v-s)}{2c} + 1, \right. \right. \\
 & \quad \left. \left. \dots, \frac{ia(m-2k) + p - b(u-2i) + 2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + e^{\frac{1}{2}i\pi(m-u)} \\
 & \left( e^{(-ia(m-2k)+p+b(u-2i)+cv)z} \left( \frac{v}{\frac{v}{2}} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k) + p + b(u-2i) + cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k) + p + b(u-2i) + cv}{2c}, \dots, \frac{-ia(m-2k) + p + b(u-2i) + cv}{2c}, v; \right. \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k) + p + b(u-2i) + cv}{2c} + 1, \dots, \frac{-ia(m-2k) + p + b(u-2i) + cv}{2c} + 1; e^{2cz} \right) + \right.
 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-i a(m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c}, \dots, \right. \\
 & \quad \left. \frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c} + 1, \right. \\
 & \quad \left. \dots, \frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(-i a(m-2k)+p+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2k)+p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c}, \right. \\
 & \quad \dots, \frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c}, v; \\
 & \quad \left. \frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c} + 1, \dots, \right. \\
 & \quad \left. \frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \Bigg) + e^{-\frac{1}{2}i\pi(m+u)} \\
 & \left( e^{(ai(m-2k)+p+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+b(u-2i)+cv}{2c}, \dots, \frac{ia(m-2k)+p+b(u-2i)+cv}{2c}, v; \right. \\
 & \quad \left. \frac{ia(m-2k)+p+b(u-2i)+cv}{2c} + 1, \dots, \frac{ia(m-2k)+p+b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+2cs+b(u-2i)}{2c}, \dots, \right. \\
 & \quad \frac{ia(m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{ia(m-2k)+p+2cs+b(u-2i)}{2c} + 1, \\
 & \quad \dots, \frac{ia(m-2k)+p+2cs+b(u-2i)}{2c} + 1; e^{2cz} \Bigg) + \\
 & \quad e^{(ai(m-2k)+p+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+b(u-2i)+2c(v-s)}{2c}, \dots, \right.
 \end{aligned}$$

$$\left. \left. \left. \frac{ia(m-2k)+p+b(u-2i)+2c(v-s)}{2c}, v; \frac{ia(m-2k)+p+b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{ia(m-2k)+p+b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \right) ; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+$$

Involving cos, sinh, exp and power

### Involving $z^n e^{pz} \cos(az) \sinh(bz) \coth^v(cz)$

01.22.21.0489.01

$$\int z^n e^{pz} \cos(az) \sinh(bz) \coth^v(cz) dz =$$

$$\begin{aligned} & \frac{1}{4} (-1)^v n! \left( -e^{(-b-ia+p+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-b-ia+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\ & \quad \left( \frac{-b-ia+p+cv}{2c}, \dots, \frac{-b-ia+p+cv}{2c}, v; \frac{-b-ia+p+cv}{2c} + 1, \dots, \frac{-b-ia+p+cv}{2c} + 1; e^{2cz} \right) - \\ & \quad e^{(-b+ia+p+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-b+ia+p+cv)^{-j-1} z^{n-j}}{(n-j)!} \\ & \quad {}_{j+2}F_{j+1} \left( \frac{-b+ia+p+cv}{2c}, \dots, \frac{-b+ia+p+cv}{2c}, v; \frac{-b+ia+p+cv}{2c} + 1, \dots, \frac{-b+ia+p+cv}{2c} + 1; e^{2cz} \right) + \\ & \quad e^{(b-ia+p+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b-ia+p+cv)^{-j-1} z^{n-j}}{(n-j)!} \\ & \quad {}_{j+2}F_{j+1} \left( \frac{b-ia+p+cv}{2c}, \dots, \frac{b-ia+p+cv}{2c}, v; \frac{b-ia+p+cv}{2c} + 1, \dots, \frac{b-ia+p+cv}{2c} + 1; e^{2cz} \right) + \\ & \quad e^{(b+ia+p+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (b+ia+p+cv)^{-j-1} z^{n-j}}{(n-j)!} \\ & \quad {}_{j+2}F_{j+1} \left( \frac{b+ia+p+cv}{2c}, \dots, \frac{b+ia+p+cv}{2c}, v; \frac{b+ia+p+cv}{2c} + 1, \dots, \frac{b+ia+p+cv}{2c} + 1; e^{2cz} \right) - \\ & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b-ia+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-ia+p+2cs}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-b-ia+p+2cs}{2c}, v; \frac{-b-ia+p+2cs}{2c} + 1, \dots, \frac{-b-ia+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \\ & \quad \left. e^{(-b-ia+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b-ia+p+2c(v-s)}{2c}, \dots, \right. \right. \end{aligned}$$



$$\begin{aligned}
 & \left. \frac{-b-ia+p+2c(v-s)}{2c}, v; \frac{-b-ia+p+2c(v-s)}{2c} + 1, \dots, \frac{-b-ia+p+2c(v-s)}{2c} + 1; e^{2cz} \right) - \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-b+ia+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+ia+p+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{-b+ia+p+2c(v-s)}{2c}, v; \frac{-b+ia+p+2c(v-s)}{2c} + 1, \dots, \frac{-b+ia+p+2c(v-s)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(-b+ia+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-b+ia+p+2cs}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{-b+ia+p+2cs}{2c}, v; \frac{-b+ia+p+2cs}{2c} + 1, \dots, \frac{-b+ia+p+2cs}{2c} + 1; e^{2cz} \right) \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(b-ia+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b-ia+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-ia+p+2cs}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{b-ia+p+2cs}{2c}, v; \frac{b-ia+p+2cs}{2c} + 1, \dots, \frac{b-ia+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \left. e^{(b-ia+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (b-ia+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b-ia+p+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{b-ia+p+2c(v-s)}{2c}, v; \frac{b-ia+p+2c(v-s)}{2c} + 1, \dots, \frac{b-ia+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(b+ia+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (b+ia+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+ia+p+2cs}{2c}, \dots, \right. \right. \\
 & \left. \left. \frac{b+ia+p+2cs}{2c}, v; \frac{b+ia+p+2cs}{2c} + 1, \dots, \frac{b+ia+p+2cs}{2c} + 1; e^{2cz} \right) + e^{(b+ia+p+2c(v-s))z} \right. \\
 & \left. \sum_{j=0}^n \frac{(-1)^j (b+ia+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{b+ia+p+2c(v-s)}{2c}, \dots, \frac{b+ia+p+2c(v-s)}{2c}, \right. \right. \\
 & \left. \left. v; \frac{b+ia+p+2c(v-s)}{2c} + 1, \dots, \frac{b+ia+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+
 \end{aligned}$$

Involving powers of cos, powers of sinh, exp and power

**Involving  $z^n e^{pZ} \cos^m(a z) \sinh^u(b z) \coth^v(c z)$**

01.22.21.0490.01

$$\int z^n e^{p z} \cos^m(a z) \sinh^u(b z) \coth^v(c z) dz =$$

$$\begin{aligned} & (-1)^v n! i^u 2^{-m-u} \binom{m}{\frac{n}{2}} \binom{u}{\frac{n}{2}} (1-m \bmod 2) (1-u \bmod 2) \left( e^{(p+cv)z} \binom{v}{\frac{n}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad {}_{j+2}F_{j+1} \left( \frac{p+cv}{2c}, \dots, \frac{p+cv}{2c}, v; \frac{p+cv}{2c} + 1, \dots, \frac{p+cv}{2c} + 1; e^{2cz} \right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs)z} \right. \\ & \quad \sum_{j=0}^n \frac{(-1)^j (p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs}{2c}, \dots, \frac{p+2cs}{2c}, v; \frac{p+2cs}{2c} + 1, \dots, \frac{p+2cs}{2c} + 1; e^{2cz} \right) + \\ & \quad \left. e^{(p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{p+2c(v-s)}{2c}, v; \frac{p+2c(v-s)}{2c} + 1, \dots, \frac{p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) + \\ & (-1)^v n! i^u 2^{-m-u} \binom{u}{\frac{n}{2}} (1-u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(-ia(m-2k)+p+cv)z} \binom{v}{\frac{n}{2}} (1-v \bmod 2) \right. \\ & \quad \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+cv}{2c}, \dots, \right. \\ & \quad \left. \frac{-ia(m-2k)+p+cv}{2c}, v; \frac{-ia(m-2k)+p+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p+cv}{2c} + 1; e^{2cz} \right) + \\ & \quad e^{(ia(m-2k)+p+cv)z} \binom{v}{\frac{n}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ia(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+cv}{2c}, \right. \\ & \quad \left. \dots, \frac{ia(m-2k)+p+cv}{2c}, v; \frac{ia(m-2k)+p+cv}{2c} + 1, \dots, \frac{ia(m-2k)+p+cv}{2c} + 1; e^{2cz} \right) + \\ & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2cs}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-ia(m-2k)+p+2cs}{2c}, v; \frac{-ia(m-2k)+p+2cs}{2c} + 1, \dots, \frac{-ia(m-2k)+p+2cs}{2c} + 1; \right. \right. \\ & \quad \left. \left. e^{2cz} \right) + e^{(-ia(m-2k)+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2c(v-s)}{2c}, \dots, \frac{-ia(m-2k)+p+2c(v-s)}{2c}, v; \right. \right. \\ & \quad \left. \left. \frac{-ia(m-2k)+p+2c(v-s)}{2c} + 1, \dots, \frac{-ia(m-2k)+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) + \end{aligned}$$

$$\begin{aligned}
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+2cs}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p+2cs}{2c}, v; \frac{ia(m-2k)+p+2cs}{2c} + 1, \dots, \frac{ia(m-2k)+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(ai(m-2k)+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \quad \left( \frac{ia(m-2k)+p+2c(v-s)}{2c}, \dots, \frac{ia(m-2k)+p+2c(v-s)}{2c}, v; \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p+2c(v-s)}{2c} + 1, \dots, \frac{ia(m-2k)+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & (-1)^v n! i^u 2^{-m-u} \binom{m}{\frac{m}{2}} (1-m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^i \binom{u}{i} \left( e^{\frac{i\pi u}{2}} \left( e^{(p-b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \right. \right. \\
 & \quad \sum_{j=0}^n \frac{(-1)^j (p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-b(u-2i)+cv}{2c}, \dots, \right. \\
 & \quad \left. \left. \frac{p-b(u-2i)+cv}{2c}, v; \frac{p-b(u-2i)+cv}{2c} + 1, \dots, \frac{p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs-b(u-2i)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{p+2cs-b(u-2i)}{2c}, v; \frac{p+2cs-b(u-2i)}{2c} + 1, \dots, \frac{p+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \quad \left( \frac{p-b(u-2i)+2c(v-s)}{2c}, \dots, \frac{p-b(u-2i)+2c(v-s)}{2c}, v; \right. \\
 & \quad \left. \left. \frac{p-b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{p-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{-\frac{1}{2}i\pi u} \left( e^{(p+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+b(u-2i)+cv}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{p+b(u-2i)+cv}{2c}, v; \frac{p+b(u-2i)+cv}{2c} + 1, \dots, \frac{p+b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs+b(u-2i)}{2c}, \dots, \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \left. \frac{p+2cs+b(u-2i)}{2c}, v; \frac{p+2cs+b(u-2i)}{2c} + 1, \dots, \frac{p+2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & e^{(p+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left( \frac{p+b(u-2i)+2c(v-s)}{2c}, \dots, \frac{p+b(u-2i)+2c(v-s)}{2c}, v; \right. \\
 & \left. \left. \left. \left. \frac{p+b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{p+b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \right) \right) + \\
 & (-1)^v n! i^u 2^{-m-u} \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^i \binom{m}{k} \binom{u}{i} \left( e^{\frac{ixu}{2}} \left( e^{(-ia(m-2k)+p-b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \right. \right. \right. \\
 & \left. \left. \left. \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \right. \right. \\
 & \left. \left. \left. \left( \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c}, \dots, \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c}, v; \right. \right. \right. \\
 & \left. \left. \left. \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \left. \left. \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \right. \\
 & \left. \left. \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c}, \dots, \right. \right. \right. \\
 & \left. \left. \left. \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c}, v; \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1, \right. \right. \right. \\
 & \left. \left. \left. \dots, \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \right. \\
 & \left. \left. \left. e^{(-ia(m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \right. \\
 & \left. \left. \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, \right. \right. \right. \\
 & \left. \left. \left. \dots, \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, v; \right. \right. \right. \\
 & \left. \left. \left. \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + 1, \dots, \right. \right. \right. \\
 & \left. \left. \left. \left. \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & e^{\frac{i\pi u}{2}} \left( e^{(ai(m-2k)+p-b(u-2i)+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p-b(u-2i)+cv}{2c}, \dots, \frac{ia(m-2k)+p-b(u-2i)+cv}{2c}, v; \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p-b(u-2i)+cv}{2c} + 1, \dots, \frac{ia(m-2k)+p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+2cs-b(u-2i)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p+2cs-b(u-2i)}{2c}, v; \frac{ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1, \right. \right. \\
 & \quad \left. \left. \dots, \frac{ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(ai(m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, v; \frac{ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + \right. \right. \\
 & \quad \left. \left. 1, \dots, \frac{ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{-\frac{1}{2}i\pi u} \left( e^{(-ia(m-2k)+p+b(u-2i)+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c}, \dots, \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c}, v; \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2cs+b(u-2i)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{-ia(m-2k)+p+2cs+b(u-2i)}{2c} + 1, \right. \right. \\
 & \quad \left. \left. \dots, \frac{-ia(m-2k)+p+2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(-ia(m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, v; \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + \right. \right. \\
 & \quad \left. \left. 1, \dots, \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) +
 \end{aligned}$$



Involving sin, cosh, exp and power

Involving  $z^n e^{pz} \sin(az) \cosh(bz) \coth^v(cz)$

01.22.21.0491.01

$$\int z^n e^{pz} \sin(az) \cosh(bz) \coth^v(cz) dz =$$

$$\frac{1}{4} (-1)^v \left(\frac{v}{2}\right) n! (1 - v \bmod 2) \left( e^{\frac{i\pi}{2} + (-b - ia + p + cv)z} \sum_{j=0}^n \frac{(-1)^j (-b - ia + p + cv)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_j F_{j+1} \left( \frac{-ia - b + p + cv}{2c}, \dots, \frac{-ia - b + p + cv}{2c}, v; \frac{-ia - b + p + cv}{2c} + 1, \dots, \frac{-ia - b + p + cv}{2c} + 1; e^{2cz} \right) +$$

$$e^{-\frac{1}{2}(i\pi) + (-b + ia + p + cv)z} \sum_{j=0}^n \frac{(-1)^j (-b + ia + p + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_j F_{j+1} \left( \frac{ia - b + p + cv}{2c}, \dots,$$

$$\frac{ia - b + p + cv}{2c}, v; \frac{ia - b + p + cv}{2c} + 1, \dots, \frac{ia - b + p + cv}{2c} + 1; e^{2cz} \right) +$$

$$e^{\frac{i\pi}{2} + (b - ia + p + cv)z} \sum_{j=0}^n \frac{(-1)^j (b - ia + p + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_j F_{j+1} \left( \frac{-ia + b + p + cv}{2c}, \dots,$$

$$\frac{-ia + b + p + cv}{2c}, v; \frac{-ia + b + p + cv}{2c} + 1, \dots, \frac{-ia + b + p + cv}{2c} + 1; e^{2cz} \right) +$$

$$e^{-\frac{1}{2}(i\pi) + (b + ia + p + cv)z} \sum_{j=0}^n \frac{(-1)^j (b + ia + p + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_j F_{j+1} \left( \frac{ia + b + p + cv}{2c}, \dots,$$

$$\frac{ia + b + p + cv}{2c}, v; \frac{ia + b + p + cv}{2c} + 1, \dots, \frac{ia + b + p + cv}{2c} + 1; e^{2cz} \right) +$$

$$\frac{1}{4} (-1)^v n! \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( e^{\frac{i\pi}{2} + (-b + 2ci - ia + p)z} \sum_{j=0}^n \frac{(-1)^j (-b + 2ci - ia + p)^{-j-1} z^{n-j}}{(n-j)!} {}_j F_{j+1} \left( \frac{-ia - b + 2ci + p}{2c}, \dots,$$

$$\frac{-ia - b + 2ci + p}{2c}, v; \frac{-ia - b + 2ci + p}{2c} + 1, \dots, \frac{-ia - b + 2ci + p}{2c} + 1; e^{2cz} \right) +$$

$$e^{\frac{i\pi}{2} + (b + 2ci - ia + p)z} \sum_{j=0}^n \frac{(-1)^j (b + 2ci - ia + p)^{-j-1} z^{n-j}}{(n-j)!} {}_j F_{j+1} \left( \frac{-ia + b + 2ci + p}{2c}, \dots,$$

$$\frac{-ia + b + 2ci + p}{2c}, v; \frac{-ia + b + 2ci + p}{2c} + 1, \dots, \frac{-ia + b + 2ci + p}{2c} + 1; e^{2cz} \right) +$$

$$e^{-\frac{1}{2}(i\pi) + (b + 2ci + ia + p)z} \sum_{j=0}^n \frac{(-1)^j (b + 2ci + ia + p)^{-j-1} z^{n-j}}{(n-j)!} {}_j F_{j+1} \left( \frac{ia + b + 2ci + p}{2c}, \dots,$$

$$\frac{ia + b + 2ci + p}{2c}, v; \frac{ia + b + 2ci + p}{2c} + 1, \dots, \frac{ia + b + 2ci + p}{2c} + 1; e^{2cz} \right) +$$

$$\begin{aligned}
 & e^{-\frac{1}{2}(i\pi)+(-b+2ci+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2ci+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+2ci+p-b}{2c}, \dots, \right. \\
 & \quad \left. \frac{ia+2ci+p-b}{2c}, v; \frac{ia+2ci+p-b}{2c} + 1, \dots, \frac{ia+2ci+p-b}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{i\pi}{2}+(-b-2ci-ia+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci-ia+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{-ia-b-2ci+p+2cv}{2c}, \dots, \frac{-ia-b-2ci+p+2cv}{2c}, v; \right. \\
 & \quad \left. \frac{-ia-b-2ci+p+2cv}{2c} + 1, \dots, \frac{-ia-b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) + \\
 & e^{-\frac{1}{2}(i\pi)+(-b-2ci+ia+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci+ia+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b-2ci+p+2cv}{2c}, \right. \\
 & \quad \left. \dots, \frac{ia-b-2ci+p+2cv}{2c}, v; \frac{ia-b-2ci+p+2cv}{2c} + 1, \dots, \frac{ia-b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) + \\
 & e^{\frac{i\pi}{2}+(b-2ci-ia+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (b-2ci-ia+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \quad \left( \frac{-ia+b-2ci+p+2cv}{2c}, \dots, \frac{-ia+b-2ci+p+2cv}{2c}, v; \frac{-ia+b-2ci+p+2cv}{2c} + 1, \right. \\
 & \quad \left. \dots, \frac{-ia+b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) + e^{-\frac{1}{2}(i\pi)+(b-2ci+ia+p+2cv)z} \\
 & \quad \sum_{j=0}^n \frac{(-1)^j (b-2ci+ia+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b-2ci+p+2cv}{2c}, \dots, \frac{ia+b-2ci+p+2cv}{2c}, \right. \\
 & \quad \left. v; \frac{ia+b-2ci+p+2cv}{2c} + 1, \dots, \frac{ia+b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+
 \end{aligned}$$



01.22.21.0492.01

$$\int z^n e^{p z} \sin(a z) \cosh(c z) \coth^v(c z) dz =$$

$$\frac{1}{4} (-1)^v i e^{c z v} \binom{v+1}{\frac{v+1}{2}} n! (1 - (v+1) \bmod 2) \left( e^{(-i a+p) z} \sum_{j=0}^n \frac{(-1)^j (-i a+p+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{c v+p-i a}{2 c}, \dots, \right. \right.$$

$$\left. \frac{c v+p-i a}{2 c}, v; \frac{c v+p-i a}{2 c} + 1, \dots, \frac{c v+p-i a}{2 c} + 1; e^{2 c z} \right) - e^{(i a+p) z} \sum_{j=0}^n \frac{(-1)^j (i a+p+c v)^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{c v+p+i a}{2 c}, \dots, \frac{c v+p+i a}{2 c}, v; \frac{c v+p+i a}{2 c} + 1, \dots, \frac{c v+p+i a}{2 c} + 1; e^{2 c z} \right) \right) +$$

$$\frac{1}{4} (-1)^v i e^{c z v} n! \sum_{s=0}^{\lfloor \frac{v}{2} \rfloor} \binom{v+1}{s} \left( -e^{(i a+p-c(-2 s+v+1)) z} \sum_{j=0}^n \frac{(-1)^j (i a+p-c(1-2 s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+i a-c(-2 s+1)}{2 c}, \right. \right.$$

$$\dots, \frac{p+i a-c(-2 s+1)}{2 c}, v; \frac{p+i a-c(-2 s+1)}{2 c} + 1, \dots, \frac{p+i a-c(-2 s+1)}{2 c} + 1; e^{2 c z} \right) +$$

$$e^{(-i a+p-c(-2 s+v+1)) z} \sum_{j=0}^n \frac{(-1)^j (-i a+p-c(1-2 s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-i a-c(-2 s+1)}{2 c}, \dots, \right.$$

$$\left. \frac{p-i a-c(-2 s+1)}{2 c}, v; \frac{p-i a-c(-2 s+1)}{2 c} + 1, \dots, \frac{p-i a-c(-2 s+1)}{2 c} + 1; e^{2 c z} \right) -$$

$$e^{(i a+p+c(-2 s+v+1)) z} \sum_{j=0}^n \frac{(-1)^j (i a+p+c(-2 s+2 v+1))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+i a+c(-2 s+1+2 v)}{2 c}, \right.$$

$$\dots, \frac{p+i a+c(-2 s+1+2 v)}{2 c}, v; \frac{p+i a+c(-2 s+1+2 v)}{2 c} + 1, \dots, \frac{p+i a+c(-2 s+1+2 v)}{2 c} + 1;$$

$$e^{2 c z} \left. \right) + e^{(-i a+p+c(-2 s+v+1)) z} \sum_{j=0}^n \frac{(-1)^j (-i a+p+c(-2 s+2 v+1))^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1} \left( \frac{p-i a+c(-2 s+1+2 v)}{2 c}, \dots, \frac{p-i a+c(-2 s+1+2 v)}{2 c}, v; \right. \right.$$

$$\left. \frac{p-i a+c(-2 s+1+2 v)}{2 c} + 1, \dots, \frac{p-i a+c(-2 s+1+2 v)}{2 c} + 1; e^{2 c z} \right) \Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of sin, powers of cosh, exp and power

**Involving  $z^n e^{p z} \sin^m(a z) \cosh^u(b z) \coth^v(c z)$**

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$$\int z^n e^{p z} \sin^m(a z) \cosh^u(b z) \coth^v(c z) dz =$$

$$(-1)^v 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2) (1 - u \bmod 2) \left( e^{(p+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+c v)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$\begin{aligned}
 & {}_{j+2}F_{j+1}\left(\frac{p+cv}{2c}, \dots, \frac{p+cv}{2c}, v; \frac{p+cv}{2c}+1, \dots, \frac{p+cv}{2c}+1; e^{2cz}\right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs)z} \right. \\
 & \sum_{j=0}^n \frac{(-1)^j (p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+2cs}{2c}, \dots, \frac{p+2cs}{2c}, v; \frac{p+2cs}{2c}+1, \dots, \frac{p+2cs}{2c}+1; e^{2cz}\right) + \\
 & e^{(p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+2c(v-s)}{2c}, \dots, \right. \\
 & \left. \frac{p+2c(v-s)}{2c}, v; \frac{p+2c(v-s)}{2c}+1, \dots, \frac{p+2c(v-s)}{2c}+1; e^{2cz}\right) \Bigg) + \\
 & (-1)^v 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1-u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{\frac{im\pi}{2}} \left( e^{(-ia(m-2k)+p+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \right. \right. \\
 & \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-ia(m-2k)+p+cv}{2c}, \dots, \right. \\
 & \left. \frac{-ia(m-2k)+p+cv}{2c}, v; \frac{-ia(m-2k)+p+cv}{2c}+1, \dots, \frac{-ia(m-2k)+p+cv}{2c}+1; e^{2cz}\right) + \\
 & \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & {}_{j+2}F_{j+1}\left(\frac{-ia(m-2k)+p+2cs}{2c}, \dots, \frac{-ia(m-2k)+p+2cs}{2c}, v; \right. \\
 & \left. \frac{-ia(m-2k)+p+2cs}{2c}+1, \dots, \frac{-ia(m-2k)+p+2cs}{2c}+1; e^{2cz}\right) + \\
 & e^{(-ia(m-2k)+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1}\left(\frac{-ia(m-2k)+p+2c(v-s)}{2c}, \dots, \frac{-ia(m-2k)+p+2c(v-s)}{2c}, v; \right. \\
 & \left. \frac{-ia(m-2k)+p+2c(v-s)}{2c}+1, \dots, \frac{-ia(m-2k)+p+2c(v-s)}{2c}+1; e^{2cz}\right) \Bigg) \Bigg) + \\
 & e^{-\frac{1}{2}im\pi} \left( e^{(ai(m-2k)+p+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1}\left(\frac{ia(m-2k)+p+cv}{2c}, \dots, \frac{ia(m-2k)+p+cv}{2c}, v; \right. \\
 & \left. \frac{ia(m-2k)+p+cv}{2c}+1, \dots, \frac{ia(m-2k)+p+cv}{2c}+1; e^{2cz}\right) +
 \end{aligned}$$

$$\begin{aligned}
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+2cs}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{ia(m-2k)+p+2cs}{2c}, v; \frac{ia(m-2k)+p+2cs}{2c} + 1, \dots, \frac{ia(m-2k)+p+2cs}{2c} + 1; e^{2cz} \right) + e^{(ai(m-2k)+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+2c(v-s)}{2c}, \dots, \frac{ia(m-2k)+p+2c(v-s)}{2c}, v; \frac{ia(m-2k)+p+2c(v-s)}{2c} + 1, \dots, \frac{ia(m-2k)+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) + \\
 & (-1)^v 2^{-m-u} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{i} \left( e^{(p-b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \right. \\
 & \quad \sum_{j=0}^n \frac{(-1)^j (p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-b(u-2i)+cv}{2c}, \dots, \right. \\
 & \quad \left. \frac{p-b(u-2i)+cv}{2c}, v; \frac{p-b(u-2i)+cv}{2c} + 1, \dots, \frac{p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(p+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+b(u-2i)+cv}{2c}, \right. \\
 & \quad \left. \dots, \frac{p+b(u-2i)+cv}{2c}, v; \frac{p+b(u-2i)+cv}{2c} + 1, \dots, \frac{p+b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\
 & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs-b(u-2i)}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{p+2cs-b(u-2i)}{2c}, v; \frac{p+2cs-b(u-2i)}{2c} + 1, \dots, \frac{p+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + e^{(p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{p-b(u-2i)+2c(v-s)}{2c}, \dots, \frac{p-b(u-2i)+2c(v-s)}{2c}, v; \frac{p-b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{p-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs+b(u-2i)}{2c}, \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \dots, \frac{p+2cs+b(u-2i)}{2c}, v; \frac{p+2cs+b(u-2i)}{2c} + 1, \dots, \frac{p+2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) \right. \\
 & e^{(p+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left( \frac{p+b(u-2i)+2c(v-s)}{2c}, \dots, \frac{p+b(u-2i)+2c(v-s)}{2c}, v; \right. \\
 & \left. \left. \frac{p+b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{p+b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & (-1)^v 2^{-m-u} n! \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \binom{u}{i} \left( e^{\frac{i\pi m}{2}} \left( e^{(-ia(m-2k)+p-b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \right. \right. \\
 & \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \left( \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c}, \dots, \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c}, v; \right. \\
 & \left. \left. \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c}, \dots, \right. \\
 & \left. \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c}, v; \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1, \right. \\
 & \left. \left. \dots, \frac{-ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{(-ia(m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, \right. \\
 & \dots, \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, v; \\
 & \left. \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + 1, \dots, \right. \\
 & \left. \left. \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & e^{-\frac{1}{2}i\pi m} \left( e^{(ai(m-2k)+p-b(u-2i)+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p-b(u-2i)+cv}{2c}, \dots, \frac{ia(m-2k)+p-b(u-2i)+cv}{2c}, v; \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p-b(u-2i)+cv}{2c} + 1, \dots, \frac{ia(m-2k)+p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+2cs-b(u-2i)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p+2cs-b(u-2i)}{2c}, v; \frac{ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1, \right. \right. \\
 & \quad \left. \left. \dots, \frac{ia(m-2k)+p+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(ai(m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, v; \frac{ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + \right. \right. \\
 & \quad \left. \left. 1, \dots, \frac{ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & e^{\frac{i\pi m}{2}} \left( e^{(-ia(m-2k)+p+b(u-2i)+cv)z} \left(\frac{v}{2}\right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c}, \dots, \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c}, v; \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right. \\
 & \quad \left. \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2cs+b(u-2i)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{-ia(m-2k)+p+2cs+b(u-2i)}{2c} + 1, \right. \right. \\
 & \quad \left. \left. \dots, \frac{-ia(m-2k)+p+2cs+b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(-ia(m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c}, v; \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + \right. \right. \\
 & \quad \left. \left. 1, \dots, \frac{-ia(m-2k)+p-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) +
 \end{aligned}$$



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$$\int z^n e^{pz} \sin^m(az) \cosh^u(cz) \coth^v(cz) dz = (-1)^v 2^{-m-u} n! e^{(p+cv)z} \left(\frac{m}{2}\right) \binom{u+v}{\frac{u+v}{2}} (1-m \bmod 2) (1-(u+v) \bmod 2)$$

$$\sum_{j=0}^n \frac{(-1)^j z^{n-j} (p+cv)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{cv+p}{2c}, \dots, \frac{cv+p}{2c}, v; \frac{cv+p}{2c} + 1, \dots, \frac{cv+p}{2c} + 1; e^{2cz} \right) +$$

$$(-1)^v 2^{-m-u} n! e^{czv} \binom{u+v}{\frac{u+v}{2}} (1-(u+v) \bmod 2)$$

$$\sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^k \binom{m}{k} \left( e^{(ai(m-2k)+p)z - \frac{im\pi}{2}} \sum_{j=0}^n \frac{1}{(n-j)!} (-1)^j z^{n-j} (ai(m-2k)+p+cv)^{-j-1} {}_{j+2}F_{j+1} \left( \frac{ai(m-2k)+p+cv}{2c}, \dots, \right. \right.$$

$$\left. \left. \frac{ai(m-2k)+p+cv}{2c}, v; \frac{ai(m-2k)+p+cv}{2c} + 1, \dots, \frac{ai(m-2k)+p+cv}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{\frac{i\pi m}{2} + (p-ia(m-2k))z} \sum_{j=0}^n \frac{1}{(n-j)!} (-1)^j z^{n-j} (-ia(m-2k)+p+cv)^{-j-1} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+cv}{2c}, \right. \right.$$

$$\left. \left. \dots, \frac{-ia(m-2k)+p+cv}{2c}, v; \frac{-ia(m-2k)+p+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p+cv}{2c} + 1; e^{2cz} \right) \right) -$$

$$(-1)^v 2^{-m-u} n! e^{czv} \binom{m}{\frac{m}{2}} (m \bmod 2 - 1) \sum_{s=0}^{\lfloor \frac{u+v-1}{2} \rfloor} \binom{u+v}{s} \left( e^{(p+c(u+2v-2s))z} \sum_{j=0}^n \frac{1}{(n-j)!} (-1)^j z^{n-j} (p+c(u+2v-2s))^{-j-1} \right.$$

$${}_{j+2}F_{j+1} \left( \frac{p+c(u+2v-2s)}{2c}, \dots, \frac{p+c(u+2v-2s)}{2c}, v; \frac{p+c(u+2v-2s)}{2c} + 1, \dots, \right.$$

$$\left. \frac{p+c(u+2v-2s)}{2c} + 1; e^{2cz} \right) + e^{(p-c(u+2v-2s))z} \sum_{j=0}^n \frac{1}{(n-j)!} (-1)^j z^{n-j} (p-c(u-2s))^{-j-1}$$

$${}_{j+2}F_{j+1} \left( \frac{p-c(u-2s)}{2c}, \dots, \frac{p-c(u-2s)}{2c}, v; \frac{p-c(u-2s)}{2c} + 1, \dots, \frac{p-c(u-2s)}{2c} + 1; e^{2cz} \right) \Bigg) +$$

$$(-1)^v 2^{-m-u} n! e^{czv} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u+v-1}{2} \rfloor} (-1)^k \binom{m}{k} \binom{u+v}{s} \left( e^{(ai(m-2k)+p-c(u+2v-2s))z - \frac{im\pi}{2}} \sum_{j=0}^n \frac{1}{(n-j)!} (-1)^j z^{n-j} (ai(m-2k)+ \right.$$

$$\left. p-c(u-2s))^{-j-1} {}_{j+2}F_{j+1} \left( \frac{ai(m-2k)+p-c(u-2s)}{2c}, \dots, \frac{ai(m-2k)+p-c(u-2s)}{2c}, \right. \right.$$

$$\left. \left. v; \frac{ai(m-2k)+p-c(u-2s)}{2c} + 1, \dots, \frac{ai(m-2k)+p-c(u-2s)}{2c} + 1; e^{2cz} \right) + \right.$$

$$\left. e^{\frac{i\pi m}{2} + (-ia(m-2k)+p+c(u+2v-2s))z} \sum_{j=0}^n \frac{1}{(n-j)!} (-1)^j z^{n-j} (-ia(m-2k)+p+c(u+2v-2s))^{-j-1} \right.$$

$${}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+c(u+2v-2s)}{2c}, \dots, \frac{-ia(m-2k)+p+c(u+2v-2s)}{2c}, v; \right.$$

$$\left. \frac{-ia(m-2k)+p+c(u+2v-2s)}{2c} + 1, \dots, \frac{-ia(m-2k)+p+c(u+2v-2s)}{2c} + 1; e^{2cz} \right) \Bigg) +$$

$$\begin{aligned}
 & e^{\frac{i\pi m}{2} + (-ia(m-2k) + p - c(u+v-2s))z} \sum_{j=0}^n \frac{1}{(n-j)!} (-1)^j z^{n-j} (-ia(m-2k) + p - c(u-2s))^{-j-1} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k) + p - c(u-2s)}{2c}, \dots, \frac{-ia(m-2k) + p - c(u-2s)}{2c}, v; \right. \\
 & \left. \frac{-ia(m-2k) + p - c(u-2s)}{2c} + 1, \dots, \frac{-ia(m-2k) + p - c(u-2s)}{2c} + 1; e^{2cz} \right) + \\
 & e^{(ai(m-2k) + p + c(u+v-2s))z - \frac{im\pi}{2}} \sum_{j=0}^n \frac{1}{(n-j)!} (-1)^j z^{n-j} (ai(m-2k) + p + c(u+2v-2s))^{-j-1} \\
 & {}_{j+2}F_{j+1} \left( \frac{ai(m-2k) + p + c(u+2v-2s)}{2c}, \dots, \frac{ai(m-2k) + p + c(u+2v-2s)}{2c}, v; \right. \\
 & \left. \frac{ai(m-2k) + p + c(u+2v-2s)}{2c} + 1, \dots, \frac{ai(m-2k) + p + c(u+2v-2s)}{2c} + 1; \right. \\
 & \left. e^{2cz} \right) \Bigg|; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$

Involving cos, cosh, exp and power

### Involving $z^n e^{p z} \cos(a z) \cosh(b z) \coth^v(c z)$

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$$\begin{aligned}
 & \int z^n e^{p z} \cos(a z) \cosh(b z) \coth^v(c z) dz = \\
 & \frac{1}{4} (-1)^v \binom{v}{\frac{v}{2}} n! (1 - v \bmod 2) \left( e^{(-b-ia+p+cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-ia+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \right. \\
 & \left. \left( \frac{-ia-b+p+cv}{2c}, \dots, \frac{-ia-b+p+cv}{2c}, v; \frac{-ia-b+p+cv}{2c} + 1, \dots, \frac{-ia-b+p+cv}{2c} + 1; e^{2cz} \right) + \right. \\
 & e^{(-b+ia+p+cv)z} \sum_{j=0}^n \frac{(-1)^j (-b+ia+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b+p+cv}{2c}, \dots, \frac{ia-b+p+cv}{2c}, v; \right. \\
 & \left. \frac{ia-b+p+cv}{2c} + 1, \dots, \frac{ia-b+p+cv}{2c} + 1; e^{2cz} \right) + e^{(b-ia+p+cv)z} \sum_{j=0}^n \frac{(-1)^j (b-ia+p+cv)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia+b+p+cv}{2c}, \dots, \frac{-ia+b+p+cv}{2c}, v; \frac{-ia+b+p+cv}{2c} + 1, \dots, \frac{-ia+b+p+cv}{2c} + 1; e^{2cz} \right) + \\
 & e^{(b+ia+p+cv)z} \sum_{j=0}^n \frac{(-1)^j (b+ia+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b+p+cv}{2c}, \dots, \frac{ia+b+p+cv}{2c}, \right. \\
 & \left. v; \frac{ia+b+p+cv}{2c} + 1, \dots, \frac{ia+b+p+cv}{2c} + 1; e^{2cz} \right) \Bigg) +
 \end{aligned}$$



$$\begin{aligned}
 & \frac{1}{4} (-1)^v n! \sum_{i=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{i} \left( e^{(-b+2ci-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2ci-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia-b+2ci+p}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{-ia-b+2ci+p}{2c}, v; \frac{-ia-b+2ci+p}{2c} + 1, \dots, \frac{-ia-b+2ci+p}{2c} + 1; e^{2cz} \right) + \right. \\
 & e^{(b+2ci-ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2ci-ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia+b+2ci+p}{2c}, \dots, \right. \\
 & \quad \left. \frac{-ia+b+2ci+p}{2c}, v; \frac{-ia+b+2ci+p}{2c} + 1, \dots, \frac{-ia+b+2ci+p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(b+2ci+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (b+2ci+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b+2ci+p}{2c}, \dots, \right. \\
 & \quad \left. \frac{ia+b+2ci+p}{2c}, v; \frac{ia+b+2ci+p}{2c} + 1, \dots, \frac{ia+b+2ci+p}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-b+2ci+ia+p)z} \sum_{j=0}^n \frac{(-1)^j (-b+2ci+ia+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+2ci+p-b}{2c}, \dots, \right. \\
 & \quad \left. \frac{ia+2ci+p-b}{2c}, v; \frac{ia+2ci+p-b}{2c} + 1, \dots, \frac{ia+2ci+p-b}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-b-2ci-ia+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci-ia+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1} \left( \frac{-ia-b-2ci+p+2cv}{2c}, \dots, \frac{-ia-b-2ci+p+2cv}{2c}, v; \right. \\
 & \quad \left. \frac{-ia-b-2ci+p+2cv}{2c} + 1, \dots, \frac{-ia-b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) + \\
 & e^{(-b-2ci+ia+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (-b-2ci+ia+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia-b-2ci+p+2cv}{2c}, \right. \\
 & \quad \left. \dots, \frac{ia-b-2ci+p+2cv}{2c}, v; \frac{ia-b-2ci+p+2cv}{2c} + 1, \dots, \frac{ia-b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) + \\
 & e^{(b-2ci-ia+p+2cv)z} \sum_{j=0}^n \frac{(-1)^j (b-2ci-ia+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia+b-2ci+p+2cv}{2c}, \right. \\
 & \quad \left. \dots, \frac{-ia+b-2ci+p+2cv}{2c}, v; \frac{-ia+b-2ci+p+2cv}{2c} + 1, \right. \\
 & \quad \left. \dots, \frac{-ia+b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) + e^{(b-2ci+ia+p+2cv)z} \\
 & \sum_{j=0}^n \frac{(-1)^j (b-2ci+ia+p+2cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia+b-2ci+p+2cv}{2c}, \dots, \frac{ia+b-2ci+p+2cv}{2c}, \right. \\
 & \quad \left. v; \frac{ia+b-2ci+p+2cv}{2c} + 1, \dots, \frac{ia+b-2ci+p+2cv}{2c} + 1; e^{2cz} \right) \Big/ ; n \in \mathbb{N} \wedge v \in \mathbb{N}^+
 \end{aligned}$$

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$$\int z^n e^{p z} \cos(a z) \cosh(c z) \coth^v(c z) dz =$$

$$\frac{1}{4} (-1)^v e^{c v z} \binom{v+1}{\frac{v+1}{2}} n! (1 - (v+1) \bmod 2) \left( e^{(-i a+p) z} \sum_{j=0}^n \frac{(-1)^j (-i a+p+c v)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ \left. {}_{j+2}F_{j+1} \left( \frac{-i a+p+c v}{2 c}, \dots, \frac{-i a+p+c v}{2 c}, v; \frac{-i a+p+c v}{2 c} + 1, \dots, \frac{-i a+p+c v}{2 c} + 1; e^{2 c z} \right) + \right. \\ \left. e^{(i a+p) z} \sum_{j=0}^n \frac{(-1)^j (i a+p+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i a+p+c v}{2 c}, \dots, \frac{i a+p+c v}{2 c}, v; \right. \right. \\ \left. \left. \frac{i a+p+c v}{2 c} + 1, \dots, \frac{i a+p+c v}{2 c} + 1; e^{2 c z} \right) \right) + \frac{1}{4} (-1)^v e^{c v z} n! \\ \sum_{s=0}^{\lfloor \frac{v}{2} \rfloor} \binom{v+1}{s} \left( e^{(-i a+p-c(-2 s+v+1)) z} \sum_{j=0}^n \frac{(-1)^j (-i a+p-c(1-2 s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i a+p-c(1-2 s)}{2 c}, \right. \right. \\ \left. \left. \dots, \frac{-i a+p-c(1-2 s)}{2 c}, v; \frac{-i a+p-c(1-2 s)}{2 c} + 1, \dots, \frac{-i a+p-c(1-2 s)}{2 c} + 1; e^{2 c z} \right) + \right. \\ \left. e^{(i a+p-c(-2 s+v+1)) z} \sum_{j=0}^n \frac{(-1)^j (i a+p-c(1-2 s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{i a+p-c(1-2 s)}{2 c}, \dots, \right. \right. \\ \left. \left. \frac{i a+p-c(1-2 s)}{2 c}, v; \frac{i a+p-c(1-2 s)}{2 c} + 1, \dots, \frac{i a+p-c(1-2 s)}{2 c} + 1; e^{2 c z} \right) + \right. \\ \left. e^{(-i a+p+c(-2 s+v+1)) z} \sum_{j=0}^n \frac{(-1)^j (-i a+p+c(-2 s+2 v+1))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-i a+p+c(-2 s+2 v+1)}{2 c}, \dots, \right. \right. \\ \left. \left. \frac{-i a+p+c(-2 s+2 v+1)}{2 c}, v; \frac{-i a+p+c(-2 s+2 v+1)}{2 c} + 1, \dots, \frac{-i a+p+c(-2 s+2 v+1)}{2 c} + 1; \right. \right. \\ \left. \left. e^{2 c z} \right) + e^{(i a+p+c(-2 s+v+1)) z} \sum_{j=0}^n \frac{(-1)^j (i a+p+c(-2 s+2 v+1))^{-j-1} z^{n-j}}{(n-j)!} \right. \\ \left. {}_{j+2}F_{j+1} \left( \frac{i a+p+c(-2 s+2 v+1)}{2 c}, \dots, \frac{i a+p+c(-2 s+2 v+1)}{2 c}, v; \right. \right. \\ \left. \left. \frac{i a+p+c(-2 s+2 v+1)}{2 c} + 1, \dots, \frac{i a+p+c(-2 s+2 v+1)}{2 c} + 1; e^{2 c z} \right) \right) / ; n \in \mathbb{N} \wedge v \in \mathbb{N}^+$$

Involving powers of cos, powers of cosh, exp and power

Involving  $z^n e^{p z} \cos^m(a z) \cosh^u(b z) \coth^v(c z)$

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$$\int z^n e^{p z} \cos^m(a z) \cosh^u(b z) \coth^v(c z) dz =$$

$$\begin{aligned} & (-1)^v 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1-m \bmod 2) (1-u \bmod 2) \left( e^{(p+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad {}_{j+2}F_{j+1} \left( \frac{p+cv}{2c}, \dots, \frac{p+cv}{2c}, v; \frac{p+cv}{2c} + 1, \dots, \frac{p+cv}{2c} + 1; e^{2cz} \right) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs)z} \right. \\ & \quad \sum_{j=0}^n \frac{(-1)^j (p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs}{2c}, \dots, \frac{p+2cs}{2c}, v; \frac{p+2cs}{2c} + 1, \dots, \frac{p+2cs}{2c} + 1; e^{2cz} \right) + \\ & \quad \left. e^{(p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2c(v-s)}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{p+2c(v-s)}{2c}, v; \frac{p+2c(v-s)}{2c} + 1, \dots, \frac{p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) + \\ & (-1)^v 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1-u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(-ia(m-2k)+p+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \right. \\ & \quad \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+cv}{2c}, \dots, \right. \\ & \quad \left. \frac{-ia(m-2k)+p+cv}{2c}, v; \frac{-ia(m-2k)+p+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p+cv}{2c} + 1; e^{2cz} \right) + \\ & \quad e^{(ia(m-2k)+p+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ia(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+cv}{2c}, \right. \\ & \quad \left. \dots, \frac{ia(m-2k)+p+cv}{2c}, v; \frac{ia(m-2k)+p+cv}{2c} + 1, \dots, \frac{ia(m-2k)+p+cv}{2c} + 1; e^{2cz} \right) + \\ & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2cs}{2c}, \dots, \right. \right. \\ & \quad \left. \left. \frac{-ia(m-2k)+p+2cs}{2c}, v; \frac{-ia(m-2k)+p+2cs}{2c} + 1, \dots, \frac{-ia(m-2k)+p+2cs}{2c} + 1; \right. \right. \\ & \quad \left. \left. e^{2cz} \right) + e^{(-ia(m-2k)+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & \quad \left. {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+2c(v-s)}{2c}, \dots, \frac{-ia(m-2k)+p+2c(v-s)}{2c}, v; \right. \right. \\ & \quad \left. \left. \frac{-ia(m-2k)+p+2c(v-s)}{2c} + 1, \dots, \frac{-ia(m-2k)+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) \Bigg) + \end{aligned}$$

$$\begin{aligned}
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(ai(m-2k)+p+2cs)z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+2cs}{2c}, \dots, \right. \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p+2cs}{2c}, v; \frac{ia(m-2k)+p+2cs}{2c} + 1, \dots, \frac{ia(m-2k)+p+2cs}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(ai(m-2k)+p+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \quad \left( \frac{ia(m-2k)+p+2c(v-s)}{2c}, \dots, \frac{ia(m-2k)+p+2c(v-s)}{2c}, v; \right. \\
 & \quad \left. \left. \frac{ia(m-2k)+p+2c(v-s)}{2c} + 1, \dots, \frac{ia(m-2k)+p+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & (-1)^v 2^{-m-u} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \binom{u}{i} \left( e^{(p-b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \right. \\
 & \quad \sum_{j=0}^n \frac{(-1)^j (p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p-b(u-2i)+cv}{2c}, \dots, \right. \\
 & \quad \left. \frac{p-b(u-2i)+cv}{2c}, v; \frac{p-b(u-2i)+cv}{2c} + 1, \dots, \frac{p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\
 & \quad e^{(p+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+b(u-2i)+cv}{2c}, \right. \\
 & \quad \left. \dots, \frac{p+b(u-2i)+cv}{2c}, v; \frac{p+b(u-2i)+cv}{2c} + 1, \dots, \frac{p+b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\
 & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs-b(u-2i)}{2c}, \right. \right. \\
 & \quad \left. \left. \dots, \frac{p+2cs-b(u-2i)}{2c}, v; \frac{p+2cs-b(u-2i)}{2c} + 1, \dots, \frac{p+2cs-b(u-2i)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad e^{(p-b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\
 & \quad \left( \frac{p-b(u-2i)+2c(v-s)}{2c}, \dots, \frac{p-b(u-2i)+2c(v-s)}{2c}, v; \right. \\
 & \quad \left. \left. \frac{p-b(u-2i)+2c(v-s)}{2c} + 1, \dots, \frac{p-b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \right) + \\
 & \quad \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(p+2cs+b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{p+2cs+b(u-2i)}{2c}, \right. \right.
 \end{aligned}$$

$$\begin{aligned} & \dots, \frac{p+2cs+b(u-2i)}{2c}, v; \frac{p+2cs+b(u-2i)}{2c} + 1, \dots, \frac{p+2cs+b(u-2i)}{2c} + 1; e^{2cz} \Big) + \\ & e^{(p+b(u-2i)+2c(v-s))z} \sum_{j=0}^n \frac{(-1)^j (p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1} \\ & \left( \frac{p+b(u-2i)+2c(v-s)}{2c}, \dots, \frac{p+b(u-2i)+2c(v-s)}{2c}, v; \frac{p+b(u-2i)+2c(v-s)}{2c} + 1, \right. \\ & \left. \dots, \frac{p+b(u-2i)+2c(v-s)}{2c} + 1; e^{2cz} \right) \Big) + (-1)^v 2^{-m-u} n! \end{aligned}$$

$$\begin{aligned} & \sum_{i=0}^{\lfloor \frac{u-1}{2} \rfloor} \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \binom{u}{i} \left( e^{(-ia(m-2k)+p-b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \right. \\ & {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c}, \dots, \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c}, v; \right. \\ & \left. \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\ & e^{(ai(m-2k)+p-b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \\ & {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p-b(u-2i)+cv}{2c}, \dots, \frac{ia(m-2k)+p-b(u-2i)+cv}{2c}, v; \right. \\ & \left. \frac{ia(m-2k)+p-b(u-2i)+cv}{2c} + 1, \dots, \frac{ia(m-2k)+p-b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\ & e^{(-ia(m-2k)+p+b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \\ & {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c}, \dots, \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c}, v; \right. \\ & \left. \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c} + 1, \dots, \frac{-ia(m-2k)+p+b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\ & e^{(ai(m-2k)+p+b(u-2i)+cv)z} \left( \frac{v}{2} \right) (1-v \bmod 2) \sum_{j=0}^n \frac{(-1)^j (ai(m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!} \\ & {}_{j+2}F_{j+1} \left( \frac{ia(m-2k)+p+b(u-2i)+cv}{2c}, \dots, \frac{ia(m-2k)+p+b(u-2i)+cv}{2c}, v; \right. \\ & \left. \frac{ia(m-2k)+p+b(u-2i)+cv}{2c} + 1, \dots, \frac{ia(m-2k)+p+b(u-2i)+cv}{2c} + 1; e^{2cz} \right) + \\ & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-ia(m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^n \frac{(-1)^j (-ia(m-2k)+p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \end{aligned}$$

$$\begin{aligned}
 & {}_{j+2}F_{j+1}\left(\frac{-i a(m-2 k)+p+2 c s-b(u-2 i)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{-i a(m-2 k)+p+2 c s-b(u-2 i)}{2 c}, v ; \frac{-i a(m-2 k)+p+2 c s-b(u-2 i)}{2 c}+1, \right. \\
 & \quad \left. \dots, \frac{-i a(m-2 k)+p+2 c s-b(u-2 i)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & e^{(-i a(m-2 k)+p-b(u-2 i)+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j(-i a(m-2 k)+p-b(u-2 i)+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1}\left(\frac{-i a(m-2 k)+p-b(u-2 i)+2 c(v-s)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{-i a(m-2 k)+p-b(u-2 i)+2 c(v-s)}{2 c}, v ; \frac{-i a(m-2 k)+p-b(u-2 i)+2 c(v-s)}{2 c}+1, \right. \\
 & \quad \left. \dots, \frac{-i a(m-2 k)+p-b(u-2 i)+2 c(v-s)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(a i(m-2 k)+p+2 c s-b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j(a i(m-2 k)+p+2 c s-b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1}\left(\frac{i a(m-2 k)+p+2 c s-b(u-2 i)}{2 c}, \dots, \frac{i a(m-2 k)+p+2 c s-b(u-2 i)}{2 c}, v ; \right. \\
 & \quad \left. \frac{i a(m-2 k)+p+2 c s-b(u-2 i)}{2 c}+1, \dots, \frac{i a(m-2 k)+p+2 c s-b(u-2 i)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & e^{(a i(m-2 k)+p-b(u-2 i)+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j(a i(m-2 k)+p-b(u-2 i)+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2}F_{j+1}\left(\frac{i a(m-2 k)+p-b(u-2 i)+2 c(v-s)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{i a(m-2 k)+p-b(u-2 i)+2 c(v-s)}{2 c}, v ; \frac{i a(m-2 k)+p-b(u-2 i)+2 c(v-s)}{2 c}+1, \right. \\
 & \quad \left. \dots, \frac{i a(m-2 k)+p-b(u-2 i)+2 c(v-s)}{2 c}+1 ; e^{2 c z}\right)+ \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(-i a(m-2 k)+p+2 c s+b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j(-i a(m-2 k)+p+2 c s+b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1}\left(\frac{-i a(m-2 k)+p+2 c s+b(u-2 i)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{-i a(m-2 k)+p+2 c s+b(u-2 i)}{2 c}, v ; \frac{-i a(m-2 k)+p+2 c s+b(u-2 i)}{2 c}+1, \right. \\
 & \quad \left. \dots, \frac{-i a(m-2 k)+p+2 c s+b(u-2 i)}{2 c}+1 ; e^{2 c z}\right)+
 \end{aligned}$$

$$\begin{aligned}
 & e^{(-i a(m-2 k)+p+b(u-2 i)+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j (-i a(m-2 k)+p+b(u-2 i)+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2} F_{j+1} \left( \frac{-i a(m-2 k)+p+b(u-2 i)+2 c(v-s)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{-i a(m-2 k)+p+b(u-2 i)+2 c(v-s)}{2 c}, v; \frac{-i a(m-2 k)+p+b(u-2 i)+2 c(v-s)}{2 c} + 1, \right. \\
 & \quad \left. \dots, \frac{-i a(m-2 k)+p+b(u-2 i)+2 c(v-s)}{2 c} + 1; e^{2 c z} \right) + \\
 & \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} \binom{v}{s} \left( e^{(a i(m-2 k)+p+2 c s+b(u-2 i)) z} \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)+p+2 c s+b(u-2 i))^{-j-1} z^{n-j}}{(n-j)!} \right. \\
 & {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)+p+2 c s+b(u-2 i)}{2 c}, \dots, \frac{i a(m-2 k)+p+2 c s+b(u-2 i)}{2 c}, v; \right. \\
 & \quad \left. \frac{i a(m-2 k)+p+2 c s+b(u-2 i)}{2 c} + 1, \dots, \frac{i a(m-2 k)+p+2 c s+b(u-2 i)}{2 c} + 1; e^{2 c z} \right) + \\
 & e^{(a i(m-2 k)+p+b(u-2 i)+2 c(v-s)) z} \sum_{j=0}^n \frac{(-1)^j (a i(m-2 k)+p+b(u-2 i)+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} \\
 & {}_{j+2} F_{j+1} \left( \frac{i a(m-2 k)+p+b(u-2 i)+2 c(v-s)}{2 c}, \dots, \frac{i a(m-2 k)+p+b(u-2 i)+2 c(v-s)}{2 c}, \right. \\
 & \quad \left. v; \frac{i a(m-2 k)+p+b(u-2 i)+2 c(v-s)}{2 c} + 1, \dots, \right. \\
 & \quad \left. \left. \frac{i a(m-2 k)+p+b(u-2 i)+2 c(v-s)}{2 c} + 1; e^{2 c z} \right) \right) / ; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$

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$$\int z^n e^{p z} \cos^m(a z) \cosh^u(c z) \coth^v(c z) dz = (-1)^v 2^{-m-u} e^{(p+c v) z} \binom{m}{\frac{m}{2}} \binom{u+v}{\frac{u+v}{2}} n! (1-m \bmod 2) (1-(u+v) \bmod 2)$$

$$\begin{aligned}
 & \sum_{j=0}^n \frac{(-1)^j z^{n-j} (p+c v)^{-j-1}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{p+c v}{2 c}, \dots, \frac{p+c v}{2 c}, v; \frac{p+c v}{2 c} + 1, \dots, \frac{p+c v}{2 c} + 1; e^{2 c z} \right) - \\
 & (-1)^v 2^{-m-u} n! e^{c z v} \binom{m}{\frac{m}{2}} (m \bmod 2 - 1) \sum_{s=0}^{\lfloor \frac{u+v-1}{2} \rfloor} \binom{u+v}{s} \left( e^{(p-c(-2 s+u+v)) z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (p+c v-c(-2 s+u+v))^{-j-1}}{(n-j)!} \right. \\
 & {}_{j+2} F_{j+1} \left( \frac{p-c(u-2 s)}{2 c}, \dots, \frac{p-c(u-2 s)}{2 c}, v; \frac{p-c(u-2 s)}{2 c} + 1, \dots, \frac{p-c(u-2 s)}{2 c} + 1; e^{2 c z} \right) + \\
 & e^{(p+c(-2 s+u+v)) z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (p+c v+c(-2 s+u+v))^{-j-1}}{(n-j)!} {}_{j+2} F_{j+1} \left( \frac{p+c(-2 s+u+2 v)}{2 c}, \dots, \right. \\
 & \quad \left. \frac{p+c(-2 s+u+2 v)}{2 c}, v; \frac{p+c(-2 s+u+2 v)}{2 c} + 1, \dots, \frac{p+c(-2 s+u+2 v)}{2 c} + 1; e^{2 c z} \right) \Big) +
 \end{aligned}$$

$$\begin{aligned}
 & (-1)^v 2^{-m-u} e^{czv} \binom{u+v}{\frac{u+v}{2}} n! (1 - (u+v) \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \left( e^{(p-ia(m-2k))z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (-ia(m-2k) + p + cv)^{-j-1}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k) + p + cv}{2c}, \dots, \frac{-ia(m-2k) + p + cv}{2c}, v; \right. \\
 & \quad \left. \left. \frac{-ia(m-2k) + p + cv}{2c} + 1, \dots, \frac{-ia(m-2k) + p + cv}{2c} + 1; e^{2cz} \right) + e^{(ai(m-2k)+p)z} \right. \\
 & \quad \left. \sum_{j=0}^n \frac{(-1)^j z^{n-j} (ai(m-2k) + p + cv)^{-j-1}}{(n-j)!} {}_{j+2}F_{j+1} \left( \frac{ia(m-2k) + p + cv}{2c}, \dots, \frac{ia(m-2k) + p + cv}{2c}, \right. \right. \\
 & \quad \left. \left. v; \frac{ia(m-2k) + p + cv}{2c} + 1, \dots, \frac{ia(m-2k) + p + cv}{2c} + 1; e^{2cz} \right) \right) + (-1)^v 2^{-m-u} e^{czv} n! \\
 & \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \sum_{s=0}^{\lfloor \frac{u+v-1}{2} \rfloor} \binom{m}{k} \binom{u+v}{s} \left( e^{(-ia(m-2k)+p-c(-2s+u+v))z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (-ia(m-2k) + p + cv - c(-2s+u+v))^{-j-1}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k) + p - c(u-2s)}{2c}, \dots, \frac{-ia(m-2k) + p - c(u-2s)}{2c}, v; \right. \\
 & \quad \left. \left. \frac{-ia(m-2k) + p - c(u-2s)}{2c} + 1, \dots, \frac{-ia(m-2k) + p - c(u-2s)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(ai(m-2k)+p-c(-2s+u+v))z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (ai(m-2k) + p + cv - c(-2s+u+v))^{-j-1}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{ia(m-2k) + p - c(u-2s)}{2c}, \dots, \frac{ia(m-2k) + p - c(u-2s)}{2c}, v; \right. \\
 & \quad \left. \left. \frac{ia(m-2k) + p - c(u-2s)}{2c} + 1, \dots, \frac{ia(m-2k) + p - c(u-2s)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(-ia(m-2k)+p+c(-2s+u+v))z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (-ia(m-2k) + p + cv + c(-2s+u+v))^{-j-1}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{-ia(m-2k) + p + c(-2s+u+2v)}{2c}, \dots, \frac{-ia(m-2k) + p + c(-2s+u+2v)}{2c}, v; \right. \\
 & \quad \left. \left. \frac{-ia(m-2k) + p + c(-2s+u+2v)}{2c} + 1, \dots, \frac{-ia(m-2k) + p + c(-2s+u+2v)}{2c} + 1; e^{2cz} \right) + \right. \\
 & \quad \left. e^{(ai(m-2k)+p+c(-2s+u+v))z} \sum_{j=0}^n \frac{(-1)^j z^{n-j} (ai(m-2k) + p + cv + c(-2s+u+v))^{-j-1}}{(n-j)!} \right. \\
 & \quad {}_{j+2}F_{j+1} \left( \frac{ia(m-2k) + p + c(-2s+u+2v)}{2c}, \dots, \frac{ia(m-2k) + p + c(-2s+u+2v)}{2c}, \right. \\
 & \quad \left. v; \frac{ia(m-2k) + p + c(-2s+u+2v)}{2c} + 1, \dots, \right. \\
 & \quad \left. \left. \frac{ia(m-2k) + p + c(-2s+u+2v)}{2c} + 1; e^{2cz} \right) \right) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+ \wedge v \in \mathbb{N}^+
 \end{aligned}$$



## Definite integration

### For the direct function itself

01.22.21.0009.01

$$\int_a^b \coth(t) dt = \log(\sinh(b)) - \log(\sinh(a))$$

### Involving the direct function

01.22.21.0010.01

$$\int_0^\infty t e^{-t} \coth(t) dt = \frac{1}{4}(\pi^2 - 4)$$

01.22.21.0011.01

$$\int_0^\infty t^2 e^{-t} \coth(t) dt = \frac{7 \zeta(3)}{2} - 2$$

### Involving related functions

01.22.21.0012.01

$$\int_0^{\frac{\pi}{4}} \log(\coth(t)) dt = \frac{1}{12} \left( -3 \log^2 \left( \coth \left( \frac{\pi}{4} \right) \right) + 6 \log \left( \coth \left( \frac{\pi}{4} \right) + 1 \right) \log \left( \coth \left( \frac{\pi}{4} \right) \right) + \pi^2 + 6 \operatorname{Li}_2 \left( 1 - \coth \left( \frac{\pi}{4} \right) \right) - 6 \operatorname{Li}_2 \left( -\tanh \left( \frac{\pi}{4} \right) \right) \right)$$

## Summation

### Finite summation

01.22.23.0001.01

$$\sum_{k=0}^{n-1} \coth^2 \left( \frac{i \pi k}{n} + z \right) = \coth^2(nz) n^2 - n^2 + n; n \in \mathbb{N}^+$$

## Representations through more general functions

### Through hypergeometric functions

01.22.26.0008.01

$$\coth(z) = \frac{2}{z} {}_3F_2 \left( 1, -\frac{iz}{\pi}, \frac{iz}{\pi}; 1 - \frac{iz}{\pi}, \frac{iz}{\pi} + 1; 1 \right) - \frac{1}{z}$$

Brychkov Yu.A. (2005)

### Through other functions

#### Involving Jacobi functions

01.22.26.0001.01

$$\coth(z) = i \operatorname{cs}(iz | 0)$$

01.22.26.0002.01

$$\coth(z) = \operatorname{ns}(z | 1)$$

01.22.26.0003.01

$$\operatorname{coth}(z) = i \operatorname{sc}\left(\frac{\pi}{2} - iz \mid 0\right)$$

01.22.26.0004.01

$$\operatorname{coth}(z) = -\operatorname{sn}\left(\frac{\pi i}{2} - z \mid 1\right)$$

### Involving Mathieu functions

01.22.26.0005.01

$$\operatorname{coth}(\sqrt{a} z) = i \frac{\operatorname{Ce}(a, 0, iz)}{\operatorname{Se}(a, 0, iz)}$$

01.22.26.0006.01

$$\operatorname{coth}(\sqrt{a} z) = -i \frac{\operatorname{Se}_z(a, 0, iz)}{\operatorname{Ce}_z(a, 0, iz)}$$

### Involving some elliptic-type functions

01.22.26.0007.01

$$\operatorname{coth}(z) = \frac{\sqrt{z^2}}{z} \sqrt{\frac{2}{3}} \sqrt{1 - \varphi\left(i \sqrt{\frac{2}{3}} z; 3, 1\right)} \quad ; i z \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

## Representations through equivalent functions

### With inverse function

01.22.27.0001.01

$$\operatorname{coth}(\operatorname{coth}^{-1}(z)) = z$$

01.22.27.0002.01

$$\operatorname{coth}(n \operatorname{coth}^{-1}(z)) = \frac{(z-1)^n + (z+1)^n}{(z+1)^n - (z-1)^n} \quad ; n \in \mathbb{N}^+$$

01.22.27.0003.02

$$\operatorname{coth}^{-1}(\operatorname{coth}(z)) = z \quad ; -\frac{\pi}{2} < \operatorname{Im}(z) < \frac{\pi}{2} \vee \left(\operatorname{Im}(z) = -\frac{\pi}{2} \wedge \operatorname{Re}(z) > 0\right) \vee \left(\operatorname{Im}(z) = \frac{\pi}{2} \wedge \operatorname{Re}(z) \leq 0\right)$$

01.22.27.0083.01

$$\operatorname{coth}^{-1}(\operatorname{coth}(z)) = z + \pi i \quad ; -\frac{3\pi}{2} < \operatorname{Im}(z) < -\frac{\pi}{2} \vee \operatorname{Im}(z) = -\frac{3\pi}{2} \wedge \operatorname{Re}(z) > 0 \vee \operatorname{Im}(z) = -\frac{\pi}{2} \wedge \operatorname{Re}(z) \leq 0$$

01.22.27.0084.01

$$\operatorname{coth}^{-1}(\operatorname{coth}(z)) = z - \pi i \quad ; \frac{\pi}{2} < \operatorname{Im}(z) < \frac{3\pi}{2} \vee \operatorname{Im}(z) = \frac{\pi}{2} \wedge \operatorname{Re}(z) > 0 \vee \operatorname{Im}(z) = \frac{3\pi}{2} \wedge \operatorname{Re}(z) \leq 0$$

01.22.27.0085.01

$$\operatorname{coth}^{-1}(\operatorname{coth}(z)) = z - \pi i k \quad ; \left(k\pi - \frac{\pi}{2} < \operatorname{Im}(z) < \pi k + \frac{\pi}{2} \vee \operatorname{Im}(z) = k\pi - \frac{\pi}{2} \wedge \operatorname{Re}(z) > 0 \vee \operatorname{Im}(z) = \pi k + \frac{\pi}{2} \wedge \operatorname{Re}(z) \leq 0\right) \wedge k \in \mathbb{Z}$$

01.22.27.0004.01

$$\coth^{-1}(\coth(z)) = z - i\pi \left[ \frac{\operatorname{Im}(z)}{\pi} + \frac{1}{2} \right] + \frac{\pi i}{2} \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} - \frac{1}{2} \rfloor + \lfloor \frac{1}{2} - \frac{\operatorname{Im}(z)}{\pi} \rfloor} \right) \theta(-\operatorname{Re}(z)); \frac{iz}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.22.27.0086.01

$$\coth^{-1}(\coth(z)) = \begin{cases} z - \pi i \left\lfloor \frac{2\operatorname{Im}(z) - \pi}{2\pi} \right\rfloor & \frac{2\operatorname{Im}(z) + \pi}{2\pi} \in \mathbb{Z} \wedge \operatorname{Re}(z) \leq 0 \\ z - \pi i \left\lfloor \frac{2\operatorname{Im}(z) + \pi}{2\pi} \right\rfloor & \text{True} \end{cases}$$

## With related functions

### Involving exp

01.22.27.0005.01

$$\coth(z) = \frac{e^z + e^{-z}}{e^z - e^{-z}}$$

01.22.27.0006.01

$$\coth(z) = \frac{2}{e^{2z} - 1} + 1$$

### Involving sin

01.22.27.0007.01

$$\coth(z) = \frac{i \sin\left(\frac{\pi}{2} - iz\right)}{\sin(iz)}$$

01.22.27.0008.01

$$\coth(z) = \frac{i \sin\left(\frac{\pi}{2} + iz\right)}{\sin(iz)}$$

01.22.27.0009.01

$$\coth(z) = \frac{i \sqrt{1 - \sin^2(iz)}}{\sin(iz)} \quad /; |\operatorname{Im}(z)| < \frac{\pi}{2}$$

01.22.27.0010.01

$$\coth(z) = \frac{i \sqrt{1 - \sin^2(iz)}}{\sin(iz)} (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} + \frac{1}{2} \rfloor} \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} + \frac{1}{2} \rfloor + \lfloor -\frac{\operatorname{Im}(z)}{\pi} - \frac{1}{2} \rfloor} \right) \theta(\operatorname{Re}(z)) \right)$$

01.22.27.0011.01

$$\coth^2(z) = \frac{\sin^2(iz) - 1}{\sin^2(iz)}$$

### Involving cos

01.22.27.0012.01

$$\coth(z) = \frac{i \cos(iz)}{\cos\left(\frac{\pi}{2} - iz\right)}$$

01.22.27.0013.01

$$\coth(z) = -\frac{i \cos(iz)}{\cos\left(\frac{\pi}{2} + iz\right)}$$

01.22.27.0014.01

$$\operatorname{coth}(z) = \frac{\sqrt{z^2}}{z} \frac{\cos(iz)}{\sqrt{\cos^2(iz) - 1}} \quad ; \quad |\operatorname{Im}(z)| < \frac{\pi}{2}$$

01.22.27.0015.01

$$\operatorname{coth}(z) = \frac{\sqrt{-z^2}}{z} \frac{\cos(iz)}{\sqrt{1 - \cos^2(iz)}} \quad ; \quad |\operatorname{Im}(z)| < \pi$$

01.22.27.0016.01

$$\operatorname{coth}(z) = -\frac{i \cos(iz)}{\sqrt{1 - \cos^2(iz)}} \quad ; \quad 0 < \operatorname{Im}(z) < \pi$$

01.22.27.0017.01

$$\operatorname{coth}(z) = \frac{i \cos(iz)}{\sqrt{1 - \cos^2(iz)}} \left( (-1)^{\lfloor -\frac{\operatorname{Im}(z)}{\pi} \rfloor} \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} \rfloor + \lfloor -\frac{\operatorname{Im}(z)}{\pi} \rfloor \right) \theta(-\operatorname{Re}(z)) \right) \right)$$

01.22.27.0018.01

$$\operatorname{coth}^2(z) = \frac{\cos^2(iz)}{\cos^2(iz) - 1}$$

### Involving tan

01.22.27.0019.01

$$\operatorname{coth}(z) = i \tan\left(\frac{\pi}{2} - iz\right)$$

01.22.27.0020.01

$$\operatorname{coth}(z) = -i \tan\left(iz + \frac{\pi}{2}\right)$$

01.22.27.0021.01

$$\operatorname{coth}(z) = \frac{i}{\tan(iz)}$$

### Involving cot

01.22.27.0022.01

$$\operatorname{coth}(z) = i \cot(iz)$$

01.22.27.0023.01

$$\operatorname{coth}(iz) = -i \cot(z)$$

### Involving csc

01.22.27.0024.01

$$\operatorname{coth}(z) = \frac{i \csc(iz)}{\csc\left(\frac{\pi}{2} - iz\right)}$$

01.22.27.0025.01

$$\operatorname{coth}(z) = \frac{i \csc(iz)}{\csc\left(\frac{\pi}{2} + iz\right)}$$

01.22.27.0026.01

$$\operatorname{coth}(z) = i e^z \csc(i z) - 1$$

01.22.27.0027.01

$$\operatorname{coth}(z) = i e^{-z} \csc(i z) + 1$$

01.22.27.0028.01

$$\operatorname{coth}(z) = \frac{i \csc(i z)}{\csc\left(\frac{\pi}{2} - i z\right)}$$

01.22.27.0029.01

$$\operatorname{coth}(z) = \frac{\sqrt{z^2}}{z} \sqrt{1 - \csc^2(i z)} \quad /; \operatorname{Re}(z) \neq 0$$

01.22.27.0030.01

$$\operatorname{coth}(z) = -i \sqrt{\csc^2(i z) - 1} \quad /; 0 < \operatorname{Im}(z) < \frac{\pi}{2}$$

01.22.27.0031.01

$$\operatorname{coth}(z) = -z \sqrt{-\frac{1}{z^2}} \sqrt{\csc^2(i z) - 1} \quad /; |\operatorname{Im}(z)| < \frac{\pi}{2}$$

01.22.27.0032.01

$$\operatorname{coth}(z) = i \sqrt{\csc^2(i z) - 1} (-1)^{\lfloor -\frac{2\operatorname{Im}(z)}{\pi} \rfloor} \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} - \frac{1}{2} \rfloor + \lfloor \frac{1}{2} - \frac{\operatorname{Im}(z)}{\pi} \rfloor} \right) \theta(-\operatorname{Re}(z)) \right) \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} \rfloor + \lfloor -\frac{\operatorname{Im}(z)}{\pi} \rfloor} \right) \theta(\operatorname{Re}(z)) \right)$$

01.22.27.0033.01

$$\operatorname{coth}^2(z) = 1 - \csc^2(i z)$$

### Involving sec

01.22.27.0034.01

$$\operatorname{coth}(z) = \frac{i \sec\left(\frac{\pi}{2} - i z\right)}{\sec(i z)}$$

01.22.27.0035.01

$$\operatorname{coth}(z) = -\frac{i \sec\left(\frac{\pi}{2} + i z\right)}{\sec(i z)}$$

01.22.27.0036.01

$$\operatorname{coth}(z) = i e^z \sec\left(\frac{\pi}{2} - i z\right) - 1$$

01.22.27.0037.01

$$\operatorname{coth}(z) = i e^{-z} \sec\left(\frac{\pi}{2} - i z\right) + 1$$

01.22.27.0038.01

$$\operatorname{coth}(z) = \frac{\sqrt{z^2}}{z} \frac{1}{\sqrt{1 - \sec^2(i z)}} \quad /; \operatorname{Re}(z) \neq 0$$

01.22.27.0039.01

$$\operatorname{coth}(z) = -\frac{i}{\sqrt{\sec^2(iz) - 1}} \quad ; 0 < \operatorname{Im}(z) < \frac{\pi}{2}$$

01.22.27.0040.01

$$\operatorname{coth}(z) = \frac{\sqrt{-z^2}}{z} \frac{1}{\sqrt{\sec^2(iz) - 1}} \quad ; |\operatorname{Im}(z)| < \frac{\pi}{2}$$

01.22.27.0041.01

$$\operatorname{coth}(z) = \frac{i}{\sqrt{\sec^2(iz) - 1}} \left( (-1)^{\lfloor -\frac{2\operatorname{Im}(z)}{\pi} \rfloor} \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} \rfloor + \lfloor -\frac{\operatorname{Im}(z)}{\pi} \rfloor \right) \theta(-\operatorname{Re}(z)) \right) \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} - \frac{1}{2} \rfloor + \lfloor \frac{1}{2} - \frac{\operatorname{Im}(z)}{\pi} \rfloor \right) \theta(\operatorname{Re}(z)) \right) \right)$$

01.22.27.0042.01

$$\operatorname{coth}^2(z) = \frac{1}{1 - \sec^2(iz)}$$

01.22.27.0043.01

$$\operatorname{coth}^2(z) = 1 - \sec^2\left(\frac{\pi}{2} - iz\right)$$

### Involving sinh

01.22.27.0044.01

$$\operatorname{coth}(z) = -\frac{i \sinh\left(\frac{i\pi}{2} - z\right)}{\sinh(z)}$$

01.22.27.0045.01

$$\operatorname{coth}(z) = -\frac{i \sinh\left(\frac{i\pi}{2} + z\right)}{\sinh(z)}$$

01.22.27.0046.01

$$\operatorname{coth}(z) = \frac{\sqrt{\sinh^2(z) + 1}}{\sinh(z)} \quad ; |\operatorname{Im}(z)| < \frac{\pi}{2}$$

01.22.27.0047.01

$$\operatorname{coth}(z) = \frac{\sqrt{\sinh^2(z) + 1}}{\sinh(z)} (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} + \frac{1}{2} \rfloor} \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} + \frac{1}{2} \rfloor + \lfloor -\frac{\operatorname{Im}(z)}{\pi} - \frac{1}{2} \rfloor \right) \theta(\operatorname{Re}(z)) \right)$$

01.22.27.0048.01

$$\operatorname{coth}^2(z) = \frac{\sinh^2(z) + 1}{\sinh^2(z)}$$

### Involving cosh

01.22.27.0049.01

$$\operatorname{coth}(z) = -\frac{i \cosh(z)}{\cosh\left(\frac{\pi i}{2} - z\right)}$$

01.22.27.0050.01

$$\operatorname{coth}(z) = \frac{i \cosh(z)}{\cosh\left(\frac{\pi i}{2} + z\right)}$$

01.22.27.0051.01

$$\operatorname{coth}(z) = \frac{\sqrt{z^2}}{z} \frac{\cosh(z)}{\sqrt{\cosh^2(z) - 1}} \quad ; \quad |\operatorname{Im}(z)| < \frac{\pi}{2}$$

01.22.27.0052.01

$$\operatorname{coth}(z) = \frac{\sqrt{-z^2}}{z} \frac{\cosh(z)}{\sqrt{1 - \cosh^2(z)}} \quad ; \quad |\operatorname{Im}(z)| < \pi$$

01.22.27.0053.01

$$\operatorname{coth}(z) = -\frac{i \cosh(z)}{\sqrt{1 - \cosh^2(z)}} \quad ; \quad 0 < \operatorname{Im}(z) < \pi$$

01.22.27.0054.01

$$\operatorname{coth}(z) = -\frac{i \cosh(z)}{\sqrt{1 - \cosh^2(z)}} \left( (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} \rfloor} \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} \rfloor + \lfloor -\frac{\operatorname{Im}(z)}{\pi} \rfloor \right) \theta(\operatorname{Re}(z)) \right) \right)$$

01.22.27.0055.01

$$\operatorname{coth}^2(z) = \frac{\cosh^2(z)}{\cosh^2(z) - 1}$$

### Involving tanh

01.22.27.0056.01

$$\operatorname{coth}(z) = -\tanh\left(\frac{\pi i}{2} - z\right)$$

01.22.27.0057.01

$$\operatorname{coth}(z) = \tanh\left(\frac{\pi i}{2} + z\right)$$

01.22.27.0058.01

$$\operatorname{coth}(z) = \tanh\left(z - \frac{\pi i}{2}\right)$$

01.22.27.0059.01

$$\operatorname{coth}(z) = \frac{1}{\tanh(z)}$$

01.22.27.0060.01

$$\operatorname{coth}(z) = \frac{\tanh^2\left(\frac{z}{2}\right) + 1}{2 \tanh\left(\frac{z}{2}\right)}$$

01.22.27.0061.01

$$\operatorname{coth}\left(\frac{\pi i}{2} + z\right) = \tanh(z)$$

01.22.27.0062.01

$$\coth\left(\frac{\pi i}{2} - z\right) = -\tanh(z)$$

01.22.27.0063.01

$$\coth(z) = \frac{2}{\tanh(2z)} - \tanh(z)$$

**Involving csch**

01.22.27.0064.01

$$\coth(z) = -\frac{i \operatorname{csch}(z)}{\operatorname{csch}\left(\frac{\pi i}{2} - z\right)}$$

01.22.27.0065.01

$$\coth(z) = -\frac{i \operatorname{csch}(z)}{\operatorname{csch}\left(\frac{\pi i}{2} + z\right)}$$

01.22.27.0066.01

$$\coth(z) = e^z \operatorname{csch}(z) - 1$$

01.22.27.0067.01

$$\coth(z) = e^{-z} \operatorname{csch}(z) + 1$$

01.22.27.0068.01

$$\coth(z) = \frac{\sqrt{z^2}}{z} \sqrt{1 + \operatorname{csch}^2(z)} \quad /; \operatorname{Re}(z) \neq 0$$

01.22.27.0069.01

$$\coth^2(z) = \operatorname{csch}^2(z) + 1$$

**Involving sech**

01.22.27.0070.01

$$\coth(z) = -\frac{i \operatorname{sech}\left(\frac{\pi i}{2} - z\right)}{\operatorname{sech}(z)}$$

01.22.27.0071.01

$$\coth(z) = \frac{i \operatorname{sech}\left(\frac{\pi i}{2} + z\right)}{\operatorname{sech}(z)}$$

01.22.27.0072.01

$$\coth(z) = -i e^z \operatorname{sech}\left(\frac{\pi i}{2} - z\right) - 1$$

01.22.27.0073.01

$$\coth(z) = 1 - i e^{-z} \operatorname{sech}\left(\frac{\pi i}{2} - z\right)$$

01.22.27.0074.01

$$\coth(z) = \frac{\sqrt{z^2}}{z} \frac{1}{\sqrt{1 - \operatorname{sech}^2(z)}} \quad /; \operatorname{Re}(z) \neq 0$$



01.22.27.0075.01

$$\operatorname{coth}(z) = -\frac{i}{\sqrt{\operatorname{sech}^2(z) - 1}} \quad ; 0 < \operatorname{Im}(z) < \frac{\pi}{2}$$

01.22.27.0076.01

$$\operatorname{coth}(z) = \frac{\sqrt{-z^2}}{z} \frac{1}{\sqrt{\operatorname{sech}^2(z) - 1}} \quad ; |\operatorname{Im}(z)| < \frac{\pi}{2}$$

01.22.27.0077.01

$$\operatorname{coth}(z) = \frac{i}{\sqrt{\operatorname{sech}^2(z) - 1}} (-1)^{\lfloor \frac{-2\operatorname{Im}(z)}{\pi} \rfloor} \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} - \frac{1}{2} \rfloor + \lfloor \frac{1}{2} - \frac{\operatorname{Im}(z)}{\pi} \rfloor} \right) \theta(\operatorname{Re}(z)) \right) \left( 1 - \left( 1 + (-1)^{\lfloor \frac{\operatorname{Im}(z)}{\pi} \rfloor + \lfloor -\frac{\operatorname{Im}(z)}{\pi} \rfloor} \right) \theta(-\operatorname{Re}(z)) \right)$$

01.22.27.0078.01

$$\operatorname{coth}^2(z) = \frac{1}{1 - \operatorname{sech}^2(z)}$$

01.22.27.0079.01

$$\operatorname{coth}^2(z) = 1 - \operatorname{sech}^2\left(\frac{\pi i}{2} - z\right)$$

**Involving trigonometric and hyperbolic functions**

01.22.27.0080.01

$$\operatorname{coth}(z) = \frac{\cosh(z)}{\sinh(z)}$$

01.22.27.0081.01

$$\operatorname{coth}(z) + \tanh(z) = 2 \operatorname{coth}(2z)$$

01.22.27.0082.01

$$\operatorname{coth}(z) - \tanh(z) = 2 \operatorname{csch}(2z)$$

**Inequalities**

01.22.29.0001.01

$$|\operatorname{coth}(x)| > 1 \quad ; x \in \mathbb{R}$$

**Zeros**

01.22.30.0001.01

$$\operatorname{coth}(z) = 0 \quad ; z = \pi i \left( k + \frac{1}{2} \right) \wedge k \in \mathbb{Z}$$

**Theorems**

**Magnetic moment**

The mean magnetic moment per dipole  $\bar{\mu}$  of a classical dipole  $\mu$  in a magnetic field  $\mathbf{H}$  is given by

$$\bar{\mu} = \mathbf{H} / |\mathbf{H}| \left| \mu \right| \left( \coth(k_B T |\mu| |\mathbf{H}|) - (k_B T |\mu| |\mathbf{H}|)^{-1} \right) |\mathbf{H}|$$

where  $k_B$  is the Boltzmann constant and  $T$  is the absolute temperature.

The mean magnetic moment per dipole  $\bar{\mu}$  of a quantum-mechanical dipole  $\mu$  generated by angular momentum  $j$  and g-factor  $g$  in a magnetic field  $\mathbf{H}$  is given by

$$\bar{\mu} = \mathbf{H}/|\mathbf{H}| |\mu| \left( \left( 1 + \frac{1}{2j} \right) \coth \left( \left( 1 + \frac{1}{2j} \right) k_B T g \mu_B j \right) - \frac{1}{2j} \coth \left( \frac{1}{2j} k_B T g \mu_B j \right) \right)$$

where  $\mu_B$  is the Bohr magneton.

## History

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–L'Abbe Sauri (1774)

The function  $\coth$  is encountered often in mathematics and the natural sciences.

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