BellB2

Notations

Traditional name

Bell polynomial

Traditional notation

$B_n(z)$

Mathematica StandardForm notation

`BellB[n, z]`

Primary definition

05.15.02.0001.01

$$B_n(z) = n! \left([t^n]\, e^{(e^t-1)z}\right) /; n \in \mathbb{N}$$

Specific values

Specialized values

For fixed *n*

05.15.03.0001.01

$$B_n(0) = 0 /; n \in \mathbb{N}^+$$

05.15.03.0002.01

$$B_n(1) = B_n$$

For fixed *z*

05.15.03.0003.01

$$B_0(z) = 1$$

05.15.03.0004.01

$$B_1(z) = z$$

05.15.03.0005.01

$$B_n(2) = z^2 + z$$

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$$B_3(z) = z^3 + 3z^2 + z$$

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$$B_4(z) = z^4 + 6 z^3 + 7 z^2 + z$$

05.15.03.0008.01

$$B_5(z) = z^5 + 10 z^4 + 25 z^3 + 15 z^2 + z$$

05.15.03.0009.01

$$B_6(z) = z^6 + 15 z^5 + 65 z^4 + 90 z^3 + 31 z^2 + z$$

05.15.03.0010.01

$$B_7(z) = z^7 + 21 z^6 + 140 z^5 + 350 z^4 + 301 z^3 + 63 z^2 + z$$

05.15.03.0011.01

$$B_8(z) = z^8 + 28 z^7 + 266 z^6 + 1050 z^5 + 1701 z^4 + 966 z^3 + 127 z^2 + z$$

05.15.03.0012.01

$$B_9(z) = z^9 + 36 z^8 + 462 z^7 + 2646 z^6 + 6951 z^5 + 7770 z^4 + 3025 z^3 + 255 z^2 + z$$

05.15.03.0013.01

$$B_{10}(z) = z^{10} + 45 z^9 + 750 z^8 + 5880 z^7 + 22\,827 z^6 + 42\,525 z^5 + 34\,105 z^4 + 9330 z^3 + 511 z^2 + z$$

General characteristics

Domain and analyticity

$B_n(z)$ is a polynomial of z and as such an analytical function of z. $B_n(z)$ is defined in the whole complex z-plane and for $n \in \mathbb{N}$.

05.15.04.0001.01

$$(n * z) \longrightarrow B_n(z) :: (\mathbb{Z} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

05.15.04.0002.01

$$B_n(\bar{z}) = \overline{B_n(z)}$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

The function $B_n(z)$ has a pole of order n at $z = \tilde{\infty}$.

05.15.04.0003.01

$$\mathcal{S}ing_z(B_n(z)) = \{\{\tilde{\infty}, n\}\}$$

Branch points

With respect to z

The function $B_n(z)$ does not have branch points.

05.15.04.0004.01

$$\mathcal{BP}_z(B_n(z)) = \{\}$$

Branch cuts

With respect to z

The function $B_n(z)$ does not have branch cuts.

05.15.04.0005.01

$$\mathcal{BC}_z(B_n(z)) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

05.15.06.0001.01

$$B_n(z) \propto B_n(z_0) + \sum_{j=0}^{n-1} (j+1)\, \mathcal{S}_n^{(j+1)}\, z_0^j (z-z_0) + \frac{1}{2} \sum_{j=0}^{n-2} (j+1)\,(j+2)\, \mathcal{S}_n^{(j+2)}\, z_0^j (z-z_0)^2 + \ldots \;/;\; (z \to z_0)$$

05.15.06.0002.01

$$B_n(z) \propto B_n(z_0) + \left(\frac{1}{z_0} B_{n+1}(z_0) - B_n(z_0)\right)(z-z_0) + \frac{1}{2}\left(B_n(z_0) - \frac{2z+1}{z^2} B_{n+1}(z_0) + \frac{1}{z^2} B_{n+2}(z_0)\right)(z-z_0)^2 + O\left((z-z_0)^3\right) \;/;\; (z \to z_0)$$

05.15.06.0003.01

$$B_n(z) = \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{j=0}^{n-k} \mathcal{S}_n^{(j+k)}\, (j+1)_k\, z_0^j\, (z-z_0)^k$$

05.15.06.0004.01

$$B_n(z) \propto B_n(z_0)\,(1 + O(z-z_0))$$

Expansions at $z = 0$

For the function itself

05.15.06.0005.01

$$B_n(z) \propto \mathcal{S}_n^{(0)} + \mathcal{S}_n^{(1)}\, z + \mathcal{S}_n^{(2)}\, z^2 + \ldots \;/;\; (z \to 0)$$

05.15.06.0006.01

$$B_n(z) \propto \mathcal{S}_n^{(0)} + \mathcal{S}_n^{(1)}\, z + \mathcal{S}_n^{(2)}\, z^2 + O\left(z^3\right)$$

05.15.06.0007.01

$$B_n(z) = \sum_{k=0}^{n} \mathcal{S}_n^{(k)}\, z^k$$

05.15.06.0008.01

$$B_n(z) \propto z\,(1 + O(z)) \,/;\, n \in \mathbb{N}^+$$

05.15.06.0009.01

$$B_n(z) \propto \begin{cases} 1 & n = 0 \\ z & \text{True} \end{cases} \,/;\, (z \to 0)$$

Asymptotic series expansions

05.15.06.0010.01

$$B_n(z) \propto z^n \sum_{k=0}^{n} \mathcal{S}_n^{(n-k)}\, z^{-k} \,/;\, (|z| \to \infty)$$

05.15.06.0011.01

$$B_n(z) \propto z^n \left(1 + O\left(\frac{1}{z}\right)\right)$$

05.15.06.0012.01

$$B_n(z) \propto z^n \left(1 + \frac{n\,(n-1)}{2\,z} + O\left(\frac{1}{z^2}\right)\right)$$

05.15.06.0013.01

$$B_n(z) \propto z^n \,/;\, (|z| \to \infty)$$

Other series representations

05.15.06.0014.01

$$B_n(z) = e^{-z} \sum_{k=0}^{\infty} \frac{k^n\, z^k}{k!} \,/;\, n \in \mathbb{N}$$

Generating functions

05.15.11.0001.01

$$B_n(z) = n!\, \left([t^n]\, e^{(e^t - 1)\, z}\right) /;\, n \in \mathbb{N}$$

Transformations

Addition formulas

05.15.16.0001.01

$$B_n(w + z) = \sum_{k=0}^{n} \binom{n}{k} B_k(z)\, B_{n-k}(w) \,/;\, n \in \mathbb{N}$$

Identities

Functional identities

05.15.17.0001.01

$$B_{n+1}(z) = z \sum_{k=0}^{n} B_k(z) \binom{n}{k} \,/;\, n \in \mathbb{N}$$

Complex characteristics

Real part

05.15.19.0001.01

$$\operatorname{Re}(B_n(x+i\,y)) = \frac{1}{2}\left(B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

05.15.19.0002.01

$$\operatorname{Im}(B_n(x+i\,y)) = \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)$$

Absolute value

05.15.19.0003.01

$$|B_n(x+i\,y)| = \sqrt{B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)}$$

Argument

05.15.19.0004.01

$$\arg(B_n(x+i\,y)) = \tan^{-1}\left(\frac{1}{2}\left(B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right),\; \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)\right)$$

Conjugate value

05.15.19.0005.01

$$\overline{B_n(x+i\,y)} = \frac{1}{2}\left(B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)-\frac{i\,x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)$$

Signum value

05.15.19.0006.01

$$\operatorname{sgn}(B_n(x+i\,y)) = \frac{\frac{i\,x}{y}\sqrt{-\frac{y^2}{x^2}}\left(B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)+B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)}{2\sqrt{B_n\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)B_n\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)}}$$

Differentiation

Low-order differentiation

05.15.20.0001.01

$$\frac{\partial B_n(z)}{\partial z} = \frac{1}{z} B_{n+1}(z) - B_n(z)$$

05.15.20.0002.01

$$\frac{\partial^2 B_n(z)}{\partial z^2} =_n (z) - \frac{2\,z+1}{z^2} B_{n+1}(z) + \frac{1}{z^2} B_{n+2}(z)$$

Symbolic differentiation

05.15.20.0003.01

$$\frac{\partial^m B_n(z)}{\partial z^m} == \sum_{k=0}^{n-m} \mathcal{S}_n^{(k+m)}\, (k+1)_m\, z^k \;/;\; m \in \mathbb{N}$$

Fractional integro-differentiation

05.15.20.0004.01

$$\frac{\partial^\alpha B_n(z)}{\partial z^\alpha} == \sum_{k=0}^{n} \frac{\mathcal{S}_n^{(k)}\, k!\, z^{k-\alpha}}{\Gamma(k-\alpha+1)}$$

Summation

Finite summation

05.15.23.0001.01

$$\sum_{k=0}^{n} \binom{n}{k} B_k(z) = \frac{1}{z} B_{n+1}(z)$$

Infinite summation

05.15.23.0002.01

$$\sum_{k=0}^{\infty} \frac{B_k(z)\, t^k}{k!} == e^{(e^t-1)\,z}$$

Representations through more general functions

Through other functions

05.15.26.0001.01

$$B_n(z) == \sum_{k=0}^{n} \mathcal{S}_n^{(k)}\, z^k \;/;\; n \in \mathbb{N}$$

Zeros

For each $n \in \mathbb{N}^+$ the equation $B_n(z) == 0$ has exactly n different nonpositive real roots.

05.15.30.0001.01

$$B_n(z) = e^{-z} \sum_{k=0}^{\infty} \frac{k^n z^k}{k!} /; n \in \mathbb{N}$$

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